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Recommendations as a result of decision evaluations based on reference examples

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ABSTRACT: The problem of evaluation of decisions is considered, which evaluation consists in selecting from the set of possible decisions those that meet the decision-maker's preferences. The added value of solving this problem lies in the reduction of the number of decisions one can choose. Evaluation of decisions is based on their complete characteristics, rather than on a pre-defined quality indicator. The basis for the quality assessment are given pattern examples of decisions made. These are decisions that the decision maker has found to be exemplary or acceptable. They are used as defining his preferences. The methods proposed in this article concern the ordering and clustering of decisions based on their characteristics. The set of decisions selected by an algorithm is interpreted as recommended for the decision maker. Presented solu-tions can find a variety of applications, for example in investment planning, routing, diagnostics or searching through multimedia databases.

KEYWORDS: evaluation, recommendation, ranking, preferences, data mining, clustering, vector optimization

1. Introduction

The issue of selecting the best decision remains ever relevant. The best known methods are based on pre-defined quality indicators. Adopting a scalar quality indicator enables decisions to be ranked according to the values of this indicator. Optimisation involves selecting the decision that is characterised by the extreme value. Determined this way, the optimal decision is often questioned by an experienced decision maker, however. The causes of dissatisfaction may be found in that ranking decisions according to a pre-determined quality indicator does not fully reflect the decision-maker's requirements. This problem can be alleviated by using a vector quality indicator. The price for a more tho-rough encompassing of the decision-maker's preferences is an optimisation procedure that only partially orders the decision set and generates a solution in the form of a subset (e.g. Pareto solutions). Obtaining a total (linear) order is possible once the indicator is made scalar, usually through a compromise that defines the relevance (weights) of the individual coordinates of the quality vector [9]. Using this optimisation method requires a more accurate knowledge of the decision-maker's preferences. It may be noted that optimisation proce-dures that select one optimal decision usually do not provide information on whether the selected value is significantly different from the values characterising the successive decisions. Showing complete information on the ranking to the decision maker is one method of solving this issue.

A major problem in evaluating decisions is obtaining an adequate representation of the decision-maker's preferences in the evaluation space, including selecting the correct evaluation space itself. Generally, the quality indicator (both scalar and vector) is defined independently from the specific act of selection. In practice, this means that the decision maker has their preferences imposed without the ability to specify additional aspects or reservations. This may lead to the decision maker considering the result of optimisation incorrect.

The drive for complete disclosure and use of the decision-maker's preferences manifests in solutions based on the decision maker comparing decisions and indicating the better one. On this basis, the partial order relation can be determined (cf. e.g. [2]). As with vector optimisation, the resulting solutions do not select the one clearly best decision in this case either, but it can in general be expected that they will correspond to the decision-maker's preferences.

Optimisation approaches that do not result in selecting single decisions can be interpreted as determining recommendations for the decision maker (decision recommendations). It follows from this reasoning that many multi-criterion optimisation methods naturally generate only recommendations as a standard solution to the problem. Recommendations can be generated from scalar optimisation as well – based on ranked decisions.

Decision evaluation based on directly defining the decision-maker's preferences encounters a major obstacle when working with the decision maker (most commonly through surveys or when formulating orders). Modern decision making systems have large datasets at their disposal, and expecting the decision maker to have the knowledge and ability to make a sufficiently large number of comparisons is unreasonable. It can, however, be assumed that the decision maker is able to provide examples of perfect decisions (or acceptable ones), usually selected from a provided set. Obtaining such examples should help to answer the question of what factors in decision evaluation are important for the decision maker, and enable decision ranking principles to be defined. This information should form the basis for preparing relevant recommendations. It can be expected that their quality will depend on the number of examples selected.

2. The issue of evaluation based on reference examples provided

The evaluation issue presented in this paper involves the reduction of a finite set of possible decisions by determining a subset (or subsets) of decisions recommended as compatible with the decision-maker's preferences. The number of identified recommended decisions should be small enough to enable the decision maker to verify the recommended decisions and make the final selection on his own.

Each decision is described by its characteristics. The basis of qualitative evaluation (recommending) of decisions is the set of decisions considered by the decision maker to be referential (perfect or acceptable). Decision evaluation is based on complete decision characteristics, not on pre-defined quality indicators (scalar or vector). Quality evaluation is done by comparing the complete characteristics of the given decision with reference characteristics. It is assumed that the references are the characteristics of the decisions specified by the decision maker or special characteristics generated by him. In both cases, the highlighted characteristics will be referred to as reference examples or exemplary patterns (examples or patterns, for short).

The methods presented in this paper concern the characteristics of decisions shown as vectors of real numbers. The difficulty in solving the problem emerges for large vector dimensions as the number of examples specified is relatively small. This situation usually occurs in the problem under discussion because all available data is generally used for evaluating decisions, even if the data is only potentially useful. This leads to the necessity to analyse large-dimension and large-manifold vectors.

The scope of applications of the problem in question is broad: it includes investment optimisation, routing, diagnostics, as well as searching multimedia databases.

3. Related studies

The subject of decision evaluation according to the decision-maker's preferences is clearly visible in designing information search systems in large datasets according to a user-specified criteria. In such tasks, the primary evaluation ideas are stated to be [8]:
1) content-based filtering,
2) collaborative filtering.

