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NEW EVENT BASED H_∞ STATE ESTIMATION FOR DISCRETE-TIME RECURRENT DELAYED SEMI-MARKOV JUMP NEURAL NETWORKS VIA A NOVEL SUMMATION INEQUALITY

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Abstract

This paper investigates the event-based state estimation for discrete-time recurrent delayed semi-Markovian neural networks. An event-triggering protocol is introduced to find measurement output with a specific triggering condition so as to lower the burden of the data communication. A novel summation inequality is established for the existence of asymptotic stability of the estimation error system. The problem addressed here is to construct an H_∞ state estimation that guarantees the asymptotic stability with the novel summation inequality, characterized by event-triggered transmission. By the Lyapunov functional technique, the explicit expressions for the gain are established. Finally, two examples are exploited numerically to illustrate the usefulness of the new methodology.

Keywords: Discrete-time neural networks, Mixed time delays, asymptotic stability, event-triggered control.

1 Introduction

Recurrent neural network (RNN) models have procured their heed over a few decades in both theory and applications in many practical areas including automatic control, image processing, combinatorial optimization, fault diagnosis, associative memory, etc [1, 2, 3, 4, 5, 6]. There has been good attention towards research from a variety of communities on the inherent features of the RNNs like the stability, synchronization, attractivity issues, analysis problems and their dynamical behaviour based on the mathematical properties which excel in approximating, clustering and learning the different concepts of RNNs [7, 8, 9]. Moreover, the rapid development of digital technology owing to the engineering significance has stirred much attention on discrete-time NNs [10, 11, 12, 13, 14, 15, 16] over the conventional continuous-time ones.

Time delays have become a universal observable fact often seen in a variety of fields resulting in divergence or oscillation or instability of the network system. Moreover, the hardware implementation of NN's gives rise to the axonal transmission delays and multitude parallel pathways. Hence, it becomes indispensable to construct a realistic NN model which includes both discrete and distributed delays. Also, in some NN's time delay exists in a stochastic fashion characterized by certain probabilistic distributions such as normal distribution and Binomial distribution [17, 18, 19].

On another research front, one of the most recurring topis emerging in the RNNs is the state estimation problem of RNNs. The most challenging task in solving a state estimation problem, is the fact that a typical RNN comprises the complexity of NNs with a large number of interconnected network nodes characterized by strong nonlinearities, high couplings reflecting the topological properties and signal transmission over the links causing time-delays. Therefore, it becomes often crucial to acquire partial information about the network state of the neuron. The state estimation algorithms, therefore become significant in both theory and in practice. It is worth noting that the event-triggered communication protocol has attracted the control community due to the uniqueness in threshold trigger and the time-triggered scheme in network bandwidth suffers from excessive consumption of limited resources due to the unnecessary signal transmissions. However, larger thresholds correspond to slower changing rate wherein smaller thresholds require a faster changing rate. Up to now, there is a wealth of literature that has focused on designing the state estimators on time delayed RNNs, see for example, [20, 21, 22, 23, 24, 25, 26, 27, 28].

Execution of RNN involves information latching which tends to become more severe when there is an increase in length of the temporal sequence, resulting the RNNs switching from time to time between finite modes or patterns which in turn reveals that the jump linear systems depend upon the sojourn-time h and has received much research attention [29, 30]. It is to be noted that some of the engineering practices of control or filtering, due to the lack of entries in the transition matrix, may not be fully accessible, leading to deficient transition probabilities. In a more general situation, jump linear system becomes the semi-Markovian jump system [31, 32, 33] wherein the transition rates are time-varying rather than being constant as in Markovian jump systems. Therefore, the semi-Markov system has a lot of advantages to that of Markovian jump systems as it has varied rates of transition on time than constants due to the relaxed distributions in probability.

On the other hand, analysis based on the systems' stability with regard to delays has become the hot research topic in the field of control theory for the past few decades. As it is well known, the Jensen inequality technique is an appropriate tool to analyze the stability in terms of LMI's by tractable derivations [34, 35]. However, the Jensen inequality introduces some unavoidable and undesirable conservatism in the stability conditions. One of the most challenging problems is to estimate the lower bound or to obtain a tighter bound of the summation term which helps in reducing the conservatism. On this basis, a novel summation inequality is established from the extended Jensen's inequality.

On account of the above discussions, we are in a practical need to consider the asymptotic stability RNNs based on H_∞ state estimation with event triggering scheme and mixed delays subject to discrete summation inequality.

The organization of the paper is as follows. Formulation of problem is in Section 2, followed by the results given in Section 3, where lemmas and summation inequalities along with the method to calculate the filter parameters are presented. Section 4 provides the efficacy of the derived results through numerical simulations, and Section 5 gives the conclusion.

2 Problem Formulation

We consider the sample space Ω with $(\Omega, \mathcal{F}, \mathcal{P})$ being the fixed space in the probability with its measure being $\mathcal{P}$ on $\mathcal{F}$. Letting the semi-Markov $\{r(k), k \geq 0\}$ to be the state in the discrete-time with a finite set $\mathbb{S} = \{1, 2, \ldots, M\}$, the process describing the evolution of $r(k)$ is governed as follows:

$$\Pr\{ \; r(k+h) = j | r(k) = l\} = \begin{cases} \gamma_{lj}(h)h + o(h) & l \neq j \\ 1 + \gamma_{lj}(h)h + o(h) & l = j \end{cases} \tag{1}$$

where $o(h)$ is $\lim_{h\to 0}(o(h)/h) = 0$ and $\gamma_{lj}(h) \geq 0$, for $l \neq j$, is the transition rate from mode l at time

k to mode j at time $k+h$ and

$$\gamma_{ll}(h) = -\sum_{j\in\mathbb{S}, j\neq l} \gamma_{lj}(h) \quad (2)$$

The discrete-time stochastic semi-Markov jump NNs with mixed time delays:

$$\begin{cases} x(k+1) &= A(r(k))x(k) + A_d(r(k))f(x(k)) \\ &+ B(r(k))Mg_{\tau_k}(x(k)) + C(r(k)) \\ &\times \sum_{s=k-d}^{k-1} h(x(s)) + J(k) + Dw(k) \\ z(k) &= Lx(k), \\ x(s) &= \psi(s), \quad s\in[-\max\{\tau_M, d\}, 0] \end{cases} \quad (3)$$

where $x(k) = \mathrm{vec}_n\{x_l(k)\} \in \mathbb{R}^n$ is the l-th neuron at k th instant time vector of the NN; $f(x(k)) = \mathrm{vec}_n\{f_l(x_l(k)\}, g(x(k)) = \mathrm{vec}_n\{g_l(x_l(k))\}, h(x(k)) = \mathrm{vec}_n\{h_l x_l(k)\}$, $g_{\tau_k}(x(k)) = \mathrm{vec}_n\{g(k-\tau_l(k))\} \in \mathbb{R}^n$ are the neuron activation functions. The state $z(k) \in \mathbb{R}$ with $J(k) = \mathrm{vec}_n\{J_l(x_l(k))\}$ being the exogenous input on the space $(\Omega, \mathcal{F}, \mathcal{P})$ with $\sigma^2 = \mathbb{E}\{\omega^2\} = 1$. $A(r(k)) = \mathrm{diag}_n\{a_l\}$ is the state feedback coefficient, $A_d = (a_{d_{lj}})_{n\times n}$ is the potential neuron with its connection weight, discrete delay and distributively delayed connection weights given respectively as $A_d(r(k)) = (a_{d_{lj}}r(k))_{n\times n}, B(r(k)) = (b_{lj}r(k))_{n\times n}$ and $C(r(k)) = (c_{lj}r(k))_{n\times n}$; $J(k) = \mathrm{vec}_n\{J_l(k)\}, L = (\mathfrak{l}_{lj})_{r\times n}$ is the known scalar, $D = \mathrm{vec}_n\{d_l\}$; $M = \mathrm{vec}_n^T\{M_l^T\}$ with $M_l = \mathrm{diag}\{\underbrace{0\ \ldots 0}_{l-1}\ 1\ \underbrace{0\ \ldots 0}_{n-l}\}$, Let d denotes the constant distributed time-delay and the positive integer $\tau_j(k)$ denotes the time-varying delay which satisfies $\tau_m \leq \tau_j(k) \leq \tau_M$ and $\tau_j(0) = \tau_M$ is been assumed and $\psi_l(s)$ is the given initial condition sequence. The following assumption is needed, throughout this paper.

Assumption 1. For every $1 \leq i \leq m$,, the activation function for stochastic NN model 3, satisfies the following conditions:

$$\delta_i^- \leq \frac{f_i(m_1) - f_i(m_2)}{m_1 - m_2} \leq \delta_i^+, \quad (4)$$

$$\beta_i^- \leq \frac{g_i(m_1) - g_i(m_2)}{m_1 - m_2} \leq \beta_i^+, \quad (5)$$

$$\gamma_i^- \leq \frac{h_i(m_1) - h_i(m_2)}{m_1 - m_2} \leq \gamma_i^+, \quad (6)$$

with $m_1, m_2 \in \mathbb{R}$ and $\delta_i^-, \delta_i^+, \beta_i^-, \beta_i^+, \gamma_i^-, \gamma_i^+$ are some constants.

