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Natural Frequencies of Small Cylinders and Tubes

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Abstract. The paper deals with modal analysis of the cylinders and tubes and their mutual comparison. Nowadays, modal analysis is a powerful method of describing the vibration properties. When we know own shapes and natural frequencies, we can predict e.g. behaviour of tubes under loading conditions. The tubes dimensions were taken as dimensions which are close to assault rifle barrels. The same dimensions were taken for cylinders. The cylinders and tubes as 3D objects were modelled and their lengths and thicknesses were modified. Modal analysis was performed by LS-DYNA software using FEM with iterative Lanczos method. In Tables 1-12, the natural frequencies for these cantilever beams and their corresponding own shapes with modification in fixing length for three different thicknesses are listed. In Figure 2 and 7-9, these natural frequencies versus modification in fixing length for three different thicknesses for corresponding modes are shown. In Figure 3-5 and 10-12, natural frequencies versus length of cylinders and tubes for three different thicknesses for corresponding modes are shown.
Keywords: mechanics, modal analysis, cylinder, tube, small weapon barrel

1. INTRODUCTION

This paper relates to natural frequencies and own shapes for three cylinders and tubes with different lengths and thicknesses and their various fixing lengths. The fixing length simulates fixation of a weapon barrel to a weapon case which is different for various weapons. The tube dimensions were taken as dimensions which are close to the barrel dimensions for assault rifles which are main weapon for individual. The same outer diameters and lengths of cylinders were taken for comparison of the behaviour of full material with material which has a hole along an axis at modal analysis. This hole simulated a weapon calibre which was chosen as 5.56 mm, which is the standard in NATO countries. For automatic shooting, it is necessary to know a value of natural frequencies. They are also important for exciting barrel vibration at a single shot, where during the shoot the barrel is affected with pressure of powder gases and moving bullet. For accurate fire we need to know barrel muzzle behaviour at the moment when the bullet leaves it. Some authors deal with this problem [1-6]. The simulation can help for research on design of assault rifles in the future.

2. LS-DYNA MODEL ANALYSIS

Very often at vibrations, the problem is solution of eigenproblem. The effective procedures for calculation of only a few eigenvalues and corresponding eigenvectors by finite element equations have been developed with Lanczos transformations based on iterations. The basic finite formulation of the problem is described by equation [7]:

$$\mathbf{K\Phi} - \omega^2\mathbf{M\Phi} = 0 \tag{1}$$

where $\mathbf{K}$ is the stiffness matrix; $\mathbf{\Phi}$ – vector of the order n (or mode shape vector); n – desired eigenmodes; ω – corresponding frequency of vibration of the vector $\mathbf{\Phi}$; $\mathbf{M}$ – the mass matrix.

For solving the problem, the geometry of the cylinders and tubes was defined in LS-DYNA [8]. The lengths and outer diameters for tubes were chosen close to the barrel dimensions for assault rifles which are main weapon for individual. The same dimensions were taken for cylinders. The first, three cylinders were designed with different lengths (448 mm, 488 mm, and 528 mm) where each of them has both three different outer diameters (15 mm, 20 mm, and 25 mm) and five fixing lengths at the one end (20 mm, 30 mm, 40 mm, 50 mm, and 60 mm). The tubes dimensions were taken the same as for cylinders and inner diameter was taken as 5.56 mm simulating a weapon calibre which is standard in NATO countries (Fig. 1). The same outer diameters and lengths of cylinders were taken for comparison of behaviour of full material with material which has a hole along axis at modal analysis. The material for both cylinders and tubes was chosen as elastic one with material parameters for steel.

The cylinders and tubes were created with 8-node hexahedral solid elements. The model for the longest cylinder has 22 737 nodes and 17 472 solid elements and shorter variations have less both nodes and elements according to their lengths. The longest model for the tube has 39 432 nodes and 31 200 solid elements and its shorter variations have less nodes and solid elements according to the length. That means that models for three different lengths for each type have the same structure of mesh.

