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# The universal validation algorithm of coordinate measuring methods

## Abstract

The paper presents the universal validation algorithm of coordinate measuring methods determined by the analysis and adaptation of appropriate validation parameters, techniques and models. The paper also provides the proposals of acceptance/rejection criteria of the analysed measurement method. These criteria are a set of mathematical formulas selected from numerous worldwide research publications and metrological studies, which have been tested, revised and adapted for the purposes of the validation.

**Keywords:** validation algorithm, validation model, coordinate measuring methods.

## 1. Validation of the laboratory methods

PN-EN ISO / IEC 17025: 2005 indicates that laboratories should prefer the current measurement methods described in the well-known international standards. However, when there is a need to develop and implement specific and individual methods, it should be done as a result of the planned activities – the validation process. There are lots of factors that affect the proper organization of the validation process such as professional laboratory staff with extensive knowledge and measuring skills, adequate resources provided by the top management of the laboratory which include, first of all, appropriate measuring machines with software and effective communication. All non-standardized methods should be agreed with a potential client, and well-validated. Validation of measuring methods, as the process of confirming that the chosen method used to perform a particular specified measurement is suitable for the intended purpose, provides the reliability and consistency of the measurements. That is why it is so important both in the scientific community as well as in production engineering.

Validation of the measuring methods is a process that is very difficult to plan and implement on the one hand, and on the other hand - an interesting issue that is rarely discussed in scientific community. Well-validated method may often become a bargaining chip for companies when choosing a partner - a laboratory for cooperation [1].

### 1.1. Validation of coordinate measuring methods

Intensive development of coordinate measuring technique requires better and better, validated methods for assessing the accuracy of measurements. Mandatory condition for allowing their daily use is their validation. For this purpose the universal validation algorithm containing all the elements required by point 5.4 of ISO 17025 [N1] and internal accreditation requirements of Polish Centre for Accreditation has been developed. These elements include, inter alia, the choice of the most important parameters of the validation and appropriate validation techniques. Validation technique must provide the best laboratory and environmental conditions for carrying out the measurements so that the results of the validation parameters were reliable and unambiguous. In the case of coordinate measuring methods two techniques of validation are considered: comparison technique with another method and interlaboratory comparisons. The algorithm was developed for calibration laboratory and takes into account the validation of three measuring methods: multi-position method, substitution method and Virtual CMM method. Specifically for the purpose of the validation, the appropriate multi-feature check was developed and manufactured,

whose selected dimensions are measured using different measuring systems, software, and the three above-mentioned coordinate measuring methods.

The issue of validation of measurement methods is the difficult subject, often overlooked and neglected but also desirable in any field of measurement. Measurements of length using coordinate technique are a research field where validation of both standardized and non-standardized methods, is highly suggested. Therefore, an adequate validation procedure has been developed. It has been verified by the countless number of measurements. The hardest part of the validation of the methods is the development or adaptation of a validation model, which is a mathematical model and enables the decision whether to adopt or reject a validated method.

Knowledge about the validation of measurement methods is reduced to the [N1] point 5.4. Most auditors of management systems in laboratories do not explore the validation area. It is only assessed by the question whether the methods used in the laboratory are developed on the basis of standards. However, most laboratories use methods only partially developed on the basis of standards and consider them to be validated.

Internationally, the issue of validation of measurement methods, in the field of general metrology, is developed by the National Measurement Institutes such as PTB, NPL or NIST.

In coordinate metrology, the effective methods of evaluation of the measurement accuracy have been developed as exemplified by the series of standards ISO 15530 but so far the universal validation model of these methods has not been presented.

## 1.2. The universal validation algorithm of coordinate measuring methods

### 1. ANALYSIS OF COORDINATE METHODS [1] [8]

#### 1.1. Substitution Method - validated, developed on the basis of ISO/TS 15530-3:2011 [N2]

Method is based on repeated measurements of a calibrated standard, whose shape and dimensions are similar to the tested object.

