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Mathematical models for simulation of layer hydrodynamics of a ship fluidized bed boiler

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Abstract
Fluidized bed boilers have been widely used for many years in energetics but their application in shipbuilding has been incidental. The hydrodynamics of a fluidized bed is a complicated issue. If the swaying of a ship on a sea wave is taken into account, this issue will be further complicated. The choice of an appropriate mathematical model is important from the viewpoint of building computer models for the simulation of a fluidized bed during disruptions that result from the ship swaying on sea waves. Fluidization is a two-phase flow process, which is described by many models such as the homogeneous-slip model or heterogeneous-slide model. The most popular models that show two-phase gas-solid or fluid-solid flow are the Euler-Euler and Euler-Lagrange models. The paper presents models that describe a ship's fluidized bed.

Nomenclature

C_D – drag force coefficient
d – diameter
F_d – drag force for a single particle
g – specific gravity force
F_i – external body force
I – unit tensor
K_{gs} – momentum exchange coefficient
m_p – mass of particles
p – pressure
t – time
V – volume of particles
α – volume of phase
ε – porosity of dense
η – dynamic viscosity
μ – shear viscosity
ρ – density
τ – shear stress tensor
υ – velocity

Subscripts

g – gas
p – particle
s – solid

Introduction

Fluidized bed boilers are being applied in many branches of industry, such as pharmaceutical, chemical, mineral-processing industrial, and metallurgy. They are also becoming increasingly popular in civil engineering; however, boilers with a circulating-fluidized bed displace boilers with a bubbling-fluidized bed because they have greater energetic efficiency and lower emission of nitrogen oxides (NO_x) (Bis, 2010). The phenomenon of fluidization, which in fact is a two-phase flow, is a complicated problem. A new problem, in the aspect of simulation modeling of layer hydrodynamics of a ship fluidized-bed

boiler, is consideration of external disturbances such as the swaying of a ship that affects the process of fluidization. This paper presents mathematical models describing the process of fluidization that are taken into account in the aspect of selection of the most appropriate model that will allow for easy adaptation while allowing for consideration of the aforementioned disturbances.

Mathematical models of two-phase flow

Two-phase flow is the common flow of two phases, continuous phase and dispersed phase, where continuous phase is a fluid or a gas and dispersed phase is the substance of any physical state. In the issue considered herein, the continuous phase represents gas and the dispersed phase will be represented by the particles of solid fuel, such as coal or solid biomass. A characteristic feature of two-phase flow is appearance of the surface of phase separation. The flow area can be treated like a space that is divided into two single-phase subareas by a boundary surface. Each of the individual phases can be formulated into an equation, like for single-phase flow. Models formulated in this way belong to the class of heterogeneous models (slide models). The second group is homogeneous models, in which each of components fill the whole volume and lose individual features, which means that both phases are treated as perfectly mixed and moving with the same velocity (slip model).

Commercial software for simulation of flow dynamics, called Computational Fluid Dynamic (CFD; for example, FLUENT), is used to create two types of mathematical models: the Euler-Euler model and Euler-Lagrange model. The approach to the simulation of two-phase flow in models mentioned above is different. In the Euler-Euler model, each phase in the mathematical approach is considered as continuous and fully interpenetrating, which may be based on the Navier-Stokes equation. Variables from the Euler equation are used in constitutive equations for all phases, while in Euler-Lagrange model, the Newton motion equation is used for each particle separately. This approach also takes into account a collision model to consider the energy dissipation caused by the non-ideal interactions of particles in the dispersed phase. The group models based on the Euler equation can be included in the Granular-Euler model. In the next part of the article, the above models will be broadly characterized (Huilin, Yurong & Gidaspow, 2003; Yang & Renken, 2003; Taghipour, Ellis & Wong, 2005; Benzarti, Mhiri & Bourhot, 2012).

Euler-Euler Model

The Euler-Euler Model is the most popular model in many types of software used for simulations and is the preferred model for simulation of a fluidized-bed layer. The basis for building the Euler-Euler Model equation is conservation of mass, energy, and momentum for each phase (Huilin, Yurong & Gidaspow, 2003; Lundberg & Halvorsen, 2008; Benzarti, Mhiri & Bourhot, 2012).