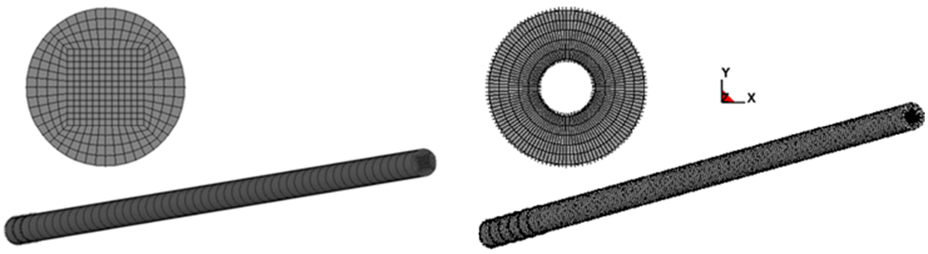

Fig. 1. The design cylinder with 20 mm fixing length
and tube with 50 mm fixing length, both on the left side

In Tables 1-3, there are listed natural frequencies for 448-mm long cylinder with three different outer diameters and modification in fixing length.

Table 1. Natural frequencies and corresponding mode shapes

Cylinder 448 mm/15 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	58.88	61.79	64.91	68.29	71.93
2.	367.49	385.56	404.99	425.93	448.53
3.	1022.8	1072.8	1126.5	1184.3	1246.7
4.	↕ 1858.3	↕ 1903.6	↕ 1951.2	↕ 2001.2	↕ 2053.9
5.	1987.4	2083.7	2187.1	2298.3	2418.1
6.	↔ 3008.9	↔ 3082.4	↔ 3159.5	↔ 3240.6	↔ 3326

Table 2. Natural frequencies and corresponding mode shapes

Cylinder 448 mm/20 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	78.49	82.37	86.54	91.03	95.88
2.	488.36	512.28	538	565.7	595.58
3.	1352.5	1418	1488.4	1564	1645.6
4.	↕ 1857.6	↕ 1902.9	↕ 1950.5	↕ 2000.5	↕ 2053.1
5.	2610.2	2734.7	2868.2	3011.6	3165.9
6.	↔ 3009.7	↔ 3083.2	↔ 3160.4	↔ 3241.5	↔ 3326.9

Table 3. Natural frequencies and corresponding mode shapes

Cylinder 448 mm/25 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	98.09	102.93	108.14	113.75	116.04
2.	607.8	637.43	669.28	703.56	718.81
3.	1673	1753.2	1839.2	1931.6	1976.4
4.	↕ 1857.3	↕ 1902.6	↕ 1950.2	↕ 2000.2	↕ 2052.8
6.	↔ 3010.5	↔ 3084	↔ 3161.2	↔ 3242.4	↔ 3263.4
5.	3202.4	3352.3	3512.9	3685	3775.9

For other two 488-mm and 528-mm long cylinders, in this paper, the results are not presented in form of table but only in a graphical form. In all tables, the sign ↕ means torsional mode and the sign ↔ means extension mode.

From Tables 1-3 and from Figures 2, there are evident the dependences of natural frequencies versus fixing length for 448-mm long cylinders with different diameters at modes from one to six. For each bending mode, it is evident its linear dependence, where modal frequencies have increasing tendency with increase in outer diameter. Very interesting is that each torsional mode 4 and extension mode 6 have almost the same values of natural frequencies at different outer diameters. However, in the cylinders which are longer or thicker, both torsional and extension mode can overtake the bending mode. It is evident e.g. from Table 3, where extension mode 6 keeps almost the same natural frequency at the outer diameter of 25 mm as at 20 mm or 15 mm, but at the bigger outer diameter, bending mode 5 has higher natural frequencies. Accordingly, each bending mode starts firstly at the axis y and next at the axis x.

Figures 3-5 show natural frequency dependences from the cylinder length. From these dependences, it is evident that with increasing length of cylinder, the modal frequencies decrease.

In Tables 4-6, there are listed natural frequencies for 448-mm long tube with three different outer diameters and modification in fixing length. Also in Tables 4-6 it is shown at which axis the bending mode starts because it is very different to determine it from behaviour of the cylinders and also from tubes.