Content-based filtering can be described as a kind of traditional valida-tion. The latter filtering type is based on identifying the user's (individual or group) preferences. Algorithms of collaboration with the user are important in this process. While the method of obtaining solutions in information search systems is based on updating and using appropriate databases, the ideas formu-lated there can also be found in other applications [10].

Among collaboration-based methods, methods utilising clustering algorithms are particularly notable [7]. Reasons exist for using such methods in information searching tasks in internet systems if user indexing is impossible. Using clustering algorithms (as a familiar method of unsupervised learning) is therefore naturally justified.

Studies related to example-based evaluation also include studies developing the idea referred to as case-based reasoning [1]. The essence of this approach to problem solving is defined by specifying a sequence of four recommendations: 1) retrieve case, 2) reuse, 3) revise, 4) retain. Implementing the first recommendation involves finding cases compatible with the examples given.

Optimisation tasks that require references to be obtained from the decision maker have been formulated in [3] and [4]. The essence of the propositions discussed there is the assumption that decision evaluation is based on using complete decision characteristics and not a specially constructed quality indicator. The basis for the optimisation procedures presented in these studies were exemplary patterns. However, these decision characteristics generally were not suitable for vector-based decision evaluation. Unlike in the problem discussed in this paper, in addition to providing exemplary patterns, the rules of evaluating the compatibility of the evaluated decisions with the reference needed to be specified.

A case where the number of given patterns was small in comparison to the number of coordinates of the characteristics vector was analysed in [6]. The paper proposes two optimisation methods based on determining the projection of characteristics vectors on reference subspaces. The first method is distinguished by the use of the distance between the characteristics vector and reference subspace. The other method involves transferring the optimisation problem to the reference subspace.

4. Feature space

Assume that the decision set to be evaluated is numbered from 1 to N . For each decision, characteristics are shown as vector of real numbers. This vector will be referred to as feature (attribute) vector. This name is arbitrary (individual coordinates of these vectors are usually obtained as raw measurement results). Decision number k will be defined as:

$$\mathbf{a}_k = \left[a_{1,k}, \quad a_{2,k}, \quad \cdots \quad a_{L,k}\right]^T, \qquad \mathbf{a}_k \in R^L \tag{1}$$

Each coordinate $a_{l,k}$ is a real number. The parameter L specifies the number of coordinates of the feature vector. These vectors form a subset:

$$\mathrm{A} = \left\{\mathbf{a}_1, \quad \mathbf{a}_2, \quad \ldots \quad \mathbf{a}_N\right\}, \qquad \mathbf{a}_k \in R^L \tag{2}$$

For convenience, feature vectors will be compiled into the following matrix:

$$\mathbf{A} = \left[\mathbf{a}_1, \quad \mathbf{a}_2, \quad \ldots \quad \mathbf{a}_N\right], \qquad \mathbf{a}_k \in R^L \tag{3}$$

Feature vector covariance (dispersion) matrix will be defined as:

$$\mathbf{R} = \tfrac{1}{N-1}\sum_{k=1}^{N}\left(\mathbf{a}_k - \overline{\mathbf{a}}\right)\left(\mathbf{a}_k - \overline{\mathbf{a}}\right)^T \tag{4}$$

where:

$$\overline{\mathbf{a}} = \tfrac{1}{N}\sum_{k=1}^{N}\mathbf{a}_k \tag{5}$$

Further, assume that:

$$\det(\mathbf{R}) \neq 0 \tag{6}$$

The distance between vectors $\mathbf{x}$ and $\mathbf{y}$ of feature space R^L will be determined in a manner that considers the magnitude of coordinate dispersion and their mutual correlation. This criterion is met by the Mahalanobis distance, defined by the following formula:

$$\mathrm{d}(\mathbf{x},\mathbf{y}) = \sqrt{(\mathbf{x}-\mathbf{y})^T\,\mathbf{R}^{-1}(\mathbf{x}-\mathbf{y})}\ , \qquad \mathbf{x},\mathbf{y} \in R^L \tag{7}$$

5. Environmental evaluation

The evaluation methods discussed in this section – ranking and clustering – will be referred to as environmental methods. This term reflects the fact that the adopted metric (7) is determined on the basis of all data analysed. It also means that when defining the metric, the decision-maker's preferences are not used.

5.1. Use of decision-maker's references for decision ranking

Examples qualified as patterns (references) will be indicated by specifying a finite set W of vectors from feature space R^L. For indicating a decision index set I_w, this set is obtained as follows:

$$\mathrm{W} = \left\{\mathbf{a}_k \in \mathrm{A} : k \in I_w\right\} \tag{8}$$

The number of elements W of a reference defined this way is defined as $N_w = \|\mathrm{W}\|$.

