Mathematical simulation of the saprapel grinding by means of the shock loads

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Abstract. The article discusses the process of grinding of the solid aggregates of the lacustrine sapropel based on the model of mechanics of deformed solid body. As the result of simulation the initially-final problem on theory of elasticity is solved by the method of integral transformations. The research presents a numerical description of the tensed and deformed states of the solid aggregates of sapropel in dependence with the rates of loadings and geometric parameters. It discovered space and time distribution of the main tangent stresses influencing sufficiently the process of destruction and degree of the given material grinding.

Key words: theory of grinding, dynamic problem, shock load, integral transformations, main tangent stress.

INTRODUCTION

The fertility of soils is a crucial factor of a highly effective agricultural production under current conditions. The main organic fertilizers provider until very recently was animal husbandry. However for the last 20 years their volumes dropped considerably and this led to the 5 times decrease of the norms of organic fertilizers application in some regions [1]. The world technologies of crop growing are oriented only on the improvement of their industrially technological systems and their optimization to receive the highest outcomes [2, 3, 4], neglecting ecological effects of economic management. On the other hand, the permanent increase of fuel costs may lead to the situation when the application of organic fertilizers of the animal origin would be ineffective. We currently get, therefore, an urgent need in seeking new types of organic fertilizers and new ways of their application. One of these may be the lacustrine sappropel found in freshwater bodies.

RECENT RESEARCH AND PUBLICATIONS ANALYSIS

The major problem of the lacustrine sapropel utilization in the natural state is its high (92 - 96%) humidity. For its further utilization as an organic fertilizer its humidity should not exceed 60 %. The sapropel humidity decrease, however, may result in the loss of nutrients [5, 6, 7]. When utilizing the lacustrine sapropel as an organic fertilizer by the surface method one should keep to the norms of 40/60 t/ha [7, 8, 9, 10]. The attemps of developing new fertilizers on the basis of sapropel faced the problems linked with the lack of industrial facilities of their manufacturing [14,15]. One of the ways of their utilization as a fertilizer, therefore, is the local application of frozen sapropels. This allows to diminish several times the norm with the simultaneous saving of the nutrient complex for the plants.

The received experimental results prove the fact that frozen lacustrine sapropels change their properties under the influence of environmental conditions. High humidity sapropels get some solid inclusions and this phenomeon should be taken into consideration when choosing the way of sapropel application in the soils. In case of local application of frozen lacustrine sapropels without any processing these solid aggregates may negatively affect the crops yielding capacity. Therefore, there appeared an urgent necessity of developing and putting into practice elementary but effective technology of sapropel grinding during its application.

Mathematical simulation of the technological process of grinding is used in different branches. Grinding is supposed to be accompanied with the destruction of inter-molecular and inter-atom connections. Therefore, to make its quantities description one may use energetic approaches [13, 14]. In particular, in stimulation of the process of grinding, the work for destructing external forces is frequently associated with the area of the surfaces formed in the result of destructing area and the volumes of the grinded materials.

Unlike the solid bodies, soils and, in particular, solid aggregates of sapropel are compound mixtures. Their destruction takes place, primarily, by the loss of adhesion of some components and is mostly determined by these components' composition, humidity, temperature, etc. In general, the mentioned above problems are referred to complex, geometrical and physical nonlinear models of mechanics of continuums. Nevertheless, quite a number of vital regularities and effects proceding the process of soil mixtures destruction and in many respects facilitating them may be found and researched with application of a linear model of the theory of elasticity [15]. The object of research is suggested to be a rectangular fragment of sapropel solid aggregate. Two opposite sides of this rectangle are high intensively loaded with an effect of self-balance. In addition, because of the effect of this material sticking to the destructive tool, its surface lacks tangent movements. Two other sides of the rectangle are considered free of loading.

SETTING OF THE PROBLEM

Let us consider the $2h \times 2l$ rectangle with x_1, and y_1 accordingly (Fig. 1). At the moment $t = 0$ the rectangle's sides $x_1 = \pm l$ are effected by normal forces $p(t)$. These sides are given the condition of the absence of tangential component of the vector of displacement. Other surfaces $y = \pm h$ for all the period of deformation stay free of loading.

