Int. J. of Applied Mechanics and Engineering, 2022, vol.27, No.2, pp.158-176
DOI: 10.2478/ijame-2022-0026

COMBINED EFFECTS OF HELICAL FORCE AND ROTATION ON STATIONARY CONVECTION OF A BINARY FERROFLUID IN A POROUS MEDIUM.

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This work studies the simultaneous effects of helical force, rotation and porosity on the appearance of stationary convection in a binary mixture of a ferrofluid and on the size of convection cells. We have determined the analytical expression of the Rayleigh number of the system as a function of the dimensionless parameters. The effect of each parameter on the system is studied. The consideration of the simultaneous effect of the basic characteristics made it possible to determine the evolution of the convection threshold in the ferrofluid and then the size of convection cells. The analyzes of the various results obtained allowed us to deduce whether the convection sets in quickly or with a delay when the various effects taken into account in the study are considered simultaneously.

Keywords: helical force; stationary convection; binary mixture of ferrofluid; porous medium; rotation.

1. Introduction

The study of the dynamic behavior of fluids is a very broad research topic and has brought out very important results. Likewise, the study of convection in fluids has made it possible to understand how the phenomena of heat transfer take place inside these fluids. These various studies have proved their importance in the use of new materials in technology and industry. Also, the nature of the fluids has a sufficient influence on the dynamic phenomena and the modes of heat transfer and therefore of convection. The typical case of ferrofluids has attracted the attention of many researchers given their applications in various fields. In the field of biotechnology, they allow the recovery and purification of proteins as well as the separation of cells. They can also detect tumors. In the field of medicine, they allow the treatment of ulcers, fistulas and the destruction of cancer cells. In the field of technology and mechanics, they are used in the design of loudspeakers, dampers and computer tools. Ferrofluids also known as magnetic fluids, are colloidal suspensions of ferromagnetic or ferrimagnetic nanoparticles of the order of 10 nanometers in a solvent or water [1, 2]. These liquids become magnetic upon application of an external magnetic field while maintaining their colloidal stability. They have both the magnetic properties and the properties of a liquid. These fluids only have a magnetic property if they are excited by an external magnetic field. When exposed to a temperature above the Curie temperature (temperature at which a material loses its magnetization), they lose their magnetic properties and behave like a fluid [1].

Ferrofluids have very strong magnetic properties. When the magnetic fluid is not subjected to any magnetic field, the magnetic moments carried by the nanoparticles are oriented randomly and the total

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magnetization of the fluid is zero. Commercial use of ferrofluids began in 1968 and fluid mechanics was recognized as a new branch of science and was named ferrohydrodynamics [2].

Heat transfer by ferrofluids is one of the fields of scientific study which has attracted the attention of many researchers due to its technological applications [1, 3, 4]. Neuringer and Rosensweig carried out the first studies on ferrofluids where they established the different equations of ferrohydrodynamics from the different principles of thermodynamics [5]. In the case of the theory of convective instability of ferrofluids, Finlayson formulated the first ideas and the first bases studying the convective instability of a layer of ferromagnetic fluid heated from below in the presence of a uniform vertical magnetic field [6]; followed later by others [7-20]. Chandrasekhar studied the classical problem of Rayleigh-Bénard convection of viscous fluids in a rotating layer [21] and showed that the rotation stabilizes the system. However, in the planetary atmosphere, the movement of turbulent convection is destined to become helical. In magnetohydrodynamics, Steenbeck *et al.* [22] discovered first the small-scale helical turbulence-generating properties. This discovery encouraged the development of the MHD theory. The publications on the hydrodynamic alpha effect were made by Levina *et al*. [23-26]. In a convective system, it has been shown that the force responsible for helical turbulence is pseudo-vector in nature, called the helical force. This force appears as the result of the mean turbulence in the mean field equations for large scale motions. The action of this force in Rayleigh-Bénard convection could induce a new type of instability, called helico-vortex instability [22]. G.V. Levina et al showed that the helical force decreases the instability threshold for three different types of boundary conditions [24]. Essoun and Chabi Orou examined the influence of the helical force on a rotating viscous fluid layer and found that the helical force has no monotonic effect on the onset of convection for all values of the Taylor number, but decreases the corresponding critical wave number for any Taylor number value [27]. P. Hounsou et al. examined the effect of the helical force on stationary convection of a binary ferrofluid and then showed that the helical force accelerates the onset of stationary convection and enlarges the size of the convective cells [28]. They subsequently studied the effect of rotation on the system and found that rotation delays the onset of stationary convection by contracting the size of convection cells [29]. These authors also studied the appearance of stationary convection in a binary mixture of a ferrofluid in a porous medium [30] and described out the effect of the porosity of the medium on convection. This literature review is shows that the case of simultaneous effects of the helical force, rotation and porosity on the stationary convection of a binary ferrofluid has not been studied yet. The study of stationary convection considering simultaneously the effects of a porous medium, the helical force and rotation on the dynamics of ferrofluids can bring to light various and very rich results in terms of stability and convection in ferrofluids. The aim of this work is therefore to study the effect of the helical force on stationary convection of a binary ferrofluid in a rotating porous medium. To this and a linear stability analysis of the conduction state is performed in the framework of Navier-Stokes equations and explicit expressions of convective thresholds in terms of the control parameters of the system are obtained.

Then, we studied the effect of the parameters in presence on the appearance of stationary convection in a binary ferrofluid and on the size of the convection cells taking into account the simultaneous effects of the helical force, rotation and porosity. To achieve the above goal, the paper is structured as follows. In section 2 the mathematical formulation of the problem is given. The linear stability analysis is illustrated in section 3. In section 4, the numerical simulations and analysis of the results are carried out. Section 5 is devoted to the conclusion.

2. Mathematical formulation of the problem

Consider a porous horizontal layer rotating at constant speed ϖ around a vertical axis, saturated by a binary ferrofluid, incompressible, confined between two horizontal plates ($z=-\frac{1}{2}d$ and $z=\frac{1}{2}d$) and heated from below in the presence of a uniform external magnetic field. The magnetic field is applied perpendicular to the plates and oriented in a direction parallel to the axis (oz). The layer is subjected to a vertical temperature gradient. A static temperature difference across the layer is imposed: $T(z=-d)=T_o+\Delta T$ and $T(z=d)=T_o$. The configuration of the problem is shown in Fig.1.

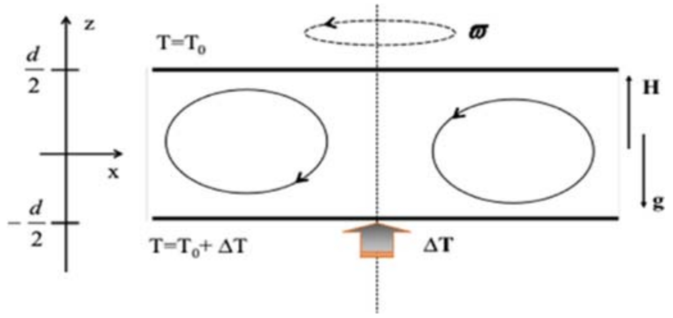

Fig.1. Vertical section through a rotating porous layer saturated with a ferrofluid.

