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Models of reliability and availability improvement of series and parallel systems related to their operation processes

Keywords

multi-state systems, reliability and availability improvement

Abstract

Integrated general models of approximate approaches of complex multi-state series and parallel systems, linking their reliability and availability improvement models and their operation processes models caused changing reliability and safety structures and components reliability characteristics in different operation states, are constructed. These joint models are applied to determining improved reliability and availability characteristics of the considered multi-state series and parallel systems related to their varying in time operation processes. The conditional reliability characteristics of the multi-state systems with hot, cold single reservation of component and the conditional reliability characteristics of the multi-state systems with reduced rate of departure by a factor of system components are defined.

1. Introduction

Most real transportation systems are very complex. Large numbers of components and subsystems and their operating complexity cause that the evaluation and optimization of their reliability improvement and availability improvement is complicated. A convenient tool for solving this problem is semi-markov modeling of the systems operation processes combining with three methods of reliability and availability improvement proposed in this paper. Therefore, the common usage of the system's reliability and availability improvement models and the semi-markov model for the system's operation process modeling in order to construct the joint general system reliability and availability improvement model related to its operation process is proposed.

2. Reliability improvement of multi-state system component in variable operation conditions

We assume that the reliability of a single system component can be improved by using hot or cold reserve of this component or by replacing this component by an improved component with the reduced rates of departure from the reliability state subset $\{u, u+1, ...z\}$ by a factor $\rho(u)$, $0 < \rho(u) < 1$, $u = 1,2,\ldots,z$. Further, we assume that the basic and reserve component have the same multi-state exponential reliability function.

If basic and reserve components of the multi-state system at the operational state z_b, $b = 1,2,...,\nu$, have the same exponential reliability functions given by

$$[R_i(t,\cdot)]^{(b)} = [1, [R_i(t,1)]^{(b)}, \ldots, [R_i(t,z)]^{(b)}], \qquad (1)$$

$$t \in (-\infty,\infty), \; b = 1,2,...,\nu,$$

where

$$[R_i(t,u)]^{(b)} = 1 \text{ for } t < 0,$$

$$[R_i(t,u)]^{(b)} = \exp[-\lambda_i^{(b)}(u)t] \text{ for } t \geq 0, \qquad (2)$$

$$\lambda_i^{(b)}(u) > 0, \; i = 1,2,...,n, \; u = 1,2,\ldots,z, \; b = 1,2,...,\nu,$$

then

(a) the reliability function of multi-state system component with a single hot reservation at the operational state z_b, $b = 1,2,...,\nu$, is respectively given by

$$[R_i^{(1)}(t,\cdot)]^{(b)} = [1,[R_i^{(1)}(t,1)]^{(b)},\ldots,[R_i^{(1)}(t,z)]^{(b)}], \quad (3)$$

$$t \in (-\infty,\infty),\ b = 1,2,...,v,$$

where

$$[R_i^{(1)}(t,u)]^{(b)} = 1-[[F_i(t,u)]^{(b)}]^2 = 1,\ t < 0,$$

$$[R_i^{(1)}(t,u)]^{(b)} = 1-[[F_i(t,u)]^{(b)}]^2$$

$$= 2\exp[-\lambda_i^{(b)}(u)t] - \exp[-2\lambda_i^{(b)}(u)t],\ t \ge 0, \quad (4)$$

$$\lambda_i^{(b)}(u) > 0,\ i = 1,2,...,n,\ u = 1,2,\ldots,z,$$

$$b = 1,2,...,v,$$

(b) the reliability function of multi-state system component with single cold reservation at the operational state z_b, $b = 1,2,...,v,$ is respectively given by

$$[R_i^{(2)}(t,\cdot)]^{(b)} = [1,[R_i^{(2)}(t,1)]^{(b)},\ldots,[R_i^{(2)}(t,z)]^{(b)}] \quad (5)$$

$$t \in (-\infty,\infty),\ b = 1,2,...,v,$$

where

$$[R_i^{(2)}(t,u)]^{(b)} = 1-[[F_i(t,u)]^{(b)} * [F_i(t,u)]^{(b)}] = 1,$$
$$t < 0,$$