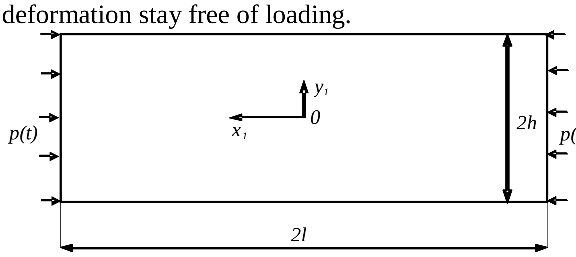

Fig. 1. Scheme of the objective

For the convenience in describing mathematical phenomena and revealing the most characteristic parameters determining dynamic tensed and deformed state, we introduce into our analysis dimensionless variables and the values $x = x_1 / l$, $y = y_1 / l$, $\tau = c_1 t / l$, $x_0 = h / l$, $\kappa^2 = c_1 / c_2 = (\lambda + 2\mu) / \mu$, where: c_1, c_2, – is the rate of spreading waves of compression and shift in the material of sapropel, λ μ – are elastic constants.

In terms of these variables when $t = 0$ and the material is in the state of rest, the objective is formulated in the following way:

- equation of the moment of elastic environment:

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \frac{\partial^2 \theta}{\partial \tau^2}. \qquad (1)$$

$$\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} = \kappa^2 \frac{\partial^2 u_y}{\partial \tau^2} - (\kappa^2 - 1)\frac{\partial \theta}{\partial y}. \qquad (2)$$

- initial conditions:

$$\theta = \frac{\partial \theta}{\partial \tau} = 0, \quad u_y = \frac{\partial u_y}{\partial \tau} = 0, \quad \tau = 0. \qquad (3)$$

- conditions on the loaded surfaces:

$$\sigma_{xx}(\pm 1, y, \tau) = \mp p(\tau), \; u_y(\pm 1, y, \tau) = 0. \qquad (4)$$

- conditions on the free surfaces:

$$\sigma_{xx}(x, \pm y_0, \tau) = 0, \; \sigma_{xy}(x, \pm y_0, \tau) = 0. \qquad (5)$$

where: $\theta(x, y, \tau) = \dfrac{\partial u_x}{\partial x} + \dfrac{\partial u_y}{\partial y}$ - is the volume expansion, $u_x(x, y, \tau)$ $u_y(x, y, \tau)$ - components of the vector of elastic displacement,

$$\sigma_{xx} = \lambda\theta + 2\mu\varepsilon_{xx}; \; \sigma_{yy} = \lambda\theta + 2\mu\varepsilon_{yy};$$

$$\sigma_{xy} = 2\mu\varepsilon_{xy}; \; \varepsilon_{xx} = \frac{\partial u_x}{\partial x}; \; \varepsilon_{yy} = \frac{\partial u_y}{\partial y};$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right);$$

- components of the tensor of stress and tensor of deformation.

- Using the condition (4) and taking into consideration:

that

$$\theta(\pm 1, y, \tau) \equiv \left.\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right)\right|_{x=\pm 1} = \left.\frac{\partial u_x}{\partial x}\right|_{x=\pm 1},$$

we receive:

$$\mu^{-1}\sigma_{xx}(\pm 1, y, \tau) = \kappa^2 \theta(\pm 1, y, \tau). \qquad (7)$$

CONSTRUCTION OF SOLVING THE PROBLEM

Let us employ the Laplace integral transformation on the temporary variable and the Fourier *cos* final construction on the variable x [17], to the equation (1).

Taking into consideration the problem's symmetry, initial zero condition (3), the correlation (7) and the conditions (4), after all transformations instead of the equation (1), we shall receive:

$$\frac{d^2 \overline{\theta}_n}{dy^2} - (\xi_n^2 + s^2)\overline{\theta}_n = (-1)^{n+1}\frac{2\xi_n}{\kappa\mu}\overline{p}(s), \qquad (8)$$

where: $\xi_n = \pi(2n+1)/2$ a $\overline{\theta}_n(y, s) =$

$$= \int_{-1}^{1} \cos(\xi_n x) \int_0^{\infty} \theta(x, y, \tau)\exp(-s\tau)\, d\tau\, dx$$ - is a representation by Laplace and Fourier.