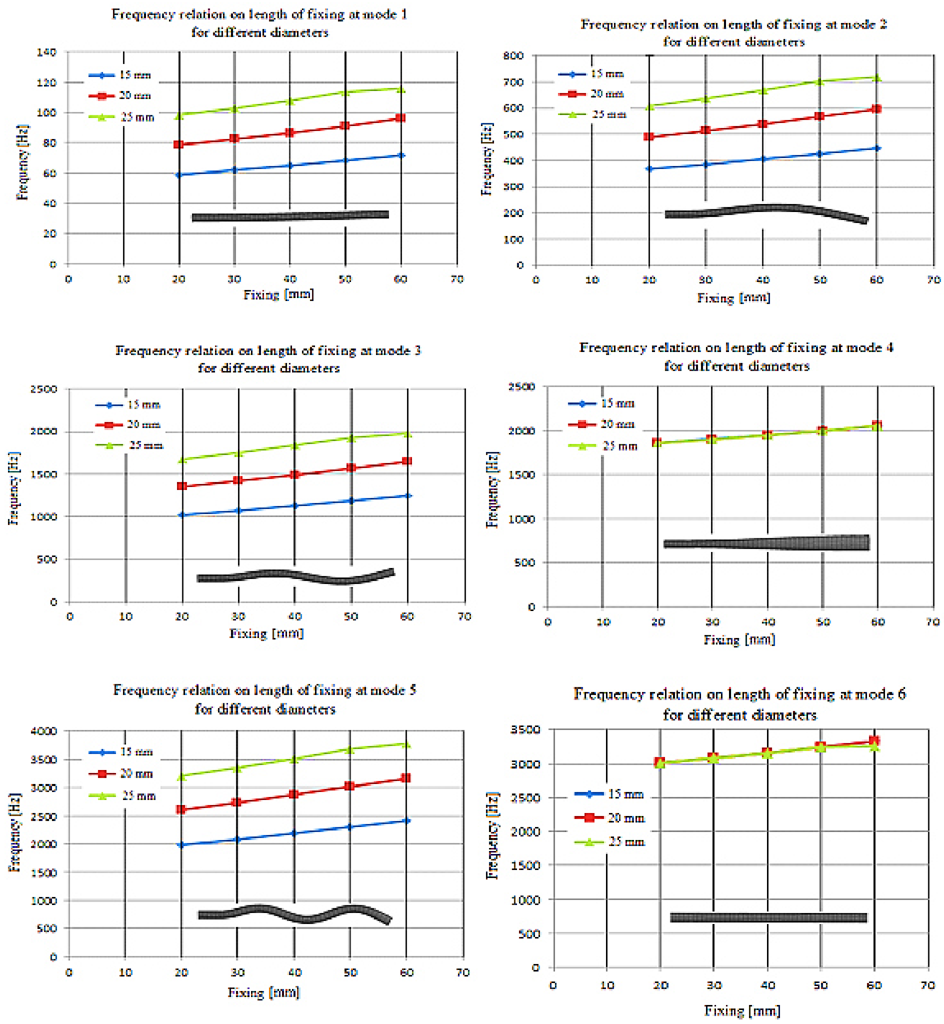

Fig. 2. Natural frequency vs fixing length for 448-mm long cylinders
with different diameters at modes 1-6

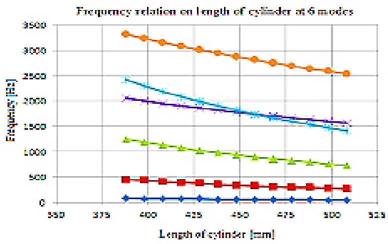

Fig. 3. Frequency vs length of
15 mm cylinder diameter

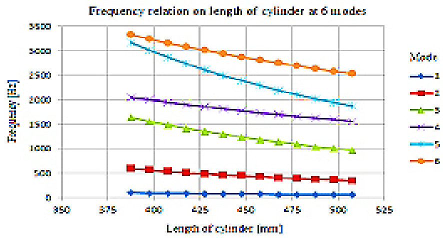

Fig. 4. Frequency vs length of
20 mm cylinder diameter

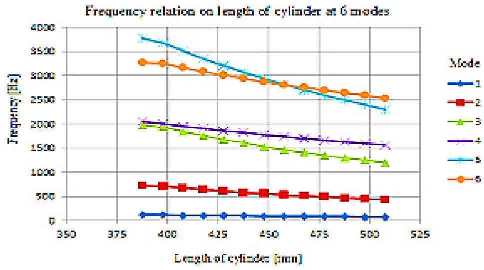

Fig. 5. Frequency vs length of
25 mm cylinder diameter

For example, 62.88/x5l means that the first number is the value of natural frequency, x5l is the direction (*x* – axis *x*) at which bending mode starts at 5° degree (5) from the axis *x* on the right. Next, it starts at 5° degree on the left (l) from the axis *y* (Fig. 6). The explanation is valid also for Tables 7-12 in this paper.