The primary problem is deciding the principles of determining the similarity of feature vector $\mathbf{x}$ to reference W (the compatibility of feature vector $\mathbf{x}$ with reference W). It is proposed that the concept of the distance between clusters be used here. The best known methods of calculating this distance are: nearest neighbour method, farthest neighbour method, mean distance method, centroid method, and Ward's method [5]. For example, choosing the centroid method, the following formula for calculating the similarity of feature vector $\mathbf{x}$ to reference W is obtained:

$$\mathrm{D}_w(\mathbf{x}) = \mathrm{d}(\mathbf{x}, \overline{\mathbf{a}}_w) = \sqrt{(\mathbf{x} - \overline{\mathbf{a}}_w)^T \mathbf{R}^{-1} (\mathbf{x} - \overline{\mathbf{a}}_w)} \tag{9}$$

where:

$$\overline{\mathbf{a}}_w = \tfrac{1}{N_w} \sum_{\mathbf{a}_j \in \mathrm{W}} \mathbf{a}_j \tag{10}$$

Using distance (9) results in an evaluation that depends on the decision-maker's preferences only through the mean value of reference features $\overline{\mathbf{a}}_w$. A greater dependence of evaluation results from the decision-maker's preferences can be expected when the mean distance of the given feature vector $\mathbf{x}$ from all references is assumed as the basis of calculation:

$$\overline{\mathrm{D}}_w(\mathbf{x}) = \tfrac{1}{N_w} \sum_{\mathbf{a}_j \in \mathrm{W}} \mathrm{d}(\mathbf{x}, \mathbf{a}_j) = \tfrac{1}{N_w} \sum_{\mathbf{a}_j \in \mathrm{W}} \sqrt{(\mathbf{x} - \mathbf{a}_j)^T \mathbf{R}^{-1} (\mathbf{x} - \mathbf{a}_j)} \tag{11}$$

For a particular metric, a decision ranking can be obtained based on the determined distances of the feature vector $\mathbf{x}$ to reference W. In both cases, i.e. for distance (9) and distance (11), decisions specified as referential will not always be placed in leading positions in the resulting ranking. This stems from the fact that the decision-maker's choices are not defined as the perfect choice, but only as examples of acceptable choices. It can be stated that the preferences revealed by the decision maker are only used to specify the core of the recommended features. The recommended decisions themselves, on the other hand, are selected depending on the distance of the individual feature vectors from this core.

5.2. Evaluation based on grouping results

Other possibilities of determining decisions corresponding to the decision-maker's preferences are provided by unsupervised learning methods based on clustering algorithms (in the method discussed previously, the concept of distance between clusters was used, but the grouping operation itself was not performed). It can be expected that grouping should result in clusters with similar elements (of limited diversity). It can be stated that the use of clustering algorithms should extract these properties of the feature set that are not obvious for the decision maker.

Step one of the evaluation involves the direct application of any clustering algorithm – with the use of the definition of distance (7) and any method of determining distances between clusters. At this stage, the decision-maker's preferences are completely ignored (as the resulting clusters are independent from the references specified). Step two involves confronting the resulting clusters with the specified reference set. Only at this stage can it be evaluated whether the grouping performed is useful for the decision maker. If there is a cluster that contains reference examples, this entire cluster can be presented as a recommendation. This recommendation will be a nontrivial solution if it contains additional vectors different than the references.

An interesting case is dichotomous grouping, i.e. grouping into two clusters, where one contains all reference examples. In this case, the action performed can be considered decision validation, although validation criteria are implicitly specified by given examples.

6. Evaluation based on reference-matched metrics

It can be expected that given reference examples by the decision maker provides information on what coordinates of the feature vector are important to the decision maker. It is suggested that this information be used by determining a metric (distance function) on the feature space not based on the covariance (dispersion) matrix of environmental features (of all analysed feature vectors), but only by reference features.

The covariance (dispersion) matrix determined on the basis of reference W is defined as follows:

$$\mathbf{R}_w = \frac{1}{N_w - 1} \sum_{\mathbf{a}_j \in \mathrm{W}} \left(\mathbf{a}_j - \overline{\mathbf{a}}_w\right)\left(\mathbf{a}_j - \overline{\mathbf{a}}_w\right)^T \tag{12}$$

where $\overline{\mathbf{a}}_w$ is calculated according to formula (10). The distance between vectors $\mathbf{x}$ and $\mathbf{y}$ of feature space R^L, defined as:

$$\mathrm{d}_w(\mathbf{x},\mathbf{y}) = \sqrt{(\mathbf{x}-\mathbf{y})^T \mathbf{R}_w^{-1}(\mathbf{x}-\mathbf{y})}\,, \qquad \mathbf{x},\mathbf{y} \in R^L \tag{13}$$

is referred to as W-reference-matched. The pre-condition for using this formula is the nonsingularity of covariance matrix $\mathbf{R}_w$. For this condition to be met, it is necessary that the number of examples specified be greater than the dimension of the feature vector.

A metric defined this way may serve to determine distances between clusters [5]. For example, if the centroid method is used, a formula for calculating the values of the functional that determines the similarity of given vector $\mathbf{x} \in R^L$ to reference W is obtained:

$$\mathrm{D}_w(\mathbf{x}) = \mathrm{d}_w(\mathbf{x},\overline{\mathbf{a}}_w) = \sqrt{(\mathbf{x}-\overline{\mathbf{a}}_w)^T \mathbf{R}_w^{-1}(\mathbf{x}-\overline{\mathbf{a}}_w)} \tag{14}$$

As with environmental evaluation, a greater dependence of the ranking from the decision-maker's preferences can be expected when the mean distance of the given vector $\mathbf{x}$ from all reference feature vectors is taken as the basis of calculation:

$$\overline{\mathrm{D}}_w(\mathbf{x}) = \tfrac{1}{N_w}\sum_{\mathbf{a}_j\in \mathrm{W}} \mathrm{d}_w(\mathbf{x},\mathbf{a}_j) = \tfrac{1}{N_w}\sum_{\mathbf{a}_j\in \mathrm{W}} \sqrt{(\mathbf{x}-\mathbf{a}_j)^T \mathbf{R}_w^{-1}(\mathbf{x}-\mathbf{a}_j)} \tag{15}$$