$$[R_i^{(2)}(t,u)]^{(b)} = 1-[[F_i(t,u)]^{(b)} * [F_i(t,u)]^{(b)}]$$

$$= [1+\lambda_i^{(b)}(u)t]]\exp[-\lambda_i^{(b)}(u)t],\ t \ge 0, \quad (6)$$

$$\lambda_i^{(b)}(u) > 0,\ i = 1,2,...,n,\ u = 1,2,\ldots,z,\ b = 1,2,...,v,$$

(c) the exponential reliability function of multi-state system component with the reduced rate of departure by a factor $\rho(u)$, $u = 1,2,...,z,$ at the operational state z_b, $b = 1,2,...,v,$ is respectively given by

$$[R_i^{(3)}(t,\cdot)]^{(b)} = [1,[R_i^{(3)}(t,1)]^{(b)},\ldots,[R_i^{(3)}(t,z)]^{(b)}] \quad (7)$$

$$t \in (-\infty,\infty),\ b = 1,2,...,v,$$

where

$$[R_i^{(3)}(t,u)]^{(b)} = 1 \text{ for } t < 0,$$

$$[R_i^{(3)}(t,u)]^{(b)} = \exp[-\lambda_i^{(b)}(u)\rho(u)t] \text{ for } t \ge 0, \quad (8)$$

$$\lambda_i^{(b)}(u) > 0,\ i = 1,2,...,n,\ u = 1,2,\ldots,z,\ b = 1,2,...,v.$$

3. Asymptotic approach to evaluation of reliability improvement of large multi-state systems in variable operation conditions

Main results concerning asymptotic approach to multi-state system reliability improvement with ageing components in fixed operation conditions are comprehensively detailed for instance [4], [5].
In order to combine the results on the reliability improvement of multi-state systems related to their operation processes and the results concerning limit reliability functions of the multi-state systems, and to obtain results of asymptotic approach to evaluation of the multi-state system reliability improvement in variable operation conditions, we use the following definition [6]. A reliability function

$$\boldsymbol{\mathfrak{R}}(t,\cdot) = [1, \boldsymbol{\mathfrak{R}}(t,1),..., \boldsymbol{\mathfrak{R}}(t,z)],\ t \in (-\infty,\infty),$$

where

$$\boldsymbol{\mathfrak{R}}(t,u) = \sum_{b=1}^{v} p_b[\boldsymbol{\mathfrak{R}}(t,u)]^{(b)},\quad u = 1,2,...,z,$$

is called a limit reliability function of a multi-state system in its operation process with reliability function

$$\boldsymbol{R}_n(t,\cdot) = [\boldsymbol{R}_n(t,0), \boldsymbol{R}_n(t,1),..., \boldsymbol{R}_n(t,z)],$$

where

$$\boldsymbol{R}_n(t,u) \cong \sum_{b=1}^{v} p_b[\boldsymbol{R}_n(t,u)]^{(b)},\ u = 1,2,...,z,$$

where

$$p_b = \lim_{t\to\infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{v}\pi_l M_l},\ b = 1,2,...,v,$$

are the limit values of the transient probabilities at the particular operation states

$$p_b(t) = P(Z(t) = z_b),\ t \in \langle 0,+\infty),\ b = 1,2,...,v,$$

and the probabilities π_b of the vector $[\pi_b]_{1xv}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases}$$

if there exist normalising constants

$$a_n^{(b)}(u) > 0,\ \ b_n^{(b)}(u) \in (-\infty, \infty),\ u = 1,2,...,z,$$

$$b = 1,2,...,v,$$

such that for $t \in C_{[\boldsymbol{\Re}(u)]^{(b)}}$, $u = 1,2,...,z,\ b = 1,2,...,v,$

$$\lim_{n\to\infty} [\boldsymbol{R}_n(a_n^{(b)}(u)t + b_n^{(b)}(u), u)]^{(b)} = [\boldsymbol{\Re}(t,u)]^{(b)}.$$

Hence, the following approximate formulae are valid

$$\boldsymbol{R}_n(t,\cdot) = [\boldsymbol{R}_n(t,0), \boldsymbol{R}_n(t,1), ..., \boldsymbol{R}_n(t,z)],$$

where

$$\boldsymbol{R}_n(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{\Re}(\frac{t - b_n^{(b)}(u)}{a_n^{(b)}(u)}, u)]^{(b)},\ t \in (-\infty, \infty),$$
$$u = 1,2,...,z.$$

The following propositions are concerned with the homogeneous exponential systems i.e. the systems which components have at operational states $z_b,\ b = 1,2,...,v,$ exponential reliability functions.