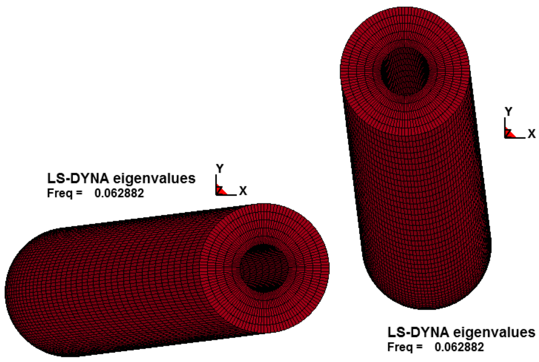

Fig. 6. Example of behaviour bending mode

From Tables 4-6 and from Figure 7, there are evident the dependences of natural frequencies versus fixing length for 448-mm long tubes with different outer diameters at modes from one to six.

For each bending mode it is evident linear dependence, where modal frequencies have increasing tendency with increase in outer diameter.

Table 4. Natural frequencies and corresponding mode shapes

Tube 448 mm/15 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	62.88/x5l	65.98/x30r	69.32/y	72.92/x10l	76.81/x30r
2.	391.86/y45l	411.09/y45r	431.77/y45l	454.04/x30l	478.08/y45r
3.	1088/y45l	1141/y45r	1197.8/y45l	1259/x15l	1325/y45r
4.	↕1861.5	↕ 1906.9	↕1954.6	↕2004.7	↕2057.4
5.	2107.3/y45l	2208.6/y45r	2317.3/y45l	2434.1/x	2560/y45r
6.	↔ 3008.7	↔ 3082.2	↔ 3159.3	↔ 3240.4	↔ 3325.8

Table 5. Natural frequencies and corresponding mode shapes

Tube 448 mm/20 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	81.55/y45l	85.57/y45r	89.90/y45l	94.57/y45l	99.61/x30r
2.	506.56/y45l	531.33/y45r	557.95/y45l	586.62/x	617.54/y45r
3.	1399.6/y45l	1467.1/y45r	1539.6/y45l	1617.5/x	1701.4/y45r
4.	↕ 1860.7	↕ 1906.1	↕ 1953.8	↕ 2003.9	↕ 2056.6
5.	2692.5/y45l	2819.9/y45r	2956.6/y45l	3103.2/x	3260.9/y45r
6.	↔ 3009.5	↔ 3083	↔ 3160.2	↔ 3241.3	↔ 3326.7

Table 6. Natural frequencies and corresponding mode shapes

Tube 448 mm/25 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	100.56/y45l	105.52/x30r	110.85/y45l	116.6/x30l	122.81/y45r
2.	622.09/y45l	652.36/y45r	684.88/y45l	719.89/x	757.63/y45r
3./4.	1708.4/y45l	1789.9/y45r	1877.3/y45l	1971.2/x	↕ 2056
4./3.	↕ 1860.2	↕ 1905.6	↕ 1953.2	↕ 2003.3	2072.3/y45r
6.	↔ 3010.3	↔ 3083.8	↔ 3161	↔ 3242.2	↔ 3327.7
5.	3260.2/y45l	3411.8/y45r	3574/y45l	3747.8/x	3934.4/y45r

Very interesting is that each torsional mode 4 and extension mode 6 have almost the same values of natural frequencies at different outer diameters. This behaviour is the same like for the cylinders. However, when the tube is thicker, torsional mode 4 at 60 mm fixing overtakes bending mode 3 which has higher natural frequency and extension mode 6 overtakes bending mode 5 which has higher natural frequency at all fixing lengths (Tab. 6).