In this paper, the measurement output signal is given as

$$y(k) \triangleq Ex(k) + Fv(k) \quad (7)$$

with the output measurement $y(k) = \mathrm{vec}_m^T\{y_l(k)\} \in \mathbb{R}^m$, the l-th entry is $y_l(k)$ and the bounded disturbance is $v(k) \in \mathbb{R}^p$ with its constraint $||v(k)||^2 \leq \bar{v}$ provided with constant matrices, $E \in \mathbb{R}^{m\times n}$ and $F \in \mathbb{R}^{m\times p}$.

2.1 The strategy of Event-Triggering

Main motive of this paper is to estimate the state of a neuron (2), by the output measurement $y(k)$ in (7). One should adhere to the strategy of event triggering, for resource-saving purpose so that the output measurement is released at different time instants to the state estimator.

Here, the mechanism of event-triggering is introduced such that the event instant series to that of the current l-th measurement component is $t_0^l = 0 < t_1^l < t_2^l < \cdots < t_l^l < \ldots$, with the latest triggering time k_p with its current sampling instant $k \in [t_{p_l}, t_{p+1})$, where for every increasing monotonic sequence t_s with $s=0$ to $s=\infty$, the event generator function is defined as $\phi_l : \mathbb{R}^3 \to \mathbb{R}, (l = 1, 2, \ldots, n)$ with

$$\phi_l(y_l(k), y_l(k_p^l)), \theta_l) \triangleq ||y_l(k) - y_l(k_p^l)||^2 - \theta_l \quad (8)$$

Based on this scheme, the data measurement of the l-th entry from the measurement device released to the estimator satisfies the condition,

$$\phi_l(y_l(k), y_l(k_p^l)) > 0 \quad (9)$$

where θ_l is the threshold which adjusts itself and decides its rate of triggering based on the required practicals. Hence, measurement output of the l-th component in the new triggering instant can be iterated as

$$t_{p+1}^l = \min\{k | k > t_p^l, \quad \phi_l(y_l(k), y_l(k_p^l), \theta_l) > 0\} \quad (10)$$

2.2 State Estimation

Let the triggering instant of the output measurement be denoted by

$$y(k_p) \triangleq [y_1(k_p^1)\ \ y_2(k_p^2)\ \ \ldots\ \ y_m(k_p^m)]^T \tag{11}$$

where k_p^i for $(i = 1,\ldots,m)$ is the measurement component with respect to the triggering instant.

Construction of the event based neuron state estimator is given by (3):

$$\begin{cases} \hat{x}(k+1) &= A_l\hat{x}(k) + A_{dl}f(\hat{x}(k)) + B_iMg_{\tau_k}(\hat{x}(k)) \\ &+\ C_l\sum_{s=k-d}^{k-1} h(\hat{x}(s)) + J(k) \\ &+\ K[y(k_p) - E\hat{x}(k)] \\ \hat{z}(k) &= L\hat{x}(k), \\ \hat{x}(s) &= \psi(s), \quad s \in [-\max\{\tau_M, d\}, 0] \end{cases} \tag{12}$$

where the estimation of $x(k)$ is $\hat{x}(k)$ and $z(k)$ is $\hat{z}(k)$ and the gain estimator is K which is to be determined.
Let us define $\varpi(k) \triangleq y(k_p) - y(k)$ which is given by (12) and is rewritten as

$$\begin{cases} \hat{x}(k+1) &= A_l\hat{x}(k) + A_{dl}f(\hat{x}(k)) + B_lMg_{\tau_k}(\hat{x}(k)) \\ &+\ C_l\sum_{s=k-d}^{k-1} h(\hat{x}(s)) + J(k) \\ &+\ K[\varpi(k) + y(k) - E\hat{x}(k)] \\ \hat{z}(k) &= L\hat{x}(k), \\ \hat{x}(s) &= 0, \quad s \in [-\max\{\tau_M, d\}, 0] \end{cases} \tag{13}$$

Moreover, noting $\tilde{x}(k+1) = x(k) - \hat{x}(k)$ and the error dynamics is $\tilde{z}(k+1) = z(k) - \hat{z}(k)$ given in (3) and (13)

$$\begin{cases} \tilde{x}(k+1) &= \tilde{A}_l\tilde{x}(k) + A_{dl}\tilde{f}(\tilde{x}(k)) + B_lM\tilde{g}_{\tau_k}(\tilde{x}(k)) \\ &+\ C_l\sum_{s=k-d}^{k-1} \tilde{h}(\tilde{x}(s)) \\ &-\ K\varpi(k) - KFv(k) + Dw(k) \\ \tilde{z}(k) &= L\tilde{x}(k), \\ \tilde{x}(s) &= \psi(s), \quad s \in [-\max\{\tau_M, d\}, 0] \end{cases} \tag{14}$$

where

$$\tilde{f}(\tilde{x}(k)) = f(x(k)) - f(\hat{x}(k))$$
$$\tilde{g}_{\tau_k}(\tilde{x}(k)) = \tilde{g}_{\tau_k}(x(k)) - \tilde{g}_{\tau_k}(\hat{x}(k))$$
$$\tilde{h}(\tilde{x}(k)) = h(x(k)) - h(\hat{x}(k)),\ \ \tilde{A} = A - KE.$$

Definition 1. The NN (14) is asymptotically and globally stable in the square mean with each of its solution $x(k)$ given by:

$$\lim_{k\to+\infty} \mathbb{E}\left\{||x(k)||^2\right\} = 0 \tag{15}$$

Lemma 1: For given $n\times n$ matrix $\mathcal{G} > 0$, with $\varsigma_1,\varsigma_2,\varsigma_3,\ldots\varsigma_n \in \mathbb{R}^n, I_R(\varsigma) = \sum_{k=0}^{n}\varsigma_k^T\mathcal{G}\varsigma_k$, the following inequality

$$I_R(\varsigma) \geq \frac{1}{n+1}\left(\sum_{k=0}^{n}\varsigma_k^T\right)\mathcal{G}\left(\sum_{k=0}^{n}\varsigma_k\right) + \frac{3}{n(n+1)(n+2)}\Lambda_0^T\mathcal{G}\Lambda_0 \tag{16}$$

where $\Lambda_0 = \sum_{k=0}^{n}(n-2k)\varsigma_k$.
Lemma 2: For a given $n\times n$ positive definite matrix $\mathcal{G} \geq 0$, and for all $u_0,u_1,u_2,\ldots u_n \in \mathbb{R}^n$, the following inequality holds;

$$\sum_{k=0}^{n}\Delta u_k^T\mathcal{G}\Delta u_k \geq \frac{1}{n+1}(u_{n+1}-u_0)^T\mathcal{G}(u_{n+1}-u_0) + \frac{3}{n+1}\Lambda_1^T\left(\frac{n+2}{n}\mathcal{G}\right)\Lambda_1 \tag{17}$$

where $\Delta u_k = u_{k+1} - u_k$ and $\Lambda_1 = u_{n+1} + u_0 - \frac{2}{n+2}\sum_{k=0}^{n+1}u_k$.
Lemma 3: For every discrete-time variable arranged in a sequence, there exists a given matrix $\mathcal{U} > 0$, with ξ in $[-\hbar, 0]\cap\mathbb{Z} \to \mathbb{R}^n$, such that

$$\sum_{i=-\hbar+1}^{0}\sum_{j=i}^{0}\hat{\xi}^T(k)\mathcal{U}\hat{\xi}(k) \geq \frac{2(\hbar+1)}{\hbar}\Upsilon_0^T\mathcal{U}\Upsilon_0 + \frac{4(\hbar+1)(\hbar+2)}{\hbar(\hbar-1)}\Upsilon_1^T\mathcal{U}\Upsilon_1 \tag{18}$$

where $\Upsilon_0 = x(0) - \frac{1}{\hbar+1}\sum_{i=-\hbar}^{0}x(i)$, $\Upsilon_1 = x(0) + \frac{2}{\hbar+1}\sum_{i=-\hbar}^{0}x(i) - \frac{6}{(\hbar+1)(\hbar+2)}\sum_{i=-\hbar}^{0}\sum_{k=i}^{0}x(k)$.