Instead of the equation (2) we, accordingly, receive:

$$\frac{d^2 \overline{v}_n}{dy^2} - (\xi_n^2 + \kappa^2 s^2)\overline{v}_n = (1 - \kappa^2)\frac{d\overline{\theta}_n}{dy}, \qquad (9)$$

where:

$$\bar{v}_n(y,s) = \int_{-1}^{1} \cos(\xi_n x) \int_0^\infty u_y(x,y,\tau) \exp(-s\tau) d\tau dx.$$

Taking into consideration that $\bar{\theta}_n(y,s)$ is a binary correlation function of a variable, the equation (7) is solved in the following way:

$$\bar{\theta}_n = A_n(s)\cosh(\gamma_1 y) + \frac{(-1)^n 2\xi_n \bar{p}(s)}{\mu\kappa^2\gamma_1^2}, \quad (10)$$

at $\gamma_1 = \sqrt{\xi_n^2 + s^2}$.

Taking into consideration (10), the equation (9) is solved in the following way:

$$\bar{v}_n = B_n(s)\sinh(\gamma_2 y) + \frac{\gamma_1}{s^2} A_n(s)\sinh(\gamma_1 y), \quad (11)$$

at $\gamma_2 = \sqrt{\xi_n^2 + \kappa^2 s^2}$.

Another component of the vector of displacement can be found taking into consideration the correlation:

$$\bar{u}_n(y,s) = \int_{-1}^{1} \sin(\xi_n x) \int_0^\infty u_x(x,y,\tau) \exp(-s\tau) d\tau dx$$

$$\bar{u}_n = \frac{1}{\xi_n}\left(\bar{\theta}_n - \frac{d\bar{v}_n}{dy}\right)$$ in the form:

$$\bar{u}_n(y,s) = -B_n(s)\xi_n^{-1}\gamma_2 \cosh(\gamma_2 y) - \frac{\xi_n}{s^2} A_n(s)\cosh(\gamma_1 y) + \frac{(-1)^n 2\bar{p}(s)}{\mu\kappa^2\gamma_1^2}. \quad (12)$$

The values $A_n(s)$ and $B_n(s)$ can be determined under the limiting conditions (5), which in transformants by Laplace and Fourier and in the terms of the found above expressions will have the following form:

$$(\kappa^2 - 2)\bar{\theta}_n + 2\frac{d\bar{u}_n}{dy} = 0,$$
$$-\xi_n \bar{v}_n + \frac{d\bar{u}_n}{dy} = 0, \quad y = \pm y_0. \quad (13)$$

Taking into consideration the expressions (10)-(12) from the conditions (13) we shall find the following:

$$A_n(s) = \frac{s^2(\xi_n^2 + \gamma_2^2)\sinh(\gamma_2 y_0)\bar{p}_n(s)}{\gamma_1^2 \Delta(\xi,s)};$$
$$B_n(s) = \frac{-2\xi_n^2 \sinh(\gamma_1 y_0)\bar{p}_n(s)}{\gamma_1 \Delta(\xi,s)} \quad (14)$$

where:

$$\Delta(\xi,s) = 4\xi_n^2\gamma_1\gamma_2 \sinh(\gamma_1 y_0)\cosh(\gamma_2 y_0) - (\xi_n^2 + \gamma_2^2)^2 \cosh(\gamma_1 y_0)\sinh(\gamma_2 y_0),$$

$$\bar{p}_n(s) = \frac{(-1)^n 2\xi_n(\kappa^2 - 2)\bar{p}(s)}{\mu\kappa^2}.$$

Finally, by the known values $A_n(s)$ i $B_n(s)$ we shall receive:

$$\bar{v}_n(y,s) = \gamma_1^{-1}\Delta^{-1}\left[(\xi_n^2 + \gamma_2^2)\sinh(\gamma_2 y_0)\sinh(\gamma_1 y) - 2\xi_n^2 \sinh(\gamma_1 y_0)\sinh(\gamma_2 y)\right]\bar{p}_n;$$
$$\bar{u}_n(y,s) = \left(\frac{1}{(\kappa^2 - 2)\xi_n} + \frac{\xi_n}{\Delta}\left[2\gamma_1\gamma_2 \sinh(\gamma_1 y_0) \times \cosh(\gamma_2 y) - (\xi_n^2 + \gamma_2^2)\sinh(\gamma_2 y_0)\cosh(\gamma_1 y)\right]\right)\frac{\bar{p}_n}{\gamma_1^2}. \quad (15)$$

