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Thermal fracture characteristics of an interface crack subjected to temperature variations

Saša M. Kalinović[1], Jelena M. Djoković[1], Ružica R. Nikolić[2], Branislav Hadzima[2]

[1] Technical Faculty of Bor, University of Belgrade, Vojske Jugoslavije 12, 19210 Bor, Serbia
[2] Research Center, University of Žilina, Univerzitna 8215/1, Žilina, Slovakia
Corresponding author e-mail: ruzicarnikolic@yahoo.com

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Abstract
Thermal fracture characteristics – the thermal energy release rate and thermal stress intensity factor of a semi-infinite crack at an interface between the two elastic isotropic materials, subjected to the temperature variations, are considered in this paper. Those characteristics are determined based on application of the linear elastic fracture mechanics (LEFM) concept. Expressions for obtained theoretical solutions are compared to solutions from literature and they are found to be more concise. Influence of the materials change on these two thermal fracture properties were observed, as well as the influence of the thickness ratio of the two layers constituting the interface.

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1. Introduction

Destruction of thin films and protective coatings, as well as peeling off of layers in layered materials, are interface phenomena. They are being paid increasing attention in practice, especially in the aviation and automotive industries. If a layered sample, made of different materials is subjected to the temperature variation, the thermal stresses would appear. They are the consequence of difference in the materials' thermal expansion coefficients and they can initiate a crack at the interface between the layers. Once such a crack is initiated, its propagation would depend on elastic and thermal properties of materials the layers are made of, as well as on the temperature change(s). The driving force for the interface fracture, in this case, is the thermal energy release rate. The heat is across the interface without a crack being transferred by conduction, while the interface can block the heat transfer or diminish it due to the heat transfer by convection.

The general case of the two-layered sample was first analyzed by Suo and Hutchinson, 1990. That solution can be used for understanding behavior of an interface crack between the two materials in conditions of the environment temperature gradient (variation). Behavior of the crack at the interface between the two materials was studied by numerous researchers. Choi, Hutchinson and Evans, 1999, have analyzed mechanisms necessary for obtaining the energy release rate that is sufficient to initiate delamination of coatings. Hutchinson and Evans, 2002, have studied susceptibility to delamination of the so-called thermo-insulating coatings (or thermal barrier coatings - TBC) subjected to the elevated temperatures. They analyzed the three possible causes of internal delamination: "*In all cases, the thermo-mechanical properties of the TBC are allowed to vary because of sintering. (a) One mechanism relates to exfoliation of an internal separation in the TBC due to a through thickness heat flux. (b) Another is concerned with edge-related delamination within a thermal gradient. (c) The third is a consequence of sintering-induced stresses. The results of these analyses, when used in combination with available properties for the TBC, strongly suggest that the second mechanism (b) predominates in all reasonable scenarios*". Itou, 2004, has analyzed the thermal stresses around the crack at the interface between the two elastic half-planes, assuming that the crack surfaces are insulated. Evans and Hutchinson, 2007, have conducted a thorough analysis of coatings' delamination by defining the relationship between the energy release rate and the mixed mode of crack propagation. They defined criteria for delamination that depend on the loading mode. Result of their work is a set of delamination maps for various types of coatings. Xue, Evans and Hutchinson, 2009, have studied delamination of coatings, which was initiated by a small crack parallel to the free surface, in conditions of the intensive heat exchange and temperature variation over the sample's thickness. Djokovic, Nikolic and Tadic, 2010, were calculating the energy release rate necessary for propagation of a crack at a two-layer interface in the case when the sample is cooled down from the joining to the room temperature. They also presented the stress distribution in terms of the temperature variation and the layer's thickness. Djoković, Nikolić and Živković, 2014, have presented the theoretical fundamentals for understanding the interface fracture in the two-layer bimaterial sample in conditions when the outside temperatures of the two layers are different. Ding, Zhou and Li, 2014, have studied influence of the materials' inhomogeneity and dimensionless heat resistance parameters on the thermal stress intensity factors in order to better understand the thermal behavior of the layered materials. Hasebe and Kato, 2014, have analyzed behavior of an interface between the two materials, exposed to the elevated temperature. They used the complex variables method and mapping of the rational function to obtain the relationship between temperature and stresses. They have defined the stress intensity of debonding (SID) and have determined its values for various crack lengths. They also analyzed influence of the joined materials on the SID. Nairn, 2019, has modelled the stress conduction across the interface using the material point method. The model of the heat conduction across the interface was obtained by interpolation of the temperature field.