7. Evaluation based on projections of features on reference subspace

7.1. Reference subspace

If matrix $\mathbf{R}_w$ is singular, calculating the matched distance (13) is impossible. In this case, it is proposed that the number of feature coordinates is reduced. Following [6], it is proposed that the feature vector dimension be reduced in a Karhunen-Loève transform value space. The basis of the Karhunen-Loève transformation are orthonormal eigenvectors $\mathbf{t}_k(\mathbf{R}_w)$ of covariance matrix $\mathbf{R}_w$ defined by formula (12). These vectors meet following equation:

$$\mathbf{R}_w\,\mathbf{t}_k(\mathbf{R}_w) = \lambda_k(\mathbf{R}_w)\,\mathbf{t}_k(\mathbf{R}_w)\,, \qquad k = 1,2,\cdots,L \tag{16}$$

where: $\mathbf{t}_k(\mathbf{R}_w) = \begin{bmatrix} t_{k,1} \\ t_{k,2} \\ \cdots \\ t_{k,L} \end{bmatrix}$, $\lambda_k(\mathbf{R}_w)$ – eigenvalues of covariance matrix $\mathbf{R}_w$.

Eigenvalues $\lambda_k(\mathbf{R}_w)$ are real numbers; assume that the values are arranged in a descending order (i.e. they decrease as index k increases). The Karhunen-Loève transform matrix can be presented as follows:

$$\mathbf{T} = \begin{bmatrix} \mathbf{t}_1^T(\mathbf{R}_w) \\ \mathbf{t}_2^T(\mathbf{R}_w) \\ \cdots \\ \mathbf{t}_L^T(\mathbf{R}_w) \end{bmatrix} \tag{17}$$

This matrix will be used to transform vector $\mathbf{x} \in R^L$ into vector $\mathbf{z} \in R^L$ as follows:

$$\mathbf{z} = \mathbf{Tx} - \mathbf{T}\overline{\mathbf{a}}_w = \mathbf{T}(\mathbf{x} - \overline{\mathbf{a}}_w), \qquad \mathbf{x} \in R^L \tag{18}$$

where: $\mathbf{Tx}$ – Karhunen-Loève transform of vector $\mathbf{x} \in R^L$, $\overline{\mathbf{a}}_w$ – vector defined by formula (10). Let M mean the number of positive eigenvalues of covariance matrix $\mathbf{R}_w$. Then, for reference vectors $\mathbf{a}_k \in \mathrm{W}$, the transformation result $\mathbf{T}(\mathbf{a}_k - \overline{\mathbf{a}}_w)$ is a vector whose first M coordinates are non-zero. Also note that $M \le min\{N_w, L\}$.

Define operator $\mathbf{P}$ functioning in space R^L in the following manner:

$$\mathbf{Pz} = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 1 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} z_{k,1} \\ \cdots \\ z_{k,M} \\ z_{k,M+1} \\ \cdots \\ z_{k,L} \end{bmatrix} = \begin{bmatrix} z_{k,1} \\ \cdots \\ z_{k,M} \\ 0 \\ \cdots \\ 0 \end{bmatrix} \tag{19}$$

Superposition $\mathbf{PT}$ defines reference subspace P_W in space R^L (Karhunen-Loève transform values) as follows:

$$P_W = \left\{ \mathbf{y} \in R^L : \mathbf{y} = \mathbf{PTx},\ \mathbf{x} \in R^L \right\} \tag{20}$$

Evaluations $\mathbf{a}_k \in R^L$ are transferred to reference subspace P_W based on the transform of difference $\mathbf{a}_k - \overline{\mathbf{a}}_w$. Vector $\mathbf{z}_k \in R^L$ of subspace P_W, corresponding to evaluation $\mathbf{a}_k \in R^L$, is determined as follows:

$$\mathbf{z}_k = \mathbf{PT}(\mathbf{a}_k - \overline{\mathbf{a}}_w), \qquad \mathbf{a}_k, \mathbf{z}_k \in R^L \tag{21}$$

Formula (20) defines the reference subspace as composed of L-dimensional vectors: $P_W \subset R^L$. Because for reference vectors $\mathbf{a}_k \in W$, the transformation result $\mathbf{T}(\mathbf{a}_k - \overline{\mathbf{a}}_w)$ is a vector whose first M coordinates only are non-zero, it is more convenient to make the calculations directly in space R^M.

For the given vector $\mathbf{x} \in R^L$, the corresponding vector of space $\boldsymbol{x} \in R^M$, obtained by substituting the first M coordinates, will be defined as follows: $\boldsymbol{x} = r(\mathbf{x})$.