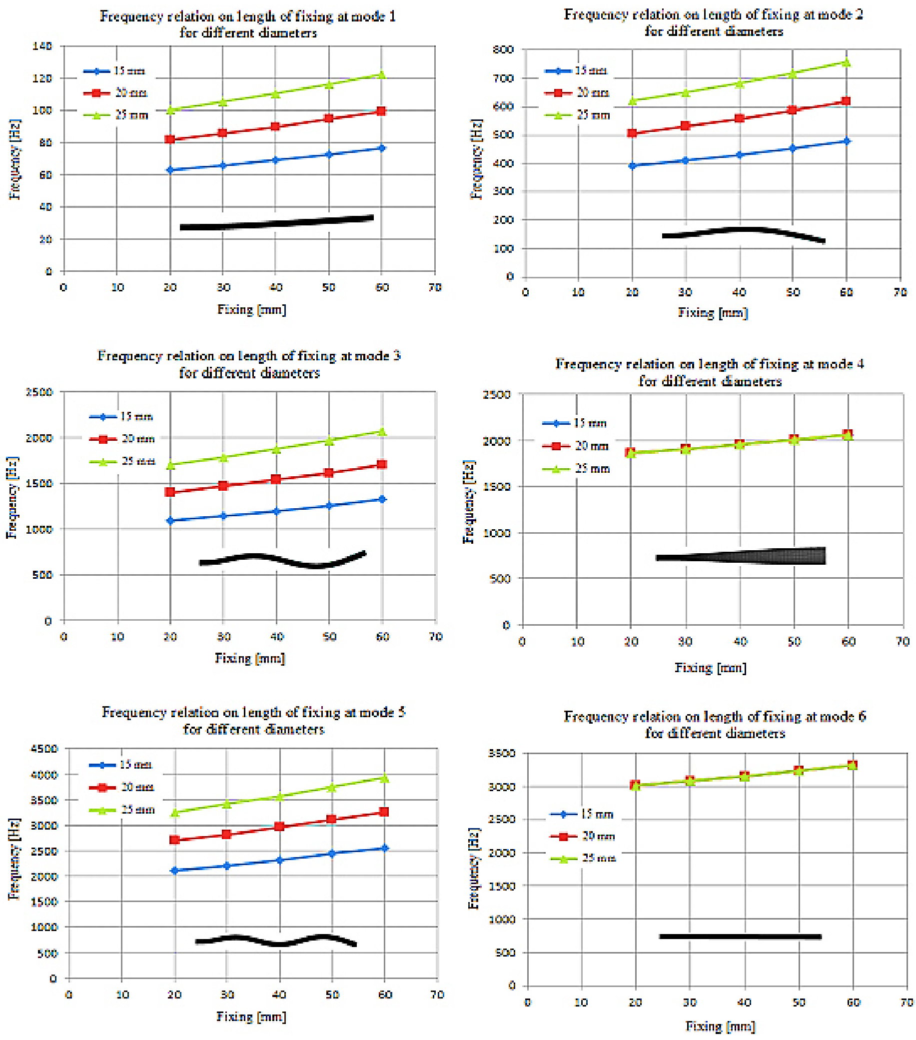

Fig. 7. Natural frequency vs fixing length for 448-mm long tubes
with different outer diameters at modes 1-6

In Tables 7-9, there are listed natural frequencies for 488-mm long tube with three different outer diameters and modification in fixing length. Also in Tables 7-9 it is shown at which axis the bending mode starts because it is very different to determine it from behaviour of the cylinders and also from tubes.

Table 7. Natural frequencies and corresponding mode shapes

Tube 488 mm/15 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	52.58/x	54.94/y	57.46/y15r	60.17/y15l	63.07/y45r
2.	327.96/y15l	342.62/y45r	358.29/y	375.05/x15l	393.02/y30r
3.	911.83/y15l	952.3/y45r	995.52/y	1041.7/x	1091.2/y30r
4.	↕1702.2	↕ 1740	↕1779.5	↕ 1820.9	↕1864.3
5.	1769.3/y30r	1847/x	1930/y15r	2018.6/x30l	2113.4/y45r
6.	↔ 2751	↔ 2812.2	↔ 2876.1	↔ 2943.1	↔ 3013.2

Table 8. Natural frequencies and corresponding mode shapes

Tube 488 mm/20 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	68.19/y	71.25/y	74.53/y	78.03/y45l	81.79/y45r
2.	424.2/y15l	443.11/y45r	463.31/y	484.91/x30l	508.06/y45r
3.	1174.5/y15l	1226.3/y45r	1281.5/y	1340.5/x30l	1403.7/y45r
4.	↕ 1701.5	↕ 1739.3	↕ 1778.8	↕ 1820.2	↕ 1863.5
5.	2265.9/y45l	2364.2/x30r	2468.9/y	2580.7/x30l	2700.2/y45r
6.	↔ 2751.6	↔ 2812.9	↔ 2876.9	↔ 2943.8	↔ 3014