3 Main Results

So as to assure the stability analysis, in this section the summation inequalities with respect to discrete and distributed delays, have been demonstrated. For simplicity, the blocked matrices are $e_q \in R^{17n\times n}(q = 1,2,\ldots,17)$. The other notations of several matrices are defined as

$\tau_{12} \triangleq \tau_M - \tau_m, \eta(k) \triangleq \tilde{x}(k+1) - \tilde{x}(k),$
$U_m \triangleq \mathrm{diag}\{l_1^-, l_2^-, \dots l_n^-\}, U_p \triangleq \mathrm{diag}\{l_1^+, l_2^+, \dots l_n^+\}$
$\hat{e}_r \triangleq \tilde{A}_l e_1 + A_{dl} e_{11} + B_l M e_{12} + C_l e_{14} - K e_{16}$
$- KF e_{17}, \zeta(k) \triangleq \Big[\tilde{x}^T(k),\ \tilde{x}^T(k-\tau_m),\ \tilde{x}^T_{\tau_k}(k),$
$\tilde{x}^T(k-\tau_M),\ \frac{1}{\tau_m+1}\sum_{s=k-\tau_m}^{k}\tilde{x}^T(s),$
$\frac{1}{\tau_k-\tau_m+1}\sum_{s=k-\tau_k}^{k-\tau_m}\tilde{x}^T(s), \frac{1}{\tau_M-\tau_k+1}\sum_{s=k-\tau_M}^{k-\tau_k}\tilde{x}^T(s),$
$\sum_{s=k-\tau_m}^{k-1}\tilde{x}^T(s) \quad \sum_{i=k-\tau_m+1}^{k-1}\sum_{s=k-\tau_m}^{i-1}\tilde{x}^T(s),$
$\sum_{i=-\tau_m+1}^{0}\sum_{s=k+i}^{k}\tilde{x}^T(s),\ \tilde{f}^T(\tilde{x}(k)), \tilde{g}^T(\tilde{x}_{\tau_k}(k))),$
$\tilde{h}^T(\tilde{x}(k)),\ \sum_{j=k-\tau_m}^{k-1}\tilde{h}^T(\tilde{x}(k)),\ \eta^T(k),\ \varpi^T(k),$
$v^T(k)\Big]$
$\Theta_0 \triangleq [\tilde{A}_l, 0,0,0,0,0,0,0,0,0,0, A_{dl}, B_l M, 0, C_l,$
$-K, -KF]\Theta_1 \triangleq [\Theta_0^T, (\tau_m+1)e_5^T - e_2^T,$
$(\tau_k - \tau_m + 1)e_6^T + (\tau_M - \tau_k + 1)e_7^T - e_3^T - e_4^T]^T,$

$$\Theta_2 \triangleq [e_1^T, (\tau_m+1)e_5^T - e_1^T, (\tau_k-\tau_m+1)e_6^T + (\tau_M-\tau_k+1)e_7^T - e_3^T - e_2^T]^T,$$

$\Theta_3 \triangleq [\tilde{A}_l - I, 0,0,0,0,0,0,0,0,0,0, A_{dl}, B_l M,$
$0, C_l, -K, -KF], \Theta_4 \triangleq [e_1^T - e_2^T,\ e_1^T + e_2^T$
$-2e_5^T]^T, \Theta_5 \triangleq [e_2^T - e_3^T, e_2^T + e_3^T - 2e_6^T, e_3^T - e_4^T,$
$e_3^T + e_4^T - 2e_7^T]^T, \Theta_{71} \triangleq e_1 - \frac{1}{\tau_m+1}(e_8 + e_1),$
$\Theta_{72} \triangleq e_1 + \left(\frac{2}{\tau_m+1} - \frac{6}{(\tau_m+1)(\tau_m+2)}\right)$
$\times(e_1+e_8) - \frac{6}{(\tau_m+1)(\tau_m+2)}e_{10},$

$\tilde{\Pi}_1(\tau(k)) \triangleq \mathrm{diag}\left\{\Pi_1^1(\tau(k)), \dots, \Pi_1^n(\tau(k))\right\},$
$\Pi_1^l(\tau(k)) \triangleq \left\{\sum_{l=1}^{n}\left\{\frac{1}{2}\mathrm{sym}\{(\Theta_1+\Theta_2)^T\right.\right.$
$\left.\left.P^l(\Theta_1+\Theta_2)\}\right\}\right\}, \tilde{\Pi}_2 \triangleq \mathrm{diag}\left\{\Pi_2^1, \dots, \Pi_2^n\right\},$
$\Pi_2 \triangleq \left\{\sum_{l=1}^{n}\left\{e_1^T Q_1^l e_1 - e_2^T Q_1^l e_2 + e_2^T Q_2^l e_2\right.\right.$
$\left.\left.-e_4^T Q_2^l e_4\right\}\right\}, \tilde{\Pi}_3 \triangleq \mathrm{diag}\left\{\Pi_3^1, \dots, \Pi_3^n\right\},$
$\Pi_3^l \triangleq \left\{\sum_{l=1}^{n}\left\{d e_{13}^T R_1^l e_{13} - e_{14} R_1^l e_{14}\right\}\right\},$
$\tilde{\Pi}_4 \triangleq \mathrm{diag}\left\{\Pi_4^1, \dots, \Pi_4^n\right\}, \Pi_4^l \triangleq \left\{\sum_{l=1}^{n}\left\{\Theta_3^T\right.\right.$
$\left.\left.\times\tau_m^2 S_1^l \Theta_3 - \Theta_4^T \begin{bmatrix} S_1^l & 0 \\ 0 & 3\frac{(\tau_m+1)}{\tau_m-1}S_1^l \end{bmatrix}^T \Theta_4\right\}\right\},$
$\tilde{\Pi}_5 \triangleq \mathrm{diag}\left\{\Pi_5^1, \dots, \Pi_5^n\right\}, \Pi_5^l \triangleq \left\{\sum_{l=1}^{n}\left\{\Theta_3^T\right.\right.$
$\left.\left.\times(\tau_{12}^2 S_2^l)\Theta_3 - \Theta_5^T \tilde{\Gamma}_1 \Theta_5\right\}\right\},$
$\tilde{\Gamma}_1 \triangleq \mathrm{diag}\left\{\Gamma_1^1, \dots, \Gamma_1^n\right\},$
$\Gamma_1^l \triangleq \left\{\sum_{l=1}^{n}\left\{\begin{bmatrix} S_2^l & 0 & Y_{11} & Y_{12} \\ 0 & 3S_2^l & Y_{21} & Y_{22} \\ Y_{11}^T & Y_{21}^T & S_2^l & 0 \\ Y_{12}^T & Y_{22}^T & 0 & 3S_2^l \end{bmatrix}\right\}\right\},$

$\tilde{\Pi}_6 \triangleq \mathrm{diag}\left\{\Pi_6^1, \dots, \Pi_6^n\right\}, \Pi_6 \triangleq \left\{\sum_{l=1}^{n}\left\{[\tau_m^2 e_1^T Z_1^l e_1\right.\right.$
$-[(\tau_m+1)e_5 - e_1]^T Z_1[(\tau_m+1)e_5 - e_1] - \frac{3}{\tau_m^2-1}$
$\times[2e_9 - (\tau_m-1)((\tau_m+1)e_5 - e_1]^T Z_1^l[2e_9$
$-(\tau_m-1)((\tau_m+1)e_5 - e_1]\}\},$
$\tilde{\Pi}_7 \triangleq \mathrm{diag}\left\{\Pi_7^1, \dots, \Pi_7^n\right\},$
$\Pi_7 \triangleq \left\{\sum_{l=1}^{n}\left\{\frac{\tau_m(\tau_m+1)}{2}e_{15}^T Z_2^l e_{15} - \frac{2(\tau_m+1)}{\tau_m}\Theta_{71}^T Z_2^l \Theta_{71}\right.\right.$
$\left.\left.-\frac{4(\tau_m+1)(\tau_m+2)}{\tau_m(\tau_m-1)}\Theta_{72}^T Z_2^l \Theta_{72}\right\}\right\},$
$\Upsilon \triangleq -\mathrm{sym}\{(e_{11} - e_1 U_m)\mathcal{Y}_1(e_{11} - e_1 U_p)^T + (e_{13} - e_1 U_m)$
$\mathcal{Y}_2(e_{13} - e_1 U_p)^T + (e_{14} - e_3 U_m)\mathcal{Y}_3(e_{14} - e_3 U_p)^T\},$
$\Pi(\tau(k)) \triangleq \Pi_1(\tau(k)) + \sum_{t=2}^{7}\Pi_t + \Upsilon$

Theorem 1: For given integers $0 \leq \tau_m \leq \tau_M$, system (14) is stable asymptotically for $\tau_m \leq \tau_M$, with matrices $P \in \mathbb{R}^{3n\times 3n}, Q_1 \in \mathbb{R}^{n\times n}, Q_2 \in \mathbb{R}^{n\times n}, R_1 \in \mathbb{R}^{n\times n}, S_1 \in \mathbb{R}^{n\times n}, S_2 \in \mathbb{R}^{n\times n}, Z_1 \in \mathbb{R}^{n\times n}, Z_2 \in \mathbb{R}^{n\times n}, > 0$ diagonal matrices $\mathcal{Y}_a \in \mathbb{R}^{n\times n} > 0, (a = 1,2,3)$, with the positive scalars $\varepsilon_1, \varepsilon_2$ and any matrices $Y_{11}, Y_{12}, Y_{21}, Y_{22} \in \mathbb{R}^{n\times n}$ with the event triggering condition and the triggering threshold θ_a satisfying the following LMIs,

$$\tilde{\Sigma}(\tau(k)) < 0, \quad \Gamma > 0, \tag{19}$$

$$\tilde{\Xi}^{\tau_m}(h) + \Omega < 0, \quad \tilde{\Xi}^{\tau_m}(h) + \Omega < 0. \tag{20}$$