This method makes possible experimental determination of realized measurement accuracy using the calibration of a reference object or a standard of size and shape similar to the measured object. Not the sources of errors are important for the measurement results performed with it, but the assessment of their effects in form of deviations from the nominal dimension.

The substitution method can be used in mass production, where from the economic point of view, it becomes profitable. However it is required then to ensure stable conditions for the measurement realization.

#### 1.2. Multi-position method - non-validated, developed on the basis of ISO/TS 15530-2 [N3]

Method is based on repeated measurements of an uncalibrated object, taking into account the various distribution of the points. The object should be measured five times at each designated position.

The analysis of the results is made on the basis of the analysis of standard deviations of the measured characteristic, separating two components of uncertainty: the impact of CMM repeatability for particular and the effect of CMM geometric errors in connection with the influence of the of measuring tip qualification process.

**1.3. Virtual CMM Method** - non-validated, developed by the laboratory scientists

This method involves the measurement simulation using a virtual machine. A simulation method is used when there is no solution to the problem in an analytical way, or if the measurement model is too complicated, and the estimation of the measurement uncertainty is difficult, or even impossible.

This method is based on constructing the simulative model of a measurement made in the considered system. The model should include all of the error sources that are known and possible to predict, and use the information about the possible error values in each place of the measuring volume of the system.

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**2. ANALYSIS OF LABORATORY COMPETENCE**

- calibrations in accordance with the requirements of the standard ISO 17025
- coordinate measuring machines with appropriate calibration and measurement capability CMC
- proficiency and qualification of laboratory staff
- housing and environmental conditions
- literature analysis

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**3. SELECTION OF VALIDATION PARAMETERS [1]**

In the case of coordinate measurements, validation parameters are expanded uncertainties  $U$  calculated for the following methods:

**Multi-position method**  
expanded measurement uncertainty  $U$

$$U = k \cdot \sqrt{u_{rep}^2 + u_{geo}^2 + u_{corrL}^2 + u_D^2 + u_{temp}^2} \quad (1)$$

where:  
 $U$  - expanded uncertainty of the measurement,  
 $k$  - coverage factor,  
 $u_{rep}$  - standard uncertainty contribution originating from repeatability of the CMM,  
 $u_{geo}$  - standard uncertainty contribution related to the geometrical errors of the CMM,  
 $u_{corrL}$  - standard uncertainty of the correction that is applied to the average measurement result on the basis of the average distance proportional error of length measurement,  
 $u_D$  - uncertainty related to the measurement of diameter,  
 $u_{temp}$  - uncertainty connected to thermal influences.

**Substitution Method**  
expanded measurement uncertainty  $U$

$$U = k \cdot \sqrt{u_{cal}^2 + u_p^2 + u_b^2 + u_w^2} \quad (2)$$

where:  
 $U$  - expanded uncertainty of the measurement  
 $k$  - coverage factor  
 $u_{cal}$  - standard uncertainty of the parameter of the calibrated workpiece or measurement standard  
 $u_p$  - standard uncertainty of the measurement procedure  
 $u_b$  - standard uncertainty of the systematic error  
 $u_w$  - standard uncertainty associated with the variations in the uncalibrated workpieces

**Virtual CMM Method**  
uncertainty standard deviation

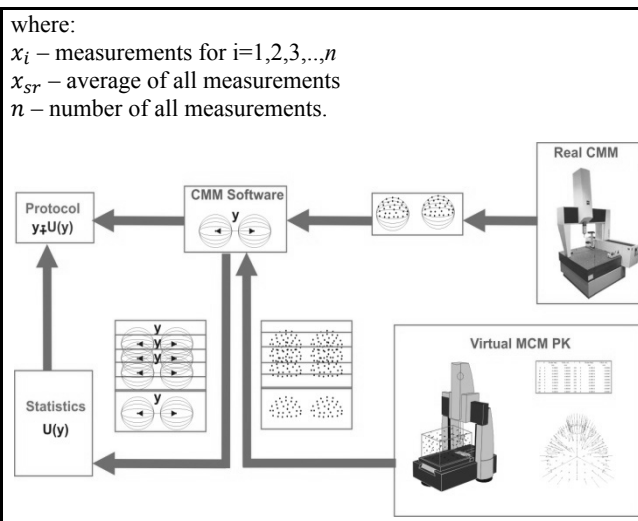
$$S_x = \sqrt{\frac{\sum(x_i - x_{sr})^2}{n - 1}} \quad (3)$$