We shall carry out transformations by Laplace using the theorem of decomposition [17]. In this case we consider the first expression (15) and find singular points of the denominator. Apparently the roots of the expression $\gamma_1 = 0$ are not the singular points of the dominator and, therefore, let us consider the equation:

$$4\xi_n^2\gamma_1\gamma_2 \sinh(\gamma_1 y_0)\cosh(\gamma_2 y_0) - (\xi_n^2 + \gamma_2^2)^2 \cosh(\gamma_1 y_0)\sinh(\gamma_2 y_0) = 0. \quad (16)$$

The roots of the characteristic equation (16) are purely imaginary and comprehensively-connected.

Therefore, it is reasonable to substitute $s = i\eta$ and, correspondingly, to receive:

$$\gamma_1 = \sqrt{\xi_n^2 - \eta^2} \quad \gamma_2 = \sqrt{\xi_n^2 - \kappa^2\eta^2}.$$

The roots $\eta_{n,k}$ apparently, depend on the discrete value ξ_n, and, therefore, there exist three cases of their arrangement :

$$0 \le |\eta_{n,k}| \le \frac{\xi_n}{\kappa}; \; \frac{\xi_n}{\kappa} < |\eta_{n,k}| \le \xi_n; \; |\eta_{n,k}| > \xi_n. \quad (17)$$

For the first interval the characteristic equation saves the form (16) and gets the final number of roads $\eta_{n,k,1}$. For the interval $\kappa^{-1}\xi_n < |\eta_{n,k}| \le \xi_n$ the characteristic equation gets the form:

$$4\xi_n^2\gamma_1\tilde{\gamma}_2 \sinh(\gamma_1 y_0)\cos(\tilde{\gamma}_2 y_0) - (\xi_n^2 + \tilde{\gamma}_2^2)^2 \cosh(\gamma_1 y_0)\sin(\tilde{\gamma}_2 y_0) = 0, \quad (18)$$

at $\tilde{\gamma}_2 = \sqrt{\kappa^2\eta^2 - \xi_n^2}$. It has the final number k_2 of the roots $\eta_{n,k,2}$.

And, accordingly, for the interval $|\eta_{n,k}| > \xi_n$ the equation:

$$4\xi_n^2\tilde{\gamma}_1\tilde{\gamma}_2 \sin(\tilde{\gamma}_1 y_0)\cos(\tilde{\gamma}_2 y_0) + (\xi_n^2 + \tilde{\gamma}_2^2)^2 \cos(\tilde{\gamma}_1 y_0)\sin(\tilde{\gamma}_2 y_0) = 0 \quad (14) \quad (19)$$

at $\tilde{\gamma}_1 = \sqrt{\eta^2 - \xi_n^2}$ has the infinite number of roots $\eta_{n,k,3}$.

For the use of the expression (15), with employment of the theorem of decomposition one must determine the value of the denominator [17]. Accordingly, we get the value of the derivative from the expressions for different intervals of arrangement of the characteristic equation roots:

$$0<|\eta|\le\frac{\xi_n}{\kappa},\ \Delta'\big|_{s=\pm\eta_{n,k,1}}\equiv\tilde{\Delta}_1(n,k)=\pm i\eta_{n,k,1}\left\{4\xi_n^2\left[\frac{\gamma_2}{\gamma_1}\sinh(\gamma_1x_0)\cosh(\gamma_2x_0)+\frac{\kappa^2\gamma_1}{\gamma_2}\times\right.\right.$$
$$\left.\times\sinh(\gamma_1x_0)\cosh(\gamma_2x_0)+x_0\gamma_2\cosh(\gamma_1x_0)\cosh(\gamma_2x_0)+\kappa^2x_0\gamma_1\sinh(\gamma_1x_0)\sinh(\gamma_2x_0)\right]-$$
$$-4\kappa^2(\xi_n^2+\gamma_2^2)\cosh(\gamma_1x_0)\sinh(\gamma_2x_0)-(\xi_n^2+\gamma_2^2)^2\frac{x_0}{\gamma_1}\sinh(\gamma_1x_0)\sinh(\gamma_2x_0)- \tag{20}$$
$$\left.-(\xi_n^2+\gamma_2^2)^2\frac{\kappa^2x_0}{\gamma_2}\cosh(\gamma_1x_0)\cosh(\gamma_2x_0)\right\}.$$