Proof: Let the Lyapunov functional for NN be

$$V(k,\tilde{x}(k)) = \sum_{b=1}^{7} V_b(k,\tilde{x}(k)), \tag{21}$$

where

$$V_1(k,\tilde{x}(k)) = \sum_{l=1}^{n} \left\{ \xi^T(k) P^l \xi(k) \right\},$$

$$\begin{aligned} V_2(k,\tilde{x}(k)) &= \sum_{l=1}^{n} \left\{ \sum_{s=k-\tau_m}^{k-1} \tilde{x}^T(s) Q_1^l \tilde{x}(s) \right. \\ &\quad \left. + \sum_{s=k-\tau_M}^{k-1} \tilde{x}^T(k) Q_2^l \tilde{x}(k) \right\}, \\ V_3(k,\tilde{x}(k)) &= \sum_{l=1}^{n} \left\{ \sum_{i=k-d}^{k-1} \sum_{j=i}^{k-1} \tilde{h}^T(\tilde{x}(j)) R_1^l \tilde{h}(\tilde{x}(j)) \right\}, \\ V_4(k,\tilde{x}(k)) &= \sum_{l=1}^{n} \left\{ \tau_m \sum_{i=-\tau_m+1}^{0} \sum_{j=k+i}^{k} \right. \\ &\quad \left. \eta^T(j-1) S_1^l \eta(j-1) \right\}, \\ V_5(k,\tilde{x}(k)) &= \sum_{l=1}^{n} \left\{ \tau_{12} \sum_{i=-\tau_M+1}^{-\tau_m} \sum_{j=k+i}^{k} \right. \\ &\quad \left. \eta^T(j-1) S_2^l \eta(j-1) \right\}, \\ V_6(k,\tilde{x}(k)) &= \sum_{l=1}^{n} \left\{ \tau_m \sum_{i=-\tau_m+1}^{0} \sum_{j=k+i}^{k} \right. \\ &\quad \left. \tilde{x}^T(j-1) Z_1^l \tilde{x}(j-1) \right\}, \\ V_7(k,\tilde{x}(k)) &= \sum_{l=1}^{n} \left\{ \sum_{i=-\tau_m+1}^{0} \sum_{u=i}^{0} \sum_{j=k+u}^{k} \eta^T(j) Z_2^l \eta(j) \right\}, \end{aligned}$$

with $\eta(k) = \tilde{x}(k+1) - \tilde{x}(k)$ and

$$\xi(k) = \left[\tilde{x}^T(k), \sum_{j=k-\tau_m}^{k-1} \tilde{x}^T(j), \sum_{j=k-\tau_M}^{k-\tau_m-1} \tilde{x}^T(j), \right]^T, \tag{22}$$

Taking up the mathematical expectations and finding the difference of $V(k)$ along the trajectories of (14), we get,

$$\mathbb{E}\{\Delta V(x(k))\} = \mathbb{E}\left\{ \sum_{b=1}^{7} \Delta V_b(k) \right\} \tag{23}$$

with

$$\begin{aligned} \mathbb{E}\{\Delta V_1(k)\} &= \mathbb{E}\left\{ \sum_{l=1}^{n} \zeta^T(k) (\Theta_1^T P^l \Theta_1 \right. \\ &\quad \left. -\Theta_2^T P^l \Theta_2) \zeta(k) \right\} \\ &= \mathbb{E}\left\{ \sum_{l=1}^{n} \zeta^T(k) (\Theta_1 + \Theta_2)^T \right. \\ &\quad \left. \times P^l (\Theta_1 - \Theta_2)^T \zeta(k) \right\} \\ &= \mathbb{E}\left\{ \sum_{l=1}^{n} \zeta^T(k) \Pi_1^l(\tau(k)) \zeta(k) \right\} \\ &= \mathbb{E}\left\{ \zeta^T(k) \tilde{\Pi}_1(\tau(k)) \zeta(k) \right\} \end{aligned} \tag{24}$$

$$\begin{aligned} \mathbb{E}\{\Delta V_2(k)\} &= \mathbb{E}\left\{ \sum_{l=1}^{n} \zeta^T(k) (e_1^T Q_1^l e_1 - e_2^T Q_1^l e_2 \right. \\ &\quad \left. + e_2^T Q_2^l e_2 - e_4^T Q_2^l e_4) \zeta(k) \right\} \\ &= \mathbb{E}\left\{ \sum_{l=1}^{n} \left\{ \zeta^T(k) \Pi_2^l \zeta(k) \right\} \right\} \\ &= \mathbb{E}\left\{ \zeta^T(k) \tilde{\Pi}_2 \zeta(k) \right\} \end{aligned} \tag{25}$$

$$\begin{aligned}\mathbb{E}\{\Delta V_3(k)\} &= \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\sum_{i=k-d}^{k}\sum_{j=i}^{k}\tilde{h}^T(\tilde{x}(j))R_1^l\tilde{h}(\tilde{x}(j))\right.\right.\\ &\quad\left.\left.-\sum_{i=k-d}^{k-1}\sum_{j=i}^{k-1}\tilde{h}^T(\tilde{x}(j))R_1^l\tilde{h}(\tilde{x}(j))\right\}\right\}\\ &\le \mathbb{E}\left\{\sum_{l=1}^{n}\left\{d\tilde{h}^T(\tilde{x}(k))R_1^l\tilde{h}(\tilde{x}(k))\right.\right.\\ &\quad\left.\left.-\sum_{j=k-d}^{k-1}\tilde{h}^T(\tilde{x}(j))R_1^l\tilde{h}(\tilde{x}(j))\right\}\right\}\\ &\le \mathbb{E}\left\{\sum_{l=1}^{n}\left\{d\tilde{h}^T(\tilde{x}(k))R_1^l\tilde{h}(\tilde{x}(k))\right.\right.\\ &\quad\left.\left.-\left[\sum_{j=k-d}^{k-1}\tilde{h}(\tilde{x}(j))\right]^T R_1^l\left[\sum_{j=k-d}^{k-1}\tilde{h}(\tilde{x}(j))\right]\right\}\right\}\\ &\le \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\zeta^T(k)(de_{13}^TR_1^le_{13}\right.\right.\\ &\quad\left.\left.-e_{14}R_1^le_{14})\right\}\zeta(k)\right\}\\ &\le \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\zeta^T(k)\Pi_3^l\zeta(k)\right\}\right\}\\ &\le \mathbb{E}\left\{\zeta^T(k)\tilde{\Pi}_3\zeta(k)\right\} \end{aligned}\tag{26}$$

Let us consider

$$\mathbb{E}\{\Delta V_4(k)\} = \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\tau_m^2\eta^T(k)S_1^l\eta(k) -\tau_m\sum_{j=k-\tau_m}^{k-1}\eta^T(k)S_1^l\eta(k)\right\}\right\}\tag{27}$$

Now, we are in a position to apply Lemma 2, to the last three negative terms of (27). Then, on one hand, we have

$$\begin{aligned}-\tau_m\sum_{j=k-\tau_m}^{k-1}\eta(k)S_1\eta(k) &\le -[\tilde{x}(k)-\tilde{x}(k-\tau_m)]^TS_1\\ &\quad\times[\tilde{x}(k)-\tilde{x}(k-\tau_m)]-3\left(\frac{\tau_m+1}{\tau_m-1}\right)\\ &\quad\left[\tilde{x}(k)+\tilde{x}(k-\tau_m)-\frac{2}{\tau_m+1}\sum_{j=k-\tau_m}^{k}\tilde{x}(j)\right]^T\\ &\quad\times S_1\left[\tilde{x}(k)+\tilde{x}(k-\tau_m)-\frac{2}{\tau_m+1}\sum_{j=k-\tau_m}^{k}\tilde{x}(j)\right]\\ &= -\zeta^T(k)\Theta_4^T\begin{bmatrix}S_1 & 0\\ 0 & 3\dfrac{(\tau_m+1)}{\tau_m-1}S_1\end{bmatrix}^T\Theta_4\zeta(k)\end{aligned}\tag{28}$$

Therefore,

$$\begin{aligned}\mathbb{E}\{\Delta V_4(k)\} &\le \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\zeta^T(k)\Theta_3^T(\tau_m^2S_1^l)\Theta_3\zeta(k)-\zeta^T(k)\right.\right.\\ &\quad\left.\left.\times\Theta_4^T\begin{bmatrix}S_1^l & 0\\ 0 & 3\dfrac{(\tau_m+1)}{\tau_m-1}S_1^l\end{bmatrix}^T\Theta_4\zeta(k)\right\}\right\}\\ &= \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\zeta^T(k)\ \Pi_4^l\zeta(k)\right\}\right\}\\ &\le \mathbb{E}\left\{\zeta^T(k)\tilde{\Pi}_4\zeta(k)\right\}\end{aligned}\tag{29}$$