Fig. 1. Virtual CMM Method

**4. SELECTION OF VALIDATION TECHNIQUE**

**4.1. Comparison with the validated method**

- multi-position method with the substitution method
- Virtual CMM method with the substitution method

**4.2. Interlaboratory comparison**

Interlaboratory comparison with a reference laboratory in accordance with the requirements specified by the standard ISO 17043 [N4] and the documents of the accreditation body. In the case of calibration laboratories, the reference laboratory should be selected in accordance with the criterion of independence and accuracy, which means the choice of a laboratory with appropriate scope of accreditation and calibration and measurement capability CMC.

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**5. SELECTION OF THE MEASURING CHECK**

Fig. 2. EUMETRON Multi feature check

Multi feature check - complex measuring standard for the purposes of measurement accuracy assessment and the determination of measurement uncertainty for nearly all features and dimensions applicable to the coordinate measuring technique.

Due to the nature of the validation process and the accuracy of coordinate systems, this standard is suitable for performing lots of measuring tasks on CMMs thanks to the diversity of its geometric structure, which includes e.g. cylindrical holes of various diameters and depths, circle hole, flat, stepped and end surfaces, short and long tapers with various opening angles.

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**6. SELECTION OF VALIDATION CRITERIA – MATHEMATICAL MODELS [2- 10]**

**Model of statistical consistency control**

The validation model is based on the concept of consistency control, which refers to classical statistics such as the weighted mean of the methods and the chi-squared test. To apply this method, the laboratory must meet three important requirements:

- measurements must be taken in adequate, environmental conditions of stability
- the measurement program must comply with the principle of independence, the measurements made by the reference method cannot affect the completed measurements made previously using the method being validated
- Gaussian distributions need to be assigned to each measurand.

This model starts with the calculation of the weighted mean (4), the so called Reference Value ( $RV$ ):

$$RV = \frac{x/(u^2(x)) + y/(u^2(y)) + z/(u^2(z))}{1/u^2(x) + 1/u^2(y) + 1/u^2(z)} \quad (4)$$

where:

$x, y, z$  - mean values of the results obtained by multi-position method  $x$ , substitution method  $y$  and virtual CMM method  $z$ ,  
 $u(x), u(y), u(z)$  - uncertainties calculated according to the particular method.

The next step of the model is to perform the chi-squared test which involves determining the probability of  $\chi_{obs}^2$  and is calculated as (5):

$$\Pr \{ \chi^2(\nu) > \chi_{obs}^2 \} < 0.05 \quad (5)$$

where:

$$\chi_{obs}^2 = \frac{(x - RV)^2 + (y - RV)^2 + (z - RV)^2}{u^2(x) + u^2(y) + u^2(z)} \quad (6)$$

where:

$\chi_{obs}^2$  - parameter describing the probability distribution of the results  $x, y, z$  referenced to the value  $RV$  in view of their individual variances of the results,

and

$\nu = N - 1$  - degrees of freedom

This test, if fails, assumes the consistency of coordinate measuring methods used. In this case, next steps have to be performed. If the test rejects the hypothesis about the consistency of the results obtained using the considered methods, the validation ends with a negative result.

The standard uncertainty associated with the reference value ( $RV$ ) needs to be calculated using (7):

$$\frac{1}{u^2(RV)} = \frac{1}{u^2(x)} + \frac{1}{u^2(y)} + \frac{1}{u^2(z)} \quad (7)$$

where:

$u(RV)$  is the standard uncertainty of determination of the reference value.

The most important is the "validation acceptance interval" for the results obtained with considered methods:

$$< RV - u(RV); RV + u(RV) > \quad (8)$$

This interval is the proposition of a mathematical range, which should overlap with the intervals that contain the true value of the measured quantity, obtained using the methods being validated.