The mass conservation equations for the gas phase (g) and for solid phase (s) can be represented by the formula [14]:

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla(\alpha_g \rho_g \vec{\upsilon}_g) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(\alpha_s \rho_s) + \nabla(\alpha_s \rho_s \vec{\upsilon}_s) = 0 \tag{2}$$

The momentum conservation equation for both phases can be presented as follows:

$$\frac{\partial}{\partial t}(\alpha_s \rho_s \vec{\upsilon}_s) + \nabla(\alpha_s \rho_s \vec{\upsilon}_s^2) = \\ = -\alpha_s \nabla p + \nabla \bar{\bar{\tau}}_s + \alpha_s \rho_s \vec{g} + K_{gs}(\vec{\upsilon}_s - \vec{\upsilon}_g) \tag{3}$$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g \vec{\upsilon}_g) + \nabla(\alpha_g \rho_g \vec{\upsilon}_g^2) = \\ = -\alpha_g \nabla p + \nabla \bar{\bar{\tau}}_g + \alpha_g \rho_g \vec{g} + K_{gs}(\vec{\upsilon}_g - \vec{\upsilon}_s) \tag{4}$$

where:
α – volume of phase fraction,
ρ – density,
υ – velocity,
p – pressure,
g – specific gravity force,
τ – shear stress tensor.

In adiabatic flows, the equation of energy conservation brings nothing new to the system of conservative equations and therefore is omitted. In many works, particular attention is paid to the drag force which is represented by the momentum-exchange coefficient K_{gs}. Depending on the model, it is variously formulated. For example, in the model proposed by Gidaspow, which is combination of two other models, Wen-Yu and Ergun (Orzechowski, 1990; Huilin, Yurong & Gidaspow, 2003; Benzarti, Mhiri & Bourhot, 2012), it is defined as follows:

$$K_{gs} = \frac{3}{4} C_D \frac{\alpha_s \alpha_g \rho_g \left|\vec{\upsilon}_s - \vec{\upsilon}_g\right|}{d_s} \alpha_g^{-2.65} \\ \text{for } \alpha_g > 0.8 \tag{5}$$

$$K_{gs} = 150\frac{\alpha_s^2 \mu_g}{\alpha_g d_s^2} + 1.75\frac{\alpha_s \rho_g \left|\vec{\upsilon}_s - \vec{\upsilon}_g\right|}{d_s} \quad \text{for } \alpha_g \le 0.8 \tag{6}$$

where:

$$C_D = \begin{cases} \dfrac{24}{\alpha_g \operatorname{Re}_S}\left[1 + 0.15\left(\alpha_g \operatorname{Re}_S\right)^{0.687}\right], & \operatorname{Re}_S < 1000 \\ 0.44, & \operatorname{Re} > 1000 \end{cases} \tag{7}$$

$$\operatorname{Re}_S = \frac{\rho_g d_S \left|\vec{\upsilon}_S - \vec{\upsilon}_g\right|}{\mu_g} \tag{8}$$

where:
C_D – drag force coefficient,
d – diameter,
μ – shear viscosity,
Re – Reynolds number.

For closure of the system of equations that creates the mathematical model, there are also needed constitutive equations that are relations between stress-state and strain-state tested material. The stress tensor is calculated based on the sum of all average values of velocities according to the Navier-Stokes equation (Kozic et al., 2011).

Granular-Euler Model

The Granular-Euler Model is another model from the group of mathematical models describing the phenomenon of two-phase flow. This model is used when the motion of particles is determined by mutual collisions. Collision of particles with the walls is also taken into account in this model.

The continuity equation of the granular phase (in this case there is no separation on a solid phase and gas phase) is represented by equation (9) (Bakker, 2008):

$$\frac{\partial}{\partial t} = (\alpha_s \rho_s) + \nabla(\alpha_s \rho_s \vec{\upsilon}_s) = \dot{m}_{fs} \tag{9}$$

where: $\dot{m}$ – mass stream.

The momentum-conservation equation, similar to the Euler-Euler Model, looks as follows (Bakker, 2008):

$$\frac{\partial}{\partial t}(\alpha_s \rho_s \vec{\upsilon}_s) + \nabla(\alpha_s \rho_s \vec{\upsilon}_s \vec{\upsilon}_s) = = -\alpha_s \nabla p_f + \nabla \bar{\tau}_s + \sum_{s=1}^{n}(\vec{R}_{fs} + \dot{m}_{fs}\vec{\upsilon}_{fs}) + \vec{F}_s \tag{10}$$

wherein: $-\alpha_s \nabla p_f$ – fluid pressure (gas); $\nabla \bar{\tau}_s$ – solid stress tensor; $\sum_{s=1}^{n}(\vec{R}_{fs} + \dot{m}_{fs}\vec{\upsilon}_{fs})$ – phase interaction term.