The objective of this work was to determine the thermal fracture characteristics – the energy release rate needed for the crack propagation along the interface, as well as the corresponding stress intensity factor(s). The crack is at the interface between the two layers made of elastic isotropic materials. There were no restrictions imposed on the crack surfaces and there were no external loads applied. All the stresses, which appeared in the sample, were consequences of the thermal loading only, i.e. the temperature variations. The solution was sought based on the concept of the linear elastic fracture mechanics (LEFM), whose assumptions are as follows: the material is isotropic and linear elastic; the small scale yielding is present; the stress field near the crack tip is calculated using the theory of elasticity; the crack would propagate if the stresses near the crack tip exceed the material fracture toughness.

2. LEFM concept approach to problem solving

Problem of the semi-infinite crack at the interface between the two elastic isotropic layers, in conditions of the temperature variation, is presented in Fig. 1. The geometry is completely defined by the layers' thicknesses h_1 and h_2. Variables E_1 and E_2 represent the Young elasticity moduli of layers 1 and 2, μ_1 and μ_2 are their Poisson's ratios and α_1 and α_2 are their thermal expansion coefficients, respectively. It is assumed that the thermal expansion coefficients do not depend on temperature.

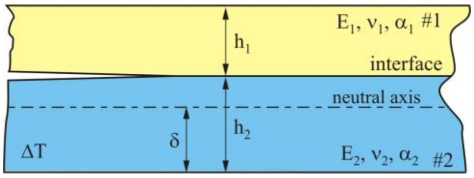

Fig. 1. Semi-infinite crack at the interface between the two elastic isotropic layers in conditions of the temperature variations.

Based on analysis of Suo and Hutchinson, 1990, the two-layered sample can be considered, far ahead of the crack tip, as a complex beam. The neutral axis lies at a distance $\delta = \Delta h$ from the bottom of the layer 2, (Figure 2) where:

$$\Delta = \frac{1 + 2\Sigma\eta + \Sigma\eta^2}{2\eta(1 + \Sigma\eta)} \tag{1}$$

where: $\eta = h_1 / h_2$ and $\Sigma = \bar{E}_1 / \bar{E}_2$, with $\bar{E}_1 = E_1 / (1 - \nu_1^2)$ and $\bar{E}_2 = E_2 / (1 - \nu_2^2)$ being valid for the plane strain state.

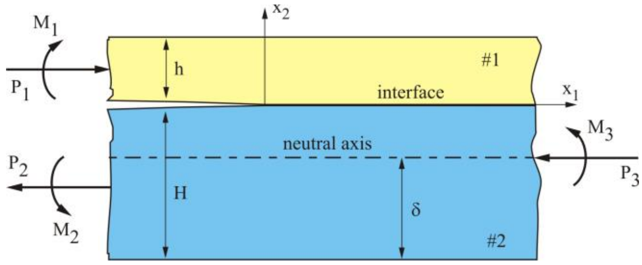

Fig. 2. Semi-infinite crack between the two layers loaded at three edges, Suo and Hutchinson, 1990.

The energy release rate can be calculated, within the plane strain state concept, as a difference between energy in the material far ahead and far behind the crack tip. The result is a positive quadratic form of *P* and *M*, which can be written as:

$$G = \frac{1}{2\bar{E}_1} \cdot \left[\frac{P^2}{Uh_1} + \frac{M^2}{Vh_1^3} + 2\frac{PM}{h_1^2\sqrt{UV}} \cdot \sin\gamma \right] \tag{2}$$

where *P* and *M* are the linear combinations of the applied loads (forces and moments), determined by Djokovic, Nikolic and Tadic, 2010:

$$\begin{aligned} P &= \frac{\eta(1+\eta^3)}{(1+\eta)^5} \bar{E}_1 (\alpha_2 - \alpha_1) \cdot \Delta T \cdot (h_1 + h_2) \\ M &= \frac{\eta^3}{2(1+\eta^3)} P (h_1 + h_2) \end{aligned} \tag{3}$$

while the geometric factors are determined as:

$$A = \frac{1}{\eta} + \Sigma \, ,$$

$$I = \Sigma \left[\left(\Delta - \frac{1}{\eta} \right)^2 - \left(\Delta - \frac{1}{\eta} \right) + \frac{1}{3} \right] + \frac{\Delta}{\eta} \left(\Delta - \frac{1}{\eta} \right) + \frac{1}{3\eta^3}$$

$$C_1 = \frac{\Sigma}{A}, \quad C_2 = \frac{\Sigma}{I}\left(\frac{1}{\eta} + \frac{1}{2} - \Delta\right), \quad C_3 = \frac{\Sigma}{12I} \tag{4}$$

$$U = \frac{1}{1+\Sigma(4\eta + 6\eta^2 + 3\eta^3)}, V = \frac{1}{12(1+\Sigma\eta^3)}$$

$$\gamma = \arcsin(6\Sigma\eta^2(1+\eta)\sqrt{UV}).$$

In this case there are no external loads applied to the two-layered sample and the stresses are results of the temperature gradient, i.e., there is the thermal loading, only (Figure 1).

By substituting equations (3) and (4) into equation (2), expression for the thermal energy release rate is obtained as:

$$G = \frac{(1+\Sigma\eta^3)\bar{E}_1 h_1(\alpha_1 - \alpha_2)^2(\Delta T)^2}{2[1+2\eta\Sigma(2+3\eta+2\eta^2)+\Sigma^2\eta^4]}. \tag{5}$$

The energy release rate determines the intensity of the singularity in the vicinity of the crack tip, but it does not determine the mixed mode of the crack propagation. That can be determined based on the complex stress intensity factor *K*, which, in accordance with the linearity of the problem and the dimensional analysis, can be written as:

$$K = K_1 + iK_2 = h_1^{-i\varepsilon}\sqrt{\frac{1-\alpha}{1-\beta^2}} \cdot \left(\frac{P}{\sqrt{2Uh_1}} - ie^{i\gamma}\frac{M}{\sqrt{2Vh_1^3}}\right)e^{i\omega} \tag{6}$$

where α and β are the two Dundurs' parameters, (Dundurs, 1969):

$$\alpha = \frac{\mu_2(\kappa_1+1) - \mu_1(\kappa_2+1)}{\mu_2(\kappa_1+1) + \mu_1(\kappa_2+1)}, \ \beta = \frac{\mu_2(\kappa_1-1) - \mu_1(\kappa_2-1)}{\mu_2(\kappa_1+1) + \mu_1(\kappa_2+1)} \tag{7}$$

where: μ_i – is the shear modulus and $\kappa_i = 3-4\nu_i$ is valid for the plane strain state, while $\kappa_i = (3-\nu_i)/(1+\nu_i)$ is valid for the plane stress state.

Angle $\omega \equiv \omega(\alpha,\beta,\eta)$ is a function of the Dundurs' parameters α and β and the ratio of the two layers' thicknesses η. This function is defined by Veljković and Nikolić, 2003, based on solving the elastic problem and processing of the tabular results in Suo and Hutchinson, 1990:

$$\omega = \frac{1-\eta}{1+\eta} \cdot \sqrt{\frac{\beta(1-\alpha)}{\alpha - \beta^2}} \tag{8}$$

Parameter ε is called the bi-elastic constant or the oscillatory index. It is a characteristics of the interface crack and is determined according to expression, Rice, 1988:

$$\varepsilon = \frac{1}{2\pi}\ln\left(\frac{1-\beta}{1+\beta}\right) \tag{9}$$

If h_1 is taken as the reference length for the considered problem, the real and imaginary parts of the complex stress intensity factor *K*, based on equation (6), can be written as:

$$K_1 = \mathrm{Re}(Kh_1^{i\varepsilon}) = \sqrt{\frac{1-\alpha}{1-\beta^2}} \cdot \left[\frac{P}{\sqrt{2Uh_1}}\cos\omega + \frac{M}{\sqrt{2Vh_1^3}}\sin(\omega+\gamma)\right] \tag{10a}$$

$$K_2 = \mathrm{Im}(Kh_1^{i\varepsilon}) = \sqrt{\frac{1-\alpha}{1-\beta^2}} \cdot \left[\frac{P}{\sqrt{2Uh_1}}\sin\omega - \frac{M}{\sqrt{2Vh_1^3}}\cos(\omega+\gamma)\right] \tag{10b}$$