7.2. Evaluation in reference subspace

The basis for the evaluation are feature vectors $\boldsymbol{a}_k \in R^M$ obtained by transforming vectors $\mathbf{a}_k \in \mathrm{A} \subset R^L$ according to formula: $\boldsymbol{a}_k = r(\mathbf{PT}(\mathbf{a}_k - \overline{\mathbf{a}}_w))$. Define the resulting set of feature vectors in space R^M as:

$$A = \left\{ \boldsymbol{a}_1,\ \ \boldsymbol{a}_2,\ \ \ldots\ \ \boldsymbol{a}_N \right\} = \left\{ \boldsymbol{y} \in R^M : \boldsymbol{y} = r(\mathbf{PT}(\mathbf{a}_k - \overline{\mathbf{a}}_w)),\ \mathbf{a}_k \in \mathrm{A} \right\} \tag{22}$$

Correspondingly, the set of reference features in space R^M is as follows:

$$W = \left\{ \boldsymbol{a}_1,\ \ \boldsymbol{a}_2,\ \ \ldots\ \ \boldsymbol{a}_{N_w} \right\} = \left\{ \boldsymbol{y} \in R^M : \boldsymbol{y} = r(\mathbf{PT}(\mathbf{a}_k - \overline{\mathbf{a}}_w)),\ \mathbf{a}_k \in \mathrm{W} \right\} \tag{23}$$

This way, a derivative problem based on evaluations in reference subspace is obtained. It can be solved using the methods discussed previously herein.

1) For environmental evaluation, the basis for selecting decisions is the following definition of distance in space R^M:

$$d(\boldsymbol{x},\boldsymbol{y}) = \sqrt{(\boldsymbol{x}-\boldsymbol{y})^T \boldsymbol{R}^{-1}(\boldsymbol{x}-\boldsymbol{y})}\ , \qquad \boldsymbol{x},\boldsymbol{y} \in R^M \tag{24}$$

where:

$$\boldsymbol{R} = \tfrac{1}{N-1}\sum_{k=1}^{N}\left(\boldsymbol{a}_k - \overline{\boldsymbol{a}}\right)\left(\boldsymbol{a}_k - \overline{\boldsymbol{a}}\right)^T , \tag{25}$$

$$\overline{\boldsymbol{a}} = \tfrac{1}{N}\sum_{k=1}^{N}\boldsymbol{a}_k \tag{26}$$

Distance (24) can be used both in clustering and ranking of decisions.

2) For evaluations based on reference-matched metrics, the basis for selecting decisions is the following relation:

$$d_w(\boldsymbol{x},\boldsymbol{y}) = \sqrt{(\boldsymbol{x}-\boldsymbol{y})^T \boldsymbol{R}_w^{-1}(\boldsymbol{x}-\boldsymbol{y})}\ , \qquad \boldsymbol{x},\boldsymbol{y} \in R^M \tag{27}$$

where:

$$\boldsymbol{R}_w = \tfrac{1}{N_w-1}\sum_{\mathbf{a}_j \in W}\left(\boldsymbol{a}_j - \overline{\boldsymbol{a}}_w\right)\left(\boldsymbol{a}_j - \overline{\boldsymbol{a}}_w\right)^T \tag{28}$$

$$\overline{\boldsymbol{a}}_w = \tfrac{1}{N_w}\sum_{\boldsymbol{a}_j \in W}\boldsymbol{a}_j \tag{29}$$

Matrix $\boldsymbol{R}_w$ is diagonal, and at the same time $\overline{\boldsymbol{a}}_w = \mathbf{0}$. Distance (27) can be used both in clustering and ranking of decisions.

7.3. Use of feature vector distances from reference subspace

The distance between given vector $\mathbf{x}$ and the reference subspace is determined in space R^L as the distance between vector $\mathbf{x}$ and its projection $\mathbf{x}_w$ on the reference subspace:

$$\mathrm{d}(\mathbf{x},\mathbf{x}_w) = \sqrt{(\mathbf{x}-\mathbf{x}_w)^T \mathbf{R}^{-1}(\mathbf{x}-\mathbf{x}_w)}\ , \qquad \mathbf{x},\mathbf{x}_w \in R^L \tag{30}$$

where:

$$\mathbf{x}_w = \mathbf{T}^{-1}\mathbf{z} + \overline{\mathbf{a}}_w , \qquad \mathbf{z} = \mathbf{PT}(\mathbf{x}-\overline{\mathbf{a}}_w) , \qquad \mathbf{x}, \overline{\mathbf{a}}_w \in R^L \tag{31}$$

Distance (30) can be used directly for ranking of decisions. It can be used as an additional similarity assessment in the ranking methods proposed previously. This operation can alleviate the consequences of ranking of decisions solely on the basis of feature projection on the reference subspace.

8. The experiment

8.1. Subject and purpose of the study

To illustrate the proposed methods, an exemplary set of decision characteristics (set of features) was analysed. The purpose of the experiment was not to determine the best method. Although it is possible and recommended for any specific problem, in this paper, the experimental results are not analysed in terms of their relevance and utility.

In the analysed dataset, an individual characteristic (feature vector) is a vector measurement result whose every coordinate was determined as a result of a single comparison test (benchmark). Each measurement was made on a different computer system, coordinates with identical indexes describe the result of the same benchmark[1]. The systems had different hardware and software configurations, i.e. had different CPUs, motherboards, operating systems, active software or network environments. The dataset used contains the results of 256 benchmarks determined for 145 systems (which gives a total of 37120 numbers).

In the analysed experiment, the direct determination of quality indicators by the decision maker is unrealistic due to the lack of specific meanings of individual characteristics (resulting, for example, from the lack of experience of the decision maker in this respect). For the purposes of the experiment, the assumption was made that the decision maker could point to examples that meet his requirements. The choice made by the decision maker allows to specify a reference set.

