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CHAOTIC STATES INDUCED BY RESETTING PROCESS IN IZHIKEVICH NEURON MODEL

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Abstract

Several hybrid neuron models, which combine continuous spike-generation mechanisms and discontinuous resetting process after spiking, have been proposed as a simple transition scheme for membrane potential between spike and hyperpolarization. As one of the hybrid spiking neuron models, Izhikevich neuron model can reproduce major spike patterns observed in the cerebral cortex only by tuning a few parameters and also exhibit chaotic states in specific conditions. However, there are a few studies concerning the chaotic states over a large range of parameters due to the difficulty of dealing with the state dependent jump on the resetting process in this model. In this study, we examine the dependence of the system behavior on the resetting parameters by using Lyapunov exponent with saltation matrix and Poincaré section methods, and classify the routes to chaos.

1 Introduction

Recently, according to the development of the brain measurement technology, it has been recognized that the information is transmitted among neurons by the spike timing instead of the firing rate of neurons. Therefore, the spiking neuron models, which can describe the spike timing, have been attracting a lot of attention. Then, many types of mechanism for generating spiking pattern have been revealed through the bifurcation analysis by the spiking neuron models [1, 2, 3]. The spiking neuron models used in these previous studies were the Hodgkin-Huxley (HH) type model [4] and simpler conductance based models such as FitzHugh-Nagumo model and Hindmarsh-Rose model [1], described by continuous ordinary differential equations. While, several hybrid neuron models, which combine continuous spike-generation mechanisms and discontinuous resetting process after spiking, have been proposed as a simple transition scheme for membrane potential between spike and hyperpolarization [5, 6, 7, 8]. As the simplest hybrid spiking neuron model, the leaky integrate-and-fire neuron model is constructed by leak ohmic conductance, membrane capacitance, refractory period and threshold for membrane potential. Its implementation cost for numerical calculation is small, but the reproduced spiking patterns are restricted to only regular spiking with class one excitability due to the sacrificing many factors of non-linear neurodynamics [1]. In contrast, Izhikevich neuron model

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as two dimensional non-linear hybrid neuron model can induce many kinds of bifurcation and reproduce almost all spiking activities observed in the actual neural systems by tuning a few parameters including the resetting process [5, 6].

As one of the characteristics of spiking activities in the brain, chaotic spiking patterns have been shown to support adaptable information processing in the function of learning and memory. Previous studies have been made on the continuous spiking neuron model, focusing on the electrical coupling strength and external input signal, and so on [3, 9, 10, 11, 12]. The reason for not adopting the above mentioned hybrid neuron model in the previous studies was the difficulty for evaluating chaos caused by state dependent jump in the resetting process. Some approaches have been made to deal with this condition by the bifurcation analysis [13, 14] and the Lyapunov exponent [15, 16] on the Poincaré section, and Lyapunov exponent with a saltation matrix on the system trajectory [17].

In this paper, using both Lyapunov exponent with saltation matrix and bifurcation analysis, we analyze the dependence of the system behavior on the resetting parameters in Izhikevich neuron model over wide range of parameters [18]. Following these results, we clarify the mechanism for generating chaos called the route to chaos.

2 Model and Methods

In this section, first, Izhikevich neuron model is explained and the typical spiking patterns are demonstrated. Second, the Lyapunov exponent with saltation matrix is defined as an evaluation index for the system state, and Poincaré section methods are introduced to analyze the structure of system trajectories.

2.1 Izhikevich neuron model

Izhikevich neuron model [5, 6] is a two-dimensional system of ordinary differential equations of the form:

$$\dot{v} = 0.04v^2 + 5v + 140 - u + I, \quad (1)$$
$$\dot{u} = a(bv - u), \quad (2)$$

with the auxiliary after-spike resetting

$$\text{if } v \geq 30[\text{mV}], \text{ then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d. \end{cases} \quad (3)$$

Here, v and u represent the membrane potential of the neuron and a membrane recovery variable, respectively. v and the time t have [mV] and [ms] scales, respectively. I is the input dc-current. The parameters a and b describe the time scale and the sensitivity of u, respectively. Spiking behaviors such as regular spiking (RS), intrinsically bursting (IB) and chattering (CH) are reproduced by using this model. In our simulation, this model is analyzed numerically by non linear differential/algebraic equation solvers of SUNDIALS library [19]

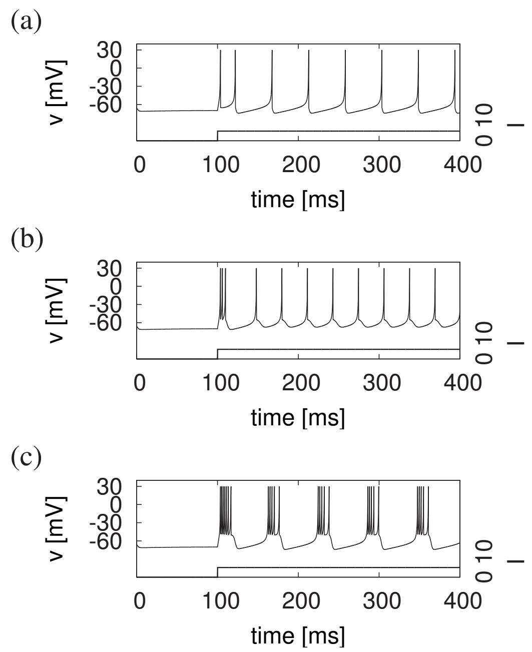

Figure 1. Spiking behaviors. (a) Regular spiking (RS) ($a = 0.02, b = 0.2, c = -65, d = 8$). (b) Intrinsically bursting (IB) ($a = 0.02, b = 0.2, c = -55, d = 4$). (c) Chattering (CH) ($a = 0.02, b = 0.2, c = -50, d = 2$).