By substituting equations (3) and (4) into (9), expressions for the thermal stress intensity factors are obtained as:

$$K_1 = \frac{\cosh(\pi\varepsilon)\bar{E}_1\sqrt{h_1}(\alpha_1-\alpha_2)\Delta T}{\sqrt{(1+\Sigma)(1+4\Sigma\eta+6\Sigma\eta^2+3\Sigma\eta^3)(1+\tan^2\omega)}} \cdot \left[\frac{\sqrt{3}\Sigma\eta^2(1+\eta)\tan\omega}{\sqrt{1+2\eta\Sigma(2+3\eta+2\eta^2)+\Sigma^2\eta^4}} - 1\right] \tag{11}$$

$$K_2 = \frac{\cosh(\pi\varepsilon)\bar{E}_1\sqrt{h_1}(\alpha_1-\alpha_2)\Delta T}{\sqrt{(1+\Sigma)(1+4\Sigma\eta+6\Sigma\eta^2+3\Sigma\eta^3)(1+\tan^2\omega)}} \cdot \left[\frac{\sqrt{3}\Sigma\eta^2(1+\eta)}{\sqrt{1+2\eta\Sigma(2+3\eta+2\eta^2)+\Sigma^2\eta^4}} + \tan\omega\right] \tag{12}$$

3. Results and discussion

Based on expressions (5), (11) and (12) one can calculate the thermal energy release rate and the thermal stress intensity factors. The following data are taken into account in calculations: $E_1 = 0.7\cdot10^5$ N/mm², $E_2 = 2.1\cdot10^5$ N/mm², $\nu_1 = \nu_2 = 0.3$, $\Delta T = 20°$. The energy release rate is normalized by $\bar{E}_1 h_1(\alpha_1-\alpha_2)^2(\Delta T)^2$, while the stress intensity factors are normalized by $\bar{E}_1\sqrt{h_1}(\alpha_1-\alpha_2)\Delta T$. The programming package *Mathematica*® was used for obtaining diagrams presented in Figs. 3 to 6.

In Fig. 3 is shown variation of the normalized energy release rate in terms of the two layers thicknesses ratio η, for different materials combinations, expressed via the ratio of the Young's elasticity moduli, Σ.

From Fig. 3 can be seen that the normalized thermal energy release rate has a tendency to decrease with increase of the layers thicknesses ratio and with increase of difference in the two materials elasticity moduli.

In Figure 4 is presented the normalized thermal energy release rate in terms of the materials combinations expressed via the Dundurs parameter α, for different layers' thicknesses ratio η.

From Figure 4 can be noticed that variation of the parameter α does not exhibit strong influence on the thermal energy release rate in the case of the very big difference in thicknesses of layers.

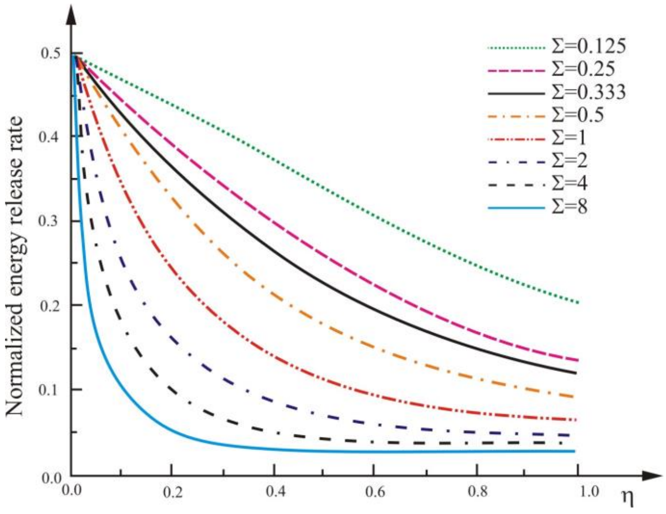

Fig. 3. Variation of the normalized thermal energy release rate in terms of the layers' thicknesses ratio for various materials' combinations

