BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 61, No. 3, 2013
DOI: 10.2478/bpasts-2013-0076

Equivalent circuits for averaged description of DC-DC switch-mode power converters based on separation of variables approach

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Abstract. Large-signal and small-signal averaged models of basic switch-mode DC-DC power converters: BUCK (step-down) and BOOST (step-up) are presented. Models are derived with the separation of variables approach and have the form of equivalent circuits, suitable for a circuit simulation. Apart from equivalent circuits, small-signal transmittances of converters for CCM and DCM modes are discussed. Parasitic resistances of all components of converters are taken into account. A few examples of simulations and measurement results of selected converter characteristics are also presented. It is shown, that neglecting parasitic resistances (often met in works of other authors) may lead to serious errors in an averaged description of converters.

Key words: switch-mode power converters, BUCK, BOOST, averaged models, equivalent circuits.

1. Introduction

DC-DC switch mode power converters find great amount of applications and are under steady development [1–5]. The considerations of this paper concern the most important group of converters based on pulse-width modulation (PWM) with constant switching frequency $f_S = 1/T_S$ (T_S – switching period). Two most popular converters are considered – step-down (BUCK) and step-up (BOOST). Basic converters consist of power stage (containing semiconductor switches and reactive components L and C) and control circuit, which controls the state of switches described usually by duty ratio $d_A = t_{\mathrm{ON}}/T_S = t_{\mathrm{ON}}/(t_{\mathrm{ON}} + t_{\mathrm{OFF}})$, where t_{ON}, t_{OFF} represent length of time segments in which the main switch is ON and OFF, respectively. Two modes of converter operation are possible: continuous conduction mode (CCM) and discontinuous conduction mode (DCM). The difference in simple converters consists in inductor current waveforms: in CCM this current is steadily nonzero, in DCM it is zero in a part of a switching period.

In the design of a control circuit of the converter, the use of adequate models of its power stage is essential. Usually the averaged models of the converter power stage are used, representing relations between currents and voltages averaged over an elementary switching period. Standard methods of an averaged model derivation, namely state-space averaging and switch pair averaging are widely described in the literature, for example [1, 2, 6–9]. The switch averaging approach is most commonly used and is described in many references as more convenient than state-space averaging [1, 2, 7 ,8]. One of the advantages of the switch averaging approach is the simplicity of obtaining equivalent circuits of converters, easy to analyze with the use of popular circuit simulators (such as SPICE). In paper [9] it is shown however, that switch averaging in the form presented in the literature leads to accurate results only in the case of ideal converters working in CCM mode. In derivation of averaged models (presented in [10], also in [1] (Ch. 7), and [2] (Ch. 10), and repeated for example in [14, 15]) the values of currents and voltages in particular subintervals (ON or OFF) are in some points mistaken with averaged values over a switching period. In derivation of averaged models for DCM [1, 2, 11–13, 16] the treatment of average voltage on an inductor is incoherent (assumed to be zero in one part of considerations and nonzero in another). As a result, some inaccuracies in averaged models of converters with parasitic resistances or working in DCM are obtained, as discussed in [9]. Another approach to the averaged model derivation is based on the separation of variables. This method, as shown in [17], is simple and gives proper results for CCM and DCM, for ideal converters as well as converters composed of non-ideal components.

Switching frequency f_S of typical PWM converter is many times greater than power stage natural frequency and frequencies of eventual perturbations of input voltage or load current. As a consequence, it may be assumed that in the course of single subinterval (ON or OFF in given switching period), currents and voltages remain constant or changes linearly with time. It is essential for further considerations to distinguish several types of variables and to use proper notation for them. Traditionally, in description of electric circuits, three types of variables are considered: (a) instantaneous, time-dependent large signal quantities, (b) steady state, quiescent values and (c) small-signal time-dependent terms of instantaneous quantities. The notation, on the example of inductor current, is: $i_L(t)$, I_L and $i_l(t)$ respectively. The s-domain representation of small-signal quantity is $I_l(s)$ or simply I_l. For switch-mode converters, additional kinds of quantities should

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be distinguished, namely – local average of large-signal quantity "w" for given subinterval, denoted by w(ON) and w(OFF) (for example i_L(ON) and i_L(OFF) eventually i_L(OFF1) and i_L(OFF2) for DCM) and average quantities over the whole switching period denoted by additional index "S" (e.g. i_{LS}). These quantities are interrelated by equation:

$$w_S = w(\mathrm{ON}) \cdot d_A + w(\mathrm{OFF}) \cdot (1 - d_A). \qquad (1)$$

The average quantities (for given switching period) should be distinguished from DC (or quiescent) currents and voltages, corresponding to steady-state conditions.

Relations between w(ON) and w(OFF) are different for two groups of variables in the converter power stage. For the first group of variables (named "A"), the relation holds:

$$w_A(\mathrm{ON}) = w_A(\mathrm{OFF}) = w_{AS} \qquad (2)$$

and for the second group (B):

$$w_B(\mathrm{ON}) \neq w_B(\mathrm{OFF}) \neq w_{BS}. \qquad (3)$$

The example of variables of the group A in BUCK converter in CCM are input and output voltage v_G, v_O and inductor current i_L. On the other hand, inductor voltage v_L, capacitor current i_C, input current i_G and currents of switches belong to group B. The principal idea of obtaining averaged models in CCM is to express each quantity of group B as a function of quantities of group A for subintervals ON and OFF and then to calculate its averaged value from Eq. (1) [17]. Averaged models of basic converters derived on the above principle are presented in [17] in the form of equation systems. Models described in present paper have the form of equivalent circuits corresponding to these equations, therefore may be directly used in circuit simulators. In Sec. 2, large-signal and small-signal averaged models of BUCK and BOOST converters in CCM mode, in the form of equivalent circuits are presented. Equivalent circuits corresponding to averaged models of converters in DCM mode are shown in Sec. 3. Models presented in Secs. 2 and 3 are based on equation systems derived in paper [17]. Several, important equations from paper [17] are repeated. Some additional equations needed for equivalent circuits (absent in [17]) are derived. Based on equivalent circuits shown in Secs. 2 and 3, some small-signal converter transmittances are also presented. In Sec. 4 the examples of simulation and measurement results of averaged characteristics of converters are shown.

2. Averaged models of BUCK and BOOST converters in continuous conduction mode (CCM)

2.1. Introduction. Power stages of BUCK and BOOST converters are depicted in Fig. 1 with parasitic resistances of transistor R_T, diode R_D, inductor R_L and capacitor R_C included.