Let us demonstrate the time evolution of $v(t)$ in the cases of RS ($a = 0.02, b = 0.2, c = -65, d = 8$), IB ($a = 0.02, b = 0.2, c = -55, d = 4$) and CH ($a = 0.02, b = 0.2, c = -50, d = 2$). When the dc-current $I = 10$ is input to RS neuron as shown in the bottom of Fig. 1 (a), the neuron fires a few spikes with short inter-spike period and then the period becomes long. This spiking behavior is the most typi-

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cal in the cortex. There are not only spiking neurons like RS neuron but also bursting neurons like IB and CH in the cortex. Figure 1 (b) shows that IB neuron bursts at the begin of $I = 10$ and then bursting shifts to spiking. On the other hand, CH neuron bursts during $I = 10$ and the inter-burst frequency can be as high as about 40 [Hz] as shown in Fig. 1 (c).

2.2 Lyapunov exponent with saltation matrix and Poincaré section methods

We quantify the chaotic activity of Izhikevich neuron model with Lyapunov exponent. On the continuous system trajectory between i-th and $(i+1)$-th spiking times: $(t_i \le t \le t_{i+1})$, the variational equations of Eqs. (1) and (2) are defined as follows:

$$\dot{\Phi}_{i+1}(t,t_i) = J(v,u,t)\Phi_{i+1}(t,t_i), \quad (4)$$
$$\Phi_{i+1}(t_i,t_i) = E, \quad (5)$$

where, Φ, J and E indicate the state transition matrix, the Jacobian and unit matrix, respectively. At $t = t_i$, the saltation matrix is given by

$$S_i = \begin{bmatrix} \frac{\dot{v}^+}{\dot{v}^-} & 0 \\ \frac{\dot{u}^+ - \dot{u}^-}{\dot{v}^-} & 1 \end{bmatrix}. \quad (6)$$

Here, (v^-,u^-) and (v^+,u^+) represent the values of (v,u) before and after spiking, respectively. In the case of the spikes arising in $[T^k : T^{k+1}]$ [ms], $\Phi^k(T^{k+1},T^k)$ $(k = 0,1,\cdots,N-1)$ [17, 20] is written by

$$\Phi^k(T^{k+1},T^k) = \Phi_{i+1}(T^{k+1},t_i)S_i\Phi_i(t_i,t_{i-1}) \cdots S_2\Phi_2(t_2,t_1)S_1\Phi_1(t_1,T^k). \quad (7)$$

By using the eigenvalues l_j^k $(j = 1,2)$ of $\Phi^k(T^{k+1},T^k)$, the Lyapunov spectrum λ_j is calculated by

$$\lambda_j = \frac{1}{T^N - T^0}\sum_{k=0}^{N-1}\log(|l_j^k|). \quad (8)$$

In our simulation, we set $T^{k+1} - T^k$ is the period of 20 spikes $(i = 20)$ or 1000 [ms] as its maximum value in the case that $T^{k+1} - T^k$ achieves 1000 [ms] before 20 spikes arise.

To investigate the structures of appeared attractors, Poincaré section method [21] is utilized. Figure 2 indicates the orbit of (v,u) on the $v-u$ plane in the case of RS. In our study, we set the Poincaré section at $v = 30$ [mV] as the resetting point given by Eq.(3) and observe the time evolutions of $(u_1,u_2,\cdots)$ which are the system behaviors of u on the the Poincaré section. The dynamics of u_i is given by Poincaré mapping $u_{i+1} = \phi(u_i)$, and then the point after m times passing through the Poincaré section from u_i becomes $u_{i+m} = \phi^m(u_i)$.

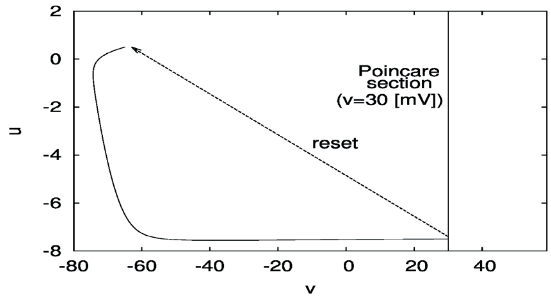

Figure 2. RS orbit of (v,u) (solid line) including state dependent jump (reset) on $v-u$ phase plane and Poincaré section utilized in this study. $(a = 0.02, b = 0.2, c = -65, d = 8, I = 10)$.

3 Results and Evaluations

In this section, we evaluate the chaotic states in two parameter regions by using λ_1 and Poincaré section methods.