Table 9. Natural frequencies and corresponding mode shapes

Tube 488 mm/25 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	84.09/y30l	87.87/y45l	91.90/y5r	96.22/y45l	100.86/y45r
2.	521.32/y30l	544.47/y45r	569.19/y	595.62/y45l	623.92/y45r
3.	1436/y30l	1498.7/y45r	1565.6/y	1637/y45l	1713.3/y45r
4.	↕1701	↕ 1738.8	↕ 1778.3	↕ 1819.7	↕ 1863
5./6.	2751.2/y45l	↔ 2813.6	↔ 2877.6	↔ 2944.6	↔ 3014.8
6./5.	↔ 2752.3	2868.8/x30r	2993.8/y	3127.1/y45l	3269.4/y45r

From Tables 7-9 and from Figure 8 there are evident the dependences of natural frequencies versus fixing length for 488-mm long tubes with different outer diameters at modes from one to six. For each bending mode it is evident linear dependence, where modal frequencies have increasing tendency with increase in outer diameter. Very interesting is that each torsional mode 4 and extension mode 6 have almost the same values of natural frequencies at different outer diameters. This behaviour is the same like at cylinders and 448-mm long tubes. However at 25-mm outer tube diameter, extension mode 6 overtake bending mode 5 which has higher natural frequencies at 30-60 mm fixing lengths (Tab. 9).

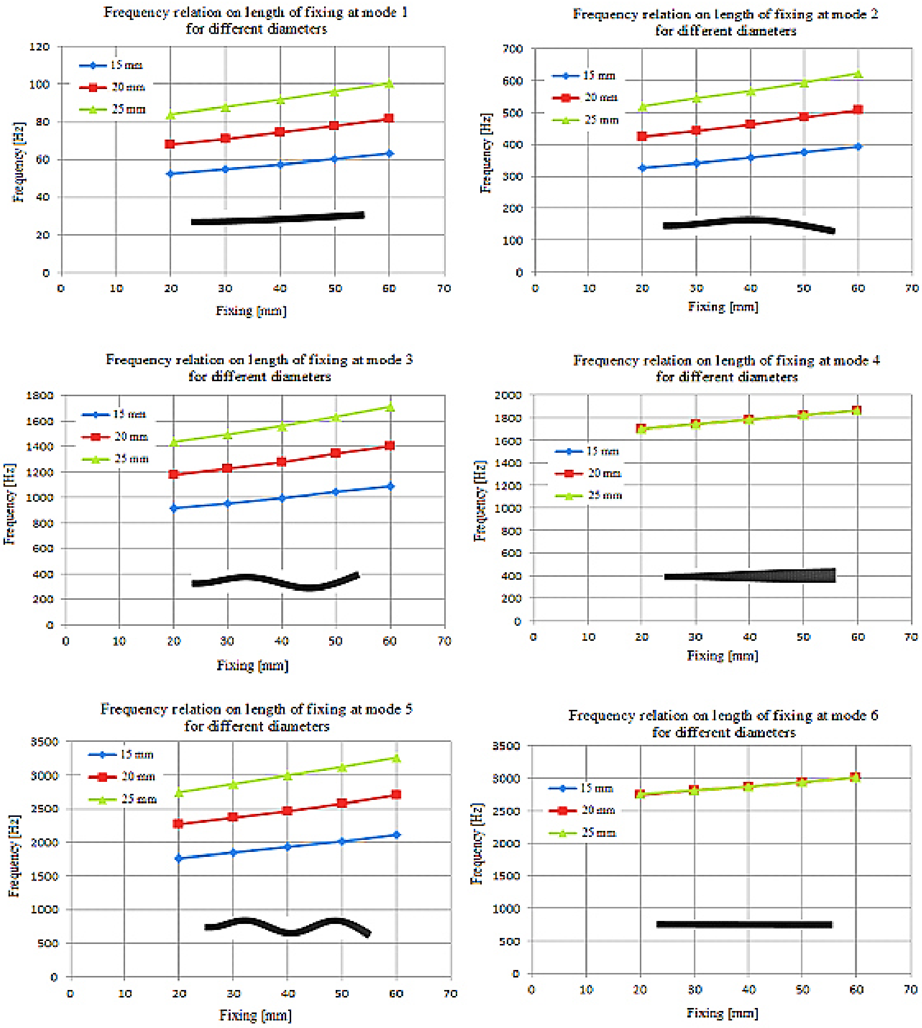

Fig. 8. Natural frequency vs fixing length for 488-mm long tubes
with different outer diameters at modes 1-6

In Tables 10-12, there are listed natural frequencies for 528-mm long tubes with three different outer diameters and modification in fixing length. Also in Tables 10-12 it is shown at which axis the bending mode starts because it is very different to determine it from behaviour of the cylinders and also from tubes.