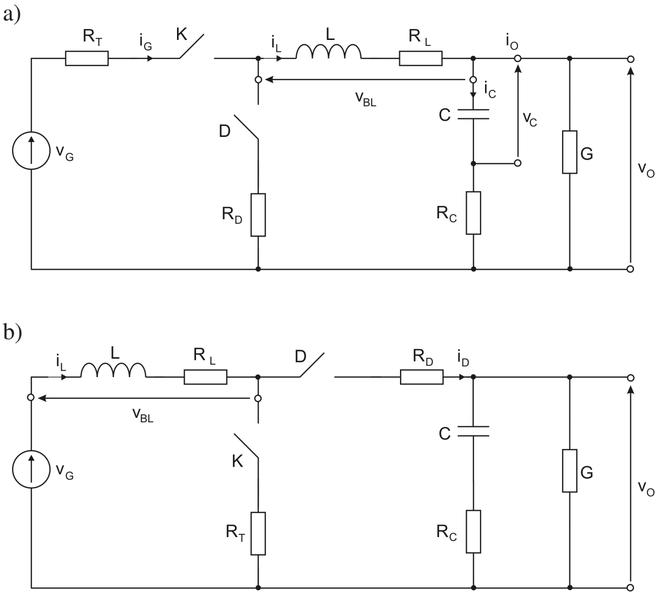

Fig. 1. Power stages of BUCK (a) and BOOST (b) converters with parasitic resistances

These resistances are assumed to be zero for ideal converters. Large signal averaged models for ideal BUCK and BOOST converters are obtained from equations given in paper [17]. The results are shown in Figs. 2 and 3. The mentioned equations are not repeated in the present paper and equivalent circuits of Figs. 2 and 3 are not discussed because they are the same as equations and equivalent circuits obtained by switch-averaging procedure described in many sources, for example in [1, 2, 7].

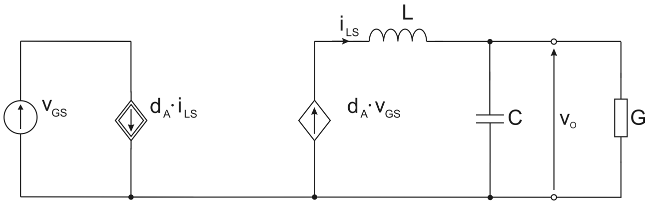

Fig. 2. Large signal averaged model of ideal BUCK converter in CCM

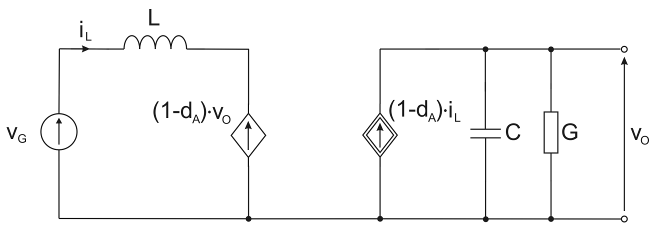

Fig. 3. Large signal averaged model of ideal BOOST converter in CCM

Small-signal equivalent circuits derived from above large-signal models are depicted in Figs. 4 and 5 and also correspond to circuits presented and discussed in the literature. Well-known formulas for input-to-output and control-to-output small-signal transmittances (see for example [1, 2, 9]) may be easily obtained from Figs. 4 and 5.

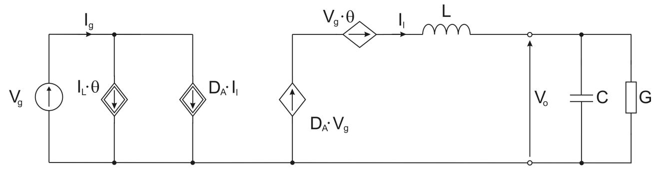

Fig. 4. Small signal averaged model of ideal BUCK converter in CCM

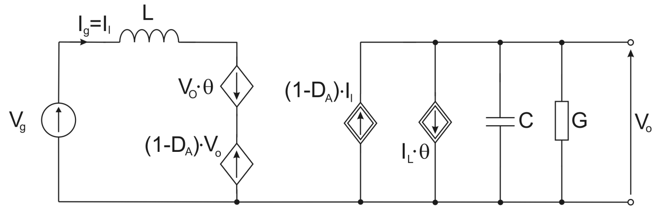

Fig. 5. Small signal averaged model of ideal BOOST converter in CCM

2.2. Large-signal models of non-ideal converters in CCM. Equations describing averaged currents and voltages in nonideal converter (with parasitic resistances), obtained by separation of variables approach, differ in some particulars from those based on switch-averaging [17]. Large-signal, averaged equation system for BUCK converter in CCM based on description presented in [17], Subsec. 5.2., is as follows (subscript "S" for averaged values is omitted):

$$v_{BL} = L \cdot \frac{di_L}{dt} + R_L \cdot i_L, \tag{4}$$

$$v_O = v_C + R_C \cdot i_C = v_C + R_C \cdot C \cdot \frac{dv_C}{dt}, \tag{5}$$

$$i_L = i_C + G \cdot v_O, \tag{6}$$

$$d_A \cdot v_G = v_{BL} + v_O + i_L \cdot R_X(d_A), \tag{7}$$

where

$$R_X(d_A) = d_A \cdot R_T + (1 - d_A) \cdot R_D, \tag{8}$$

$$i_G = d_A \cdot i_L. \tag{9}$$

Large-signal averaged equivalent circuit of nonideal BUCK in CCM corresponding to Eqs. (4)–(9) is shown in Fig. 6.

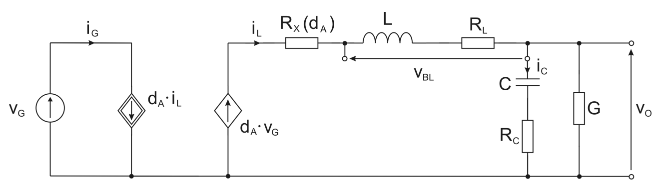

Fig. 6. Large signal averaged model of BUCK converter with parasitic resistances in CCM

A large signal averaged model of BOOST converter is obtained from (4), (5) above and following equations [17] (Subsec. 5.2.2):

$$i_D = G \cdot v_O + i_C = G \cdot v_O + C \cdot \frac{dv_C}{dt}, \tag{10}$$

$$i_D = (1 - d_A) \cdot i_L, \tag{11}$$

$$v_G = v_{BL} + i_L \cdot R_X + (1 - d_A) \cdot v_O, \tag{12}$$

where R_X is given by (8). The resulting large-signal equivalent circuit for BOOST in CCM is shown in Fig. 7.

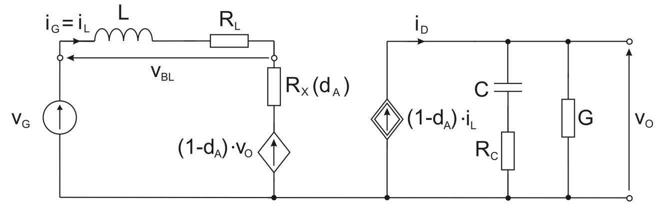

Fig. 7. Large signal averaged model of BOOST converter with parasitic resistances in CCM

Symbol $R_X(d_A)$ in Figs. 6 and 7 denotes resistance controlled by duty ratio d_A and may be expressed as:

$$R_X = R_D + d_A \cdot (R_T - R_D). \tag{8'}$$

The voltage drop v_X across R_X is:

$$v_X = R_D \cdot i_L + v_Y, \tag{13}$$

where

$$v_Y = (R_T - R_D) \cdot d_A \cdot i_L = f_Y(d_A, i_L). \tag{14}$$

The element R_X in Figs. 6 and 7 (with current i_L flowing in it) may be therefore replaced by subcircuit shown in Fig. 8.