Under the Boussinesq approximation [21], the equilibrium equations are:

$$\nabla.\boldsymbol{u}=0\,, \tag{2.1}$$

$$\begin{gathered}\rho_o\varepsilon^{-1}\left[\frac{\partial \boldsymbol{u}}{\partial t}+\varepsilon^{-1}(\boldsymbol{u}.\nabla)\boldsymbol{u}\right]=\\ =-\nabla P_{\text{eff}}+\rho\boldsymbol{g}+\mu_e\nabla^2\boldsymbol{u}+(\boldsymbol{M}.\nabla)\boldsymbol{H}+2\rho_o\varepsilon^{-1}\boldsymbol{u}\wedge\boldsymbol{\omega}+\rho_o\varepsilon^{-1}a\omega\mathrm{d}\boldsymbol{f}-\frac{\mu}{k_p}\boldsymbol{u},\end{gathered} \tag{2.2}$$

$$\frac{C_H}{T_o}\left[\frac{\partial \boldsymbol{T}}{\partial t}+(\boldsymbol{u}.\nabla)\boldsymbol{T}\right]+\chi_T\boldsymbol{H_o}\cdot\left[\frac{\partial \boldsymbol{H}}{\partial t}+(\boldsymbol{u}.\nabla)\boldsymbol{H}\right]=\bar{k}\,\nabla^2\boldsymbol{T}+D_s\gamma_H\nabla^2\boldsymbol{C}+D_s\chi_c\boldsymbol{H_o}.\nabla^2\boldsymbol{H}\,, \tag{2.3}$$

$$\left[\frac{\partial \boldsymbol{C}}{\partial t}+(\boldsymbol{u}.\nabla)\boldsymbol{C}\right]=D_c\nabla^2\boldsymbol{C}+D_s\nabla^2\boldsymbol{T}+\frac{D_c\chi_c}{\gamma_H}\boldsymbol{H_o}.\nabla^2\boldsymbol{H} \tag{2.4}$$

where $\boldsymbol{f}=\boldsymbol{e}(curl\boldsymbol{u})_z-\dfrac{\partial(\boldsymbol{e}\wedge\boldsymbol{u})}{\partial z}$; $\boldsymbol{e}=(0,0,1)$; **u** represents the filtration rate field; P_{eff} is the effective pressure which contains the static pressure, the hydrodynamic pressure and the gradient of the magnetic force; ρ the density; ρ_o is the reference density, μ is the dynamic viscosity; μ_e is the effective viscosity; $\boldsymbol{M}$ is the magnetization field; $\boldsymbol{H}$ is the magnetic field; C_H is the specific heat at constant magnetic field; T is the temperature, T_o is the reference temperature; χ_T is the pyromagnetic coefficient, $\bar{k}$ is the thermal diffusivity coefficient; C is the concentration of the magnetic particle; D_s is the Soret coefficient; D_c is the diffusion coefficient, a is the dimensionless number, $a\omega\mathrm{d}$ is the magnitude of the helical force.

The equation of state of the total density is linearized on the temperature T, on the magnetic field **H** and on the concentration C of the magnetic particles and gives:

$$\rho=\rho_o\left[1-\alpha_T(T-T_o)+\alpha_c(C-C_o)+\alpha_H\boldsymbol{H_o}.(\boldsymbol{H}-\boldsymbol{H_o})\right] \tag{2.5}$$

where α_T, α_c and α_H denote, respectively, the thermal, mass and magnetic expansion coefficients. T_o, C_o and $\boldsymbol{H_o}$ denote, respectively, the average values of the temperature, the concentration of the magnetic particles and the intensity of the magnetic field. Since the system cannot conduct (non-conductor) apart from the displacement of magnetic particles, the magnetic field **H** and the magnetic induction **B** are given by the simplified forms of Maxwell's equations

$$\nabla \wedge \boldsymbol{H} = 0 . \tag{2.6}$$

From equation (2.6), we can write that $\boldsymbol{H} = -\nabla\varphi$ where φ represents the scalar magnetic potential

$$\nabla.\boldsymbol{B} = 0 . \tag{2.7}$$

The magnetic field **H**, the magnetization $\boldsymbol{M}$ and the magnetic induction $\boldsymbol{B}$ are related by $\boldsymbol{B} = \boldsymbol{M} + \boldsymbol{H}$ where $\boldsymbol{M} = M(T, C, H)\hat{\boldsymbol{H}}$.

The magnetic equation of state is linearized with respect to the average temperature $\mathrm{T_o}$, to the average concentration $\mathrm{C_o}$ of magnetic particles and to the average magnetic field $\mathrm{H_o}$ and is given by [21]

$$M(T,C,H) = \chi_o \mathrm{H_o} + \mathrm{H_o}\left[\chi_T (T - T_o) + \chi_c (C - C_o) + \boldsymbol{H_o}.(\boldsymbol{H} - \boldsymbol{H_o})\right] . \tag{2.8}$$

In order for the system to be closed, appropriate edge conditions are used. For the limit at the temperature edges, we have assumed that the temperature is constant on each border: $T\left(z = -\frac{1}{2}d\right) = T_o + \Delta T$ and $T\left(z = \frac{1}{2}d\right) = T_o$. For the concentration of magnetic particles, we used the condition of impermeability to the walls:

$$\hat{\boldsymbol{n}}.\nabla\left(\mathrm{C} + (D_s / D_c)\mathrm{T}\right) + (\chi_c / \gamma_H)\boldsymbol{H_o}.(\hat{\boldsymbol{n}}.\nabla)\boldsymbol{H} = 0$$

where $\hat{\boldsymbol{n}}$ is a unit vector orthogonal to the plates.
Continuity conditions are applied to the magnetic field and to the magnetic induction: $\hat{\boldsymbol{n}} \wedge (\boldsymbol{H_{in}} - \boldsymbol{H_{ex}}) = 0$ and $\hat{\boldsymbol{n}}.(\boldsymbol{B_{in}} - \boldsymbol{B_{ex}}) = 0$. For the velocity field, we have the rigid boundary condition.

Using Eqs (2.1)-(2.8) and the boundary conditions, the basic state of the system is given by:

$$\boldsymbol{u_{con}} = 0 , \tag{2.9}$$

$$T_{con}(z) = -\beta z + \overline{T} , \tag{2.10}$$

$$C_{con} = \overline{\mathrm{C}} , \tag{2.11}$$

$$H_{con}(z) = (1 + \lambda\beta z) + H_o \tag{2.12}$$

where

$$\beta = \Delta T / d \quad \text{and} \quad \lambda = \chi_T / (1 + \chi) \quad \text{with} \quad \chi = \chi_o + \chi_H H_o^2 .$$

The next step is to derive the equations for the disturbance of the conductive state of the system and to introduce the dimensionless parameters before studying its stability. To do this, let us disrupt the conductive state of the system in the form:

$$\mathrm{u}=\boldsymbol{u'}, \qquad p=p_{con}(z)+p', \qquad C=C_{con}(z)+C', \qquad T=T_{con}(z)+\theta,$$

$$\rho=\rho_{con}(z)+\rho', \qquad \boldsymbol{H}=\boldsymbol{H_{con}}(z)+\boldsymbol{H}', \qquad \boldsymbol{M}=\boldsymbol{M_{con}}(z)+\boldsymbol{M}'$$

where $\boldsymbol{u'}=(u,v,w)$, ρ', p', C', θ, $\boldsymbol{H'}=\left(H_1',H_2',H_3'\right)$ and $\boldsymbol{M'}=\left(M_1',M_2',M_3'\right)$ denote respectively the disturbed velocity field, the disturbed density, the disturbed pressure, the disturbed concentration, the disturbed temperature, the disturbed magnetic field and the disturbed magnetization. By using these perturbed variables and by introducing the characteristic scales : d for the length, $\frac{d^2}{k}$ for the time $\left(k=\frac{\bar{\mathrm{k}}\,T_o}{C_H}\right)$, $\frac{k}{d}$ for speed, βd for temperature, $\frac{\chi_T\beta d^2 H_o}{1+\chi}$ for the magnetic potential, $\frac{\mu k}{d^2}$ for pressure and $\frac{\beta\bar{D}_T\rho_o d}{\tilde{D}\gamma_H}$ for the concentration in the Eqs (2.1)-(2.12), we get the following dimensionless equations:

$$\nabla.\boldsymbol{u}=0, \tag{2.13}$$

$$\begin{gathered}\varepsilon^{-1}P^{-1}\left[\frac{\partial\boldsymbol{u}}{\partial t}+\varepsilon^{-1}(\boldsymbol{u}.\nabla)\boldsymbol{u}\right]=-\nabla\mathrm{P_{eff}}+Ra(1+\mathrm{M}_1)\theta\vec{k}-\mathrm{Ra}(\psi+\mathrm{M}_1\psi_m)C\vec{k}+\\ -\mathrm{Ra}(\mathrm{M}_1-\mathrm{M}_5)\frac{\partial\phi}{\partial z}\vec{k}-Ra\mathrm{M}_1(\theta-\psi_m C)\nabla\left(\frac{\partial\phi}{\partial z}\right)+Ta^{\frac{1}{2}}\left(\boldsymbol{u}\wedge\vec{k}\right)+\Lambda\nabla^2\boldsymbol{u}+\mathrm{S}\boldsymbol{f}-Da^{-1}\boldsymbol{u},\end{gathered} \tag{2.14}$$

$$\left[\frac{\partial\theta}{\partial t}+(\boldsymbol{u}.\nabla)\theta\right]-\mathrm{M}_4\left[\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right)+(\boldsymbol{u}.\nabla)\frac{\partial\phi}{\partial z}\right]=(1-\mathrm{M}_4)w+\nabla^2\theta+F\nabla^2\left[C-\mathrm{M}_2\frac{\partial\phi}{\partial z}\right], \tag{2.15}$$

$$\left[\frac{\partial C}{\partial t}+(\boldsymbol{u}.\nabla)C\right]=L\nabla^2\left[C+\theta-\mathrm{M}_2\frac{\partial\phi}{\partial z}\right], \tag{2.16}$$

$$\left[\frac{\partial^2\phi}{\partial z^2}+\mathrm{M}_3\nabla_\perp^2\phi\right]=\frac{\partial}{\partial z}[\theta-\psi_m C], \tag{2.17}$$

$$\nabla^2_{\phi_{ext}}=0 \tag{2.18}$$

where

$$\nabla_\perp^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$$

represents the horizontal nabla operator.
From Eqs (2.13)-(2.18), the following dimensionless numbers appear:

(a) Ordinary fluids in porous media:

The thermal Rayleigh number which depends on the temperature gradient is $Ra = \alpha_T \rho_o g \beta d^4 / \mu k$, the Prandlt number is $P = \mu / \rho_o k$ which is the ratio between the kinematic viscosity and the coefficient of thermal diffusivity, the Darcy number is $Da = k_p / d^2$ which characterizes the permeability of the medium; the Brinkman number is $\Lambda = \mu_e / \mu$ which is the ratio between the effective viscosity and the dynamic viscosity and the intensity of the helical force is $S = a\varpi d^2 \varepsilon^{-1} / \nu$.

(b) Magnetic fluids:

The ratio of the magnetic force to the gravitational force is $M_1 = \beta\chi_T^2 H_o^2 \left(\alpha_T \rho_o g (1+\chi)\right)$, the magnetophoretic number which gives rise to a dependence between the heat current and concentration fields is $M_2 = \tilde{D}\chi_c \chi_T H_o^2 / \rho_o \bar{D}_T (1+\chi)$, the non-linearity of the magnetization is $M_3 = \left(1+\chi_o + \chi_H H_o^2\right)/(1+\chi)$, the force relating to the thermal dependence of magnetic susceptibility is $M_4 = \chi_T H_o^2 T_o / C_H (1+\chi)$, the magnetic and thermal buoyancy ratio is $M_5 = \alpha_H \chi_T H_o^2 / \alpha_T (1+\chi)$ and the magnetic separation ratio $\psi_m = -\rho_o \chi_c \bar{D}_T / \chi_T \tilde{D}\gamma_H = -\chi_c D_s / \chi_T D_c$.

(c) Binary mixtures:

The Lewis number is $L = D_c / k$ which represents the ratio between thermal and mass diffusion times, the separation report is $\psi = \rho_o \bar{D}_T \alpha_c / \alpha_T \tilde{D}\gamma_H$ and the Dufour number is $F = D_s \bar{D}_T \rho_o T_o / k C_H \tilde{D} = \bar{D}_T^2 / \tilde{D}\bar{k}$ with $D_c = \tilde{D}\gamma_H / \rho_o^2$ and $D_s = \bar{D}_T / \rho_o$.

(d) Rotation in ordinary fluids:

The Taylor number is $Ta = \left(2\rho_o \varepsilon^{-1} \varpi d^2 / \mu\right)^2$.

The magnetic numbers used have the following orders of magnitude: $M_1 = 10^{-4} - 10$, $M_3 \simeq 1.1$, $M_4 \simeq M_5 \simeq 10^{-6}$ [21].

The values of the numbers M_4, M_5 and F are very small. This is why they will be neglected in the calculations which follow.

3. Linear stability analysis

To study the linear stability of the system, we need the linear parts of equations (2.13)-(2.17). The effective pressure and the two components of the velocity field can be easily eliminated by applying $\nabla\Lambda$ et $\nabla\Lambda\nabla\Lambda$ to the Navier-Stokes equations and considering the vertical components of the resulting equations. After some calculations, we obtain the following linear equations:

$$\begin{aligned} \varepsilon^{-1} P^{-1} \frac{\partial}{\partial t}\nabla^2 w &= \Lambda\nabla^4 w + Ra\nabla_\perp^2\left[(1+\mathrm{M}_1)\theta - (\psi + \mathrm{M}_1\psi_m)C - \mathrm{M}_1\frac{\partial\phi}{\partial z}\right] + \\ &+ \mathrm{S}\left[\nabla_\perp^2\xi - \frac{\partial^2\xi}{\partial z^2}\right] - Ta^{\frac{1}{2}}\frac{\partial\xi}{\partial z} - Da^{-1}\nabla^2 w, \end{aligned} \tag{3.1}$$

$$\varepsilon^{-1}P^{-1}\frac{\partial\xi}{\partial t}=\Lambda\nabla^2\xi+\mathrm{S}\frac{\partial^2 w}{\partial z^2}+Ta^{\frac{1}{2}}\frac{\partial w}{\partial z}-Da^{-1}\xi\ , \tag{3.2}$$

$$\frac{\partial\theta}{\partial t}=\nabla^2\theta+w\ , \tag{3.3}$$

$$L^{-1}\frac{\partial C}{\partial t}=\nabla^2\left[C+\theta-\mathrm{M}_2\frac{\partial\phi}{\partial z}\right], \tag{3.4}$$

$$\left[\frac{\partial^2\phi}{\partial z^2}+\mathrm{M}_3\nabla_\perp^2\phi\right]=\frac{\partial}{\partial z}\left[\theta-\psi_m C\right]. \tag{3.5}$$

By following the standard techniques described by Chandrasekhar [21] the spatio-temporal dependencies are separated by the decomposition in normal mode

$$(\theta,C,\xi,w,\phi)(\boldsymbol{r},t)=\left[\Theta(z),\psi(z),\mathrm{Z}(\mathrm{z}),\mathrm{W}(\mathrm{z}),\Phi(\mathrm{z})\right]exp\left(i\boldsymbol{k}.\boldsymbol{r}_\perp+st\right) \tag{3.6}$$

where $\boldsymbol{k}$ represents the vector of the horizontal wavenumber of the perturbations, $\boldsymbol{r}_\perp$ is the horizontal position vector and $s=\sigma+i\omega$ is a complex number which represents the rate of growth of the perturbations with σ the disturbance growth factor and ω is the disturbance frequency.

By replacing Eq.(3.6) in Eqs (3.1)-(3.5), we obtain the following ordinary differential equations

$$\begin{gathered}\varepsilon^{-1}P^{-1}s\left(D^2-k^2\right)W=\Lambda\left(D^2-k^2\right)^2W-Rak^2\left[(1+\mathrm{M}_1)\Theta+\right.\\ \left.-(\psi+\mathrm{M}_1\psi_m)\psi-\mathrm{M}_1D\Phi\right]+S\left(D^2+k^2\right)-Ta^{\frac{1}{2}}D\mathrm{Z}-Da^{-1}\left(D^2-k^2\right)W\ ,\end{gathered} \tag{3.7}$$

$$\varepsilon^{-1}P^{-1}s\,Z=Ta^{\frac{1}{2}}D\mathrm{W}+\Lambda\left(D^2-k^2\right)Z-Da^{-1}Z+S\,D^2W\ , \tag{3.8}$$

$$s\Theta=\left(D^2-k^2\right)\Theta+\mathrm{W}\ , \tag{3.9}$$

$$sL^{-1}\psi=\left(D^2-k^2\right)\left(\psi+\Theta-\mathrm{M}_2D\Phi\right), \tag{3.10}$$

$$D^2\Phi=\mathrm{M}_3k^2\Phi+\mathrm{D}\Theta-\psi_m\mathrm{D}\psi\ . \tag{3.11}$$