Table 10. Natural frequencies and corresponding mode shapes

Tube 528 mm/15 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	44.61/x30l	46.45/x30r	48.41x	50.49/x	52.71/x
2.	278.49/y30l	289.92/x30r	302.07/y30r	315/y15l	328.78/x30r
3.	775.09/y30l	806.72/x30r	840.32/y10l	876.05/y20l	914.1/x30r
5./4.	1506.1/y30l	1567.1/x30r	1631.7/y10l	↕1668.1	↕1704.3
4./5.	↕1568	↕ 1600	↕1633.3	1700.5/y10l	1773.6/x15r
7./6.	2463.6/y15l	2662.2/y45r	↔2639.6	↔ 2695.8	↔ 2754.5
6./7.	↔ 2533.9	↔ 2585.7	2666.8/y10l	2777.8/y5l	2895.9/x5r

Table 11. Natural frequencies and corresponding mode shapes

Tube 528 mm/20 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	57.86/y45l	60.25/x30l	62.79/y45r	65.48/x30l	68.36/x
2.	360.37/y15l	375.13/x	390.81/y10r	407.49/y30l	425.27/x
3.	999.44/y20l	1040/x	1083/y10l	1128.7/y30l	1177.4/x
4.	↕1567.4	↕ 1599.3	↕1632.7	↕1667.4	↕1703.6
5.	1932.5/y30l	2009.8/x	2091.8/y20l	2178.9/y20l	2271.4/x30l
6.	↔ 2534.5	↔ 2586.3	↔ 2640.2	↔ 2696.4	↔ 2755.1

Table 12. Natural frequencies and corresponding mode shapes

Tube 528 mm/25 mm					
Mode	Fixing length [mm]				
	20	30	40	50	60
1.	71.36/x	74.30/x	77.43/x	80.76/y45l	84.30/x
2.	443.13/y10l	461.22/x	480.43/y	500.87/y45l	522.63/x
3.	1223.5/y30l	1272.8/x	1325/y15l	1380.5/y30l	1439.5/x
4.	↕1566.9	↕ 1598.9	↕1632.2	↕1666.9	↕1703.2
5./6.	2351.6/y30l	2444.4/x	2542.7/y20l	2647.1/y30l	↔ 2755.8
6./5.	↔ 2535.1	↔ 2586.9	↔ 2640.8	↔ 2697.1	2757.9/x30l

From Tables 10-12 and from Figure 9, there are evident the dependences of natural frequencies versus fixing length for 528-mm long tubes with different outer diameters at modes from one to six. For each bending mode, it is evident linear dependence, where modal frequencies have increasing tendency with increase in outer diameter. Very interesting is that each torsional mode 4 and extension mode 6 have almost the same values of natural frequencies at different outer diameters like at all previous events.

However, at 15-mm outer tube diameter, bending mode 5 overtakes torsional mode 4 at fixing length of 20-40 mm which has higher natural frequencies and bending mode 7 overtakes extension mode 6 at fixing lengths 20-30 mm (Tab. 10). As well at 25-mm outer tube diameter, extension mode 6 overtakes bending mode 5 at fixing length 60 mm (Tab. 12).