If all the three intervals  $< x - u(x); x + u(x) >$ ,  $< y - u(y); y + u(y) >$ ,  $< z - u(z); z + u(z) >$  have the common part with the validation acceptance interval then the validation ends with the positive result and two methods that were under validation may now be considered as the validated methods.

### Model of metrological compatibility

$$\frac{|x_i - x_R|}{\sqrt{u^2(x_i) + u^2(x_R) - 2r(x_i, x_R)u(x_i)u(x_R)}} \leq \kappa \quad (9)$$

where:

$x_i, u(x_i)$  - values being validated with the standard uncertainties,  
 $x_R, u(x_R)$  - reference values with the standard uncertainties,  
 $r(x_i, x_R)$  - correlation coefficient  $R(X_i, X_R)$  between the variables  $X_i$  and  $X_R$

$$r(x_i, x_R) = \frac{\sum_{i=1}^n [(x_i - \bar{x})(x_R - \bar{x})]}{\{[\sum_{i=1}^n (x_i - \bar{x})^2] \cdot [\sum_{i=1}^n (x_R - \bar{x})^2]\}^{\frac{1}{2}}} \quad (10)$$

and

$\kappa$  - the threshold determined on the basis of the measurements and calculations for coordinate metrology.

The number in the denominator (9) is related to the correlation of pdfs represented by  $[x_i, u(x_i)]$  and  $[x_R, u(x_R)]$ .

The application of this model is possible if the measurements (measured value with its associated standard uncertainty) are traceable to the same reference that gives the VIM's definition of metrological comparability [N5, definition 2.46]. According to another definition of metrological traceability found in VIM [N5], the results must be related to a practical realization of the reference through a documented chain of unbroken calibrations, each contributing to the total measurement uncertainty [N5, definition 2.41].

The result of the metrological compatibility test makes it possible to determine whether measurement results with their corresponding uncertainties, obtained using the validated coordinate method  $[x_i, u(x_i)]$  are sufficiently comparable with the set of results obtained from the reference method  $[x_R, u(x_R)]$ , and can thus be considered acceptable.

The answer is positive, because the fulfilled test of metrological compatibility demonstrates the insignificance of differences between the measured values  $x_i$  and  $x_R$ , in view of their uncertainties  $u(x_i)$  and  $u(x_R)$ .

Taking into account the adopted economic consequences, the conventional value of  $\kappa$  is 2, but the threshold may be adjusted to a different set value for various purposes. In this case, the threshold  $\kappa$  is chosen from the set of values of calculated metrological compatibility models.

The lower the threshold  $\kappa$ , the better to prove the consistency and reliability of the results of measurements.

### 7. DEVELOPMENT AND APPROVAL OF "VALIDATION PROGRAM OF COORDINATE MEASURING METHODS"

The validation program of coordinate measuring methods specifies the exact people and dates for the various stages of validation including measurements and statistical studies.

The validation program should be approved by the top management of the laboratory to provide adequate financial resources for the implementation of this program.

### 8. MEASUREMENTS

Measuring chosen values of the multi feature check using three multi-position, substitution and Virtual CMM methods on the coordinate measuring machine PMM 12106 intended for calibration spatial reference objects, with proven accuracy CMC =  $0.6 \mu\text{m} + 0.7L \mu\text{m/m}$  and calibrated length standards that ensures the accuracy of the performed measurements.

Due to the specific nature of coordinate measurements, the standard described in Section 5 possesses many distinct geometrical features and dimensions (Fig. 2). Apart from different types of dimensions (from a metrological standpoint),

e.g. external, internal, mixed and intermediate, all geometrical features, such as: shape, direction, position, and even radial and axial runout were taken into account in the measurements.

Measurements should be made by laboratory staff with proven measurement proficiency, selected from accredited personnel, who meet, specified by the management, eligibility criteria for performing calibration, results authorization and operation of the specified coordinate system.