The average linkage method was taken as the basis for calculating the distances between clusters. According to this method, the following relation was used in the clustering algorithms [5]:

[1] Measurement results shared by their author, Artur Miktus (artur.miktus@wat.edu.pl), were used in the calculations.

$$dist(G_r, G_s) = \frac{1}{N_r N_s} \sum_{k=1}^{N_r} \sum_{l=1}^{N_s} d(\mathbf{x}_{rk}, \mathbf{x}_{sl}) \tag{32}$$

where: $dist(G_r, G_s)$ – distance between clusters G_r and G_s, with:

$$G_r = \{\mathbf{x}_{r1}, \mathbf{x}_{r2}, \ldots, \mathbf{x}_{rN_r}\},\ G_s = \{\mathbf{x}_{s1}, \mathbf{x}_{s2}, \ldots, \mathbf{x}_{sN_s}\}, \qquad \mathbf{x}_{rk}, \mathbf{x}_{sl} \in R^L .$$

In the ranking algorithms, this relation takes the form of (15) or (11), respectively.

As it is attempted to take into account the dispersion of measurement vector coordinates and their correlations when calculating distances, it is preferable to use Mahalanobis distances, i.e. formula (7), or when using a reference-matched metric, formula (13). This involves the necessity to ensure that the corresponding covariance matrices, defined by formulas (4) and (12), respectively, are nonsingular. This requires the number of measurement vector coordinates to be reduced. In the experiment, this reduction was made in the principal component space. Only those components whose insufficient variance (dispersion) could cause numerical calculation errors were ignored.

8.2. Environmental evaluation results

The data source was a matrix made up of result vectors from testing 145 systems. Each coordinate of these vectors defined a measurement result obtained from the same test. The original number of coordinates was reduced so that the covariance matrix of all system features was nonsingular. The feature vector dimension was reduced by calculating Karhunen-Loève transforms for the original vectors, then discarding these transform coordinates that had zero or insufficient variance (i.e. causing numerical calculation errors when determining the covariance matrix inverse). As a result, an feature matrix (3) composed of 145 10-dimensional vectors (with a nonsingular feature covariance matrix) was obtained.

The resulting vectors $\mathbf{a}_k \in R^L$ constitute the characteristics of the analysed systems, with: $k \in \{1, 2, \ldots, N\}$, $N = 145$, $L = 10$. The coordinates of the calculated feature vector are interpreted as principal components of the measurement result.

The first variant of calculations involved using a covariance (dispersion) matrix determined on the basis of environmental features, i.e. calculated for all systems according to formula (4). The results of the ranking evaluation are shown in Fig. 1. The lists of recommended systems (decisions) were determined based on an initial section of the ranking list that contained all reference elements. Expansions to the basic ranking were determined, adding N_d further elements to it. The clustering method evaluation was based on determining the least numerous cluster that contained the reference set as a subset. This result was achieved by performing the grouping on the largest number of clusters possible (in the experiment, the number was 16), among which there is a cluster containing all the specified reference vectors. The recommended decision set was expanded by adopting a correspondingly smaller number of clusters. The numbers of clusters for which the visualised recommendations were obtained are stated in the description to Fig. 2.

The second variant of calculations involved using a covariance (dispersion) matrix determined on the basis of given patterns. In the experiment under consideration, the feature space is L-dimensional, with $L = 10$. The rank of the covariance matrix calculated according to the formula (12) is not more than 2 because the number of pattern examples $N_w = 3$. So the covariance matrix is singular. This variant of calculations could not be applied.

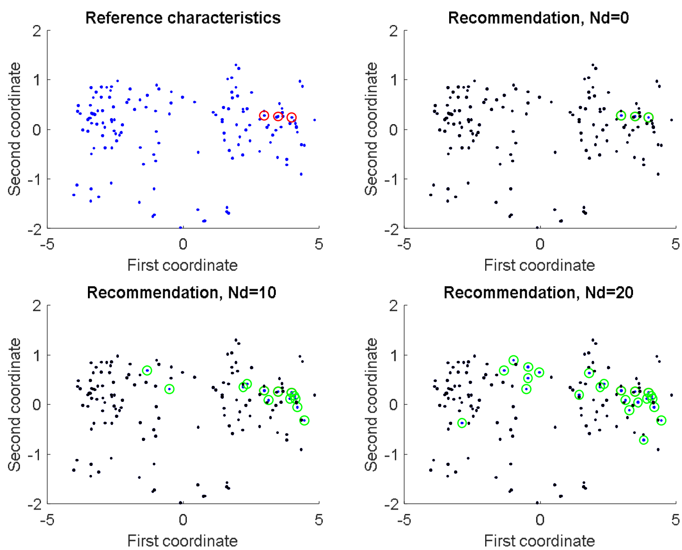

Fig. 1. Visualisation of environmental evaluation. Recommended decision sets are determined on the basis of a ranking list. Reference characteristics are marked with red circles. Recommended set characteristics, with green circles