Proposition 3.1
If components of the homogeneous multi-state series system at the operational state z_b have exponential reliability functions and the system have:
(a) a single hot reservation of system components and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)\sqrt{n}},\ \ b_n^{(b)}(u) = 0,$$

$$u = 1,2,...,z,\ b = 1,2,...,v,$$

then

$$\overline{\boldsymbol{\Re}}^{(1)}(t,\cdot) = [1, \overline{\boldsymbol{\Re}}^{(1)}(t,1), \ldots, \overline{\boldsymbol{\Re}}^{(1)}(t,z)], \tag{9}$$

$$t \in (-\infty,\infty),$$

where

$$\overline{\boldsymbol{\Re}}^{(1)}(t,u) = 1 \ \text{ for } t < 0,$$

$$\overline{\boldsymbol{\Re}}^{(1)}(t,u) = \sum_{b=1}^{v} p_b \exp[-t^2] \ \text{ for } t \geq 0, \tag{10}$$

is its unconditional multi-state limit reliability function. Hence, the following exact formulae is valid

$$\overline{\boldsymbol{R}}_n^{(1)}(t,\cdot) = [1, \overline{\boldsymbol{R}}_n^{(1)}(t,1), ..., \overline{\boldsymbol{R}}_n^{(1)}(t,z)], \tag{11}$$

where

$$\overline{\boldsymbol{R}}_n^{(1)}(t,u) = 1 \ \text{ for } t < 0,$$

$$\overline{\boldsymbol{R}}_n^{(1)}(t,u) = \sum_{b=1}^{v} p_b \exp[-(\lambda^{(b)}(u)\sqrt{n}t)^2],\ t \geq 0, \tag{12}$$

$$u = 1,2,...,z,$$

(b) a single cold reservation of system components and

$$a_n^{(b)}(u) = \frac{\sqrt{2}}{\lambda^{(b)}(u)\sqrt{n}},\ \ b_n^{(b)}(u) = 0,\ u = 1,2,...,z,$$

$$b = 1,2,...,v,$$

then

$$\overline{\boldsymbol{\Re}}^{(2)}(t,\cdot) = [1, \overline{\boldsymbol{\Re}}^{(2)}(t,1), \ldots, \overline{\boldsymbol{\Re}}^{(2)}(t,z)], \tag{13}$$

$$t \in (-\infty,\infty),$$

where

$$\overline{\boldsymbol{\Re}}^{(2)}(t,u) = 1 \ \text{ for } t < 0,$$

$$\overline{\boldsymbol{\Re}}^{(2)}(t,u) = \sum_{b=1}^{v} p_b \exp[-t^2] \ \text{ for } t \geq 0, \tag{14}$$

$$u = 1,2,...,z,$$

is its unconditional multi-state limit reliability function. Hence the following approximate formulae is valid

$$\overline{\boldsymbol{R}}_n^{(2)}(t,\cdot) = [1, \overline{\boldsymbol{R}}_n^{(2)}(t,1), ..., \overline{\boldsymbol{R}}_n^{(2)}(t,z)], \tag{15}$$

where

$$\overline{\boldsymbol{R}}_n^{(2)}(t,u) = 1 \ \text{ for } t < 0,\ u = 1,2,...,z,$$

$$\overline{\boldsymbol{R}}_n^{(2)}(t,u) \cong \sum_{b=1}^{v} p_b \exp[-(\frac{\lambda^{(b)}(u)\sqrt{n}}{\sqrt{2}} t)^2], \qquad (16)$$

$t \ge 0,\ u = 1,2,...,z,$

(c) components with the reduced rate of departure by a factor $\rho(u),\ u = 1,2,...,z,$ and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)\rho(u)n},\ \ b_n^{(b)}(u) = 0,$$

$u = 1,2,...,z,\ b = 1,2,...,v,$

then

$$\overline{\mathfrak{R}}^{(3)}(t,\cdot) = [1, \overline{\mathfrak{R}}^{(3)}(t,1), \ldots, \overline{\mathfrak{R}}^{(3)}(t,z)], \qquad (17)$$

$t \in (-\infty,\infty),$

where

$$\overline{\mathfrak{R}}^{(3)}(t,u) = 1 \text{ for } t < 0,$$

$$\overline{\mathfrak{R}}^{(3)}(t,u) = \sum_{b=1}^{v} p_b \exp[-t],\ t \ge 0, u = 1,2,...,z, \qquad (18)$$

is its unconditional multi-state limit reliability function. Hence the following exact formulae is valid

$$\overline{\boldsymbol{R}}_n^{(3)}(t,\cdot) = [1, \overline{\boldsymbol{R}}_n^{(3)}(t,1), ..., \overline{\boldsymbol{R}}_n^{(3)}(t,z)], \qquad (19)$$

where

$$\overline{\boldsymbol{R}}_n^{(3)}(t,u) = 1 \text{ for } t < 0,$$

$$\overline{\boldsymbol{R}}_n^{(3)}(t,u) = \sum_{b=1}^{v} p_b \exp[-\lambda^{(b)}(u)\rho^{(b)}(u)nt, \qquad (20)$$