$$\frac{\xi_n}{\kappa}<|\eta|\le\xi_n,\ \Delta'\big|_{s=\pm\eta_{n,k,2}}\equiv\tilde{\Delta}_2(n,k)=\mp\eta_{n,k,2}\left\{4\xi_n^2\left[\frac{\tilde{\gamma}_2}{\gamma_1}\sinh(\gamma_1x_0)\cos(\tilde{\gamma}_2x_0)-\right.\right.$$
$$\left.-\frac{\kappa^2\gamma_1}{\tilde{\gamma}_2}\sinh(\gamma_1x_0)\cos(\tilde{\gamma}_2x_0)+x_0\tilde{\gamma}_2\cosh(\gamma_1x_0)\cos(\tilde{\gamma}_2x_0)+\kappa^2x_0\gamma_1\sinh(\gamma_1x_0)\sin(\tilde{\gamma}_2x_0)\right]-$$
$$-4\kappa^2(\xi_n^2-\tilde{\gamma}_2^2)\cosh(\gamma_1x_0)\sin(\tilde{\gamma}_2x_0)-(\xi_n^2-\tilde{\gamma}_2^2)^2\frac{x_0}{\gamma_1}\sinh(\gamma_1x_0)\sin(\tilde{\gamma}_2x_0)+ \tag{21}$$
$$\left.+(\xi_n^2-\tilde{\gamma}_2^2)^2\frac{\kappa^2x_0}{\tilde{\gamma}_2}\cosh(\gamma_1x_0)\cos(\tilde{\gamma}_2x_0)\right\}.$$

$$|\eta|>\xi_n,\ \Delta'\big|_{s=\pm\eta_{n,k,3}}\equiv\tilde{\Delta}_3(n,k)=\mp\eta_{n,k,3}\left\{4\xi_n^2\left[\frac{\tilde{\gamma}_2}{\tilde{\gamma}_1}\sin(\tilde{\gamma}_1x_0)\cos(\tilde{\gamma}_2x_0)+\right.\right.$$
$$\left.+\frac{\kappa^2\tilde{\gamma}_1}{\tilde{\gamma}_2}\sin(\tilde{\gamma}_1x_0)\cos(\tilde{\gamma}_2x_0)+x_0\tilde{\gamma}_2\cos(\tilde{\gamma}_1x_0)\cos(\tilde{\gamma}_2x_0)-\kappa^2x_0\tilde{\gamma}_1\sin(\tilde{\gamma}_1x_0)\sin(\tilde{\gamma}_2x_0)\right]-$$
$$-4\kappa^2(\xi_n^2-\tilde{\gamma}_2^2)\cos(\tilde{\gamma}_1x_0)\sin(\tilde{\gamma}_2x_0)-(\xi_n^2-\tilde{\gamma}_2^2)^2\frac{x_0}{\tilde{\gamma}_1}\sin(\tilde{\gamma}_1x_0)\sin(\tilde{\gamma}_2x_0)+ \tag{22}$$
$$\left.+(\xi_n^2-\tilde{\gamma}_2^2)^2\frac{\kappa^2x_0}{\tilde{\gamma}_2}\cos(\tilde{\gamma}_1x_0)\cos(\tilde{\gamma}_2x_0)\right\}.$$

Apart from the roots of the characteristic equitation (16) as singular points of at denominator, the second expression (15) has the roots of equation $\gamma_1=0$: $s=\pm i\xi$.