By eliminating Z between the Eqs (3.7) and (3.8) we obtain:

$$\begin{aligned}&\varepsilon^{-2}P^{-2}s^2\left(D^2-k^2\right)W+\varepsilon^{-1}P^{-1}s\,Da^{-1}\left(D^2-k^2\right)W-2\varepsilon^{-1}P^{-1}s\Lambda\left(D^2-k^2\right)^2W=\\&=-\Lambda^2\left(D^2-k^2\right)^3W+2\Lambda Da^{-1}\left(D^2-k^2\right)^2W+\Lambda Rak^2\left(D^2-k^2\right)\left[\left(1+\mathrm{M}_1\right)\Theta+\right.\\&-\left(\psi+\mathrm{M}_1\psi_m\right)\psi-\mathrm{M}_1D\Phi\left.\right]+-Rak^2\left(Da^{-1}+\varepsilon^{-1}P^{-1}s\right)\left[\left(1+\mathrm{M}_1\right)\Theta-\left(\psi+\mathrm{M}_1\psi_m\right)\psi+\right.\\&-\mathrm{M}_1D\Phi\left.\right]-S^2D^2\left(D^2+k^2\right)W-Ta\,D^2W+\\&-Da^{-2}\left(D^2-k^2\right)W-Da^{-1}\varepsilon^{-1}P^{-1}s\left(D^2-k^2\right)W.\end{aligned}\tag{3.12}$$

In order to solve Eqs (3.9)-(3.12) analytically, the boundary conditions below are used at $z=-\frac{1}{2}d$ and $z=\frac{1}{2}d$

$$\Theta=D^2W=W=D\Phi=\psi=0\,.\tag{3.13}$$

The exact solutions which satisfy the boundary conditions (3.13) are [21]:

$$W=\boldsymbol{A}\cos(\pi z),\ \ \Theta=\boldsymbol{B}\cos(\pi z),\ \ D\Phi=\boldsymbol{C}\cos(\pi z),\ \ \Phi=\frac{\boldsymbol{C}}{\pi}\sin(\pi z),\ \ \psi=\boldsymbol{E}\cos(\pi z)\tag{3.14}$$

where **A**, **B**, **C** and **E** are constants. By replacing Eq.(3.13) in Eqs (3.9)-(3.12), we get:

$$\boldsymbol{A}-\left[s+\left(\pi^2+k^2\right)\right]\boldsymbol{B}=0\,,\tag{3.15}$$

$$\left(\pi^2+k^2\right)\boldsymbol{B}-\mathrm{M}_2\left(\pi^2+k^2\right)\boldsymbol{C}+\left[sL^{-1}+\left(\pi^2+k^2\right)\right]\boldsymbol{E}=0\,,\tag{3.16}$$

$$\pi^2\boldsymbol{B}-\left(\pi^2+\mathrm{M}_3k^2\right)\boldsymbol{C}-\pi^2\psi_m\boldsymbol{E}=0\,,\tag{3.17}$$

$$\begin{aligned}&-\left[\varepsilon^{-2}P^{-2}s^2\delta^2+2\varepsilon^{-1}P^{-1}Da^{-1}s\delta^2+2\varepsilon^{-1}P^{-1}s\Lambda\delta^4+\Lambda^2\delta^6+2\Lambda Da^{-1}\delta^4+\right.\\&\left.-\pi^2S^2\left(\pi^2-k^2\right)+\pi^2Ta+Da^{-2}\delta^2\right]\boldsymbol{A}+Rak^2\left(1+\mathrm{M}_1\right)\left[\Lambda\delta^2+\left(Da^{-1}+\varepsilon^{-1}P^{-1}s\right)\right]\boldsymbol{B}\\&-Rak^2\left(\psi+\mathrm{M}_1\psi_m\right)\left[\Lambda\delta^2+\left(Da^{-1}+\varepsilon^{-1}P^{-1}s\right)\right]\boldsymbol{E}+\\&-Rak^2\mathrm{M}_1\left[\Lambda\delta^2+\left(Da^{-1}+\varepsilon^{-1}P^{-1}s\right)\right]\boldsymbol{C}=0\,.\end{aligned}\tag{3.18}$$

The system formed by Eqs (3.15)-(3.18) admits a non-trivial solution only if the determinant of the matrix whose elements are the coefficients of the constants $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{E}$ is equal to zero.
Thus, the eigenvalue problem leads to the following characteristic polynomial:

$$p(s)=a_o+a_1s+a_2s^2+a_3s^3+a_4s^4=0\tag{3.19}$$

where the coefficients a_o, a_1, a_2, a_3 and a_4 are based on system settings with:

$$a_4 = L^{-1}\varepsilon^{-2}P^{-2}\left(\pi^2 + k^2\right)\left(\pi^2 + \mathrm{M}_3 k^2\right) \ , \tag{3.20}$$

$$\begin{aligned} a_3 =& \varepsilon^{-1}P^{-1}\left(\pi^2 + k^2\right)^2\Big[\varepsilon^{-1}P^{-1}\pi^2\mathrm{M}_2\psi_m + \varepsilon^{-1}P^{-1}\left(\pi^2 + \mathrm{M}_3 k^2\right)\left(1 + L^{-1}\right) + \\ &+ 2\Lambda L^{-1}\left(\pi^2 + \mathrm{M}_3 k^2\right)\Big] + 2\varepsilon^{-1}P^{-1}L^{-1}Da^{-1}\left(\pi^2 + k^2\right)\left(\pi^2 + \mathrm{M}_3 k^2\right), \end{aligned} \tag{3.21}$$

$$\begin{aligned} a_2 =& \varepsilon^{-2}P^{-2}\left(\pi^2 + k^2\right)^3\Big[\left(\pi^2 + \mathrm{M}_3 k^2\right) + \pi^2\mathrm{M}_2\psi_m\Big] + 2\varepsilon^{-1}P^{-1}\left(\pi^2 + k^2\right)^2\Big[\Big(\pi^2 + \\ &+ \mathrm{M}_3 k^2\Big)\left(1 + L^{-1}\right) + \pi^2\mathrm{M}_2\psi_m\Big]\Big[Da^{-1} + \Lambda\left(\pi^2 + k^2\right)\Big] + L^{-1}\left(\pi^2 + \mathrm{M}_3 k^2\right)\times \\ &\times\Big[\Lambda^2\left(\pi^2 + k^2\right)^3 + 2\Lambda Da^{-1}\left(\pi^2 + k^2\right)^2 + Da^{-2}\left(\pi^2 + k^2\right) - \pi^2 S^2\left(\pi^2 - k^2\right) + \pi^2 Ta\Big] + \\ &- Rak^2\varepsilon^{-1}P^{-1}L^{-1}\Big[\left(1 + \mathrm{M}_1\right)\left(\pi^2 + \mathrm{M}_3 k^2\right) - \pi^2\mathrm{M}_1\Big], \end{aligned} \tag{3.22}$$

$$\begin{aligned} a_1 =& \left(\pi^2 + k^2\right)^2\Big[\left(\pi^2 + \mathrm{M}_3 k^2\right)\left(1 + L^{-1}\right) + \pi^2\mathrm{M}_2\psi_m\Big]\Big[\Lambda^2\left(\pi^2 + k^2\right)^3 + \\ &+ 2\Lambda Da^{-1}\left(\pi^2 + k^2\right)^2 + Da^{-2}\left(\pi^2 + k^2\right) - \pi^2 S^2\left(\pi^2 - k^2\right) + \pi^2 Ta\Big] + \\ &+ 2\varepsilon^{-1}P^{-1}\left(\pi^2 + k^2\right)^3\Big[\left(\pi^2 + \mathrm{M}_3 k^2\right) + \pi^2\mathrm{M}_2\psi_m\Big]\Big[Da^{-1} + \Lambda\left(\pi^2 + k^2\right)\Big] + \\ &- Rak^2\Big[L^{-1}\left(1 + \mathrm{M}_1\right)\left(\pi^2 + \mathrm{M}_3 k^2\right)\left(Da^{-1} + \Lambda\left(\pi^2 + k^2\right)\right) + \\ &+ \varepsilon^{-1}P^{-1}\left(\pi^2 + k^2\right)\left(\left(\pi^2 + \mathrm{M}_3 k^2\right) + \pi^2\mathrm{M}_2\psi_m\right) + \\ &- \pi^2\mathrm{M}_1\left(\varepsilon^{-1}P^{-1}\left(\pi^2 + k^2\right)\left(1 + \psi_m\right) + L^{-1}\left(Da^{-1} + \Lambda\left(\pi^2 + k^2\right)\right)\right) + \\ &+ \varepsilon^{-1}P^{-1}\left(\pi^2 + k^2\right)\left(\psi + \mathrm{M}_1\psi_m\right)\left(\left(\pi^2 + \mathrm{M}_3 k^2\right) - \pi^2\mathrm{M}_2\right)\Big], \end{aligned} \tag{3.23}$$

$$\begin{aligned} a_o =& \left(\pi^2 + k^2\right)^2\Big[\left(\pi^2 + \mathrm{M}_3 k^2\right) + \pi^2\mathrm{M}_2\psi_m\Big]\Big[\Lambda^2\left(\pi^2 + k^2\right)^3 + 2\Lambda Da^{-1}\left(\pi^2 + k^2\right)^2 + \\ &+ Da^{-2}\left(\pi^2 + k^2\right) - \pi^2 S^2\left(\pi^2 - k^2\right) + \pi^2 Ta\Big] - Rak^2\left(\pi^2 + k^2\right)\Big[Da^{-1} + \Lambda\left(\pi^2 + k^2\right)\Big]\times \\ &\times\Big[\pi^2\left(1 + \psi + \mathrm{M}_2\left(\psi_m - \psi\right)\right) + \mathrm{M}_3 k^2\left(1 + \psi + \mathrm{M}_1\left(1 + \psi_m\right)\right)\Big] \end{aligned} \tag{3.24}$$