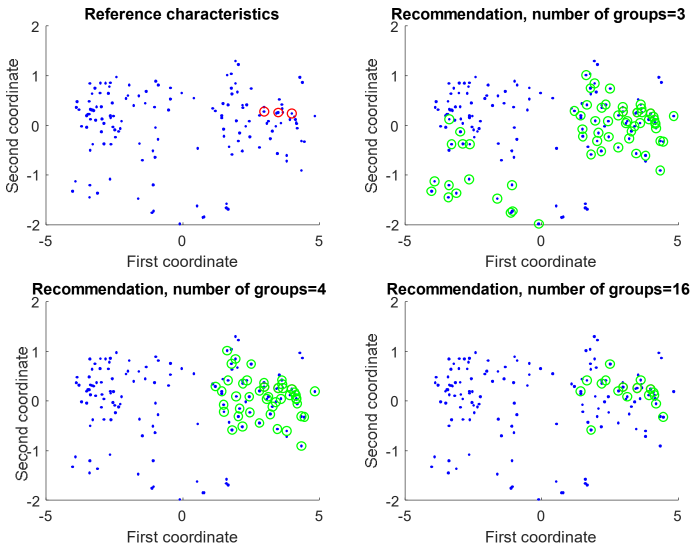

Fig. 2. Visualisation of environmental evaluation. Recommended decision sets are determined on the basis of the grouping results. Reference characteristics are marked with red circles. Recommended set characteristics, with green circles

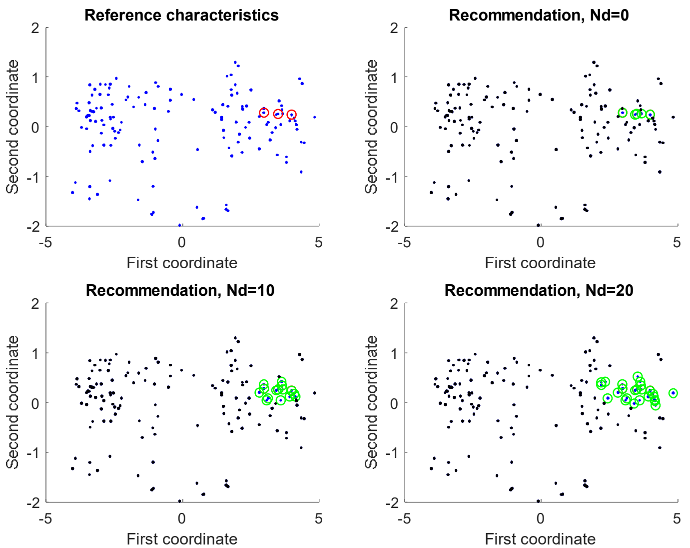

Fig. 3. Visualisation of environmental evaluation in reference subspace. Recommended decision sets are determined on the basis of a ranking list. Reference characteristics are marked with red circles. Recommended set characteristics, with green circles

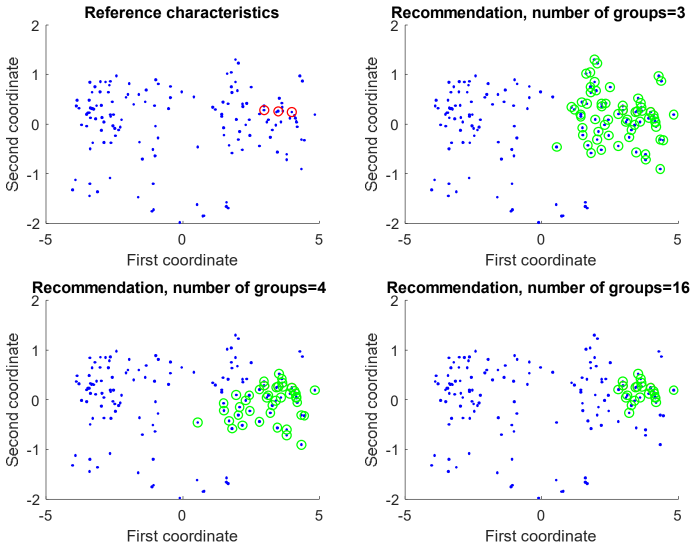

Fig. 4. Visualisation of environmental evaluation in reference subspace. Recommended decision sets are determined on the basis of the grouping results. Reference characteristics are marked with red circles. Recommended set characteristics, with green circles

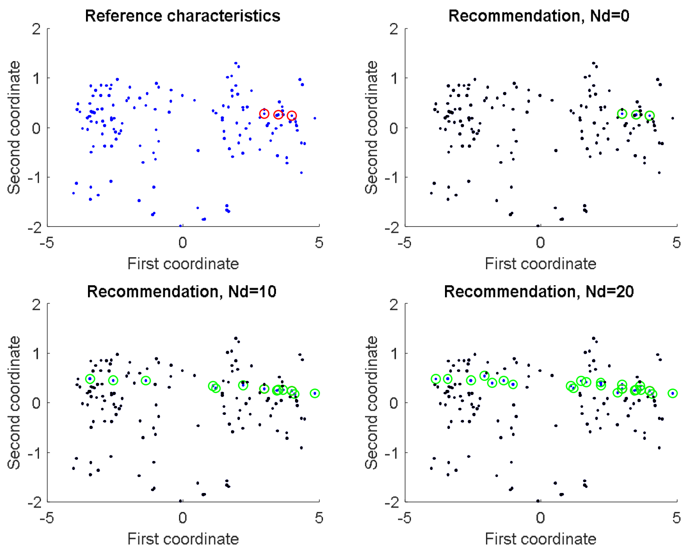

Fig. 5. Visualisation of evaluation in reference subspace for a reference-matched metric. Recommended decision sets are determined on the basis of a ranking list. Reference characteristics are marked with red circles. Recommended set characteristics, with green circles