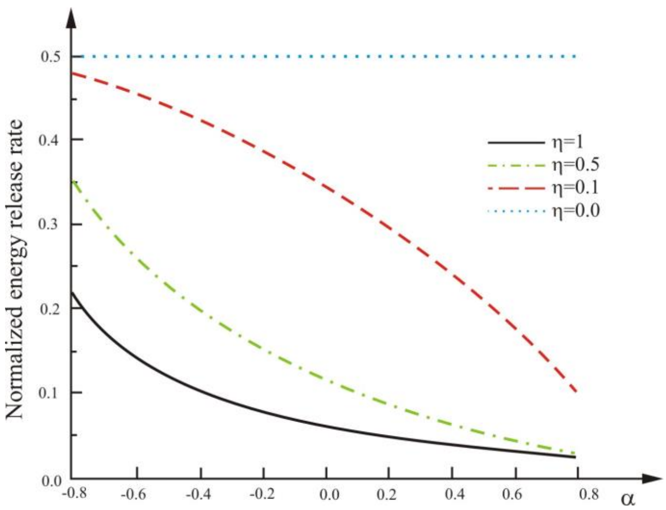

Fig. 4. Variation of the normalized thermal energy release rate in terms of the Dundurs parameter α for different thicknesses of layers.

In Fig. 5 is shown variation of the normalized thermal stress intensity factor for the Mode 1 of the crack propagation in terms of the two layers thicknesses ratio η, for different materials' combinations, expressed via the ratio of the Young's elasticity moduli, Σ.

From Fig. 5 can be seen that the normalized thermal stress intensity factor for the Mode 1 crack propagation has a tendency to increase with increase of the layers thicknesses ratio and with increase of difference in the two materials elasticity moduli.

In Fig. 6 is shown variation of the normalized thermal stress intensity factor for the Mode 2 crack propagation in terms of the two layers thicknesses ratio η and for different materials combinations, expressed via the ratio of the Young's elasticity moduli, Σ.

From Fig. 6 can be seen that the normalized thermal stress intensity factor for the Mode 2 crack propagation has a tendency to decrease with increase of the layers thicknesses ratio

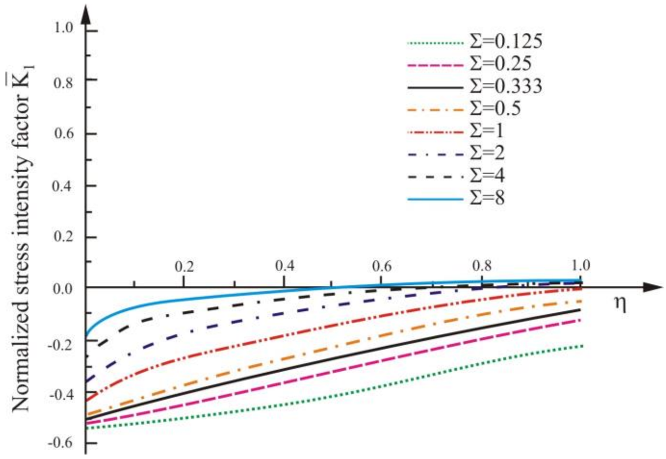

Fig. 5. Variation of the normalized thermal stress intensity factor for the Mode 1 crack propagation in terms of the layers' thicknesses ratio for various materials combinations

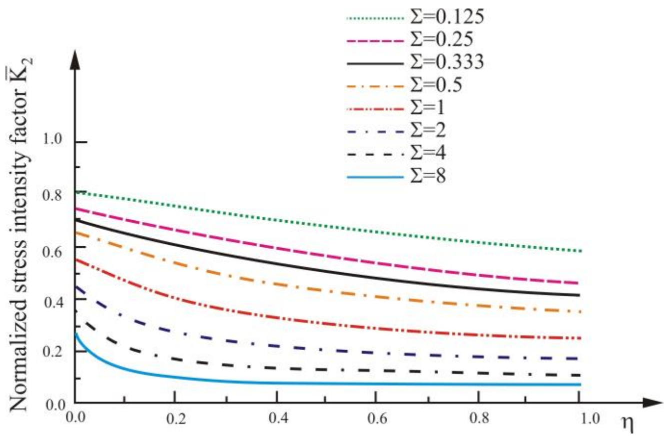

Fig. 6. Variation of the normalized thermal stress intensity factor for the Mode 2 crack propagation in terms of the layers' thicknesses ratio for various materials combinations

and with increase of difference in the two materials elasticity moduli.