3.1 Period-doubling bifurcation route to chaos

In parameter sets for RS, IB and CH, the parameters a and b are common $(a = 0.02, b = 0.2)$ and the parameters on the resetting process c and d are set differentially ($(c,d) = (-65,8)$ for RS, $(-55,4)$ for IB, $(-50,2)$ for CH). That is, Izhikevich neuron model can reproduce the various spiking patterns by tuning the parameters regarding the resetting process. This section focuses on the chaotic system behavior around these parameter sets for RS, IB and CH.

At first, Fig. 3 shows the dependence of λ_1 on the parameters of c and d around the parameter region for the parameter sets of RS, IB and CH [5]. Here, the other parameters are fixed $(a = 0.02, b = 0.2, I = 10)$. As a result, it is confirmed that the chaotic state $(\lambda_1 > 0)$ exists in the region: $-59 \lesssim c \lesssim -40$, $d \approx 1.0$. Furthermore, under the condition $c = -55$ [mV], we demonstrate the dependence of the system behavior on the parameter d

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around the value inducing chaotic spiking ($d \approx 1.0$). Figure 4 shows the time series of $v(t)$ [mV] (upper) and the orbit of (v,u) (lower) indicated black solid line. Here, as shown in the lower figures, v-nullcline ($\dot{v} = 0$ indicated by red dotted line) and u-nullcline ($\dot{u} = 0$ indicated by green dashed line) do not have the intersection point. Therefore, the neuron exhibits the spiking state regardless of the value of c and d. At $d = 0.8$ (Fig. 4 (a)), the orbit indicates the period-1 state. Then, as the value of d increases, the system state transits to period-2 ($d = 0.85$ in Fig.4 (b)), period-4 ($d = 0.89$ in Fig.4 (c)) and chaos ($\lambda_1 \approx 0.043$) ($d = 0.93$ in Fig.4 (d)) through period-doubling bifurcation. The duration for this chaotic spiking activity is divided into the duration for hyperpolarization ($v \approx -60$) [mV] and the duration for arising several spikes from $v \approx -50$ [mV], i.e., this chaotic spike pattern has the bursting characteristic.

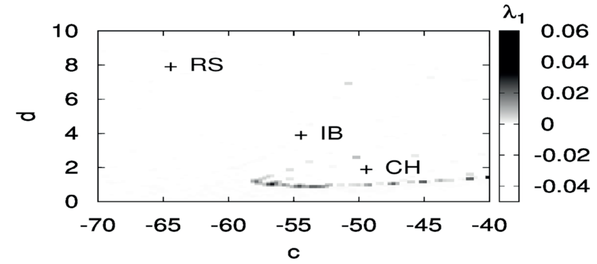
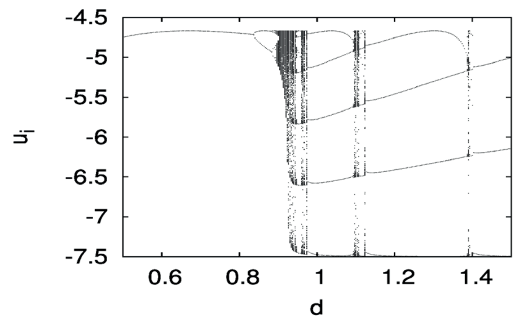


Figure 3. Dependence of maximum Lyapunov exponent λ_1 on parameters of d and c for resetting process. The symbols of (+) indicate the parameter sets for RS, IB and CH given by Fig.1. ($a = 0.02, b = 0.2, I = 10$)

Next, the system behavior indicated in Fig.4 is analyzed by Poincaré section method in detail. According to the time series of u_i in Fig.5 (upper), it is confirmed that the orbits of period-1, period-2, period-4 and chaos in Fig. 4 correspond to fixed point ((a)), period-1 ((b)) period-3 ((c)) and chaos ((d)) on Poincaré section, respectively. In order to investigate the relationship between u_i and u_{i+1}, the return map of u_i is shown in Fig.5 (lower). Here, the black solid, red dotted and green dashed lines indicate the orbit of u_i, Poincaré map $u_{i+1} = \phi(u_i)$ and $u_{i+1} = u_i$, respectively. In Fig. 5 (a) ($d = 0.8$), u_i stays at the stable fixed point $(u_i, u_{i+1}) = (-4.7, -4.7)$ as the intersection between $u_{i+1} = \phi(u_i)$ and $u_{i+1} = u_i$. As increasing d, the period-1 ((b)), period-3 ((c)) and chaotic ((d)) states appear through the period-doubling bifurcation. Also, the shapes of ϕ in Figs. 5 (a) to (d) exhibit the stretching and folding structure as a feature of non-linear map.

Furthermore, in order to examine the change of system behavior against the parameter d, continuously, we investigate the bifurcation by changing d with bifurcation diagram of u_i ((a)) and λ_1 ((b)) in Fig. 6. Here, this evaluating parameter region includes the values of $d = 0.8$ to 0.93 used in Fig.4 and Fig. 5. Form these results, in the range of $0 \lesssim d \lesssim 0.8$, u_i is on the fixed point, and then through the period-doubling bifurcation as the familiar route to chaos in the spiking neuron models [1, 3, 22], the chaotic state ($\lambda_1 > 0$) appears at $d \approx 0.93$.