Taking it into consideration, the final expression for the component vector of displacement will get the following form:

$$u_x(x,y,\tau)=-\frac{4}{\mu}\left(1-\frac{2}{\kappa^2}\right)\sum_{n=0}^{\infty}(-1)^n\xi_n^2\sin(\xi_nx)\times$$
$$\times\left\{\sum_{k=1}^{k_1}\frac{2\gamma_1\gamma_2\sinh(\gamma_1y_0)\cosh(\gamma_2y)-\left(2\xi_n^2-\kappa^2\eta_{n,k,1}^2\right)\sinh(\gamma_2y_0)\cosh(\gamma_1y)}{\gamma_1^2\tilde{\Delta}_1(n,k)}f(\eta_{n,k,1},\tau)+\right.$$
$$+\sum_{k=1}^{k_2}\frac{2\gamma_1\tilde{\gamma}_2\sinh(\gamma_1y_0)\cos(\tilde{\gamma}_2y)-\left(2\xi_n^2-\kappa^2\eta_{n,k,2}^2\right)\sin(\tilde{\gamma}_2y_0)\cosh(\gamma_1y)}{\gamma_1^2\tilde{\Delta}_2(n,k)}f(\eta_{n,k,2},\tau)+ \tag{23}$$
$$\left.+\sum_{k=1}^{\infty}\frac{2\tilde{\gamma}_1\tilde{\gamma}_2\sin(\tilde{\gamma}_1y_0)\cos(\tilde{\gamma}_2y)+\left(2\xi_n^2-\kappa^2\eta_{n,k,3}^2\right)\sin(\tilde{\gamma}_2y_0)\cos(\tilde{\gamma}_1y)}{\tilde{\gamma}_1^2\tilde{\Delta}_3(n,k)}f(\eta_{n,k,3},\tau)\right\}$$

$$u_y(x,y,\tau) = -\frac{4}{\mu}\left(1-\frac{2}{\kappa^2}\right)\sum_{n=0}^{\infty}(-1)^n \xi_n \cos(\xi_n x)\times$$
$$\times\left\{\sum_{k=1}^{k_1}\frac{\left(2\xi_n^2-\kappa^2\eta_{n,k,1}^2\right)\sinh(\gamma_2 y_0)\sinh(\gamma_1 y)-2\xi_n^2\sinh(\gamma_1 y_0)\sinh(\gamma_2 y)}{\gamma_1\tilde{\Delta}_1(n,k)}f(\eta_{n,k,1},\tau)+\right.$$
$$+\sum_{k=1}^{k_2}\frac{\left(2\xi_n^2-\kappa^2\eta_{n,k,2}^2\right)\sin(\tilde{\gamma}_2 y_0)\sinh(\gamma_1 y)-2\xi_n^2\sinh(\gamma_1 y_0)\sin(\tilde{\gamma}_2 y)}{\gamma_1\tilde{\Delta}_2(n,k)}f(\eta_{n,k,2},\tau)+ \tag{24}$$
$$\left.+\sum_{k=1}^{\infty}\frac{\left(2\xi_n^2-\kappa^2\eta_{n,k,3}^2\right)\sin(\tilde{\gamma}_2 y_0)\sin(\tilde{\gamma}_1 y)-2\xi_n^2\sin(\tilde{\gamma}_1 y_0)\sin(\tilde{\gamma}_2 y)}{\tilde{\gamma}_1\tilde{\Delta}_3(n,k)}f(\eta_{n,k,3},\tau)\right\},$$

where:

$$f(\eta,\tau) = \int_0^{\tau} p(\tau-t)\sin(\eta t)dt\,. \tag{25}$$

The component of the tensor of deformation and the tensor of stress are calculated by the formula (6) at the known components of the vector of displacement (23). One may also show that all the series of the solution (23) uniformly converge and, therefore, differentiating operations at determining component-tensor of deformations and tensor of stresses can be carried out under the sign of a sum.

Under conditions simulating the sapropel grinding the dynamic loadings is determined and depends upon the velocity of rotor blades rotation. It increases monotoneously from the zero to its maximum values. When carrying out the numerical calculations we employed the following dependence $p(t) = p*(1-\exp(-at))^2$ which for the unlimited time τ will have the form:

$$p(\tau) = p^*(1-\exp(-\tau_0\tau))^2, \tag{26}$$

at: $\tau_0 = (l\cdot a)/c_1$.