$t \ge 0,\ u = 1,2,...,z.$

Proposition 3.2
If components of the homogeneous multi-state parallel system at the operational state z_b have exponential reliability functions and the system have
(a) a single hot reservation of system components and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)},\ b_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)} \log 2n,$$

$u = 1,2,...,z,\ b = 1,2,...,v,$

then

$$\mathfrak{R}^{(1)}(t,\cdot) = [1, \mathfrak{R}^{(1)}(t,1), \ldots, \mathfrak{R}^{(1)}(t,z)], \qquad (21)$$

$t \in (-\infty,\infty),$

where

$$\mathfrak{R}^{(1)}(t,u) = 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-t]], \qquad (22)$$

$t \in (-\infty,\infty),\ u = 1,2,...,z,$

is its unconditional multi-state limit reliability function. Hence the following approximate formulae is valid

$$\boldsymbol{R}_n^{(1)}(t,\cdot) = [1, \boldsymbol{R}_n^{(1)}(t,1), ..., \boldsymbol{R}_n^{(1)}(t,z)], \qquad (23)$$

where

$$\boldsymbol{R}_n^{(1)}(t,u)$$

$$\cong 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-(\lambda^{(b)}(u)t - \log 2n)]] \qquad (24)$$

$t \in (-\infty,\infty),\ u = 1,2,...,z,$

(b) a single cold reservation of system components and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)},\ \frac{\exp[\lambda^{(b)}(u) b_n^{(b)}(u)]}{\lambda^{(b)}(u) b_n^{(b)}(u)} = n, \qquad (25)$$

$u = 1,2,...,z,\ b = 1,2,...,v,$

then

$$\mathfrak{R}^{(2)}(t,\cdot) = [1, \mathfrak{R}^{(2)}(t,1), \ldots, \mathfrak{R}^{(2)}(t,z)], \qquad (26)$$

$t \in (-\infty,\infty),$

where

$$\mathfrak{R}^{(2)}(t,u) = 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-t]], \qquad (27)$$

$t \in (-\infty,\infty),\ u = 1,2,...,z,$

is its unconditional multi-state limit reliability function, hence, the following approximate formulae is valid

$$\boldsymbol{R}_n^{(2)}(t,\cdot) = [1, \boldsymbol{R}_n^{(2)}(t,1), ..., \boldsymbol{R}_n^{(2)}(t,z)], \qquad (28)$$

where

$$\boldsymbol{R}_n^{(2)}(t,u) \cong 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-(\lambda^{(b)}(u)t - \lambda^{(b)}(u)b_n^{(b)}(u))]], (29)$$

$t \in (-\infty,\infty)$, $u = 1,2,...,z$,

(c) components with the reduced rate of departure by a factor $\rho(u)$, $u = 1,2,...,z$, and

$$a_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)\rho(u)},$$

$$b_n^{(b)}(u) = \frac{1}{\lambda^{(b)}(u)\rho(u)} \log n, \ u = 1,2,...,z,$$

$b = 1,2,...,v$,

then

$$\mathfrak{R}^{(3)}(t,\cdot) = [1, \mathfrak{R}^{(3)}(t,1), \ldots, \mathfrak{R}^{(3)}(t,z)], \qquad (30)$$

$t \in (-\infty,\infty)$,

where

$$\mathfrak{R}^{(3)}(t,u) = 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-t]], \qquad (31)$$

$t \in (-\infty,\infty)$, $u = 1,2,...,z$,

is its unconditional multi-state limit reliability function hence, the following approximate formulae is valid

$$\boldsymbol{R}_n^{(3)}(t,\cdot) = [1, \boldsymbol{R}_n^{(3)}(t,1), ..., \boldsymbol{R}_n^{(3)}(t,z)], \qquad (32)$$

where

$$\boldsymbol{R}_n^{(3)}(t,u) \cong 1 - \sum_{b=1}^{v} p_b \exp[-\exp[-(\lambda^{(b)}(u)\rho^{(b)}(u)t - \log n)]], (33)$$

$t \in (-\infty,\infty)$, $u = 1,2,...,z$.