Measurements should be taken under normal laboratory work with appropriate supervision of environmental conditions. All the intermediate data associated with the preparation, measurements and analysis of the results must be protected in order to ensure full traceability of the validation process.

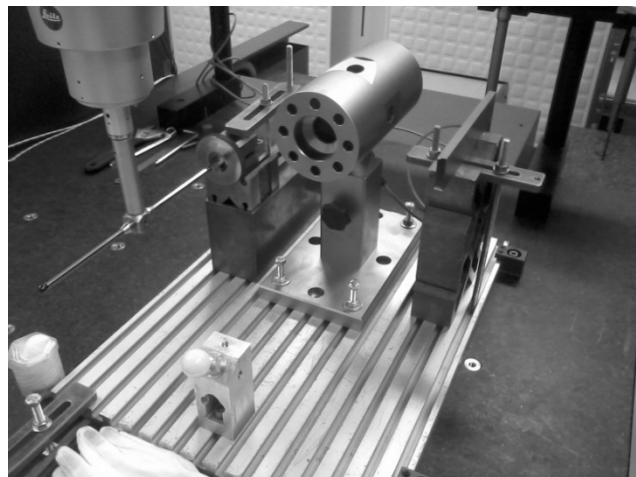


Fig. 3. Multi-feature check mounted in the measuring space of PMM 12106

## 9. STATISTICAL DEVELOPMENT OF VALIDATION PARAMETERS

Calculations of measuring uncertainties in accordance with Section 3.

## 10. APPLICATION OF VALIDATION MODELS

Calculation of the mathematical models in accordance with Section 6.

Presentation of the results in tabular form and graphically.

If the calculated validation parameters, uncertainties, do not meet the conditions specified in the established models, a detailed analysis of the validation and measuring process and possible change of parameters from Sections 3 to 6 are required.

## 11. ACCEPTANCE OR REJECTION OF COORDINATE MEASURING METHOD

The decision on acceptance or rejection of the results obtained from the measurements using methods to be validated.

The results of the validation (positive or negative) should be included in a detailed report. This report, in addition to the main decisions, should include the data from the various stages of validation according to the present algorithm.

This report, since its approval, becomes a part of the supervised document in the management system in the laboratory and, therefore, is a subject to the verification by the Polish Centre for Accreditation.

## 2. Model of validation of coordinate measuring methods

As a part of the presented algorithm, the validation models were also developed. They are the basis for accepting or rejecting new/changed measuring methods and are the innovation in the field of coordinate metrology. They are universal models that contain the

mathematical analysis of the estimation of the results and their uncertainty according to numerous global publications and metrological studies. These models are confirmed by the validation of two – developed and improved in the Laboratory of Coordinate Metrology (LCM) methods: Multi-position method based on repeated measurements of an uncalibrated object and the matrix method used to build the system of virtual measuring machine to assess the accuracy of the measurement *on Line*. The results of validation of the mentioned above method using the model of statistical consistency control are described in details in [10].

The presented algorithm and the developed models of validation of measurement methods in calibration laboratories are the response to current market requirements and the designation of a new, broad research area. The recipients of the model can be accredited calibration laboratories, research institutions and national conformity assessment bodies. The model may also be, in the future, used in the validation of measurement methods in production conditions, because the calculation of measurement uncertainty, as the basis for judgments on the conformity with the specifications of the product geometry, will become the standard.

## 3. Conclusions

The presented algorithm is purposely developed for the validation of coordinate measuring methods. It can be used to meet the requirements of the standard ISO 17025 (point 5.4) with the rational choice of a validation model. These presented models of validation are the proposition of methodology that may be used for validating the new methods that have recently been developed, internal laboratory procedures and methods that are rarely used in practice and cannot be considered as validated. Fulfilling the criterion presented in this paper is the necessary condition for assuming the considered method to be a validated one. The works on improvement of the models are now carried out in LCM.

With small amendments, the presented validation algorithm and models may also be used in other fields of science and may contribute to significant facilitation in demonstrating that the used methods are validated and produce correct results.

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