$p(s) = 0$ leads

$$Ra = \frac{\mathrm{A}}{k^2\mathrm{B}} \tag{3.25}$$

with

$$\mathrm{A} = \left[s+\left(\pi^2+k^2\right)\right]\left[\left(\pi^2+\mathrm{M}_3k^2\right)\left(L^{-1}s+\left(\pi^2+k^2\right)\right)+\pi^2\left(\pi^2+k^2\right)\mathrm{M}_2\psi_m\right]\times$$
$$\times\left[\varepsilon^{-2}P^{-2}\left(\pi^2+k^2\right)s^2+2\varepsilon^{-1}P^{-1}\Lambda\left(\pi^2+k^2\right)^2s+\Lambda^2\left(\pi^2+k^2\right)^3+\right.$$
$$\left.+2\varepsilon^{-1}P^{-1}Da^{-1}\left(\pi^2+k^2\right)s+2\Lambda Da^{-1}\left(\pi^2+k^2\right)^2-\pi^2S^2\left(\pi^2-k^2\right)+\pi^2Ta+Da^{-2}\left(\pi^2+k^2\right)\right]$$

and

$$\mathrm{B}=\left[\Lambda\left(\pi^2+k^2\right)+Da^{-1}+\varepsilon^{-1}P^{-1}s\right]\left[(1+\mathrm{M}_1)\left(\left(\pi^2+\mathrm{M}_3k^2\right)\left(L^{-1}s+\left(\pi^2+k^2\right)\right)+\right.\right.$$
$$+\pi^2\left(\pi^2+k^2\right)\mathrm{M}_2\psi_m\Big)+\left(\pi^2+k^2\right)(\psi+\mathrm{M}_1\psi_m)\left(\left(\pi^2+\mathrm{M}_3k^2\right)-\pi^2\mathrm{M}_2\right)+$$
$$\left.-\pi^2\mathrm{M}_1\left(L^{-1}s+\left(\pi^2+k^2\right)(1+\psi_m)\right)\right].$$

We know that $s=\sigma+i\omega$. To study system instability, we set $\sigma=0$ and then $s=i\omega$.

Stationary convection

In the case of stationary convection $\omega=0$, we find the marginal stability curve between the Rayleigh number (Ra) and the wave number (k). The Rayleigh number is given by the expression:

$$Ra=\left\{\left(\pi^2+k^2\right)\left[\pi^2(1+\mathrm{M}_2\psi_m)+\mathrm{M}_3k^2\right]\left[\Lambda^2\left(\pi^2+k^2\right)^3+2\Lambda Da^{-1}\left(\pi^2+k^2\right)^2+\right.\right.$$
$$\left.-\pi^2S^2\left(\pi^2-k^2\right)+\pi^2Ta+Da^{-2}\left(\pi^2+k^2\right)\right]\left\{k^2\left[\Lambda\left(\pi^2+k^2\right)+Da^{-1}\right]\times\right. \tag{3.26}$$
$$\left.\left.\times\left[\pi^2\left(1+\psi+\mathrm{M}_2(\psi_m-\psi)\right)+\mathrm{M}_3k^2\left(1+\psi+\mathrm{M}_1(1+\psi_m)\right)\right]\right\}^{-1}.$$

From Eq. (3.26), we recognized some known results in literature. If $S=0$, $Ta=0$, $\Lambda=1$, $Da=\infty$ and $\psi=\psi_m=0$, we find the expression of the Rayleigh number in the case of convective instability in a horizontal layer of a ferrofluid heated from below in the presence of a uniform magnetic field [6]. If $S=0$, $\Lambda=1$, $Da=\infty$ and $\psi=\psi_m=0$, we find the expression of the Rayleigh number in the case of convective instability in a horizontal layer of a rotating ferrofluid heated from below in the presence of a uniform magnetic field [21]. If $\Lambda=1$, $Da=\infty$ and $Ta=0$, we find the expression of the Rayleigh number in the case of the effect of the helical force on stationary convection of a binary ferrofluid [28]. If $\Lambda=1$ and $Da=\infty$, we find the expression of the Rayleigh number in the case of the effect of the helical force on stationary convection of a rotating binary ferrofluid [29]. If $S=0$ and $Ta=0$, we find the expression of the Rayleigh number in the case of stationary convection of a binary ferrofluid in a porous medium [30].

The critical Rayleigh number R_{ac} and the critical wave number k_c are determined numerically. The equation verifying the critical wave number k_c is obtained by minimizing the expression of the Rayleigh number R_a with respect to the wave number k (i.e. $\frac{\partial \mathrm{Ra}}{\partial \mathrm{k}}=0$). The determined critical wave number k_c is then replaced in the expression of the Rayleigh number to find the critical Rayleigh number R_{ac}.

The Rayleigh number R_a reaches its critical value R_{ac} at $k^2 = k_c^2$ where k_c^2 satisfies the equation

$$b_7(k_c^2)^7 + b_6(k_c^2)^6 + b_5(k_c^2)^5 + b_4(k_c^2)^4 + b_3(k_c^2)^3 + b_2(k_c^2)^2 + b_1\left(k_c^2\right) + b_0 = 0 \tag{3.27}$$

where

$$b_0 = -\pi^8\left(1+\mathrm{M}_2\psi_m\right)\left(\pi^2\Lambda + Da^{-1}\right)\left(1+\psi+\mathrm{M}_2\left(\psi_m-\psi\right)\right)\Big[\pi^4\Lambda^2 +$$
$$+2\pi^2\Lambda Da^{-1} - \pi^2 S^2 + Ta + Da^{-2}\Big],$$
$$b_1 = -2\pi^6\left(1+\mathrm{M}_2\psi_m\right)\Big[\mathrm{M}_3\left(\pi^2\Lambda + Da^{-1}\right)\left(1+\psi+\mathrm{M}_1\left(1+\psi_m\right)\right) + \pi^2\Lambda\left(1+\psi+\right.$$
$$\left.+\mathrm{M}_2\left(\psi_m-\psi\right)\right)\Big]\Big[\pi^4\Lambda^2 + 2\pi^2\Lambda Da^{-1} - \pi^2 S^2 + Ta + Da^{-2}\Big],$$