In addition, from Fig. 5 can be seen that the normalized thermal stress intensity factor for the Mode 1 crack propagation has negative values, what would be favorable. However, the real sign is being determined based on the sign of the variable that was used for normalizing the stress intensity factor, namely $\bar{E}_1\sqrt{h_1}(\alpha_1-\alpha_2)\Delta T$. If both ($\alpha_1 - \alpha_2$) and ΔT were positive (or both negative), the real sign of the normalized stress intensity factor for the Mode 1 would be negative, in the opposite case (when those two variables are of different signs) it would be positive. If one assumes that the interfacial crack surfaces are without restrictions and if the thermal expansion coefficient of material 1 is greater than the same variable of material 2, the temperature increase ($\Delta T > 0$) would be favorable, while the temperature decrease ($\Delta T < 0$) would be harmful, since in the former case the stress intensity factor would be negative and in the latter case it would be positive.

For the sake of comparison with results from literature, the two layered sample with following characteristics $\alpha = 0.4$, $\beta = 0.15$ and $\eta = 0.5$ was considered. Based on equation (5) one obtains: $G / \bar{E}_1 h_1 (\alpha_1 - \alpha_2)^2 (\Delta T)^2 = 0.0605$. Based on equations (11) and (12) one obtains:

$$K_1 / \bar{E}_1 \sqrt{h_1} (\alpha_1 - \alpha_2) \Delta T = -0.0591$$

and

$$K_2 / \bar{E}_1 \sqrt{h_1} (\alpha_1 - \alpha_2) \Delta T = 0.1834$$

For the same values of parameters α, β and η, Suo and Hutchinson, 1990, have obtained the following values:

$$K_1 / \bar{E}_1 \sqrt{h_1} (\alpha_1 - \alpha_2) \Delta T = -0.0593$$

and

$$K_2 / \bar{E}_1 \sqrt{h_1} (\alpha_1 - \alpha_2) \Delta T = 0.1835$$

while the value for the normalized thermal energy release rate was identical, 0.0605.

For the case of $\alpha = 0.4$, $\beta = 0.15$ and $\eta = 1.0$, values of the thermal stress intensity factors, obtained by equations (10) and (11) were $\bar{K}_1 = 0.0216$ and $\bar{K}_2 = 0.1604$, while Suo and Hutchinson, 1990, have obtained the following values: $\bar{K}_1 = 0.0216$ and $\bar{K}_2 = 0.1603$.

As can be seen, the agreement of values is excellent for both examples.

4. Conclusions

A problem of a semi-infinite crack at the interface between the two elastic isotropic materials, subjected to the temperature variation, is considered in this paper. Expressions for the thermal fracture characteristics (thermal energy release rate and stress intensity factors) were determined, based on the linear elastic fracture mechanics concept (LEFM). There were no restrictions imposed on the interface crack surfaces and no external load was applied. The stresses that appeared in the two-layered sample were results of the temperature gradient and difference in the thermal expansion coefficients of the two materials, only. The driving force for the crack propagation is the thermal energy release rate.

Solutions obtained for expressions for the normalized thermal energy release rate and the normalized thermal stress intensity factors are more concise than expressions for the same variables that were earlier obtained by different authors.

Results obtained for the normalized thermal energy release rate and stress intensity factors for the two illustrative examples of the two-layered sample excellently agree with results obtained by Suo and Hutchinson, 1990.

Based on equations (5), (10) and (11), one can calculate the thermal characteristics of the interface crack subjected to temperature variations, simply and efficiently, for the given geometrical parameters, specified material combination and set temperature change. Diagrams obtained based on those equations present the review of behavior of the thermal stress intensity factors and thermal energy release rate, which can be useful in optimization of combination of layers' materials and thicknesses in order to decrease the stress concentration.

Note: *The shorter version of this research was presented at The 25th International Seminar of PhD Students SEMDOK2020 in Zuberec, Slovakia, reference: Kalinović, Djoković and Nikolić, 2020.*

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温度变化引起的界面裂纹的热断裂特性

關鍵詞
界面裂纹
两层样品
温度变化
断裂特性
LEFM 概念

摘要
本文考虑了热断裂特性－两种弹性各向同性材料之间界面处的半无限大裂纹的热能释放速率和热应力强度因子，其温度随温度的变化而变化。 这些特性是根据线性弹性断裂力学（LEFM）概念的应用确定的。 将获得的理论解的表达式与文献中的解进行比较，发现它们更为简洁。 观察到材料变化对这两个热断裂性能的影响，以及构成界面的两层的厚度比的影响。