From large-signal models presented above, one may obtain DC models, i.e. models for quiescent values (V_G, V_O, D_A, I_L) and small-signal dynamic models. Derivation of DC models is straightforward – in Figs. 6 and 7, the inductors should be replaced by short-circuit, capacitors – by open circuit, and symbols v_G, i_L etc. by V_G, I_L, etc.

2.3. Small-signal equivalent circuits. Small-signal averaged models of nonideal BUCK and BOOST converters are obtained by proper linearization of large-signal models. Some components in Figs. 6 and 7 are linear (L, R_L, C, R_C, G). Nonlinear elements are described by products of variables, for example $d_A \cdot i_L$ and $d_A \cdot v_G$ in Fig. 6. Small-signal equivalents of such nonlinear elements, obtained according to well-known procedure (e.g. [1, 9]) are, for example:

$$d_A \cdot v_G \rightarrow V_G \cdot \theta(s) + D_A \cdot V_g(s), \tag{15}$$

where $\theta(s)$ and $V_g(s)$ are the s-domain representation of small-signal components and D_A, V_G are D.C. steady-state values of d_A and v_G respectively [9, 17]. Small-signal equivalents of $d_A \cdot i_L$, $(1 - d_A) \cdot v_O$, $(1 - d_A) \cdot i_L$ are obtained similarly. Small-signal equivalent of resistance R_X is obtained (with the use of Fig. 8) in the form depicted in Fig. 9, where

$$R_E = R_T \cdot D_A + R_D \cdot (1 - D_A). \tag{16}$$

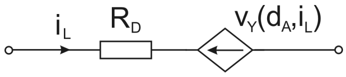

Fig. 8. Equivalent subcircuit for R_X

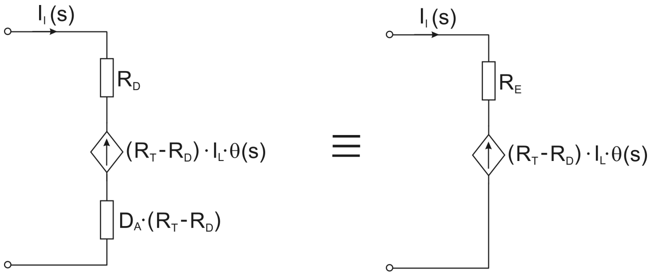

Fig. 9. Small-signal equivalent subcircuit for R_X

The resulting small-signal averaged models of nonideal BUCK and BOOST converters working in CCM are shown in Figs. 10 and 11. Equivalent model in Fig. 10a) is obtained from Fig. 6 with the use of Eqs. (8') and (13)–(16).

a)

b)

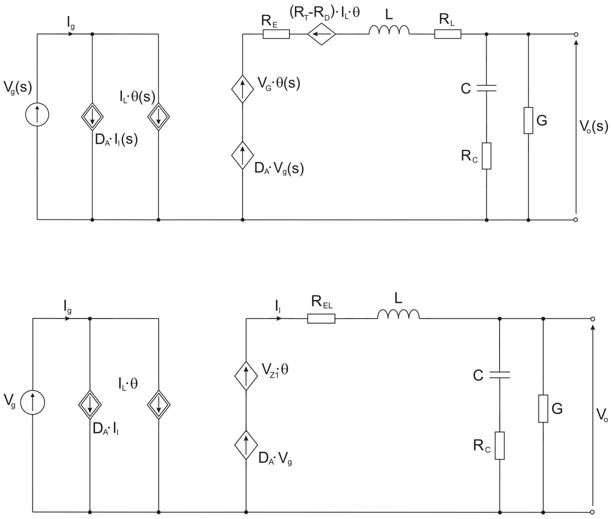

Fig. 10. Small-signal averaged model of non-ideal BUCK converter in CCM: a) as obtained from above formulas; b) after reduction of elements

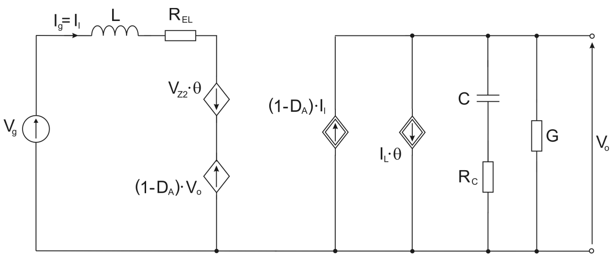

Fig. 11. Small-signal averaged model of non-ideal BOOST converter in CCM

In Fig. 10b the notation:

$$R_{EL} = R_E + R_L \tag{17}$$

and

$$V_{Z1} = V_G + (R_D - R_T) \cdot I_L \tag{18}$$

is introduced and term (s) at the symbols of small-signal s-domain variables is omitted. Small-signal equivalent circuit of ideal BUCK converter in CCM is easily obtained from Fig. 10 by setting $R_L = R_C = R_T = R_D = 0$ and is identical with models presented elsewhere (e.g. [1, 9]).

It should be pointed out, that in Figs. 10a and 11 the equivalent models of real inductor and capacitor in the form of L, R_L and C, R_C are present. It is possible to improve model accuracy by introducing other models of inductor and capacitor including, for example stray capacitance of inductor or parasitic inductance of capacitor.

The derivation of a small-signal model of non-ideal BOOST in CCM, shown in Fig. 11, is similar to that of BUCK model. In particular, expressions (16) and (17) are valid and symbol V_{Z2} denotes:

$$V_{Z2} = V_O + (R_D - R_T) \cdot I_L. \tag{19}$$

Models in the form of equivalent circuits derived in this subsection differ from models presented typically in the literature. Large-signal and small-signal averaged models in widely used textbook of Erickson and Maksimovic [1] are valid only for ideal converters, without parasitic resistances. It is a very serious simplification, because parasitic resistances of converter components strongly influence converter behavior, as shown for example in [18] and [19]. Small-signal models of converters, useful in designing control circuits, may be found in application notes of controller manufacturers. Typically, in those sources, parasitic resistances are only partially included. For example, in [20–22], only parasitics of capacitor and inductor are taken into account. In textbook of Kazimierczuk [2] (and related papers of the same author [23, 24]), one may find models including properly parasitics of inductor and capacitor. Parasitic resistances of semiconductor switches are introduced on "energy conservation principle", not properly justified in this application. As a result, those models differ slightly from models described in present paper. Differences in large signal models of BUCK and BOOST consist in description of resistance R_X (see Figs. 6 and 7). Small-signal models in [2] do not include controlled source $(R_T - R_D) \cdot I_L \cdot \theta$ for BUCK and proper term of $V_{Z2} \cdot \theta$ for BOOST.