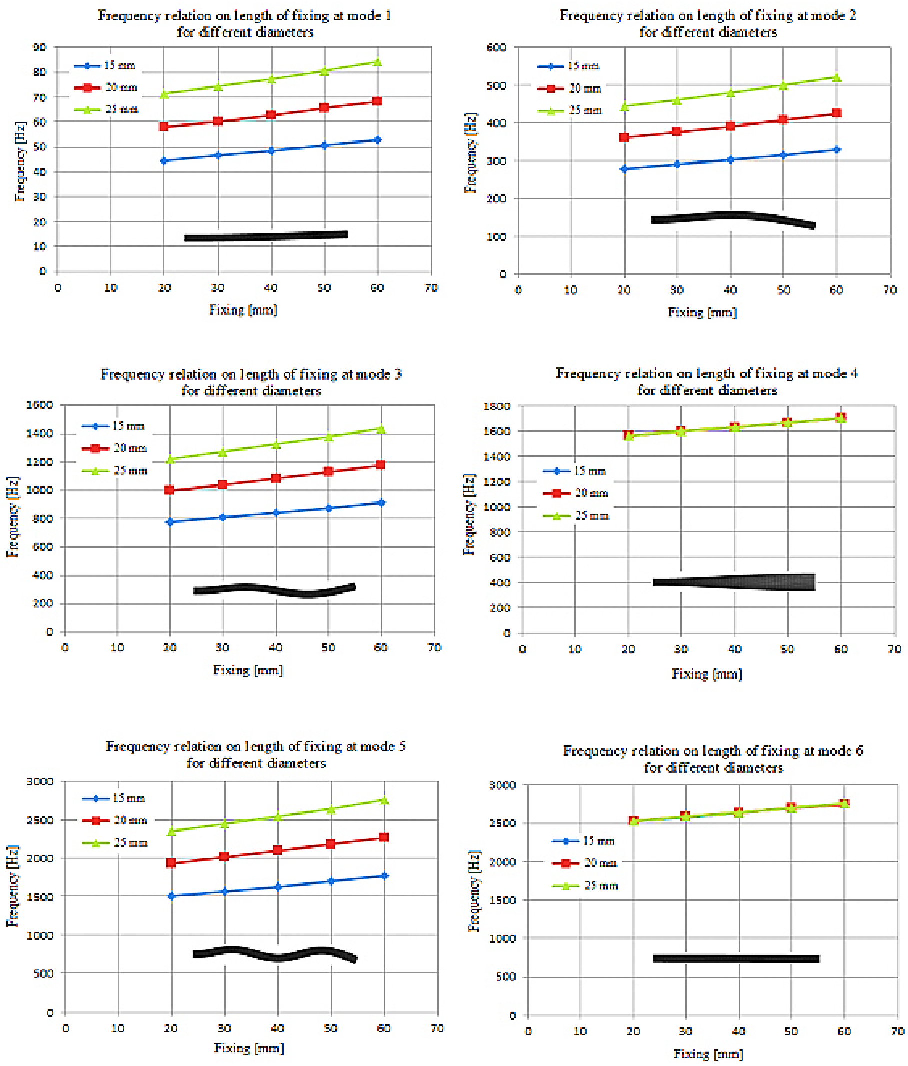

Fig. 9. Natural frequency vs fixing length for 528-mm long tubes with different outer diameters at modes 1-6

Figures 10-12 show natural frequency dependences from the tube length. From these dependences, it is evident, that with increasing length of tube, the modal frequencies decrease.

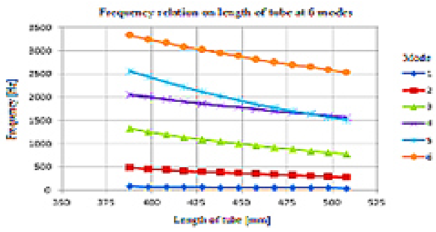

Fig. 10. Frequency vs length of 15 mm tube outer diameter

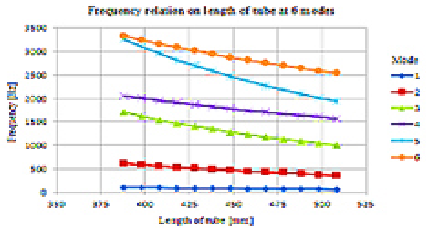

Fig. 11. Frequency vs length of 20 mm tube outer diameter

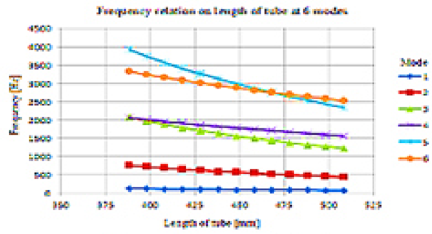

Fig. 12. Frequency vs length of 25 mm tube outer diameter

3. CONCLUSION

The results show that natural frequencies:

- increase with the length of the cylinder and tube fixing on one side (simulating fixation of weapon barrel to weapon case), i.e. that the longer fixing the barrel, the higher natural frequencies are;
- decrease when the cylinder or tube is longer, i.e. that the longer barrel will be excited at lower natural frequencies than the shorter barrel;
- increase with larger wall thicknesses of cylinder and tube (barrel) and they are inversely proportional to the cylinder and tube (barrel) lengths;
- in the investigated range of thicknesses, at the same length and different thicknesses of cylinder and tube (barrel), there are approximately the same in torsional modes 4 and also the same in extension modes 6.

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