Calculating the expectation of $\Delta V_5(k)$ gives,

$$\begin{aligned}\mathbb{E}\{\Delta V_5(k)\} &= \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\tau_{12}^2\eta^T(k)S_2^l\eta(k) -\tau_{12}\sum_{j=k-\tau_M}^{k-\tau_m-1}\eta^T(k)S_2^l\eta(k)\right\}\right\}\\ &= \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\tau_{12}^2\eta^T(k)S_2^l\eta(k) -\tau_{12}\sum_{j=k-\tau_M}^{k-\tau(k)-1}\eta^T(k)S_2^l\eta(k)\right.\right.\\ &\quad\left.\left.-\tau_{12}\sum_{j=k-\tau(k)}^{k-\tau_m-1}\eta^T(k)S_2^l\eta(k)\right\}\right\}\end{aligned}\tag{30}$$

Further, it follows,

$$-\tau_{12}\sum_{j=k-\tau_M}^{k-\tau(k)-1}\eta^T(k)S_2\eta(k)\times S_2\left[\tilde{x}(k-\tau(k))+\tilde{x}(k-\tau_M))-\frac{2}{\tau_M-\tau(k)+1}\sum_{j=k-\tau_M}^{k-\tau(k)}\tilde{x}(j)\right]\Bigg\}$$

$$\leq -\frac{\tau_{12}}{\tau_M-\tau(k)}\left\{[\tilde{x}(k-\tau(k))-\tilde{x}(k-\tau_M))]^T\times S_2[\tilde{x}(k-\tau(k))-\tilde{x}(k-\tau_M))]-\frac{3\tau_{12}}{\tau_M-\tau(k)}\left[\tilde{x}(k-\tau(k))+\tilde{x}(k-\tau_M))-\frac{2}{\tau_M-\tau(k)+1}\sum_{j=k-\tau_M}^{k-\tau(k)}\tilde{x}(j)\right]^T S_2\left[\tilde{x}(k-\tau(k))+\tilde{x}(k-\tau_M)-\frac{2}{\tau_M-\tau(k)+1}\sum_{j=k-\tau_M}^{k-\tau(k)}\tilde{x}(j)\right]\right\} \quad (31)$$

In a similar way,

$$-\tau_{12}\sum_{j=k-\tau(k)}^{k-\tau_m-1}\eta^T(k)S_2\eta(k) \leq -\frac{\tau_{12}}{\tau(k)-\tau_m}\left\{[\tilde{x}(k-\tau_m)-\tilde{x}(k-\tau(k))]^T\times S_2[\tilde{x}(k-\tau_m)-\tilde{x}(k-\tau(k))]-\frac{3\tau_{12}}{\tau(k)-\tau_m}\times\left[\tilde{x}(k-\tau_m)+\tilde{x}(k-\tau(k))-\frac{2}{\tau(k)-\tau_m+1}\times\sum_{j=k-\tau(k)}^{k-\tau_m}\tilde{x}(j)\right]^T S_2\left[\tilde{x}(k-\tau_m)+\tilde{x}(k-\tau(k))-\frac{2}{\tau(k)-\tau_m+1}\sum_{j=k-\tau(k)}^{k-\tau_m}\tilde{x}(j)\right]\right\} \quad (32)$$

Under the conditions (19) and applying Lemma 3 to (31)-(32), it yields

$$\sum_{l=1}^{n}\left\{-\tau_{12}\sum_{j=k-\tau_M}^{k-\tau(k)-1}\eta^T(k)S_2^l\eta(k)-\tau_{12}\sum_{j=k-\tau(k)}^{k-\tau_m-1}\eta^T(k)S_2^l\eta(k)\right\}\leq\sum_{l=1}^{n}\left\{-\zeta^T(k)\Theta_5^T\Gamma_1^l\Theta_5\zeta(k)\right\} \quad (33)$$

Therefore, the expectation of $\Delta V_5(k)$ becomes,

$$\begin{aligned}\mathbb{E}(\Delta V_5(k)) &\leq \sum_{l=1}^{n}\left\{\zeta^T(k)\Theta_3^T(\tau_{12}^2S_2^l)\Theta_3\zeta(k)-\zeta^T(k)\Theta_5^T\Gamma_1^l\Theta_5\zeta(k)\right\}\\ &=\sum_{l=1}^{n}\left\{\zeta^T(k)\Pi_5^l\zeta(k)\right\}\\ &\leq\mathbb{E}\left\{\zeta^T(k)\tilde{\Pi}_5\zeta(k)\right\} \end{aligned} \quad (34)$$

Calculating $\Delta V_6(k)$, we get

$$\begin{aligned}\mathbb{E}\{\Delta V_6(k)\} &= \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\tau_m^2\tilde{x}^T(k)Z_1^l\tilde{x}(k)-\tau_m\sum_{j=k-\tau_m+1}^{k}\tilde{x}^T(j-1)Z_1^l\tilde{x}(j-1)\right\}\right\}\\ &=\mathbb{E}\left\{\sum_{l=1}^{n}\left\{\tau_m^2\tilde{x}^T(k)Z_1^l\tilde{x}(k)-\tau_m\sum_{j=k-\tau_m}^{k-1}\tilde{x}^T(j)Z_1^l\tilde{x}(j)\right\}\right\}\end{aligned} \quad (35)$$

By Lemma 1, we get

$$\begin{aligned}-\tau_m\sum_{j=k-\tau_m}^{k-1}\tilde{x}^T(j)Z_1\tilde{x}(j) &\leq -\left[\sum_{j=k-\tau_m}^{k-1}\tilde{x}(j)\right]^T Z_1\left[-\sum_{j=k-\tau_m}^{k-1}\tilde{x}(j)\right]\\ &-\frac{3}{\tau_m^2-1}\left[2\sum_{i=k-\tau_m+1}^{k-1}\sum_{j=k-\tau_m}^{i-1}\tilde{x}^T(j)-(\tau_m-1)\left(\sum_{j=k-\tau_m}^{k-1}\tilde{x}^T(j)\right)\right]^T Z_1\left[2\sum_{i=k-\tau_m+1}^{k-1}\sum_{j=k-\tau_m}^{i-1}\tilde{x}^T(j)-(\tau_m-1)\left(\sum_{j=k-\tau_m}^{k-1}\tilde{x}^T(j)\right)\right]\end{aligned}$$

$$\leq -\zeta^T(k)\{[(\tau_m+1)e_5-e_1]^T Z_1[(\tau_m+1)e_5 -e_1] - \frac{3}{\tau_m^2-1}[2e_9-(\tau_m-1)((\tau_m+1)e_5-e_1]^T Z_1[2e_9-(\tau_m-1)((\tau_m+1)e_5-e_1]\zeta(k)\} \quad (36)$$

Also, we have

$$\mathbb{E}\{\Delta V_6(k)\} \leq \sum_{l=1}^{n}\Big\{\zeta(k)\Big\{[\tau_m^2 e_1^T Z_1^l e_1 - [(\tau_m+1)e_5 -e_1]^T Z_1^l[(\tau_m+1)e_5-e_1] - \frac{3}{\tau_m^2-1} [2e_9-(\tau_m-1)((\tau_m+1)e_5-e_1]^T Z_1^l \times[2e_9-(\tau_m-1)((\tau_m+1)e_5-e_1]\zeta(k)\}\} = \sum_{l=1}^{n}\Big\{\zeta^T(k)\Pi_6^l\zeta(k)\Big\} \leq \mathbb{E}\{\zeta^T(k)\tilde{\Pi}_6\zeta(k)\} \quad (37)$$

Calculating $\Delta V_7(k)$, we get

$$\mathbb{E}\{\Delta V_7(k)\} = \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\frac{\tau_m(\tau_m+1)}{2}\eta^T(k)Z_2\eta(k) - \sum_{s=-\tau_m+1}^{0}\sum_{u=k+s}^{k}\eta^T(u)Z_2\eta(u)\right\}\right\} \quad (38)$$

From Lemma 2, we have

$$-\sum_{s=-\tau_m+1}^{0}\sum_{u=k+s}^{k}\eta^T(u)Z_2\eta(u) \leq -\frac{2(\tau_m+1)}{\tau_m}\psi_3^T Z_2\psi_3 - \frac{4(\tau_m+1)(\tau_m+2)}{\tau_m(\tau_m-1)} \left[\tilde{x}(k)+\frac{2}{\tau_m+1}\sum_{s=k-\tau_m}^{k}\tilde{x}(s) - \frac{6}{(\tau_m+1)(\tau_m+2)} \sum_{s=-\tau_m}^{0}\sum_{u=k+s}^{k}\tilde{x}^T(u)Z_2\tilde{x}(u)\right]^T Z_2\left[\tilde{x}(k)+\frac{2}{\tau_m+1}\sum_{s=k-\tau_m}^{k}\tilde{x}(s) - \frac{6}{(\tau_m+1)(\tau_m+2)}\sum_{s=-\tau_m}^{0}\sum_{u=k+s}^{k}\tilde{x}^T(u)Z_2\tilde{x}(u)\right]$$
$$= -\frac{2(\tau_m+1)}{\tau_m}\psi_3^T Z_2\psi_3 - \frac{4(\tau_m+1)(\tau_m+2)}{\tau_m(\tau_m-1)}\psi_4^T Z_2\psi_4 = \zeta^T(k)\left(-\frac{2(\tau_m+1)}{\tau_m}\Theta_{71}^T Z_2^l\Theta_{71} - \frac{4(\tau_m+1)(\tau_m+2)}{\tau_m(\tau_m-1)}\Theta_{72}^T Z_2^l\Theta_{72}\right)\zeta(k) \quad (39)$$

where

$$\psi_3 = \tilde{x}(k) - \frac{1}{\tau_m+1}\sum_{u=k+s}^{k}\tilde{x}(u),$$
$$\psi_4 = \tilde{x}(k) + \left(\frac{2}{\tau_m+1} - \frac{6}{(\tau_m+1)(\tau_m+2)}\right)\sum_{s=k-\tau_m}^{k}\tilde{x}(s) - \frac{6}{(\tau_m+1)(\tau_m+2)}\sum_{s=-\tau_m+1}^{0}\sum_{u=k+s}^{k}\tilde{x}(u).$$