2.4. Examples of small-signal transmittances for CCM. Small-signal transmittances, input-to-output H_g and control-to-output H_d for both converters and their output impedance Z_{out} are derived below from the equivalent circuits shown in Figs. 10b and 11. Transmittances H_g and H_d are involved in the expression for small-signal term of output voltage:

$$V_o = H_g \cdot V_g + H_d \cdot \theta. \tag{20}$$

Formal definitions of H_g and H_d result directly from (20) (see for example [9] or [1]).

A small-signal output impedance of DC-DC converter, especially BUCK, is an important characteristic in designing converters applied to supplying modern processors in computer equipment [25–28]. For derivation of this characteristic, it is assumed, that a dummy current source I_{out} is connected to output terminals of converter and the resulting small-signal term V_o of output voltage is calculated. The output impedance is then defined as:

$$Z_{out} = \left.\frac{V_o}{I_{out}}\right|_{\substack{V_g=0\\ \theta=0}}. \tag{21}$$

In deriving H_g, H_d and Z_{out} for nonideal BUCK converter in CCM, the input part of circuit shown in Fig. 10b may be neglected.

The equivalent circuit of BUCK for this situation is depicted in Fig. 12a. Symbols Z_L and Y_{GC} are explained in Figs. 12b and c and by Eqs. (22), (23).

$$Z_L = R_{EL} + sL, \tag{22}$$

$$Y_{GC} = \frac{s \cdot C_Z + G}{s \cdot C \cdot R_C + 1}, \tag{23}$$

$$C_Z = C \cdot (1 + R_C \cdot G). \tag{24}$$

For calculations of given transmittance, the particular sources in Fig. 12a are set to zero. Input-to-output transmittance H_g is obtained for $\theta = 0$ and $I_{out} = 0$. Output voltage is:

$$V_o = D_A \cdot V_g \cdot \frac{1/Y_{GC}}{1/Y_{GC} + Z_L} \tag{25}$$

therefore

$$H_g(BUCK, CCM) = \frac{D_A}{Z_L \cdot Y_{GC} + 1}. \tag{26}$$

Transmittance H_d is obtained for $I_{out} = 0$ and $V_g = 0$

$$H_d(BUCK, CCM) = \frac{V_{Z1}}{Z_L \cdot Y_{GC} + 1}, \tag{27}$$

where V_{Z1} is given by Eq. (18).

Z_{out}, according to definition (21) is:

$$Z_{out}(BUCK, CCM) = \frac{1}{Y_{GC} + 1/Z_L} = \frac{Z_L}{Z_L \cdot Y_{GC} + 1}. \tag{28}$$

By introducing (22) and (23) into (26)–(28) one obtains:

$$H_g(BUCK, CCM) = \frac{D_A \cdot (s \cdot C \cdot R_C + 1)}{a^*}, \tag{29}$$

$$H_d(BUCK, CCM) = \frac{V_{Z1} \cdot (s \cdot C \cdot R_C + 1)}{a^*}, \tag{30}$$

$$Z_{out}(BUCK, CCM) = \frac{s^2 \cdot L \cdot C \cdot R_C + s \cdot (L + C \cdot R_C \cdot R_{EL}) + R_{EL}}{a^*}, \tag{31}$$

where

$$a^* = s^2 \cdot L \cdot C_Z + s \cdot (G \cdot L + C_Z \cdot R_{EL} + C \cdot R_C) + R_{EL} \cdot G + 1.$$

For calculation of transmittances H_g, H_d, and Z_{out} of nonideal BOOST converter in CCM, symbols Z_L, Y_{GC} and I_{out} are used to modify circuit of Fig. 11, as seen in Fig. 13.

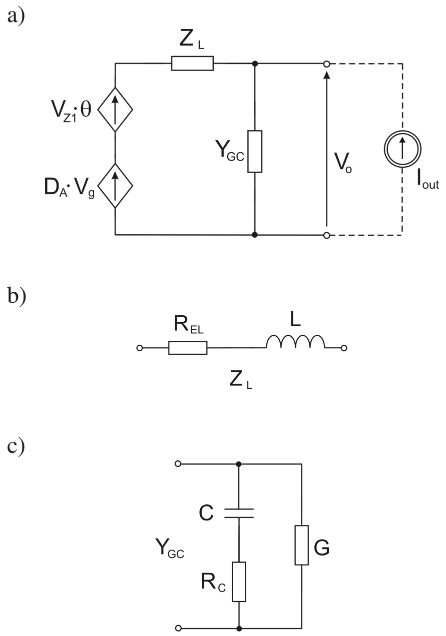

Fig. 12. Model of non-ideal BUCK in CCM for calculations of H_g, H_d and Z_{out}

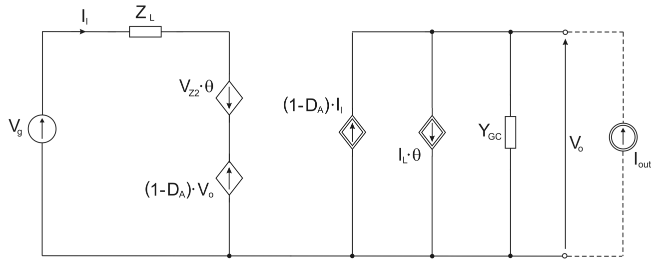

Fig. 13. Model of non-ideal BOOST in CCM for calculations of H_g, H_d and Z_{out}

For derivation of transmittance H_g, sources $V_{Z2} \cdot \theta$, $I_L \cdot \theta$ and I_{out} are set to zero. For calculation of H_d, sources V_g and I_{out} are set to zero. The resulting transmittances H_g and H_d for nonideal BOOST in CCM are given in [17]. For calculation of Z_{out}, according to (21), the circuit of Fig. 13 is modified to obtain scheme shown in Fig. 14.