$$b_2 = \pi^4\Big[\left(1+\mathrm{M}_2\psi_m + \mathrm{M}_3\right)\left(3\pi^4\Lambda^2 + 4\pi^2\Lambda Da^{-1} + \pi^2 S^2 + Da^{-2}\right) + \mathrm{M}_3\left(\pi^4\Lambda^2 +\right.$$
$$\left.+2\pi^2\Lambda Da^{-1} - \pi^2 S^2 + Ta + Da^{-2}\right) + \pi^2\Lambda\left(1+\mathrm{M}_2\psi_m\right)\left(3\pi^2\Lambda + 2Da^{-1}\right)\Big]\Big[\left(\pi^2\Lambda +\right.$$
$$\left.+Da^{-1}\right)\left(1+\psi+\mathrm{M}_2\left(\psi_m-\psi\right)\right)\Big] - \pi^4\Big[\left(1+\mathrm{M}_2\psi_m + \mathrm{M}_3\right)\left(\pi^4\Lambda^2 + 2\pi^2\Lambda Da^{-1} +\right.$$
$$\left.-\pi^2 S^2 + Ta + Da^{-2}\right) + \left(1+\mathrm{M}_2\psi_m\right)\left(3\pi^4\Lambda^2 + 4\pi^2\Lambda Da^{-1} + \pi^2 S^2 + Da^{-2}\right)\Big]\times$$
$$\times\Big[\mathrm{M}_3\left(\pi^2\Lambda + Da^{-1}\right)\left(1+\psi+\mathrm{M}_1\left(1+\psi_m\right)\right) + \pi^2\Lambda\left(1+\psi+\mathrm{M}_2\left(\psi_m-\psi\right)\right)\Big] +$$
$$-3\pi^6\Lambda\mathrm{M}_3\left(1+\mathrm{M}_2\psi_m\right)\Big[1+\psi+\mathrm{M}_1\left(1+\psi_m\right)\Big]\Big[\pi^4\Lambda^2 + 2\pi^2\Lambda Da^{-1} - \pi^2 S^2 + Ta + Da^{-2}\Big],$$

$$b_3 = 2\pi^2\Big[\pi^2\Lambda\left(1+\mathrm{M}_2\psi_m + \mathrm{M}_3\right)\left(3\pi^2\Lambda + 2Da^{-1}\right) + \mathrm{M}_3\left(3\pi^4\Lambda^2 + 4\pi^2\Lambda Da^{-1} + \pi^2 S^2 +\right.$$
$$\left.+Da^{-2}\right) + \pi^4\Lambda^2\left(1+\mathrm{M}_2\psi_m\right)\Big]\Big[\left(\pi^2\Lambda + Da^{-1}\right)\left(1+\psi+\mathrm{M}_2\left(\psi_m-\psi\right)\right)\Big] - 2\pi^4\Lambda\mathrm{M}_3\Big[\left(1+\psi+\right.$$
$$\left.+\mathrm{M}_1\left(1+\psi_m\right)\right)\Big]\Big[\left(1+\mathrm{M}_2\psi_m + \mathrm{M}_3\right)\left(\pi^4\Lambda^2 + 2\pi^2\Lambda Da^{-1} - \pi^2 S^2 + Ta + Da^{-2}\right) +$$
$$+\left(1+\mathrm{M}_2\psi_m\right)\left(3\pi^4\Lambda^2 + 4\pi^2\Lambda Da^{-1} + \pi^2 S^2 + Da^{-2}\right)\Big],$$

$$b_4 = -\Big[\Lambda\mathrm{M}_3\left(1+\psi+\mathrm{M}_1\left(1+\psi_m\right)\right)\Big]\Big[\pi^2\left(1+\mathrm{M}_2\psi_m + \mathrm{M}_3\right)\left(3\pi^4\Lambda^2 + 4\pi^2\Lambda Da^{-1} +\right.$$
$$\left.+\pi^2 S^2 + Da^{-2}\right) + \pi^2\mathrm{M}_3\left(\pi^4\Lambda^2 + 2\pi^2\Lambda Da^{-1} - \pi^2 S^2 + Ta + Da^{-2}\right) +$$
$$+\pi^4\Lambda\left(1+\mathrm{M}_2\psi_m\right)\left(3\pi^2\Lambda + 2Da^{-1}\right)\Big] + \Big[\pi^2\Lambda\left(1+\mathrm{M}_2\psi_m + \mathrm{M}_3\right)\left(3\pi^2\Lambda + 2Da^{-1}\right) +$$
$$+\mathrm{M}_3\left(3\pi^4\Lambda^2 + 4\pi^2\Lambda Da^{-1} + \pi^2 S^2 + Da^{-2}\right) + \pi^4\Lambda^2\left(1+\mathrm{M}_2\psi_m\right)\Big]\Big[\mathrm{M}_3\left(Da^{-1} +\right.$$
$$\left.+\pi^2\Lambda\right)\left(1+\psi+\mathrm{M}_1\left(1+\psi_m\right)\right) + \pi^2\Lambda\left(1+\psi+\mathrm{M}_2\left(\psi_m-\psi\right)\right)\Big] + 3\pi^2\Lambda\Big[\left(Da^{-1} +\right.$$
$$\left.++\pi^2\Lambda\right)\left(1+\psi+\mathrm{M}_2\left(\psi_m-\psi\right)\right)\Big]\Big[\pi^2\Lambda\left(1+\mathrm{M}_2\psi_m + \mathrm{M}_3\right) + \mathrm{M}_3\left(3\pi^2\Lambda + 2Da^{-1}\right)\Big],$$

$$b_5 = 2\left[\pi^2\Lambda^2\left(1+M_2\psi_m + M_3\right) + M_3\Lambda\left(3\pi^2\Lambda + 2Da^{-1}\right)\right]\left[M_3\left(Da^{-1}+\pi^2\Lambda\right)\left(1+\psi+\right.\right.$$
$$\left.\left.+M_1\left(1+\psi_m\right)\right)+\pi^2\Lambda\left(1+\psi+M_2\left(\psi_m-\psi\right)\right)\right]+$$
$$+4M_3\pi^2\Lambda^2\left(Da^{-1}+\pi^2\Lambda\right)\left[1+\psi+M_2\left(\psi_m-\psi\right)\right],$$

$$b_6 = M_3\Lambda^2\left[\pi^2\Lambda\left(1+M_2\psi_m+M_3\right)+M_3\left(3\pi^2\Lambda+2Da^{-1}\right)\right]\left[1+\psi+M_1\left(1+\psi_m\right)\right]+$$
$$+3M_3\Lambda^2\left[M_3\left(Da^{-1}+\pi^2\Lambda\right)\left(1+\psi+M_1\left(1+\psi_m\right)\right)+\pi^2\Lambda\left(1+\psi+M_2\left(\psi_m-\psi\right)\right)\right],$$

$$b_7 = 2M_3^2\Lambda^3\left[1+\psi+M_1\left(1+\psi_m\right)\right].$$

4. Main results and discussion

Our study consists in analysing theoretically the criterion for the appearance of stationary convection in a horizontal layer saturated with a binary mixture of a ferrofluid in the presence of an external magnetic field and heated from below in a rotating porous medium and in the presence of helical force, with the effects of porosity, rotation and helical force being taken into account simultaneously. The study consists in comparing the onset of convection as obtained in the works [28], [29], [30] where the effects of porosity, rotation and helical force were considered separately, with the onset of convection when the three effects are considered simultaneously.

Figures 2, 3, 4 and 5 present, respectively, the curves of marginal stability in the plane (k, Ra) for distinct values of the Taylor number, the intensity of the helical force, the Darcy number and the ratio viscosities. The coordinates of the lowest point of each of these curves (i.e. the convection threshold) denote the critical values of the wave number (k) and the Rayleigh number (Ra). Then we take a value of these parameters and we see on which side the stability threshold moves when the action of the helical force, rotation and porosity is simultaneous. We then obtain Figs 6, 7, 8 and 9.

Figures 2 and 5 show that the Rayleigh number (Ra) increases with the Taylor number (Ta) and the viscosity ratio. This therefore means that the Taylor number and the viscosity ratio delay the onset of stationary convection and stabilize the system. The instability zone decreases in favor of the system stability zone.

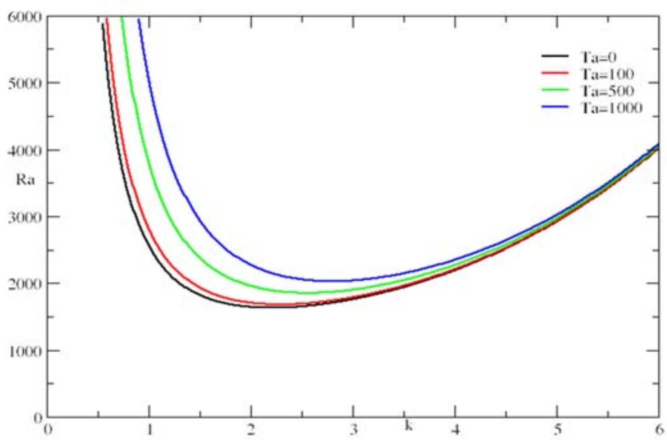

Fig.2. Marginal stability curves with $M_1 = 0.1$, $M_2 = 1$, $M_3 = 1.1$, $\psi = 1$, $\psi_m = 3$, $Da = 0.05$, $\Lambda = 2$ and $S = 10$.