Hence,

$$\mathbb{E}\{\Delta V_7(k)\} \leq \mathbb{E}\left\{\sum_{l=1}^{n}\left\{\zeta^T(k)\Pi_7^l\zeta(k)\right\}\right\} \leq \mathbb{E}\{\zeta^T(k)\tilde{\Pi}_7\zeta(k)\}. \quad (40)$$

From Assumption 1, $\mathcal{Y}_a = \text{diag}\{y_{ai}\} > 0$ for $a = 1,2,3$ and $i = 1,\dots m$, the following inequality holds:

$$0 \leq -2\sum_{i=1}^{m} y_{1i}(\tilde{f}(\tilde{x}_i(k)) - \delta_i^-\tilde{x}_i(k))(\tilde{f}(\tilde{x}_i(k)) - \delta_i^+\tilde{x}_i(k)) - 2\sum_{i=1}^{m} y_{2i}(\tilde{g}_{\tau_k}(\tilde{x}_i(k)) - \beta_i^-(\tilde{x}_{\tau_k})) \times(\tilde{g}_{\tau_k}(\tilde{x}_i(k)) - \beta_i^+(\tilde{x}_{\tau_k})) - 2\sum_{i=1}^{m} y_{3i}(\tilde{h}(\tilde{x}_i(k)) - \gamma_i^-\tilde{x}_i(k))(\tilde{h}(\tilde{x}_i(k)) - \gamma_i^+\tilde{x}_i(k)) \triangleq \zeta^T(k)\Upsilon\zeta(k). \quad (41)$$

Now, we take the bounded disturbance along with triggering condition (8), then

$$\Delta_1 \triangleq \varpi^T(k)\varpi(k) - \theta \leq 0, \Delta_2 \triangleq v^T(k)v(k) - \bar{v} \leq 0, \quad (42)$$

Then, it follows from (21)-(42), and adding all the inequalities along with the Assumption 1, we get

$$\mathbb{E}\{\Delta V(\tilde{x}(k))\} \le \mathbb{E}\left\{\zeta^T(k)(\Pi(\tau(k))+\Omega)\zeta(k)\right\}, \triangleq \mathbb{E}\left\{\zeta^T(k)(\Xi(\tau(k)))\zeta(k)\right\}, \quad (43)$$

where $\Xi(\tau(k)) = \Pi(\tau(k))+\Omega$.
Moreover, if for

$$\Phi(\tau(k)) \triangleq \begin{bmatrix} \Xi(\tau(k)) & \hat{\Xi}_2 & \hat{\Xi}_3 & \hat{\Xi}_4 \\ \star & \hat{\Xi}_5 & 0 & 0 \\ \star & \star & \hat{\Xi}_6 & 0 \\ \star & \star & \star & \hat{\Xi}_7 \end{bmatrix}, \quad (44)$$

where $\hat{\Xi}_2 \triangleq \hat{e}_r, \hat{\Xi}_3 \triangleq -PK, \hat{\Xi}_4 \triangleq -PKF, \hat{\Xi}_5 \triangleq -P, \hat{\Xi}_6 \triangleq -\varepsilon_1 I, \hat{\Xi}_7 \triangleq -\varepsilon_2 I$, we have, $\Delta V(\tilde{x}(k)) \le \Phi(\tau(k))$. Then, obviously if, $\Phi(\tau(k)) < 0$ and for $\zeta(k) \ne 0$, we get $\Delta V\tilde{x}(k) < 0$, which indicates that the error system is asymptotically mean square stable when $w(k) = 0$.
Next, for all non-zero $w(k)$, we get,

$$\begin{aligned}&\Delta V(\tilde{x}(k)) + \tilde{z}^T(k)\tilde{z}(k) - \gamma^2 w^T(k)w(k)\\ &\le \zeta^T(k)\Phi(\tau(k))\zeta(k) + \tilde{x}^T(k)L_i^T L_i\tilde{x}(k)\\ &\quad -\gamma^2 w^T(k)w(k)\\ &\le \zeta_1(k)\tilde{\Phi}(\tau(k))\zeta_1(k) \end{aligned} \quad (45)$$

where $\zeta_1^T(k) \triangleq [\zeta^T(k) \quad w^T(k)]$ where $\Phi(\tau(k))$ is given in LMI (44).
Hence, it follows that, $\Delta V(\tilde{x}(k)) + \tilde{z}^T(k)\tilde{z}(k) - \gamma^2 w^T(k)w(k) < 0$, that is

$$\tilde{\Sigma}(\tau(k)) \triangleq \begin{bmatrix} \Phi(\tau(k)) & 0 & \hat{e}_r P_i \\ \star & -\gamma^2 I & D_i^T P_i \\ \star & \star & -P_i \end{bmatrix}.$$

Now, the index is established with the H_∞ performance:

$$\mathcal{J}(s) \triangleq \mathbb{E}\sum_{k=0}^{s}\left\{||\tilde{z}(k)||^2 - \gamma^2||w(k)||^2\right\} \quad (46)$$

By zero initial condition,

$$\begin{aligned}\mathcal{J}(s) &\triangleq \mathbb{E}\sum_{k=0}^{s}\left\{||\tilde{z}(k)||^2 - \gamma^2||w(k)||^2 + \Delta V(k)\right.\\ &\qquad \left. -\mathbb{E}\{V(s+1)\}\right\}\\ &\le \mathbb{E}\sum_{k=0}^{s}\left\{||\tilde{z}(k)||^2 - \gamma^2||w(k)||^2 + \Delta V(k)\right\}\\ &\le \mathbb{E}\sum_{k=0}^{s}\{\zeta_1^T(k)\tilde{\Phi}(\tau(k))\} < 0. \end{aligned} \quad (47)$$

Letting $s \to \infty$, we obtain,

$$\sum_{k=0}^{\infty}\mathbb{E}\left\{||\tilde{z}(k)||^2\right\} \le \gamma^2\sum_{k=0}^{\infty}||w(k)||^2$$

This proves that the error system is (14) is asymptotically stable.

Remark 2. Due to the existence of the time-varying term $\sum_{j\in\mathbb{S}}\gamma_{lj}(h)P_j$, it is observed that (19) is not an LMI and is therefore hard to solve. Also in [36], the transition rate $\gamma_{lj}(h)$ is bounded and $\gamma_{lj}{}^- \le \gamma_{lj}(h) \le \gamma_{lj}{}^+$, where γ_{lj}^- and γ_{lj}^+ are constants. Therefore, $\gamma_{lj}(h)$ is given by [36, 37].

$$\gamma_{lj}(h) = \sum_{k=1}^{\mathcal{K}}\hat{\varphi}_k\gamma_{lj,k}, \quad \sum_{k=1}^{\mathcal{K}}\hat{\varphi}_k = 1, \quad \hat{\varphi}_k \ge 0, \quad (48)$$

and

$$\gamma_{lj,k} = \begin{cases} \gamma_{lj}^- + (k-1)\frac{\gamma_{ij}^- - \gamma_{lj}^+}{\mathcal{K}-1}, & i \ne j, \ j \in \mathbb{S} \\ \gamma_{lj}^+ - (k-1)\frac{\gamma_{ij}^- - \gamma_{lj}^+}{\mathcal{K}-1}, & i = j, \ j \in \mathbb{S} \end{cases}, \quad (49)$$

Remark 3. Sufficient condition given by Theorem 1 provides the error dynamics (14) to achieve the estimation performance. To be specific, the guaranteed feasibility of the constraints is achieved by the values of the triggering thresholds θ_l.

Now, we will determine parameters of filter (13) based on the LMIs established in Theorem 1.