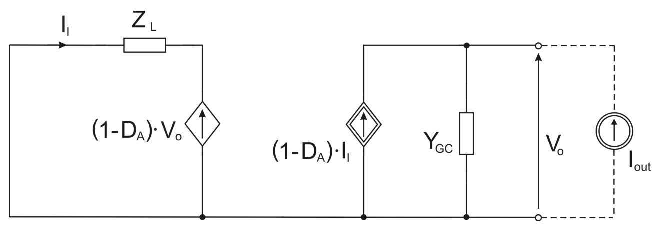

Fig. 14. Equivalent circuit for calculations of Z_{out} (BOOST, CCM)

From the input loop, current I_l is:

$$I_l = -\frac{V_o \cdot (1 - D_A)}{Z_L}. \tag{32}$$

After introducing it into description of output loop, one obtains:

$$V_o = \left(I_{out} - \frac{(1 - D_A)^2}{Z_L} \cdot V_o\right) \cdot \frac{1}{Y_{GC}} \tag{33}$$

and

$$Z_{out} = \frac{Z_L}{Y_{GC} \cdot Z_L + (1 - D_A)^2}. \quad (34)$$

After introducing (22) and (23) into (34) we get finally:

$$Z_{out}(BOOST, CCM) = \frac{s^2 \cdot L \cdot C \cdot R_C + s \cdot (C \cdot R_C \cdot R_{EL} + L) + R_{EL}}{b^*}, \quad (35)$$

where

$$b^* = s^2 \cdot L \cdot C_Z + s \cdot [C_Z \cdot R_{EL} + G \cdot L + C \cdot R_C(1 - D_A)^2] + R_{EL} \cdot G + (1 - D_A)^2.$$

3. Averaged models of BUCK and BOOST converters in DCM

Equations describing large-signal and small-signal dynamic models of BUCK and BOOST in discontinuous current mode (DCM), derived on the base of separation of variables approach and presented in [17] differ from equations described in the literature, derived on the switch-averaging. As it is discussed in [9] and [17], models of ideal as well as nonideal converters presented for example in [1, 7, 12], obtained on switch averaging approach are of the second order, whereas models based on the separation of variables exhibit first order dynamics (assuming the same description of converter components). Informalities in deriving averaged models for DCM, based on switch averaging approach consist among others in mistaking D.C. (i.e. quiescent point) quantities with quantities averaged over switching period, that leads to contradictions in description of averaged value of voltage v_L over inductance [9].

Contrary to contents of Sec. 2, the averaged models of ideal as well as non-ideal converters in DCM are discussed here in details. It seems to be necessary, because even models of ideal converters in DCM presented in the literature arouse serious objections.

3.1. Ideal converters. Large-signal description of ideal BUCK in DCM contains simple equations resulting from I-st Kirchhoff law for inductor current and current-voltage relationships defining ideal inductance and capacitance. Further equations for averaged values, derived in [17], are repeated below:

$$v_L = 0, \quad (36)$$

$$i_L = f_L(d_A, v_G, v_O) = G_Z \cdot d_A^2 \cdot \frac{v_G}{v_O} \cdot (v_G - v_O), \quad (37)$$

$$i_G = f_G(d_A, v_G, v_O) = G_Z \cdot d_A^2 \cdot (v_G - v_O), \quad (38)$$

where

$$G_Z = \frac{T_S}{2L} \quad (39)$$

and T_S is switching period.

The equivalent circuit corresponding to above equations is depicted in Fig. 15. Because of condition (36) for averaged value of inductor voltage, the capacitor C is only reactive element in this model.

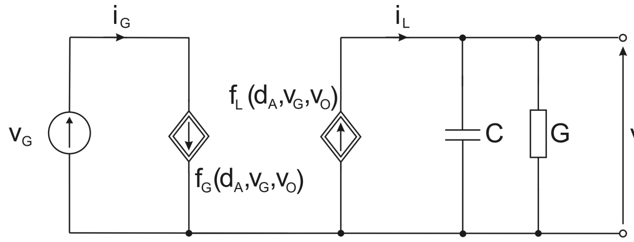

Fig. 15. Large-signal averaged model of ideal BUCK converter in DCM. Functions f_L and f_G are defined by Eqs. (37) and (38)

Steady-state model is obtained from Fig. 15 by replacing the capacitor C by open circuit and quantities d_A, v_G, v_O in description of functions f_G and f_L by corresponding D.C. values D_A, V_G, V_O.

Large-signal averaged model of ideal BOOST in DCM has been presented in [17]. In particular, according to [17], the average voltage v_L on inductor is zero, similarly as for BUCK converter. Equivalent circuit corresponding to equations presented in [17] is shown in Fig. 16.

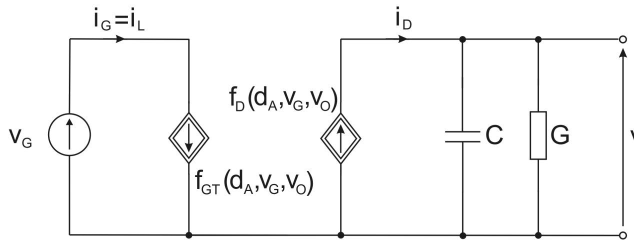

Fig. 16. Large-signal averaged model of ideal BOOST converter in DCM

The controlled sources in this model are described as follows:

$$f_{GT}(d_A, v_G, v_O) = G_Z \cdot d_A^2 \cdot \frac{v_G \cdot v_O}{v_O - v_G}, \quad (40)$$

$$f_D(d_A, v_G, v_O) = G_Z \cdot d_A^2 \cdot \frac{v_G^2}{v_O - v_G}. \quad (41)$$

Large-signal models of ideal BUCK and BOOST in DCM are represented by equivalent circuits having the same structure, but different description of current sources.

In small-signal models of ideal converters in DCM, the linearized equivalents of functional dependencies f_G, f_L, f_{GT} and f_D on variables d_A, v_G, v_O have to be used (see [17]). Expressions for small-signal terms of inductor current I_l and input current I_g in BUCK converter are as follows:

$$I_l = \alpha_{l1} \cdot \theta + \alpha_{l2} \cdot V_g + \alpha_{l3} \cdot V_o, \quad (42)$$

$$I_g = \alpha_{g1} \cdot \theta + \alpha_{g2} \cdot V_g + \alpha_{g3} \cdot V_o, \quad (43)$$

where coefficients α are respective partial derivatives of functions f_L and f_G (Eqs. (37) and (38)). Circuit representation of small-signal model of ideal BUCK converter in DCM is shown in Fig. 17.

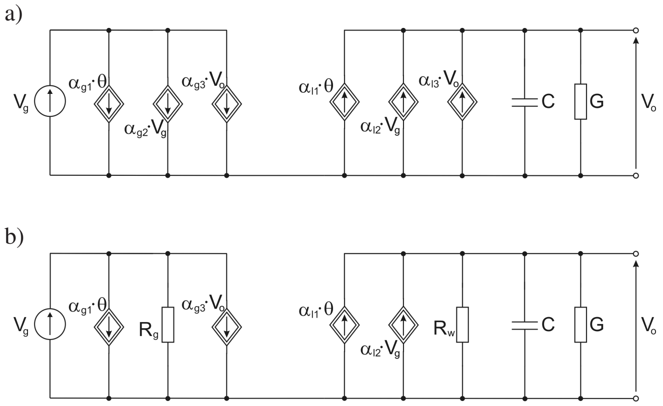

Fig. 17. Small-signal averaged models of ideal BUCK converter in DCM. Two current sources from Fig. a) are replaced by proper resistances in Fig. b)

Components of equivalent circuit in Fig. 17 are described below (see also [17]):

$$\alpha_{g1} = 2G_Z \cdot D_A \cdot (V_G - V_O), \tag{44}$$

$$R_g = \frac{1}{\alpha_{g2}} = \frac{1}{G_Z \cdot D_A^2}, \tag{45}$$

$$\alpha_{g3} = -G_Z \cdot D_A^2, \tag{46}$$

$$\alpha_{l1} = 2 \cdot G_Z \cdot V_G \cdot D_A \cdot \left(\frac{V_G}{V_O} - 1\right), \tag{47}$$

$$\alpha_{l2} = G_Z D_A^2 \cdot \left(\frac{2V_G}{V_O} - 1\right), \tag{48}$$

$$R_w = -\frac{1}{\alpha_{l3}} = G_Z \cdot D_A^2 \cdot \frac{V_G^2}{V_O^2}. \tag{49}$$

The structure of small-signal model of ideal BOOST converter in DCM is shown in Fig. 18 and is the same as model of BUCK in Fig. 17.