The granular-temperature equation – an equation implemented in the Granular Model, which is proportional to the kinetic energy of the random motion of the particles – looks as follow (Bakker, 2008):

$$\frac{3}{2}\left[\frac{\partial}{\partial t}\left(\alpha_s \rho_s \Theta_s\right) + \nabla\left(\alpha_s \rho_s \Theta_s \vec{\upsilon}_s\right)\right] = \bar{\tau}_s : \nabla \vec{\upsilon}_s + \nabla(k_{\Theta s} \nabla \Theta_s) - \gamma_s + \phi_{ss} + \phi_{gs} \tag{11}$$

where: $\bar{\tau}_s : \nabla \vec{\upsilon}_s$ – generation of energy by the solid stress tensor; $\nabla(k_{\Theta s} \nabla \Theta_s)$ – diffusion of energy; $\phi_{ss} + \phi_{gs}$ – represent the energy exchange among solid phase (*ss*) and exchange between gas and solid phase (*gs*).

To solve the complete granular-temperature equation, Syamlal proposed an algebraic form to this equation (Benzarti, Mhiri & Bourhot, 2012). He assumed that the energy of the granular phase is steady state and dissipates locally, which is why convection and diffusion terms can be neglected. Equation (11) can be represented as:

$$0 = \left(-P_s I + \tau_s\right) : \nabla \upsilon_s - \gamma \tag{12}$$

where: γ – dissipation of energy due to inelastic collisions.

The basic difference between the momentum-conservation equation in the Euler-Euler Model (3, 4) and that of the Granular-Euler Model (10) is the presence in the first of them two equations that show the momentum-conservation equation separately for the gas phase and solid phase, while in the Granular-Euler Model there is only a single equation; another difference is the presence of a momentum-exchange coefficient mentioned above. In the second of them an interaction is described between phases that is compared to zero, with present balance between states. This model is based on the kinetic theory of gases, which makes it difficult to apply to the computer simulations because there is needed a heat-exchange analysis and mass-exchange analysis between phases.

Euler-Lagrange Model

The Euler-Lagrange Model approaches the problem differently than the Euler-Euler Model. For the solid phase, there are equations of energy balance while the dispersed phase is treated like a set of

single particles, in which each of them is described using Lagrange's variables – variables of motion for the single particle. This theory mainly concerns hydrodynamics of molecules and it can be applied where is a small quantity of particles (Di Felice & Kehlenbeck, 2000; Kudela & Lewtak, 2002; Huilin, Yurong & Gidaspow, 2003; Lundberg & Halvorsen, 2008; Benzarti, Mhiri & Bourhot, 2012). Accepting the assumptions of the Euler-Lagrange Model that each particle is treated separately and for each of them there is a determined trajectory of motion using the Newton motion equation, the equation of Euler-Lagrange Model can be represented as follows (Sobieski, 2009):

For the continuous phase:

- continuity equation:

$$\frac{\partial\left(\varepsilon\rho_g\right)}{\partial t}+\varepsilon\rho_g\vec{\upsilon}_g=0 \tag{13}$$

- momentum equation:

$$\frac{\partial\left(\varepsilon\rho_g\right)}{\partial t}+\nabla\varepsilon\rho_g\vec{\upsilon}\vec{\upsilon}= =-\varepsilon\nabla p-\varepsilon\nabla\vec{\tau}_g+\varepsilon\rho_g\vec{g}-\frac{1}{V_p}\sum_{i=1}^{N_p}\left[\frac{K_{gs}V_p}{1-\varepsilon}\left(\vec{\upsilon}_g-\vec{\upsilon}_p\right)\right] \tag{14}$$

For the dispersed phase:

To describe the dispersed phase using the Lagrange approach, it is necessary to take into account the huge quantity of particles and treat them like a set of particles described using a series of differential equations:

- location of particle:

$$\frac{\partial\vec{x}_p}{\partial t}=\vec{\upsilon}_p \tag{15}$$

- velocity of particle:

$$m_p\frac{\partial\vec{\upsilon}_p}{\partial t}=\sum\vec{F}_i \tag{16}$$

- angular velocity of particle:

$$I_p\frac{\partial\vec{\omega}_p}{\partial t}=\vec{T} \tag{17}$$

where:

ε – porosity of fraction,
V – volume,
F_i – external body force,
m – mass.