Theorem 2. For given integers $0 \le \tau_m \le \tau_M$, (14) is asymptotically stable for $\tau_m \le \tau_M$, if there exists matrices $P \in \mathbb{R}^{3n\times 3n}, Q_1 \in \mathbb{R}^{n\times n}, Q_2 \in \mathbb{R}^{n\times n}, R_1 \in \mathbb{R}^{n\times n}, S_1 \in \mathbb{R}^{n\times n}, S_2 \in \mathbb{R}^{n\times n}, Z_1 \in \mathbb{R}^{n\times n}, Z_2 \in \mathbb{R}^{n\times n} > 0$, diagonal matrices $\mathcal{Y}_a \in \mathbb{R}^{\kappa\times\kappa} > 0, (a = 1,2,3)$, positive scalars $\varepsilon_1, \varepsilon_2$ and any matrices $Y_{11}, Y_{12}, Y_{21}, Y_{22} \in \mathbb{R}^{n\times n}$ with the event triggering condition (8) and the triggering thresholds $\theta_l, (l = 1,2,\ldots,n)$ which satisfies the LMIs,

$$\Lambda(\tau(k)) \triangleq \tilde{\Sigma}(\tau(k)) + \mathrm{Sym}\{\Upsilon_2\Upsilon_3\} < 0, \ \Gamma > 0, \quad (50)$$

with $\Upsilon_2 \triangleq X(\tilde{A}_l e_1 + A_{dl}e_{11} + B_l M e_{12} + C_l e_{14} + De_{17} - Y(e_{15} + Fe_{16}), \Upsilon_3 \triangleq e_1 + e_{15}$ then, $K = X^{-1}Y$ is the gain estimator.

Proof: By applying the zero inequality, gain matrix K designing is done by the parameters given in Theorem 1, for any matrix X of appropriate dimension:

$$\begin{aligned} 0 =& 2(\tilde{x}(k)+\eta(k))X[\tilde{A}_l\tilde{x}(k)+A_{dl}\tilde{f}(\tilde{x}(k)) \\ &+B_lM\tilde{g}_{\tau_k}(\tilde{x}(k))+C_l\sum_{s=k-d}^{k-1}\tilde{h}(\tilde{x}(s))-K\varpi(k) \\ &-KFv(k)+Dw(k)-\tilde{x}(k+1)] \quad (51) \\ \triangleq& \zeta_1(k)\mathrm{Sym}\{\Upsilon_2\Upsilon_3\}\zeta_1(k) \quad (52) \end{aligned}$$

Combining all the inequalities, from (24) to (40),

$$\Delta V(k) \leq \zeta_1(k)\Lambda(\tau(k))\zeta_1(k). \quad (53)$$

Obviously, if $\Lambda(\tau(k))<0$ and $\zeta_1\neq 0$, then $\Delta V(k)<0$, that is error system is asymptotically stable with the estimator gain K.

4 Numerical Simulation

Two numerical simulations are provided to illustrate the usefulness of the obtained theoretical results for a class of discrete-time NNs.

Example 1. The system parameters of the semi-Markovian NNs are set with:

$$A_1 = \begin{bmatrix} -1.32 & -0.13 \\ -0.96 & -1.0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1.28 & -0.35 \\ -0.59 & -0.8 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -1.02 & -0.06 \\ 0.058 & -2.0 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.24 & -0.03 \\ -0.07 & -0.6 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.12 & -0.09 \\ -1.09 & -0.8 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.36 & -0.36 \\ -0.95 & -0.7 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.26 & -0.05 \\ -0.09 & -0.8 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -0.29 & -0.12 \\ -0.08 & -0.9 \end{bmatrix},$$

$$J(k) = \begin{bmatrix} 0.1\cos(k/2) \\ 0.05\sin(k/2) \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.05 \\ 0.04 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.04 \\ 0.03 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix},$$

$$\tau_m = 4, \tau(k) = 6, \tau_M = 15$$

The neuron activation functions are assumed to be

$$\begin{aligned} f(x(s)) &= \begin{bmatrix} \tanh\left(\frac{4x_1(s)}{10}\right) & \tanh\left(\frac{5x_2(s)}{10}\right) \end{bmatrix}^T \\ g(x(s)) &= \begin{bmatrix} \tanh\left(\frac{3x_1(s)}{10}\right) & \tanh\left(\frac{4x_2(s)}{10}\right) \end{bmatrix}^T \\ h(x(s)) &= \begin{bmatrix} \tanh\left(\frac{-5x_1(s)}{10}\right) & \tanh\left(\frac{3x_2(s)}{10}\right) \end{bmatrix}^T. \end{aligned}$$

from which it is easy to verify that $\delta_1 = \beta_1 = \gamma_1 = 0, \delta_2 = \mathrm{diag}\{-0.2,-0.25\}, \beta_2 = \mathrm{diag}\{-0.15,-0.2\}$,
$\gamma_2 = \mathrm{diag}\{0.25,-0.15\}$ The output measurement of the NN (7) is modeled with:

$$E = \begin{bmatrix} 2.2 & 0.5 \\ 0.3 & 1.2 \end{bmatrix}, F = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}$$

The transition rate matrix is chosen as $0.1 \leq \theta_{12} \leq 2.0$, $0.8 \leq \theta_{21} \leq 1.7$. From equations (48) and (49), we have $\theta_{12,1} = 0.1, \theta_{12,2} = 2.0, \theta_{21,1} = 0.8, \theta_{21,2} = 1.7$. Also $v(k) = \bar{v}sin(k)$ with $\bar{v} = 1.5$. The individual triggering thresholds is taken into consideration in the event-triggering transmission protocol and the value of threshold θ corresponding to the output measurement is obtained as $\Theta = 142.4114$. The asymptotic stability is achieved by solving the LMIs in Theorem 2, to obtain the filter gains by using the semi-Markovian generalized NNs for the designed filters as,

$$K_1 = \begin{bmatrix} -0.2547 & 0.0031 \\ -0.1690 & -1.5868 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.3097 & -0.0800 \\ -0.0823 & -1.0630 \end{bmatrix},$$

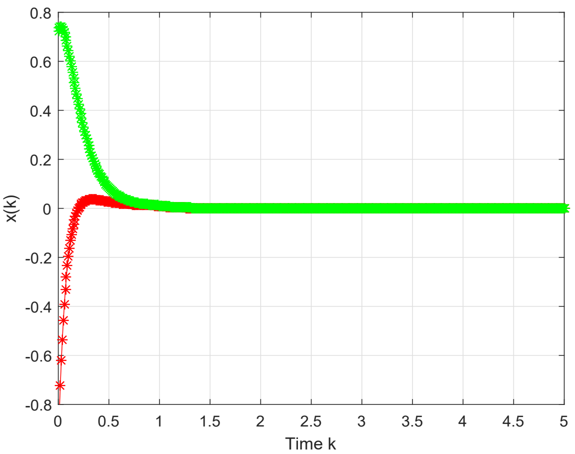

Figure 1. State responses of the system (14)

From the Figure 1, the state responses of the considered NN converge to zero and hence can be concluded that the proposed estimation done via event-triggering performs better than the time-triggered scheme which also reduces the triggering frequency.

Example 2. The system parameters of the semi-Markovian neural networks are set with the following parameters:

$$A_1 = \begin{bmatrix} -6.2 & 0 & 0 \\ 0 & -2.5 & 0 \\ 0 & 0 & 0.45 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 6.4 & 0 & 0.1 \\ 0.2 & 3.2 & 2.1 \\ 2.0 & 3.2 & 4.1 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -1.02 & 0.06 & 0.01 \\ 0.058 & -2 & 0.5 \\ 0.07 & 0.3 & -0.6 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.24 & -0.03 & 1.2 \\ -0.07 & -0.6 & 0.8 \\ -2.3 & 2.4 & 2.5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -0.8 & 0.90.1 & \\ 0.6 & 1.5 & 0.3 \\ 0.2 & 0.4 & 0.5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.7 & 0.6 & 0 \\ -0.58 & 0.7 & 0 \\ 070 & 0.4 & \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.02 & 0.09 & 0 \\ -0.03 & 0.09 & 0.01 \\ 0.4 & -0.2 & 0.01 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -0.03 & 0.07 & 0.03 \\ 0.03 & 0.07 & 0.01 \\ 0.01 & 0.2 & 0.4 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0.05 \\ 0.04 \\ 0.06 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.04 \\ 0.03 \\ 0.02 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

$$\tau_m = 6, \tau(k) = 8, \tau_M = 15$$

The activation functions in neuron are the same as in Example 1, with the same kind of the transition matrix, then the event triggering threshold $\theta_1 = 429.6158$.

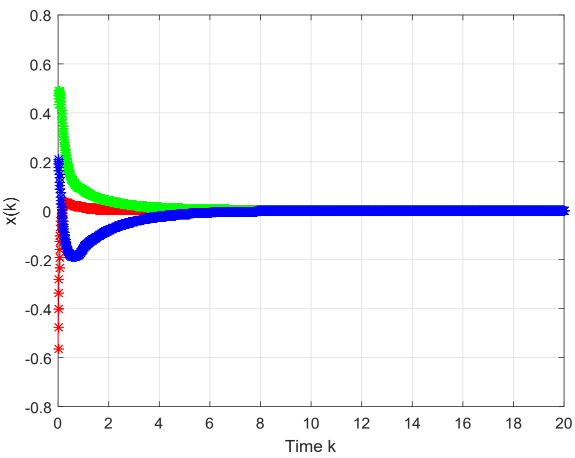

Figure 2. State responses of the system (14)

Through MATLAB simulation, the LMI (14) is feasible with the gain matrices given by

$$K_1 = \begin{bmatrix} -1.9967 & -0.7646 & 0.3471 \\ 0.0527 & -4.6057 & -2.4581 \\ 1.3421 & -3.2971 & 0.7651 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -1.4446 & -0.4484 & -0.2345 \\ -0.0447 & -3.3331 & -2.3485 \\ 0.0325 & -2.3485 & -1.7835 \end{bmatrix}.$$

Figure 2. depicts the convergence dynamics of the system of (14).