(a)



(b)

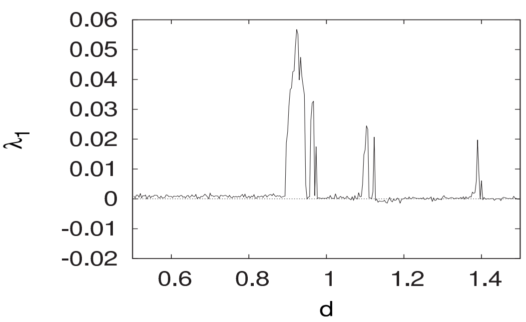


Figure 6. (a) Bifurcation diagram. (b) Dependence of maximum Lyapunov exponent λ_1 on d. ($a = 0.02, b = 0.2, c = -55, I = 10$)

3.2 Intermittent route to chaos

We evaluate the chaotic behavior in the region around the parameter set ($a = 0.2, b = 2, c = -56, d = -16, I = -99$) which is proposed by Izhikevich [6] as the parameter set producing the chaotic behavior. Note that route to chaos for this parameter set has not been investigated, so far.

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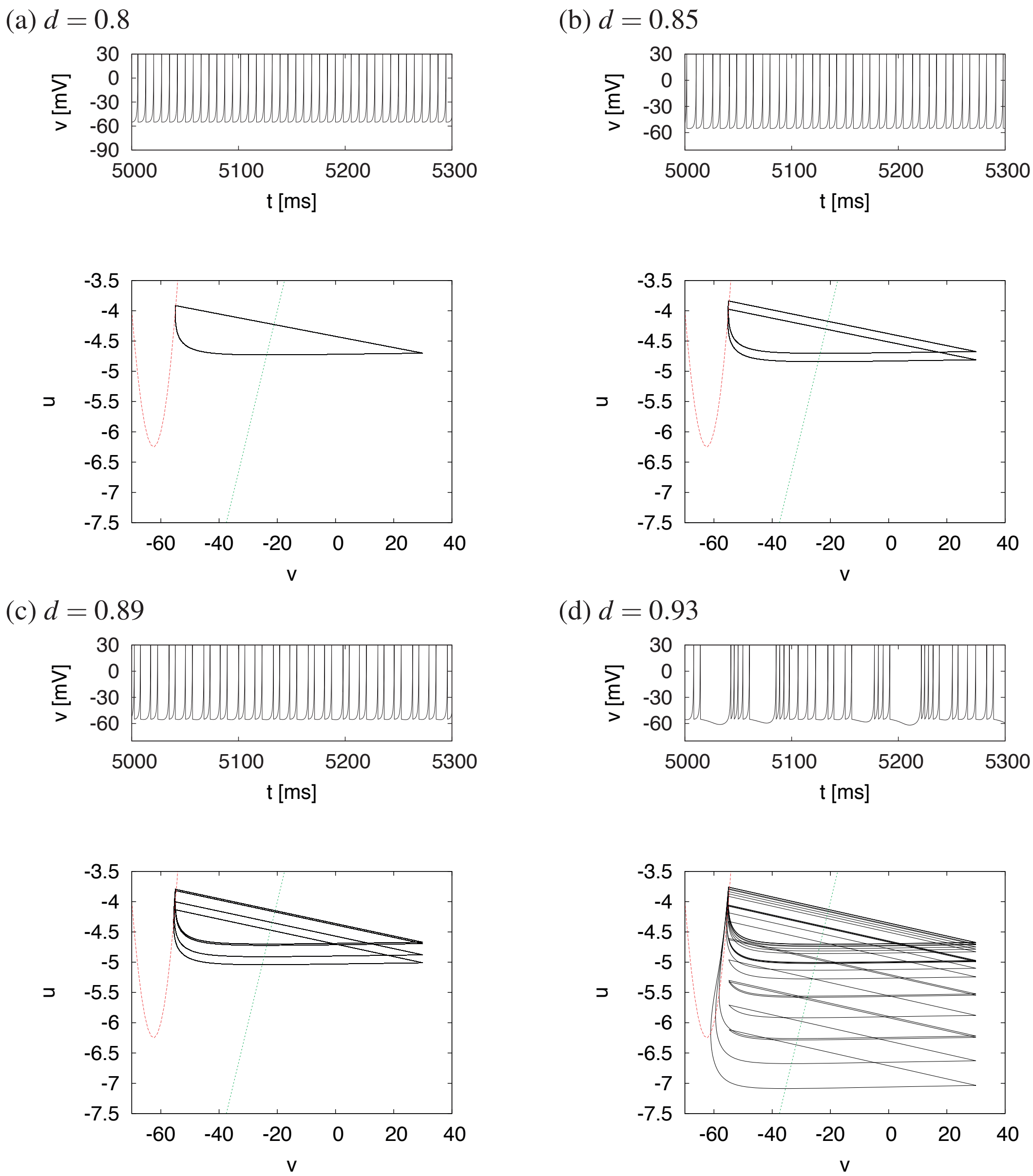