4. Availability of multi-state series and parallel systems in variable operation conditions

There is presented a combination of reliability, availability improvement models of multi-state renewal systems only with non-ignored time of renovation in a model of variable in time operation processes. On the basis of those joined models, with assumption, that systems' improved conditional reliability functions dependent on variable in time operation states are the same as improved limit reliability functions of the exponential non-renewal multi-state series and parallel systems, improved availability characteristics of the systems are determined.

4.1 Multi-state systems with non-ignored time of renovation in variable operation conditions

We assume similarly as in non renewal systems considered in Point 3 that the changes of the process *Z*(*t*) states have an influence on the multi-state system reliability structure. The main characteristics of multi-state renewal system with hot, cold single reservation of system components and system with improved component's reliability related to their operation process can be approximately determined by taking account described system's operation process properties.

Proposition 4.1
If components of the multi-state renewal system with non-ignored time of renovation at the operational states z_b, $b = 1,2,...,v$, have exponential reliability functions and the time of the system renovation has the mean value $[\mu_o(r)]^{(b)}$ and the standard deviation $[\sigma_o^2(r)]^{(b)}$, then:

i) the distribution function of the time $\overline{\overline{S}}_N{}^{(k)}(r)$, $k = 1,2,3$, until the *Nth* system's renovation, for sufficiently large *N*, has approximately normal distribution

$$N(N(\mu^{(k)}(r) + \mu_o(r)), \sqrt{N({\sigma^{(k)}}^2(r) + \sigma_o^2(r))})$$

i.e.,

$$\overset{=}{F}{}^{(N)^{(k)}}(t,r) = P(\overset{=}{S}_N{}^{(k)}(r) < t)$$

$$\cong F_{N(0,1)}{}^{(k)}\left(\frac{t-N(\mu^{(k)}(r)+\mu_o(r))}{\sqrt{N({\sigma^{(k)}}^2(r)+\sigma_o^2(r))}}\right),$$

$t\in(-\infty,\infty)$, $N=1,2,\ldots$, $r\in\{1,2,\ldots,z\}$,

ii) the expected value and the variance of the time $\overline{\overline{S}}_N{}^{(k)}(r)$, $k=1,2,3,$ until the *Nth* system's renovation take respectively forms

$$E[\overline{\overline{S}}_N{}^{(k)}(r)]\cong N(\mu^{(k)}(r)+\mu_o(r)),$$

$$D[\overline{\overline{S}}_N{}^{(k)}(r)]\cong N({\sigma^{(k)}}^2(r)+\sigma_o^2(r)),$$

$r\in\{1,2,\ldots,z\}$,

iii) the distribution function of the time $\overline{S}_N{}^{(k)}(r)$, $k=1,2,3,$ until the *Nth* exceeding the reliability critical state *r* of this system takes form

$$\overline{F}^{(N)(k)}(t,r)=P(\overline{S}_N{}^{(k)}(r)<t)$$

$$\cong F_{N(0,1)}{}^{(k)}\left(\frac{t-N(\mu^{(k)}(r)+\mu_o(r))+\mu_o(r)}{\sqrt{N({\sigma^{(k)}}^2(r)+\sigma_o^2(r))-\sigma_o^2(r)}}\right),$$

$t\in(-\infty,\infty)$, $N=1,2,\ldots$, $r\in\{1,2,\ldots,z\}$,

iv) the expected value and the variance of the time $\overline{S}_N{}^{(k)}(r)$, $k=1,2,3,$ until the *Nth* exceeding the reliability critical state *r* of this system take respectively forms

$$E[\overline{S}_N{}^{(k)}(r)]\cong N\mu^{(k)}(r)+(N-1)\mu_o(r),$$

$$D[\overline{S}_N{}^{(k)}(r)]\cong N{\sigma^{(k)}}^2(r)+(N-1)\sigma_o^2(r),$$

$r\in\{1,2,\ldots,z\}$,

v) the distribution of the number $\overline{\overline{N}}^{(k)}(t,r)$, $k=1,2,3,$ of system's renovations up to the moment $t, t\geq 0,$ is of the form

$$P(\overline{\overline{N}}^{(k)}(t,r)=N)\cong$$

$$F_{N(0,1)}{}^{(k)}\left(\frac{N(\mu^{(k)}(r)+\mu_o(r))-t}{\sqrt{\frac{t}{\mu^{(k)}(r)+\mu_0(r)}({\sigma^{(k)}}^2(r)+\sigma_o^2(r))}}\right)$$

$$-F_{N(0,1)}{}^{(k)}\left(\frac{(N+1)(\mu^{(k)}(r)+\mu_o(r))-t}{\sqrt{\frac{t}{\mu^{(k)}(r)+\mu_0(r)}({\sigma^{(k)}}^2(r)+\sigma_o^2(r))}}\right),$$