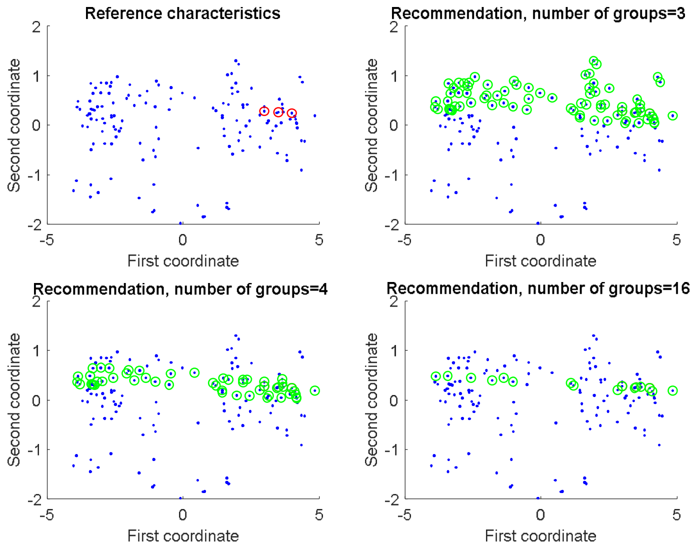

Fig. 6. Visualisation of evaluation in reference subspace for a reference-matched metric. Recommended decision sets are determined on the basis of the grouping results. Reference characteristics are marked with red circles. Recommended set characteristics, with green circles

8.3. Results of evaluation in reference subspace

As with performing calculations in the complete environment, calculations in the reference subspace were made for Karhunen-Loève transforms. The basis for the transformation was a covariance matrix (12). The reference subspace was determined by leaving the principal components with non-zero variance. As a result of reduction, a nonsingular matrix of 2-dimensional feature vectors was obtained. The coordinates of the calculated feature vector $\boldsymbol{a}_k \in R^M$ are interpreted as principal components of the measurement result (with: $k \in \{1, 2, \ldots, N\}$, $N = 145$, $M = 2$).

The first variant of calculations involved using a covariance (dispersion) matrix determined on the basis of environmental features, i.e. calculated for all systems. The results of the ranking evaluation are shown in Fig. 5. The evaluation results obtained by grouping in the reference subspace are shown in Fig. 6. The recommended decision sets were determined identically as described in section 8.2.

The second variant of calculations involved using a covariance (dispersion) matrix determined on the basis of reference features, i.e. calculated for reference systems according to formula (12). The results of the ranking evaluation are shown in Fig. 7. The evaluation results obtained by grouping in the reference subspace are shown in Fig. 8.

8.4. Conclusions from the experiment

The calculation results lead to the following conclusions:

1) Using the ranking method enables a recommended decision set to be obtained containing virtually any number of elements. It is the property of the method itself. Determining recommendations based on the results of grouping gives no such guarantee. However, a useful diversity of the numbers of resulting recommendations is achievable.
2) The irrelevance of the recommendations from the decision-maker's perspective does not necessarily disqualify this method. The incompatibility of the resulting recommendation with the decision-maker's expectations may be the consequence of the inconsistency of the specified reference source set. Verification of this set should enable satisfactory recommendations to be reach.

9. General conclusions

1) The proposed evaluation methods are versatile and can be used wherever decision characteristics can be represented as real number vectors. As recommended decisions are defined here as those whose characteristics are close to the references, it is natural that the methods discussed can be used in one-class classification problem.
2) If only a small number of reference examples (compared to the dimension of the feature vector) are available, it is important to use the principal components. If a sufficient number of reference examples are available, the reference subspace becomes the feature space.
3) The computation methods presented enable evaluations to be performed in situations where decision characteristics are not selected for their utility in

a specific act of decision. This particularly applies to selections based on automatically generated data, often intended for other purposes.

4) All of the methods proposed enable flexible narrowing or expanding of the recommended decision sets. For ranking-based methods, this can be achieved by directly shortening or expanding the ranking list used. For methods utilising clustering algorithms, this aim can be achieved by increasing or reducing the specified number of clusters.
5) The quality of the evaluation results depends on the consistency of the reference specifications. Trivial, multipartite or excessively large sets of recommended decisions indicate an inconsistency of the references specified by the decision maker. If such solutions are obtained, verification of the reference set is suggested.

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Rekomendacje jako wynik ewaluacji decyzji bazującej na wzorcowych przykładach

STRESZCZENIE: Rozpatrywany jest problem ewaluacji decyzji polegający na wytypowaniu spośród możliwych decyzji tych, które spełniają preferencje decydenta. Użyteczność rozwiązania problemu polega na zredukowaniu liczby możliwych do wyboru decyzji. Ewaluacja decyzji bazuje na ich kompletnych charakterystykach, a nie na wcześniej zdefiniowanym wskaźniku jakości. Podstawą oceny jakości są wzorcowe przykłady decyzji. Są to decyzje, które decydent uznał za doskonałe lub akceptowalne. Wskazane przez decydenta przykłady są wykorzystywane jako określające jego preferencje. Proponowane w artykule metody dotyczą porządkowania i grupowania decyzji na podstawie ich charakterystyk. Wytypowany zbiór decyzji jest interpretowany jako rekomendowany dla decydenta. Przedstawione rozwiązania mogą znaleźć różnorakie zastosowania, np. w planowaniu inwestycji, trasowaniu, diagnostyce czy przeszukiwaniu multimedialnych baz danych.

SŁOWA KLUCZOWE: ewaluacja, rekomendacja, ranking, preferencje, eksploracja danych, grupowanie, optymalizacja wektorowa

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