Figure 4. System behavior in periodic-1 ((a) $d = 0.8$), periodic-2 ((b) $d = 0.85$), periodic-4 ((c) $d = 0.89$) and chaotic ((d) $d = 0.93$) states. Upper figures indicate time series of $v(t)$ [mV] and lower figures indicate orbit of (v, u), v-nullcline ($\dot{v} = 0$) and u-nullcline ($\dot{u} = 0$) given by black solid, red dotted and green dashed lines, respectively. ($a = 0.02, b = 0.2, c = -55, I = 10$)

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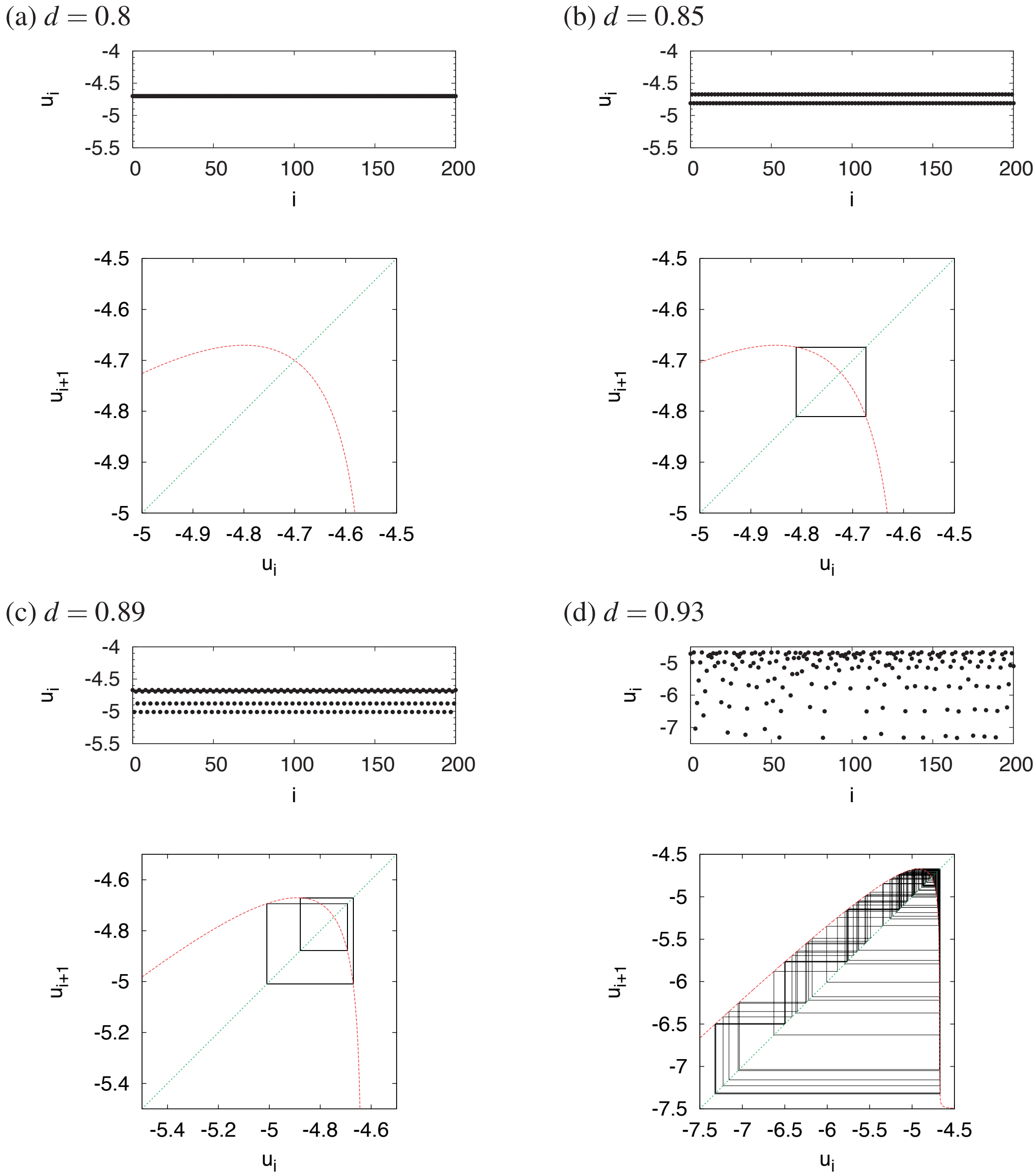

Figure 5. System behaviors on Poincaré section. Time series of u_i (upper). Return maps between u_i and u_{i+1} (lower), where red solid line is the orbit for u_i, red dotted line is Poincaré map $u_{i+1} = \phi(u_i)$, and green dashed line is $u_{i+1} = u_i$. The values of parameter d correspond to Fig. 4 from (a) to (d). $(a = 0.02, b = 0.2, c = -55, I = 10)$

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First, Fig. 7 shows the $c-d$ map of λ_1 in the case of $(a = 0.2, b = 2, I = -99)$. The chaotic state $(\lambda_1 > 0)$ exists in the region $d \lesssim -13$. Under the condition $c = -56$ [mV], we demonstrate the dependence of the system behavior on d. In Fig. 8, the time series of v[mV] (upper) and the orbit of (v, u) (indicated by black solid line) (lower) are shown. Here, v-nullcline ($\dot{v} = 0$; indicated by red dotted line) and u-nullcline ($\dot{u} = 0$; indicated by green dashed line) have two intersection points at $(-57.0, -114.0)$ and $(-17.9, -35.9)$ as unstable fixed points, thus, the neuron exhibits the spiking state. At $d = -11$ (Fig. 8 (a)), the orbit indicates the 1-period state. Then, as the value of d decreases, the system state transits to chaotic state in Figs.8 (b), (c) and (d). Here, it can be observed that the durations of the apparently periodic spiking seem to be shortened in these chaotic behaviors with decrease in d.