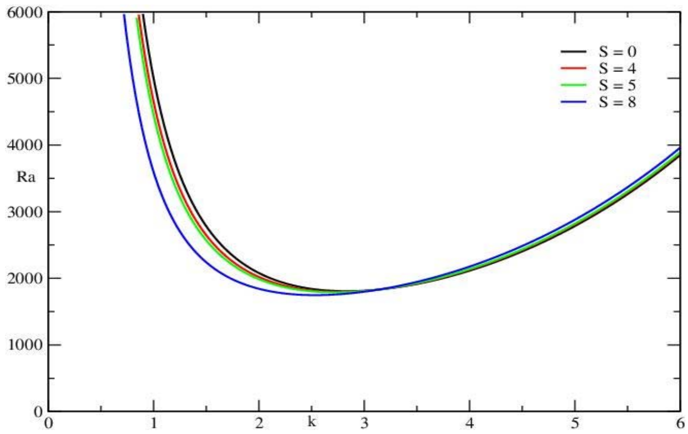

Fig.3. Marginal stability curves with $M_1 = 0.1$, $M_2 = 1$, $M_3 = 1.1$, $\psi = 1$, $\psi_m = 3$, $Da = 0.05$, $\Lambda = 2$ and $Ta = 100$.

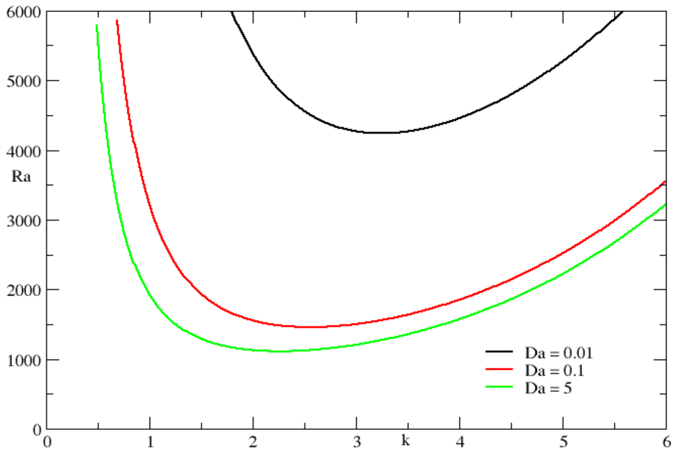

Fig.4. Marginal stability curves with $M_1 = 0.1$, $M_2 = 1$, $M_3 = 1.1$, $\psi = 1$, $\psi_m = 3$, $\Lambda = 2$, $S = 5$ and $Ta = 100$.

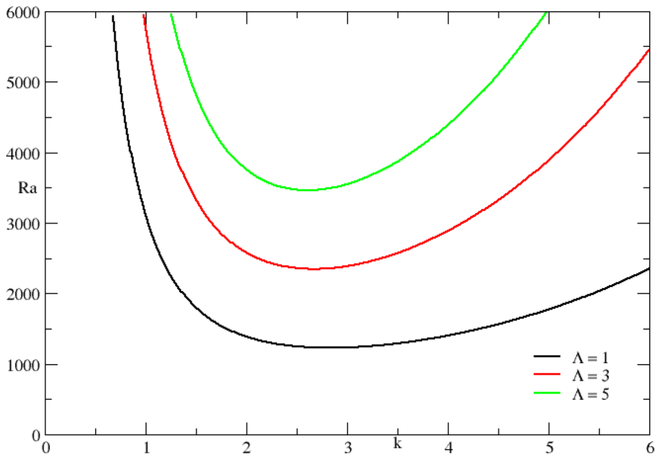

Fig.5. Marginal stability curves with $M_1 = 0.1$, $M_2 = 1$, $M_3 = 1.1$, $\psi = 1$, $\psi_m = 3$, $Da = 0.05$, $S = 5$ and $Ta = 100$.

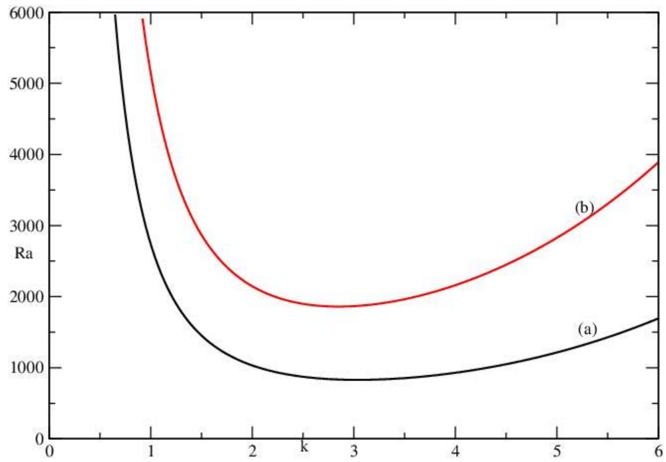

Fig.6. Marginal stability curves with $M_1 = 0.1$, $M_2 = 1$, $M_3 = 1.1$, $\psi = 1$, $\psi_m = 3$, $S = 5$ and $Ta = 300$; (a): non-porous medium, (b): porous medium with $Da = 0.05$ and $\Lambda = 2$.

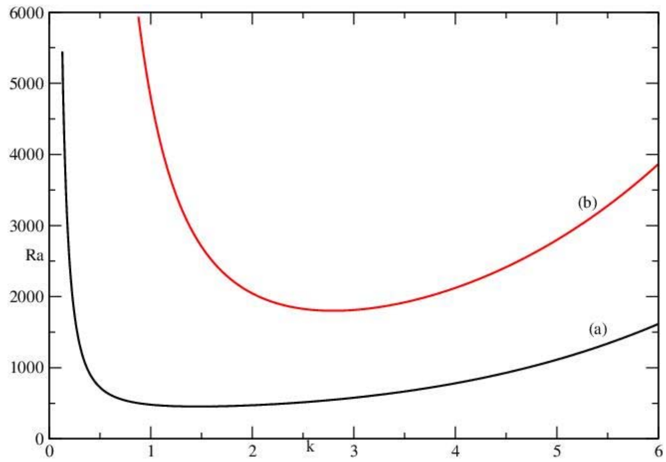

Fig.7. Marginal stability curves with $M_1 = 0.1$, $M_2 = 1$, $M_3 = 1.1$, $\psi = 1$, $\psi_m = 3$, $S = 3$.(a): non-porous and non-rotating medium, (b): porous and rotating medium with $Ta = 100$, $Da = 0.05$ and $\Lambda = 2$.

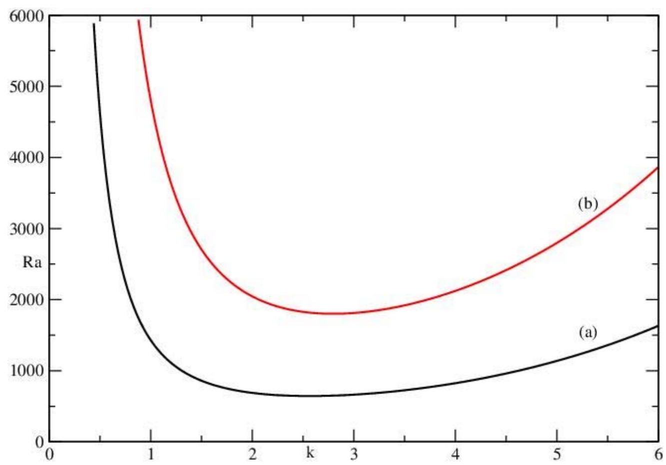

Fig.8. Marginal stability curves with $M_1 = 0.1$, $M_2 = 1$, $M_3 = 1.1$, $\psi = 1$, $\psi_m = 3$, $S = 3$, $Ta = 100$; (a): non-porous medium, (b): porous medium with $Da = 0.05$ and $\Lambda = 2$.