$N=1,2,\ldots$, $r\in\{1,2,\ldots,z\}$,

vi) the expected value and the variance of the number $\overline{\overline{N}}^{(k)}(t,r)$, $k=1,2,3,$ of system's renovations up to the moment $t, t\geq 0,$ take respectively forms

$$\overline{H}^{(k)}(t,r)\cong\frac{t}{\mu^{(k)}(r)+\mu_o(r)},$$

$$\overline{D}^{(k)}(t,r)$$

$$\cong\frac{t}{(\mu^{(k)}(r)+\mu_o(r))^3}({\sigma^{(k)}}^2(r)+{\sigma_o}^2(r)),$$

$r\in\{1,2,\ldots,z\}$,

vii) the distribution of the number $\overline{N}^{(k)}(t,r)$, $k=1,2,3,$ of exceeding the reliability critical state *r* of this system up to the moment $t, t\geq 0,$ is of the form

$$P(\overline{N}^{(k)}(t,r)=N)\cong$$

$$F_{N(0,1)}{}^{(k)}\left(\frac{N(\mu^{(k)}(r)+\mu_o(r))-t-\mu_0(r)}{\sqrt{\frac{t+\mu_0(r)}{\mu^{(k)}(r)+\mu_0(r)}({\sigma^{(k)}}^2(r)+\sigma_o^2(r))}}\right)$$

$$-F_{N(0,1)}{}^{(k)}\left(\frac{(N+1)(\mu^{(k)}(r)+\mu_o(r))-t-\mu_0(r)}{\sqrt{\frac{t+\mu_0(r)}{\mu^{(k)}(r)+\mu_0(r)}({\sigma^{(k)}}^2(r)+\sigma_o^2(r))}}\right),$$

$N=1,2,\ldots$, $r\in\{1,2,\ldots,z\}$,

viii) the expected value and the variance of the number $\overline{N}^{(k)}(t,r)$, $k=1,2,3,$ of exceeding the reliability critical state *r* of this system up to the moment $t, t \geq 0,$ for sufficiently large t, are approximately respectively given by

$$\bar{H}^{(k)}(t,r) \cong \frac{t+\mu_0(r)}{\mu^{(k)}(r)+\mu_o(r)},$$

$$\bar{D}^{(k)}(t,r)$$

$$\cong \frac{t+\mu_0(r)}{(\mu^{(k)}(r)+\mu_o(r))^3}(\sigma^{(k)^2}(r)+\sigma_o{}^2(r)),$$

$r \in \{1,2,...,z\},$

ix) the availability coefficient of the system at the moment t is given by the formula

$$K^{(k)}(t,r) \cong \frac{\mu^{(k)}(r)}{\mu^{(k)}(r)+\mu_o(r)}, \ t \geq 0, \ r \in \{1,2,...,z\},$$

x) the availability coefficient of the system in the time interval $< t, t+\tau), \tau > 0,$ is given by the formula

$$K^{(k)}(t,\tau,r) \cong \frac{1}{\mu^{(k)}(r)+\mu_o(r)}\int_{\tau}^{\infty}\boldsymbol{R}_n^{(k)}(t,r)dt,$$

$t \geq 0, \ \tau > 0, \ r \in \{1,2,...,z\},$

where for $u = r,$ and $\mu^{(k)}(r)$ and $\sigma^{(k)}(r)$, $k = 1,2,3,$ are given by:

- for a homogeneous series system
(a) with a single hot reservation of system components

$$\mu^{(1)}(r) \cong \sum_{b=1}^{v} p_b \frac{\sqrt{\pi}}{2\lambda^{(b)}(r)n}, \ r \in \{1,2,...,z\}, \quad (34)$$

$$[\sigma^{(1)}(r)]^2 = \int_{-\infty}^{+\infty} t^2 d\overline{\boldsymbol{F}}_n{}^{(1)}(t,r) - [\mu^{(1)}(r)]^2,$$