Such dependence allows to coordinate the initial and final conditions and, in many a case, to approach rather accurately the real dependence of dynamic loading on the time (Fig. 2).

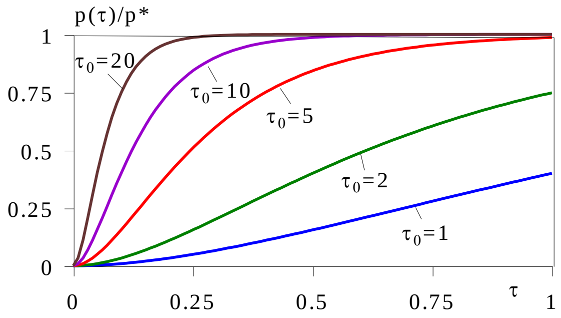

Fig. 2. Dependence of loading upon the time

The integral calculation (25) for the dependence of loading (26) allows to determine the function $f(\eta,\tau)$:

$$f(\eta,\tau) = \frac{1}{\eta} - \frac{2\eta\exp(-\tau_0\tau)}{\eta^2+\tau_0^2} + \frac{\eta\exp(-2\tau_0\tau)}{\eta^2+4\tau_0^2} +$$
$$+\cos(\eta\tau)\left(\frac{2\eta}{\eta^2+\tau_0^2} - \frac{\eta}{\eta^2+4\tau_0^2} - \frac{1}{\eta}\right) + \tag{26}$$
$$+\sin(\eta\tau)\left(\frac{2\tau_0}{\eta^2+4\tau_0^2} - \frac{2\tau_0}{\eta^2+\tau_0^2}\right).$$

For such type of loading we calculated the tensed state of the sapropel solid aggregates of a rectangular form for different values τ_0 at different correlations of the width and length of a rectangle. Fig. 3-5 illustrate the results of calculating the tensed state of the sapropel solid aggregate of the quadratic form ($l = h$) at different values of the rates of loading. Fig. 3-5 illustrate the results of calculating the stained state of the sapropel solid aggregate of the quadratic form ($l = h$) at different values of the rates of loading.

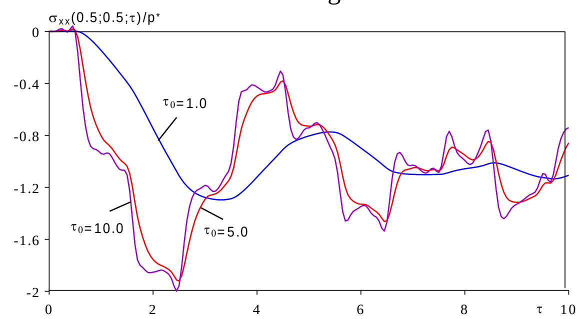

Fig. 3. Prolonged stress in sapropel depending on the rate of loading

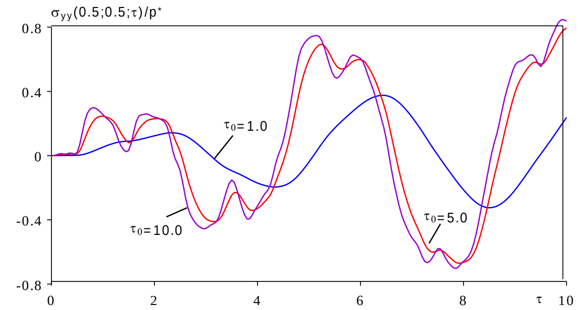

Fig.4. Transversal stress in sapropel depending on the rate of loading

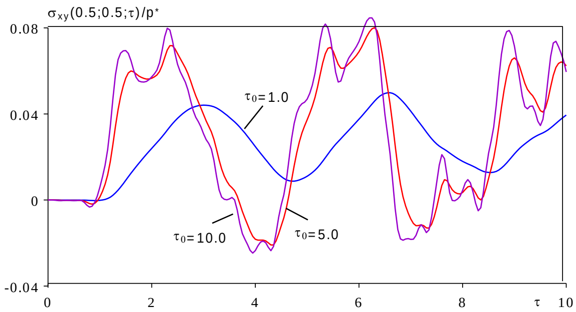

Fig.5. Tangent stress in sapropel depending on the rate of loading

Fig. 3-5 allow to make a conclusion that at the adequate rate of loading ($\tau_o \geq 5$) the prolonged dynamic stress is two times higher than the corresponding static value by an amplitude.