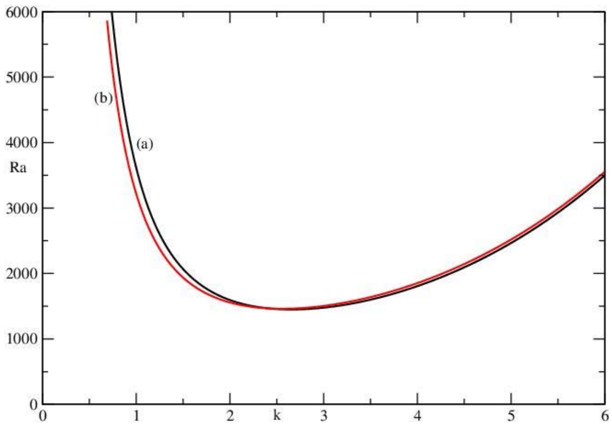

Fig.9. Marginal stability curves with $M_1 = 0.1$, $M_2 = 1$, $M_3 = 1.1$, $\psi = 1$, $\psi_m = 3$, $\Lambda = 2$, $Da = 0.1$; (a): Non-rotating porous medium without helical force , (b): porous medium with $S = 5$ and $Ta = 100$.

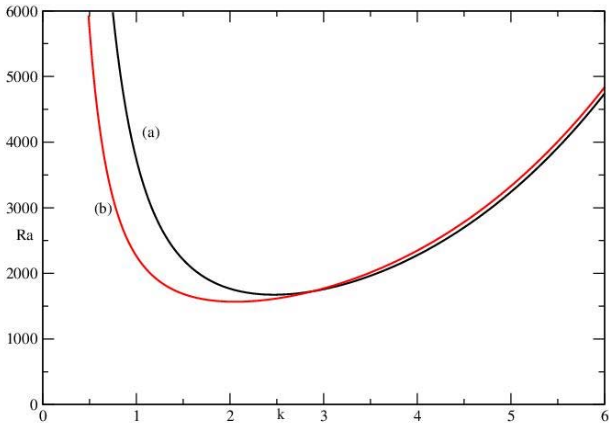

Fig.10. Marginal stability curves with $M_1 = 0.1$, $M_2 = 1$, $M_3 = 1.1$, $\psi = 1$, $\psi_m = 3$, $\Lambda = 3$, $Da = 5$; (a): Non-rotating porous medium without helical force, (b): porous medium with $S = 8$ and $Ta = 100$.

In Figs 3 and 4, it can be seen that the Rayleigh number (Ra) decreases with the Darcy number (Da) and intensity of the helical force (S). So the Darcy number (Da) and the intensity of the helical force (S) accelerate the onset of stationary convection and destabilize the system. This means that the more the porous medium is permeable to the flow of ferrofluid, the more the system is destabilized and the appearance of stationary convection is accelerated. The zone of stability decreases in favor of the zone of instability.

The results known on each of these parameters when they are studied separately and their effects on the onset of convection in the ferrofluid have thus just been confirmed in the present case where their effects are studied simultaneously. By comparing Fig.2 of the present work with Fig.3 of [29] and looking at the lowest points for the same values of Ta (Fig.6), we easily notice that the simultaneous consideration of the effects of helical force, porosity and rotation drastically displaces the onset of convection by delaying it to a large extent.

Comparing Fig.3 of the present work with Fig.2 of [28] and [29] and looking at the lowest points for the same values of S (Figs 7 and 8), we easily notice that the simultaneous consideration of the effects of helical force, porosity and rotation drastically displaces the onset of convection by delaying it to a large extent.

Concerning the Darcy number and the viscosity ratio which are two parameters related to the porous medium, we note that the convection threshold is not displaced in the same way as previously, but in reverse and more slowly (Figs 9 and 10). We notice through all these figures that the effect of porosity prevails over the effect of helical force and the effect of rotation. As soon as the effect of porosity is taken into account, the convection threshold is delayed.

We then conclude that the simultaneous consideration of the effects leads to a better stability of the ferrofluid because this considerably delays the onset of convection in the case of the helical force and rotation. Note that when the critical Rayleigh number (R_{ac}) increases the system is stable. On the other hand, when increasing a parameter causes the critical Rayleigh number to decrease, the system is destabilized and the appearance of stationary convection is accelerated. The size of convection cells (convective structures) is linked to the evolution of the critical wave number (k_c). When the critical wave number increases as soon as a parameter increases, the size of convective cells (convective structures) is contracted. On the other hand, when the critical wave number decreases when a parameter increases, the size of the convective cells is enlarged.

5. Conclusion

In this work, linear stability theory is used to study the combined effects of helical force, rotation and porosity on the appearance of stationary convection in a binary mixture of a ferrofluid and on the size of convection cells. We determined the analytical expression of the thermal Rayleigh number according to the dimensionless parameters. The results known on each of these parameters when they are studied separately on the onset of convection in the ferrofluid have thus just been confirmed in the present case where their effects are studied simultaneously. We easily notice that the simultaneous consideration of the effects of helical force, porosity and rotation drastically displaces the onset of convection by delaying it to a large extent compared to the consideration of each effect separately in the case of helical force and rotation and this behavior depends on the values given to the parameters.

Acknowledgments

The authors thank The World Academy of Sciences (TWAS) for financial support through Research Grant Agreement N° 20-307 RG/PHYS/AF/AC_G-FR3240314170. The authors would like to thank very much the anonymous referees whose useful criticisms, comments and suggestions have helped improve the content and the quality of the paper.

Nomenclature

$\boldsymbol{B}$ − magnetic induction vector

C − magnetic particles concentration

C_o − concentration of magnetic particles at temperature T_o

C_H − specific heat at constant magnetic field, $\left(\mathrm{J.kg^{-1}.K^{-1}}\right)$)

d − thickness of the fluid layer, (m)

$D = \frac{d}{dz}$ − differential operator

Da − Darcy number

D_c − diffusion coefficient

D_s − Soret coefficient

F − Dufour number

$\boldsymbol{g}$ − vector acceleration of gravity, $\left(m.\,\mathrm{s}^{-2}\right)$

$\boldsymbol{H}$ – magnetic field vector $\left(A.\mathrm{m}^{-1}\right)$

$\boldsymbol{H_o}$ – reference magnetic field vector

$\boldsymbol{k} = (k_x, k_y, k_z)$ – wave number vector

k – wave number

k_p – permeability

L – Lewis number

$\boldsymbol{M}$ – magnetization vector $\left(A.\mathrm{m}^{-1}\right)$

M_1 – ratio of magnetic force to buoyancy

M_2 – magnetophoretic number

M_3 – non-linearity of the magnetization

M_4 – strength of the thermal dependence of magnetic susceptibility

M_5 – ratio of magnetic and thermal buoyancy

P – Prandlt number

$\mathrm{P_{eff}}$ – effective pressure (Pa)

$\boldsymbol{r}_\perp = (x, y)$ – horizontal vector position.

Ra – thermal Rayleigh number.

$s = \sigma + i\omega$ – complex eigenvalue.

S – intensity of the helical force.

t – time.

T – temperature of fluid particle, (K)

$\mathrm{T_o}$ – reference temperature

$\boldsymbol{u} = (u, v, w)$ – fluid particle velocity vector, $\left(m.\,\mathrm{s}^{-1}\right)$

w – dependence in z of the vertical component of the velocity

$(x, y, z))$ – cartesian coordinates

α_c – mass expansion coefficient

α_H – magnetic expansion coefficient

α_T – thermal expansion coefficient, $\left(\mathrm{K}^{-1}\right)$

β – temperature gradient

ρ – density of fluid particle, $\left(\mathrm{kg}.m^{-3}\right)$

ρ_o – density of fluid particle at temperature $\mathrm{T_o}$

$\nabla = (\partial x, \partial y, \partial z)$ – nabla operator

$\nabla_\perp = (\partial x, \partial y)$ – horizontal nabla operator

ΔT – temperature difference between top and bottom plates

k – thermal diffusivity of the fluid particle, $\left(m^2.\,\mathrm{s}^{-1}\right)$

ϕ – scalar magnetic potential

Φ – dependence in z of magnetic potential

ε – porosity of the porous medium

μ – dynamic viscosity, $(\mathrm{Pa}.s)$

μ_e – effective viscosity

θ – fluctuations of temperature

Θ – dependence in z of temperature

χ – magnetic susceptibility

χ_T – pyromagnetic coefficient, $\left(\mathrm{K}^{-1}\right)$

ψ – separation ratio

ψ_m – magnetic separation ratio

Ψ – dependence in z of concentration

Λ – viscosity ratio

con – conduction

in – intern

ex – extern

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Received: December 27, 2021
Revised: March 27, 2022