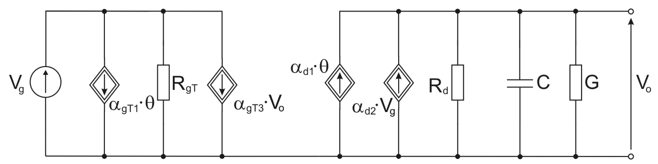

Fig. 18. Small-signal averaged model of ideal BOOST converter in DCM

Parameters of equivalent circuit shown in Fig. 18 are described by the following expressions:

$$\alpha_{gT1} = \frac{\partial f_{GT}}{\partial d_A} = 2 \cdot G_Z \cdot D_A \cdot \frac{V_G \cdot V_O}{V_O - V_G}, \tag{50}$$

$$R_{gT} = \frac{1}{\dfrac{\partial f_{GT}}{\partial v_G}} = \frac{(V_O - V_G)^2}{G_Z \cdot D_A^2 \cdot V_O^2}, \tag{51}$$

$$\alpha_{gT3} = \frac{\partial f_{GT}}{\partial v_O} = -\frac{G_Z \cdot D_A^2 \cdot V_G^2}{(V_O - V_G)^2}, \tag{52}$$

$$\alpha_{d1} = \frac{\partial f_D}{\partial d_A} = 2 \cdot D_A \cdot G_Z \cdot \frac{V_G^2}{V_O - V_G}, \tag{53}$$

$$\alpha_{d2} = \frac{\partial f_D}{\partial v_G} = G_Z \cdot D_A^2 \cdot \frac{V_G \cdot (2V_O - V_G)}{(V_O - V_G)^2}, \tag{54}$$

$$R_{wT} = -\frac{1}{\dfrac{\partial f_D}{\partial v_O}} = \frac{(V_O - V_G)^2}{G_Z \cdot D_A^2 \cdot V_G^2}. \tag{55}$$

3.2. Non-ideal converters. Before presenting equivalent circuits of non-ideal BUCK and BOOST in DCM it should be observed, that the distribution of currents and voltages in output the subcircuit of both converters, presented in Fig. 19, remains the same in CCM and DCM, in each subinterval (ON, OFF1, OFF2). For BUCK converter with parasitic resistances working in DCM, the respective equations given in Subsec. 3.4 of paper [17], for quantities averaged over switching period T_S should be applied.

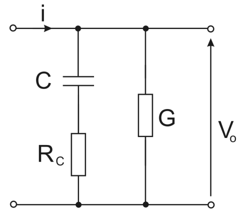

Fig. 19. Output subcircuit of non-ideal BUCK and BOOST converters. Current "i" equals inductor current i_L in BUCK or diode current i_D in BOOST

Taking into account the above remarks concerning output subcircuit, one obtains large-signal averaged equivalent circuit for non-ideal BUCK in DCM in the form depicted in Fig. 20.

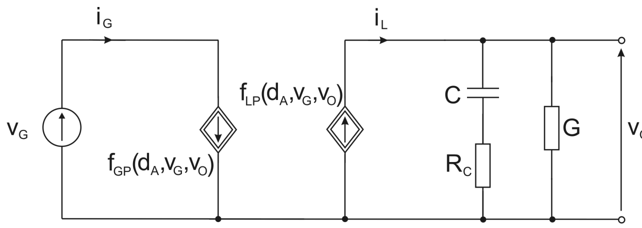

Fig. 20. Large-signal averaged model of non-ideal BUCK converter in DCM

Functions f_{GP} and f_{LP} describing controlled sources are:

$$f_{GP}(d_A, v_G, v_O) = \frac{(v_G - v_O) \cdot d_A^2}{R_G + R_P \cdot d_A}, \tag{56}$$

$$f_{LP}(d_A, v_G, v_O) = \frac{(v_G - v_O) \cdot v_G \cdot d_A^2}{v_O \cdot R_G + v_G \cdot R_P \cdot d_A}. \tag{57}$$

Expressions (56), (57) are based on [17], with notation:

$$R_G = \frac{1}{G_Z} = \frac{2L}{T_S}. \tag{58}$$

It was shown in Subsec. 5.4 of paper [17], that even in the presence of parasitic resistances, the average value of voltage v_L on inductance L is zero. A simplified model of ideal BUCK in DCM shown in Fig. 15, may be obtained by neglecting R_C in Fig. 20 and replacing equations (56), (57) by (37), (38).

The non-ideal BOOST converter in DCM has not been considered in [17], therefore the derivation of its averaged model is outlined below. Equivalent circuits of BOOST with parasitic resistances for ON, OFF1 and OFF2 subintervals are shown in Fig. 21 and waveforms of inductor current i_L and voltage v_L and diode current i_D – in Fig. 22.

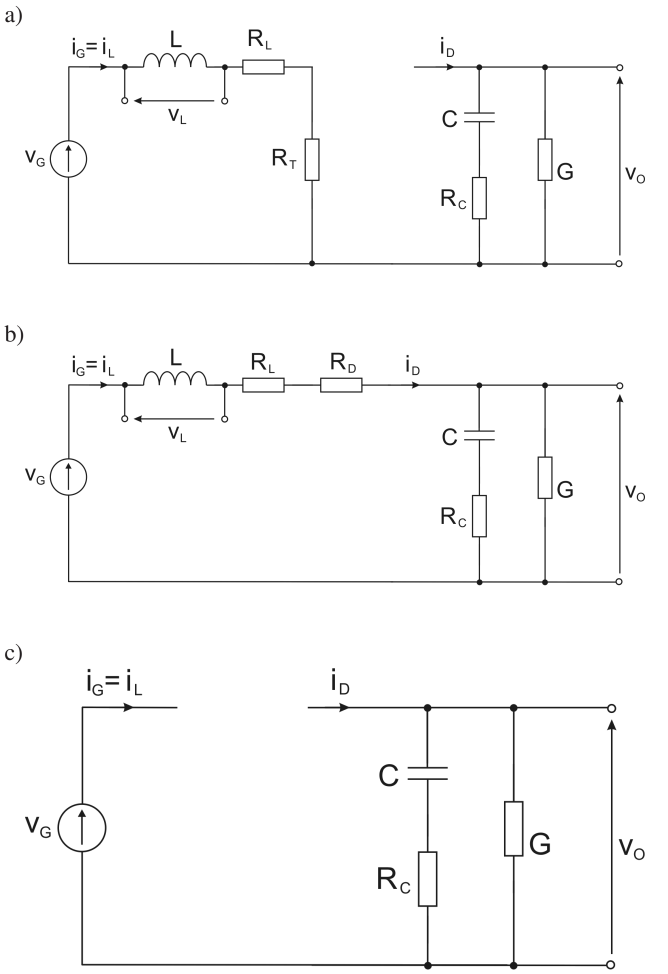

Fig. 21. Equivalent circuits of BOOST converter for ON (a), OFF1 (b) and OFF2 (c) subintervals