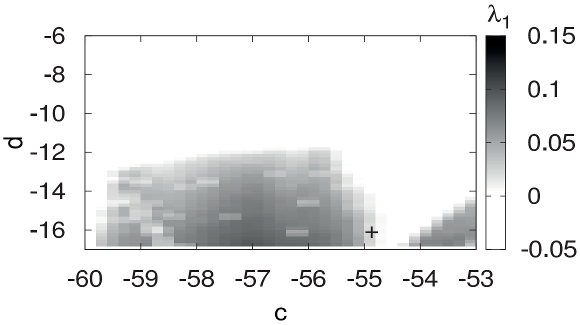

Figure 7. Dependence of maximum Lyapunov exponent λ_1 on parameters of d and c. The symbol of (+) indicates the chaotic parameter set proposed in Ref.[6]. $(a = 0.2, b = 2, I = -99)$

Next, by using Poincaré section method, the system behavior indicated in Fig.8 is evaluated. As shown in Fig.9 (a) (upper), u_i stays at the fixed point ($u_i \approx -98.6$). At $d = -12$ (Fig.9 (b) (upper)), u_i begins to oscillate with a focus on $u_i \approx -98.6$, the amplitude of this oscillation expands to $-102 \lesssim u_i \lesssim -90$, and then u_i returns to around -98.6, again. Here, the periodic oscillation disappears gradually with decrease of d as shown in Figs.9 (b) ($d = -12$), (c) ($d = -13$) and (d) ($d = -16$). Now, to investigate the feature of u_i oscillation, we use the return map of u_i and u_{i+2}. Lower figures in Fig. 9 indicate the orbit of u_i (black solid line), Poincaré map $u_{i+2} = \phi^2(u_i)$ (red dotted line) and $u_{i+2} = u_i$ (green dashed line). In the case of $d = -11$ ((a)), the orbit of u_i stays at the intersection point ($\approx (-98.5, -98.5)$) between $u_{i+2} = \phi^2(u_i)$ and $u_{i+2} = u_i$. However, at $d = -12$ ((b)), the orbit of u_i exhibits sluggish movement (laminar mode) in the region where the slope of ϕ^2 around 1.0 ($-102 \lesssim u_i \lesssim -94$) and irregularly active movement (turbulent mode) in the other region having larger slope ($\gg 1$). This chaotic dynamics alternating between the laminar and turbulent modes is called intermittency chaos [23, 24]. As the value of d decreases, the region producing the laminar mode, where the ϕ^2 slope indicates around 1.0, reduces and then the turbulent mode is dominant in the dynamics as shown in Figs.9 (b) to (d).

Furthermore, we investigate the bifurcation in detail by enlarging the parameter region on d including the value of $d = -11$ to -16 used in Fig.8 and Fig.9. Figures 10 (a) and (b) indicate the bifurcation diagram of u_i and λ_1, respectively. In the region $d \gtrsim -11.9$, u_i stays at the fixed point ($\lambda_1 \approx 0$), but in $d \lesssim -11.9$ the system state transits to chaos ($\lambda_1 \gtrsim 0$) not to undergo the period-doubling bifurcation.

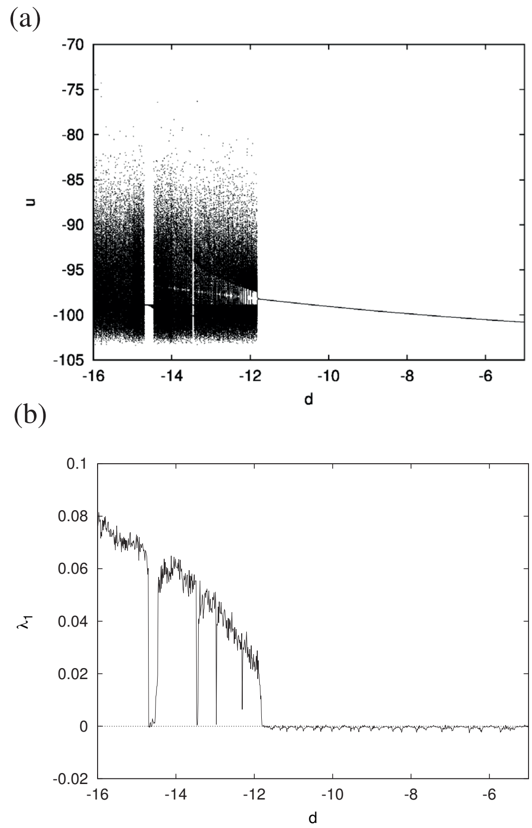

Figure 10. (a) Bifurcation diagram. (b) Dependence of maximum Lyapunov exponent λ_1 on d. $(a = 0.2, b = 2, c = -56, I = -99)$