$r \in \{1,2,...,z\},$

where $\mu^{(1)}(r)$ is given by the formula (34)

$$\overline{\boldsymbol{F}}_n{}^{(1)}(t,r) = 1 - \overline{\boldsymbol{R}}_n{}^{(1)}(t,r), \ t \in (-\infty,\infty),$$

and $\overline{\boldsymbol{R}}_n{}^{(1)}(t,r)$ is given by the formulae (11)-(12) for $u = r, \ r \in \{1,2,...,z\}$,

(b) with a single cold reservation of system components

$$\mu^{(2)}(r) \cong \sum_{b=1}^{v} p_b \frac{\sqrt{\pi}}{\lambda^{(b)}(r)\sqrt{2n}}, \ r \in \{1,2,...,z\}, \quad (35)$$

$$[\sigma^{(2)}(r)]^2 = \int_{-\infty}^{+\infty} t^2 d\overline{\boldsymbol{F}}_n{}^{(2)}(t,r) - [\mu^{(2)}(r)]^2,$$

$r \in \{1,2,...,z\},$

where $\mu^{(2)}(r)$ is given by the formula (35)

$$\overline{\boldsymbol{F}}_n{}^{(2)}(t,r) = 1 - \overline{\boldsymbol{R}}_n{}^{(2)}(t,r), \ t \in (-\infty,\infty),$$

and $\overline{\boldsymbol{R}}_n{}^{(2)}(t,r)$ is given by the formulae (15)-(16) for $u = r, \ r \in \{1,2,...,z\}$,

(c) with the reduced rate of departure by a factor $\rho(u), \ u = 1,2,...,z,$ of its components

$$\mu^{(3)}(r) \cong \sum_{b=1}^{v} p_b \frac{1}{\lambda^{(b)}(r)\rho(r)n}, \ r \in \{1,2,...,z\}, \quad (36)$$

$$[\sigma^{(3)}(r)]^2 = \int_{-\infty}^{+\infty} t^2 d\overline{\boldsymbol{F}}_n{}^{(3)}(t,r) - [\mu^{(3)}(r)]^2,$$

$r \in \{1,2,...,z\},$

where $\mu^{(3)}(r)$ is given by the formula (36)

$$\overline{\boldsymbol{F}}_n{}^{(3)}(t,r) = 1 - \overline{\boldsymbol{R}}_n{}^{(3)}(t,r), \ t \in (-\infty,\infty),$$

and $\overline{\boldsymbol{R}}_n{}^{(3)}(t,r)$ is given by the formulae (19)-(20) for $u = r, \ r \in \{1,2,...,z\}$,

- for a homogeneous parallel system
(a) with a single hot reservation of system components

$$\mu^{(1)}(r) \cong \sum_{b=1}^{v} p_b (C/\lambda^{(b)}(r) + \log 2n/\lambda^{(b)}(r)), \ (37)$$

$r \in \{1,2,...,z\},$

where $C \cong 0.5772$ is Euler's constant

$$[\sigma^{(1)}(r)]^2 = \int_{-\infty}^{+\infty} t^2 d\overline{\boldsymbol{F}}_n^{\,(1)}(t,r) - [\mu^{(1)}(r)]^2,$$

$$r \in \{1,2,...,z\},$$

where $\mu^{(1)}(r)$ is given by the formula (37)

$$\boldsymbol{F}_n^{\,(1)}(t,r) = 1 - \boldsymbol{R}_n^{\,(1)}(t,r), \ \ t \in (-\infty,\infty),$$

and $\boldsymbol{R}_n^{\,(1)}(t,r)$ is given by the formulae (23)-(24) for $u = r, \ r \in \{1,2,...,z\}$,

(b) with a single cold reservation of system components

$$\mu^{(2)}(r)$$

$$\cong \sum_{b=1}^{v} p_b (C/\lambda^{(b)}(r) + \log(n\lambda^{(b)}(r) b_n^{(b)}(r)), \qquad (38)$$

$$r \in \{1,2,...,z\},$$

where $C \cong 0.5772$ is Euler's constant and

$$\frac{\exp[\lambda^{(b)}(r) b_n^{(b)}(r)]}{\lambda^{(b)}(r) b_n^{(b)}(r)} = n$$

$$[\sigma(r)]^2 = \int_{-\infty}^{+\infty} t^2 d\overline{\boldsymbol{F}}_n(t,r) - [\mu(r)]^2, \ \ r \in \{1,2,...,z\},$$

where $\mu^{(2)}(r)$ is given by the formula (38)

$$\boldsymbol{F}_n^{\,(2)}(t,r) = 1 - \boldsymbol{R}_n^{\,(2)}(t,r), \ \ t \in (-\infty,\infty),$$

and $\boldsymbol{R}_n^{\,(2)}(t,r)$ is given by the formulae (25)-(26) for $u = r, \ r \in \{1,2,...,z\}$,