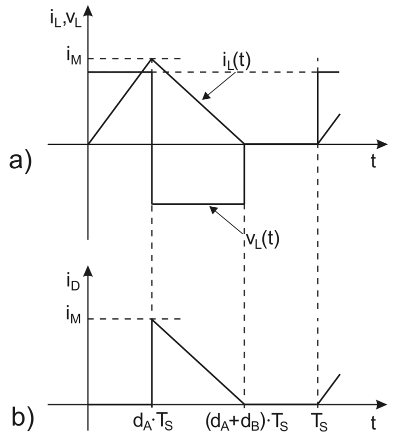

Fig. 22. Waveforms of inductor current i_L and voltage v_L on ideal inductance (a) and diode current (b)

The local average of voltage v_L on an ideal inductance L and inductor current i_L in ON subinterval are:

$$v_L(\mathrm{ON}) = v_G - i_L(\mathrm{ON}) \cdot (R_T + R_L) = v_G - i_L(\mathrm{ON}) \cdot R_P, \tag{59}$$

$$i_L(\mathrm{ON}) = \frac{1}{2} \cdot \frac{v_L(\mathrm{ON})}{L} \cdot d_A \cdot T_S = d_A \cdot v_L(\mathrm{ON}) \cdot G_Z. \tag{60}$$

The resulting formula for $i_L(\mathrm{ON})$ is:

$$i_L(\mathrm{ON}) = \frac{d_A \cdot G_Z \cdot v_G}{1 + d_A \cdot G_Z \cdot R_P}. \tag{61}$$

Similar formulas for OFF1 subinterval are:

$$v_L(\mathrm{OFF1}) = v_G - v_O - i_L(\mathrm{OFF1}) \cdot R_P, \tag{62}$$

$$i_L(\mathrm{OFF1}) = \frac{d_B \cdot G_Z}{1 - d_B \cdot G_Z \cdot R_P} \cdot (v_O - v_G). \tag{63}$$

Currents $i_L(\mathrm{ON})$ and $i_L(\mathrm{OFF1})$ as local averages for respective intervals have to be equal (see Fig. 22a). From this condition, a formula for d_B is obtained:

$$d_B = d_A \cdot \frac{v_G}{v_O \cdot (1 + d_A \cdot R_P \cdot G_Z) - v_G}. \tag{64}$$

The value of v_L averaged over whole switching period is:

$$v_{LS} = d_A \cdot v_L(\mathrm{OFF1}) + d_B \cdot v_L(\mathrm{OFF2}) \tag{65}$$

and, as a result:

$$v_{LS} = 0. \tag{66}$$

Condition (66) is the same as for ideal converters and for non-ideal BUCK.

Formulas for values of input and inductor current $i_G = i_L$ as well as diode current i_D averaged over switching period T_S are obtained from the above equations (subscript "S" is omitted in further formulas).

$$i_L = i_G = f_{LT}(d_A, v_G, v_O) = \frac{d_A^2 \cdot G_Z \cdot v_G \cdot v_O}{v_O \cdot (1 + d_A \cdot K) - v_G}, \tag{67}$$

$$i_D = f_{DT}(d_A, v_G, v_O) = \frac{d_A^2 \cdot G_Z \cdot v_G^2}{(v_O \cdot (1 + d_A \cdot K) - v_G) \cdot (1 + d_A \cdot K)}, \tag{68}$$

where

$$K = G_Z \cdot R_P. \tag{69}$$

The introductory remarks of Subsec. 3.2, together with the condition (66) lead to the structure of circuit representation of large-signal averaged model of BOOST converter in DCM, shown in Fig. 23.

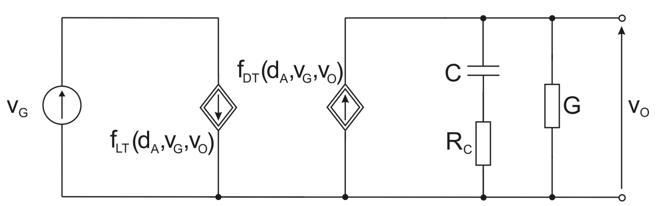

Fig. 23. Large-signal averaged model of non-ideal BOOST converter

Current sources f_{LT} and f_{DT} are described by Eqs. (67) and (68). It is seen, that the structure of averaged nonideal models of BUCK and BOOST converters is the same. Difference in models consists in different descriptions of controlled current sources.

Models of BUCK and BOOST for D.C. steady-state values may be obtained from Figs. 20 and 23 in simple and obvious way (similarly as for CCM). To obtain small-signal dynamic models of nonideal BUCK and BOOST working in DCM mode, similar procedure should be applied to large-signal models of Figs. 20 and 23 as of Figs. 15 and 16 in an ideal case. The resulting small-signal models of BUCK and BOOST have a unified structure presented in Figs. 24a) and b) and differ only in description of their components, as expressed by Eqs. (70)–(81).

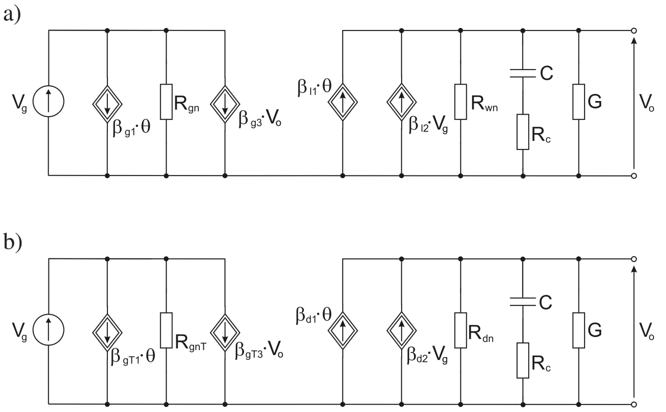

Fig. 24. Structure of small-signal averaged models of BUCK (a) and BOOST (b) converters with parasitic resistances in DCM