In the Euler-Lagrange Model, the equation of motion is solved for each particle separately, which requires considerable computing power. During equation solving, also considered is the energy dissipation caused by the non-ideal interactions of particles.

Foscolo-Gibilaro and Richardson-Zaki Model

Analyzing the behavior of the fluidized bed also allows for the Foscolo and Gibilaro Model to be used (Joseph, 1990; Lattieri et al., 2001; Lundberg & Halvorsen, 2008). In this model, it is assumed that the layer is built of two continuous ones penetrating each other from the gas phase and phase of solid particles. Both phases are treated as incompressible.

The conservation equations in the Foscolo-Gibilaro Model, written as one dimensional, are represented as (Joseph, 1990):

- mass conservation equation:

$$\frac{\partial\varepsilon}{\partial t}+\frac{\partial\left(\varepsilon\upsilon_g\right)}{\partial z}, \text{ for fluid} \tag{18}$$

$$\frac{\partial(1-\varepsilon)}{\partial t}+\frac{\partial\left((1-\varepsilon)\upsilon_p\right)}{\partial z}, \text{ for particles} \tag{19}$$

- momentum conservation equation:

$$\frac{\partial\varepsilon}{\partial t}+\frac{\partial\left(\varepsilon\upsilon_g\right)}{\partial z}\varepsilon\rho_f\left(\frac{\partial\upsilon_g}{\partial t}+u_f\frac{\partial\upsilon_g}{\partial z}\right)+ +\varepsilon\rho_g g+F_I+\frac{\partial p_g}{\partial z}=0, \text{ for fluid} \tag{20}$$

$$(1-\varepsilon)\rho_p\left(\frac{\partial\upsilon_p}{\partial t}+u_p\frac{\partial\upsilon_p}{\partial z}\right)+(1-\varepsilon)\rho_p g+ -F_I+\frac{\partial p_p}{\partial z}=0, \text{ for particles} \tag{21}$$

where F_I is the strength of interaction between the fluid and particle. Force is the sum of the contribution: drag force and buoyancy force for a single particle.

Buoyancy force in this model looks as follows:

$$F_d=\frac{\pi d_p^3}{6}\left(\rho_u-\rho_g\right)g\left[\frac{\upsilon}{\upsilon_p}\right]^{\frac{4.8}{n}}\varepsilon^{-3.8} \tag{22}$$

where: $\upsilon=\upsilon_p\varepsilon^n$; υ_p – terminal fall velocity.

Coefficient n depends on the Reynolds number and in correlation proposed by Richardson-Zaki: $n=n(\mathrm{Re}_t)$:

$$\begin{cases} n=4.8 & \text{for } \mathrm{Re}_t<0.2 \\ n=4.6\,\mathrm{Re}_t^{-0.03} & \text{for } 0.2<\mathrm{Re}_t<1 \\ n=4.6\,\mathrm{Re}_t^{-0.1} & \text{for } 1<\mathrm{Re}_t<500 \end{cases} \tag{23}$$

where:

$$\mathrm{Re}_t = \frac{d_p \rho_g \upsilon_t}{\mu_g} \tag{24}$$

Conclusions

Mathematical modeling of fluidized-bed layers is a complicated issue, due to complexity of the phenomenon of two-phase flow. The choice of mathematical models needs to be fitted to the assumed simulation conditions, such as the diameter of particles and the concentration of the material.

For the purposes of simulation of the fluidized-bed boiler, where there are additional forces caused by a swaying ship on a wave, it is recommended to simplify the model while maintaining the highest accuracy. If in the model, forces acting on other particles are not considered, the best model will be the Euler-Euler Model; this is a model with the widest range of applications. The model treats each of the phases separately, like a continuous phase, facilitating work and in preparing simulation. Another selection criterion is the adoption of an appropriate model for the description of the momentum-exchange coefficient. In this case, there are also several possibilities that can be distinguished by the model proposed by Gidaspow, or models by Sinclair or Syamlal-O'Brien, not described here. These models differ from each other in coefficients that are taken into account, where dividing is based on the volume fraction of the fluidized column that is filled with gas. The Gidaspow Model is a model that is the best in the case of a simulation of dense fluidized-bed layers. The Sinclair Model is a model that is applicable when the simulation concerns linear pneumatic transport (Bakker, 2008).

Homogeneous models may be especially useful where the motion of a single particle, in a two-phase mixture, is not analyzed and the most important parameter to be determined is drag forces of the mixture. In this study, we used averaged values of the continuous phase and dispersed phase that reduces the complexity of the model and calculations.

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