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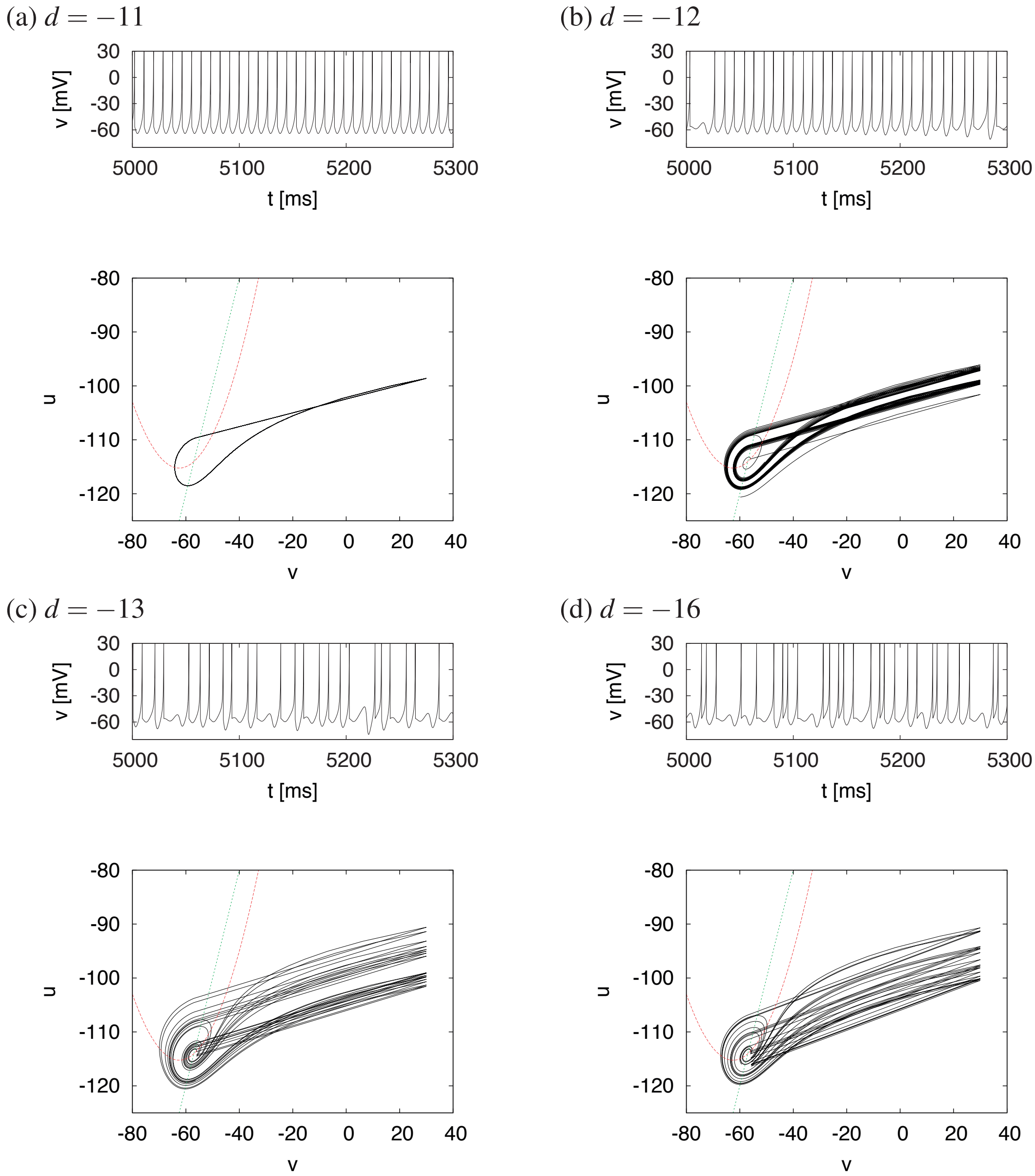

Figure 8. System behavior in periodic-1 ((a) $d = -11$) and chaotic ((b) $d = -12$, (c) $d = -13$ and (d) $d = -16$) states. Upper figures indicate time series of $v(t)$ [mV] and lower figures indicate orbit of (v, u), v-nullcline ($\dot{v} = 0$) and u-nullcline ($\dot{u} = 0$) given by black solid, red dotted and green dashed lines, respectively. ($a = 0.2, b = 2, c = -56, I = -99$)