5 Conclusion

In this article, investigation of state estimation via H_∞ approach is carried over with mixed time delays for discrete-time stochastic NNs under the event-triggered communication scheme. The transmission of the measurement component is done only when the corresponding triggering condition is satisfied. New summation inequalities are established which extends the discrete Jensen's inequality effectively. Asymptotic stability analysis of delayed discrete-time NNs with the H_∞ performance $\gamma > 0$ is established as an application of the summation inequality. Two simulation results are presented for illustration of the proposed methodologies. In future research, we plan to expand the proposed methodology to continuous-time stochastic systems.

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References

[1] L. Rutkowski, Adaptive probabilistic neural networks for pattern classification in time-varying environment, IEEE Transactions on Neural Networks, 15, 2004, 811–827.

[2] Y. Xia and J. Wang, A general projection neural network for solving monotone variational inequalities and related optimization problems, IEEE Transactions on Neural Networks 15, 2004, 318–328.

[3] H. K. Kwan and Y. Cai, A fuzzy neural network and its application to pattern recognition, IEEE Transactions on Fuzzy Systems, 2, 1994, 185-–193.

[4] W. Gao and R. R. Selmic, 'Neural network control of a class of nonlinear systems with actuator saturation,' IEEE Transactions on Neural Networks, 17, 2006, 147-–156.

[5] H. K. Kwan and Y. Cai, A fuzzy neural network and its application to pattern recognition, IEEE Transactions on Fuzzy Systems, 2, 1994, 185—193.

[6] A. U. Levin and K. S. Narendra, Control of nonlinear dynamical systems using neural networks: Controllability and stabilization, IEEE Transactions on Neural Networks, 4, 1993, 192-–206.

[7] Z. Wang, Y. Liu, K. Fraser, and X. Liu, Stochastic stability of uncertain Hopfield neural networks with discrete and distributed delays, Physica Letters A, 354, 2006, 288–-297.

[8] S. Wen, Z. Zeng, and T. Huang, Exponential stability analysis of memristor-based recurrent neural networks with time-varying delays, Neurocomputing, 97, 2012, 233-–240.

[9] H. Zhang, Z. Wang, and D. Liu, A comprehensive review of stability analysis of continuous-time recurrent neural networks, IEEE Transactions on Neural Networks Learning Systems, 25, 2014, 1229-– 1262.

[10] H. Gao and T. Chen, New results on stability of discrete-time systems with time-varying state delay, IEEE Transactions on Automatic Control, 52, 2007, 328—334.

[11] K.L. Mak, J.G. Peng, Z.B. Xu and K.F.C. Yiu, A new stability criterion for discrete-time neural networks: Nonlinear spectral radius, Chaos, Solitons and Fractals, 31, 2007, 424–436.

[12] Y. He, M. Wu, and J. H. She, An improved global asymptotic stability criterion for delayed cellular neural networks, IEEE Transactions on Neural Networks, 17, 2006, 250-–252.

[13] C. Maharajan, R. Raja, J. Cao, G. Rajchakit and A. Alsaedi, Novel results on passivity and exponential passivity for multiple discrete delayed neutral-type neural networks with leakage and distributed time-delays, Chaos, Solitons and Fractals, 115, 2018, 268–282.

[14] W. Xiong and J. Cao, Global exponential stability of discrete-time Cohen-Grossberg neural networks, Neurocomputing, 64, 2005, 433—446.

[15] J. Cao, R. Rakkiyappan, K. Maheswari, and A. Chandrasekar, Exponential H_∞ filtering analysis for discrete-time switched neural networks with random delays using sojourn probabilities, Science China Technological Sciences, 59 2016, 387-–402.

[16] R. Rakkiyappan, K. Maheswari and K. Sivaranjani, Non-weighted state estimation for discrete-time switched neural networks with persistent dwell-time switching regularities based on Finsler's lemma, Neurocomputing, 260, 2017, 131–141.

[17] Z. Wu, P. Shi, H. Su and J. Chu, Dissipativity analysis for discrete-time stochastic neural networks with time-varying delays, IEEE Transactions Neural Networks and Learning Systems, 24, 2013, 345-–355.

[18] O. Kwon and J. Park, On improved delay-dependent robust control for uncertain time-delay systems, IEEE Transactions on Automatic Control, 49, 2004, 1991-–1995.

[19] O. Kwon, Stability criteria for uncertain stochastic dynamic systems with time-varying delays, International Journal Robust Nonlinear Control, 21, 2011, 338-–350.

[20] D. Ding, Z. Wang, B. Shen, and H. Dong, Event-triggered distributed H_∞ state estimation with packet dropouts through sensor networks, IET Control Theory Applications, 9, 2015, 1948-–1955.

[21] X. Jia, X. Chi, Q. Han, and N. Zheng, Event-triggered fuzzy H_∞ control for a class of nonlinear networked control systems using the deviation bounds of asynchronous normalized membership functions, Information Sciences, 259, 2014, 100-–117.

[22] L. Li, D. W. C. Ho, and S. Xu, A distributed event-triggered scheme for discrete-time multi-agent consensus with communication delays, IET Control Theory Applications, 8, 2014, 830-–837.

[23] Q. Li, B. Shen, Y. Liu, and F. E. Alsaadi, Event-triggered H_∞ state estimation for discrete-time stochastic genetic regulatory networks with Markovian jumping parameters and time-varying delays, Neurocomputing, 174, 2016, 912-–920.

[24] J. Zheng and B. Qui, State estimation of chaotic Lurie system with logarithmic quantization, Chaos, Solitons and Fractals, 112, 2018, 141–148.

[25] C. Peng and Q.L. Han, A novel event-triggered transmission scheme and L_2 control co-design for sampled-data control systems, IEEE Transactions on Automatic Control, 58, 2013, 2620—2626.

[26] D. Shi, T. Chen, and L. Shi, Event-triggered maximum likelihood state estimation, Automatica, 50, 2014, 247-–254.

[27] W. Zhang, Z. Wang, Y. Liu, D. Ding, and F. E. Alsaadi, Event-based state estimation for a class of complex networks with time-varying delays: A comparison principle approach, Physica Letter A, 381, 2017, 10-–18.

[28] L. Wang, Z. Wang, G. Wei and F. E. Alsaadi, Finite-state estimation for recurrent delayed neural networks with component based event-triggering protocol, IEEE Transactions on Neural Networks and Learning Systems, 29, 2018, 1046-1057.

[29] H. Liu, D.W.C. Ho and F. Sun, Design of H_∞ filter for Markov jumping linear systems with non-accessible mode information, Automatica, 44, 2008, 2655-–2660.

[30] J. Xiong, J. Lam, H. Gao and D.W.C. Ho, On robust stabilization of Markovian jump systems with uncertain switching probabilities, Automatica, 41, 2005, 897-–903.

[31] Z. Hou, J. Luo, P. Shi and S.K. Nguang, Stochastic stability of Itô differential equations with semi-Markovian jump parameters, IEEE Transactions on Automatomatic Control, 51, 2006, 1838—1842.

[32] E. Shmerling and, K.J. Hochberg, Stability of stochastic jump-parameter semi-Markov linear systems of differential equations, Stochastics, 80, 2008, 513-–518.

[33] X. Zhang and Z. Hou, The first-passage times of phase semi-Markov processes, Statistics in Probability Letters, 82, 2012, 40-–48.

[34] X. G. Liu, F. X. Wang and Y. J. Shu, A novel summation inequality for stability analysis of discrete-time neural networks, Journal of Computational and Applied Mathematics, 304, 2016, 160-–171.

[35] S. B. Qiu, X. G. Liu, Y. J. Shu, A study on state estimation for discrete-time recurrent neural networks with leakage delay and time-varying delay, Advances in Difference Equations, 234, 2016, doi: 10.1186/s13662-016-0958-4.

[36] J. Huang and Y.Shi, Stochastic stability and robust stabilization of semi-Markovian jump linear systems, International Journal of Robust and Nonlinear Control, 23, 2013, 2028–2043.

[37] F. Li, L. Wu and P. Shi, Stochastic stability of semi-Markovian jump systems with mode-dependent delays, International Journal of Robust and Nonlinear Control, 24, 2014, 3317–3330.

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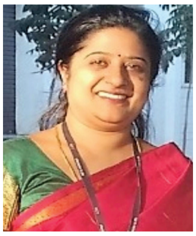

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