At some moments the transversal stresses are about 80% of the static ones. As to the tangent stresses, they are only about 8% of them. The similar results are received in calculating the stressed states at other points and other correlations of the heights and width of the sapropel solid aggregate. Moreover, any variations in the correlation between the height and width of the solid aggregate lead to dropping of the tangent stresse levels.

This fact allows to state that in such a way of formulating the objective the axes of the chosen systems of coordinates may be considered the principal axes of the tensor of stress at any points of the right-angled area. In its turn, this sufficiently simplifies the calculation of the main tangent stress in the material of sapropel.

In such case the maximum tangent stresses by module are known to act on the sites inclined to the axes of coordinates under the angle of 45º[3] and form:

$$\mathrm{T}_{\max} = \left| \frac{\sigma_{xx} - \sigma_{yy}}{2} \right|. \tag{27}$$

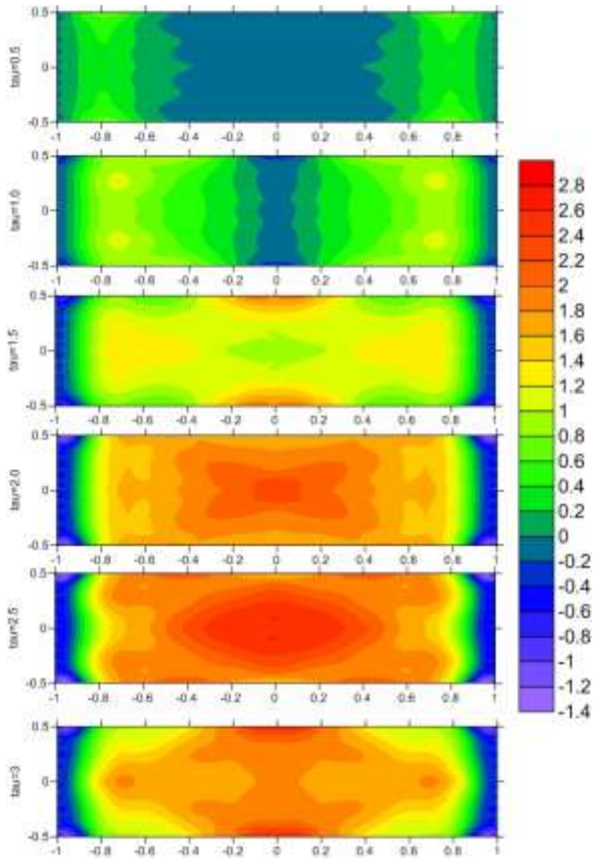

Fig. 6. Distribution of the main tangent stress in sapropel at $h = 0.5l$

The Fig. 6 presents the results of calculating spatial and time distribution of refered to p^* of the main tangent stresses in the element of the element of sapropel which has the rectangular form with correlation of the hight and width 2 : 1, at $\tau_0 = 5$ for different values of infinite time.

CONCLUSIONS

Our research resulted in the following qualitative positions. At first moments after the load application, zones of maximum tangent forces are concentrated in the local areas. Their disposal allows to forecast the pattern's breaking down into 6-8 segments under condition of the tangent forces getting their maximum values. At these momentit it is correlated with the value of external loading. If these forces are not sufficient enough for destruction, they have the chance to increase twice at τ_0=1. (t = 0.2mc) because of the climbed waves covering. In case of the reflected waves arrival these forces increase three times comparing with the value of applicated loading. Nevertheless, zone of these forces action is concentrated closer to the pattern's centre which gives the opportunity to forecast the pattern's breaking down into 2-4 segments. The analogous results are also received for other geometric correlations between the pattern's height and width. Thus, taking into consideration simulation constructions and numerical calculations one may state that the maximum value of the main tangent stress is reached in the areas situated closer to the patterns centre and make up about 300% of the external loading level. But for the more accurate grinding one must lead external loading to the level of maximum static value of the net destruction which can be determined experimentally.

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