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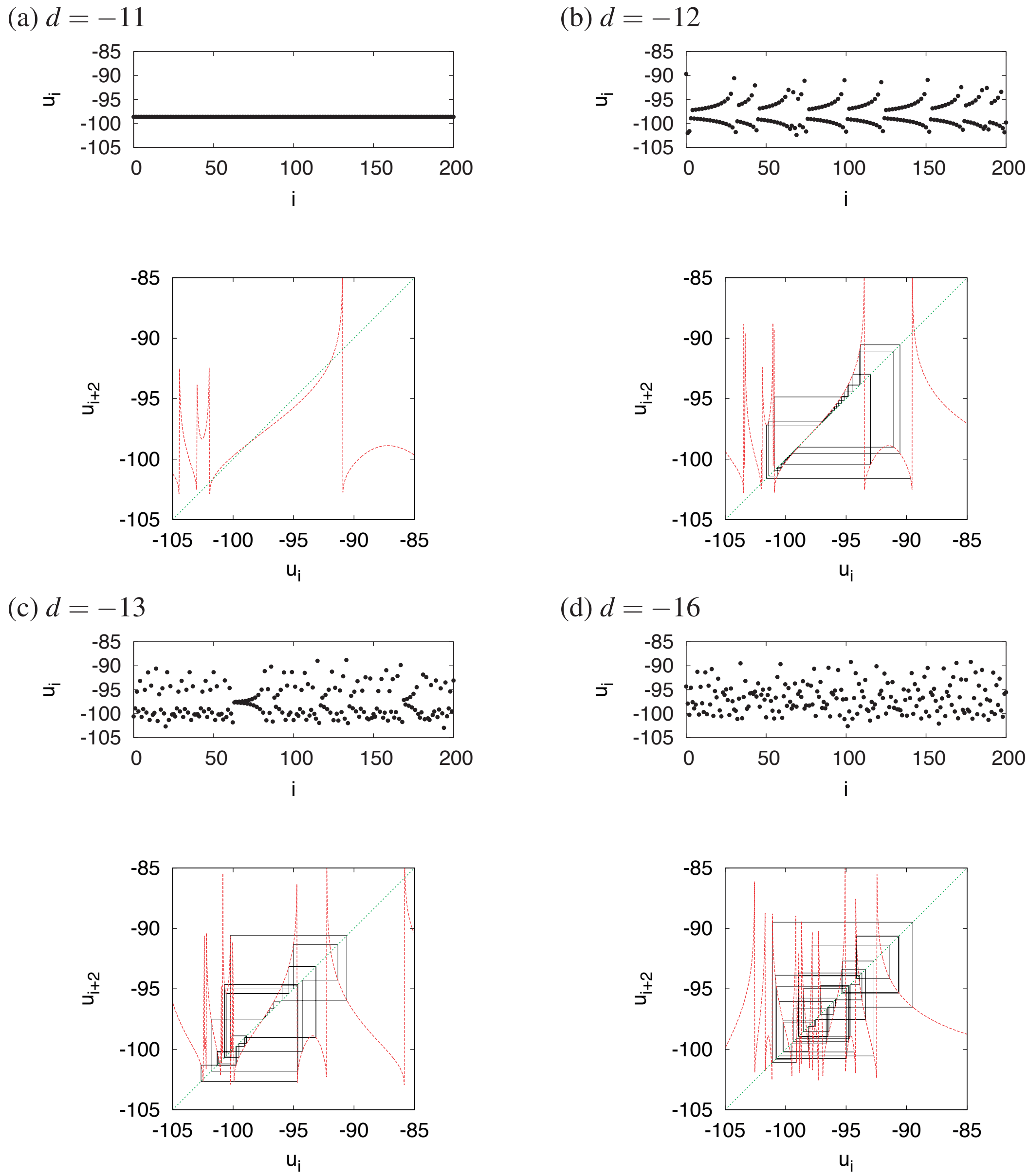

Figure 9. System behavior on Poincaré section. Time series of u_i (upper). Return map of u_i and u_{i+2} (lower), where black solid line is the orbit for u_i, red dotted line is Poincaré map $u_{i+2} = \phi^2(u_i)$, and green dashed line is $u_{i+2} = u_i$. The values of parameter d correspond to Fig. 8 from (a) to (d). $(a = 0.2, b = 2, c = -56, I = -99)$

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As a result, the period-doubling bifurcation is not observed in the parameter region around ($a = 0.2, b = 2, c = -56, d = -16, I = -99$) proposed by Izhikevich as the parameter set producing the chaotic behavior. Alternatively, it is confirmed that in this region the periodic state transits to chaos through the intermittency chaos, i.e., the intermittent route to chaos exists in this region.

4 Conclusion

We have examined the chaotic behaviors of Izhikevich neuron model from the viewpoint of the route to chaos rigorously by using the Lyapunov exponent with saltation matrix and Poincaré section methods in two representative parameter regions. The first is parameter region around the parameter sets for RS, IB and CH neurons. The second is the region around the parameter set for chaotic spiking proposed by Izhikevich. From these results, it was revealed that in the former region, the chaotic state appears through the period-doubling bifurcation route as the familiar route to chaos in the spiking neuron models, and this chaotic spiking pattern has the bursting characteristic. On the other hand, in the latter region the chaotic state is induced through the intermittent route to chaos, and this intermittency chaos is divided into laminar and turbulent modes.

Therefore, Izhikevich neuron model can produce various chaotic spiking patterns by tuning the parameters for resetting process. The transition between spike and hyperpolarization, which is described as the resetting process in Izhikevich neuron model, may play an important role of the function to generate various chaotic spiking patterns from several routes to chaos in the actual neural systems.

Further research based on this study would be to evaluate the chaotic system behaviors in the Izhikevich neuron assemblies jointed by electric synapses and chemical synapses. Besides, we would investigate the functionality of chaos for signal transmission and information processing such as chaotic resonance.

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