(c) with the reduced rate of departure by a factor $\rho(u), \ u = 1,2,...,z,$ of its components

$$\mu^{(3)}(r)$$

$$\cong \sum_{b=1}^{v} p_b (C/(\lambda^{(b)}(r)\rho(r)) + \log n/(\lambda^{(b)}(r)\rho(r))), \qquad (39)$$

$$r \in \{1,2,...,z\},$$

where $C \cong 0.5772$ is Euler's constant

$$[\sigma^{(3)}(r)]^2 = \int_{-\infty}^{+\infty} t^2 d\overline{\boldsymbol{F}}_n^{\,(3)}(t,r) - [\mu^{(3)}(r)]^2,$$

$$r \in \{1,2,...,z\},$$

where $\mu^{(3)}(r)$ is given by the formula (39)

$$\boldsymbol{F}_n^{\,(3)}(t,r) = 1 - \boldsymbol{R}_n^{\,(3)}(t,r), \ \ t \in (-\infty,\infty),$$

and $\boldsymbol{R}_n^{\,(3)}(t,r)$ is given by the formulae (32)-(33) for $u = r, \ r \in \{1,2,...,z\}$.

5. Effects of comparison on reliability and availability improvement

In order to determine the value of the coefficient $\rho(r), \ r \cong \{1,2,...,z\},$ by which it is necessary to reduce (multiply by) the failure rates of departure of the reliability states subsets of basic components of the non renewal system or renewal with non-ignored time of renovation in order to receive the system having the mean value of the number of exceeding the critical reliability state during the time t as the mean value of the number of exceeding the critical reliability state during the time t of the system with either hot or cold single reserve of basic components we can solve either the equation

$$\mu^{(1)}(r) = \mu^{(3)}(r), \ \ t \ge 0, \ \ r \in \{1,2,...,z\}, \qquad (40)$$

or respectively the equation

$$\mu^{(2)}(r) = \mu^{(3)}(r), \ \ t \ge 0, \ \ r \in \{1,2,...,z\}, \qquad (41)$$

and if the system is the system with not ignored time of renovation in order to receive the system having the same availability as the availability of the system with either hot or cold single reserve of basic components we can solve either the equation

$$K^{(1)}(r) = K^{(3)}(r), \ \ t \ge 0, \ \ r \in \{1,2,...,z\}, \qquad (42)$$

or respectively the equation

$$K^{(2)}(r) = K^{(3)}(r), \ \ t \ge 0, \ \ r \in \{1,2,...,z\}. \qquad (43)$$

Proposition 5.1

If components of the multi-state non renewal system or renewal system with non ignored time of renovation at the operational states z_b, $b = 1,2,...,v,$ have exponential reliability functions then the coefficient $\rho(r)$, $r \in \{1,2,...,z\},$ is in the form

- for a homogeneous series system

in the first case by (40) , after considering (34) and (36), we get

$$\rho(r) = \frac{2}{\sqrt{\pi n}}, \; r \in \{1,2,...,z\}, \quad (44)$$

in the second case by (41), after considering (35) and (36), we get

$$\rho(r) = \frac{\sqrt{2}}{\sqrt{\pi n}}, \; r \in \{1,2,...,z\}, \quad (45)$$

- for a homogeneous parallel system

in the first case by (40) , after considering (36) and (38), we get

$$\rho(r) = \frac{C + \log n}{C + \log 2n}, \; r \in \{1,2,...,z\}, \quad (46)$$

in the second case by (41), after considering (37) and (38), we get

$$\rho(r) = \frac{C + \log n}{C + \log \lambda^{(b)}(r) b_n^{(b)}(r) n}, \; r \in \{1,2,...,z\}, \; (47)$$

where $C \cong 0.5772$ is Euler's constant and

$$\frac{\exp[\lambda^{(b)}(r) b_n^{(b)}(r)]}{\lambda^{(b)}(r) b_n^{(b)}(r)} = n, \; r \in \{1,2,...,z\}.$$

6. Conclusion

In the paper the multi-state approach to the improvement of systems' reliability, and availability for series and parallel systems has been presented. Constructed in this paper the final integrated, general analytical models of complex systems reliability and availability improvement related to their operation processes are very important for the further optimization of system operation costs.

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