Parameters of BUCK converter model in DCM are:

$$\beta_{g1} = \frac{\partial f_{GP}}{\partial d_A} = D_A \cdot (V_G - V_O) \cdot \frac{2R_G + D_A \cdot R_P}{(R_G + R_P \cdot D_A)^2}, \quad (70)$$

$$R_{gn} = \frac{1}{\dfrac{\partial f_{GP}}{\partial v_G}} = \frac{R_G + D_A \cdot R_P}{D_A^2}, \quad (71)$$

$$\beta_{g3} = \frac{\partial f_{GP}}{\partial v_O} = -\frac{D_A^2}{R_G + R_P \cdot D_A}, \quad (72)$$

$$\beta_{l1} = \frac{\partial f_{LP}}{\partial d_A} = D_A \cdot (V_G - V_O) \cdot V_G \cdot \frac{2V_O \cdot R_G + V_G \cdot D_A \cdot R_P}{(V_O \cdot R_G + V_G \cdot D_A \cdot R_P)^2}, \quad (73)$$

$$\beta_{l2} = \frac{\partial f_{LP}}{\partial v_G} = D_A^2 \cdot \frac{2V_G \cdot V_O \cdot R_G + V_G^2 \cdot D_A \cdot R_P - V_O^2 \cdot R_G}{(V_O \cdot R_G + V_G \cdot D_A \cdot R_P)^2}, \quad (74)$$

$$R_{wn} = -\frac{1}{\dfrac{\partial f_{LP}}{\partial v_O}} = \frac{(V_O \cdot R_G + V_G \cdot D_A \cdot R_P)^2}{D_A^2 \cdot V_G^2 \cdot (D_A \cdot R_P + R_G)}. \quad (75)$$

Parameters of BOOST converter model for DCM:

$$\beta_{gT1} = \frac{\partial f_{LT}}{\partial d_A} = D_A \cdot G_Z \cdot V_G \cdot V_O \cdot \frac{V_O \cdot (2 + D_A \cdot K) - 2V_G}{[V_O \cdot (1 + D_A \cdot K) - V_G]^2}, \quad (76)$$

$$R_{gnT} = \frac{1}{\dfrac{\partial f_{LT}}{\partial v_G}} = \frac{[V_O \cdot (1 + D_A \cdot K) - V_G]^2}{D_A^2 \cdot G_Z \cdot V_O^2 \cdot (1 + D_A \cdot K)}, \quad (77)$$

$$\beta_{gT3} = \frac{\partial f_{LT}}{\partial v_O} = -\frac{D_A^2 \cdot G_Z \cdot V_G^2}{[V_O \cdot (1 + D_A \cdot K) - V_G]^2}, \quad (78)$$

$$\beta_{d1} = \frac{\partial f_{DT}}{\partial d_A} = D_A \cdot G_Z \cdot V_G^2 \cdot \frac{2V_O \cdot (1 + D_A \cdot K) - V_G \cdot (2 + D_A \cdot K)}{[V_O \cdot (1 + D_A \cdot K) - V_G]^2 \cdot (1 + D_A \cdot K)^2}, \quad (79)$$

$$\beta_{d2} = \frac{\partial f_{DT}}{\partial v_G} = \frac{D_A^2 \cdot G_Z \cdot V_G}{1 + D_A \cdot K} \cdot \frac{2V_O \cdot (1 + D_A \cdot K) - V_G}{[V_O \cdot (1 + D_A \cdot K) - V_G]^2}, \quad (80)$$

$$R_{dn} = -\frac{1}{\dfrac{\partial f_{DT}}{\partial v_O}} = \frac{[V_O \cdot (1 + D_A \cdot K) - V_G]^2}{D_A^2 \cdot G_Z \cdot V_G^2}. \quad (81)$$

Formulas for various small-signal transmittances of converters may be derived from the above models. Such transmittances may be used in the design of control circuit of given converter to meet the assumed demands on converter behavior.

3.3. Examples of small-signal transmittances for DCM. Small-signal input-to-output H_g, control-to-output H_d transmittances and output impedance Z_{out} for nonideal BUCK and BOOST converters working in DCM mode are obtained in this subsection from models presented in Figs. 24a) and b). It may be observed, that the input part of equivalent circuits of Figs. 24a) and b) does not influence calculations of the characteristics listed above.

Equivalent circuit for calculations H_g, H_d and Z_{out} has an unified form depicted in Fig. 25. Parameters β_{l1}, β_{l2} and R$_{wn}$ given by Eqs. (73)–(75) should be substituted for β_1, β_2 and R_n in the case of BUCK; the respective parameters for BOOST are β_{d1}, β_{d2} and R_{dn} (Eqs. (79)–(81)).

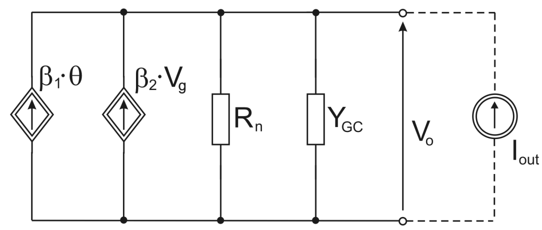

Fig. 25. Unified model for calculations of H_g, H_d and Z_{out} of BUCK and BOOST converters in DCM

According to definitions of H_g, H_d and Z_{out}, the respective current sources should be set to zero for given calculations. Formulas for H_g, H_d and Z_{out} may be expressed in general form from Fig. 25. For calculation of H_g, $I_{out} = 0$ and $\theta = 0$ conditions are assumed, therefore:

$$V_o = \frac{\beta_2 \cdot V_g}{1/R_n + Y_{GC}} \tag{82}$$

and

$$H_g = \frac{\beta_2}{1/R_n + Y_{GC}}, \tag{83}$$

where Y_{GC} is defined by Eqs. (23), (24).

Other transmittances are obtained from Fig. 25 in similar way:

$$H_d = \frac{\beta_1}{1/R_n + Y_{GC}}, \tag{84}$$

$$Z_{out} = \frac{1}{1/R_n + Y_{GC}}. \tag{85}$$

The explicit forms of transmittances for particular converter are obtained by substituting proper expressions for β_1, β_2, R_n and Y_{GC} into Eqs. (83)–(85). For BUCK converter, from (83)–(85), (74), (75), (23), we get:

$$H_g(BUCK, DCM) = \frac{[M_V \cdot R_G \cdot (2 - M_V) + D_A \cdot R_P] \cdot (s \cdot C \cdot R_C + 1)}{s \cdot (C \cdot R_C \cdot R_{PG} + C_Z \cdot R_Y^2) + R_{PG} + G \cdot R_Y^2}, \tag{86}$$

where

$$R_{PG} = D_A \cdot R_P + R_G, \tag{87}$$

$$R_Y = R_G \cdot M_V / D_A + R_P, \tag{88}$$

M_V is steady-state voltage transfer function of converter:

$$M_V = \frac{V_O}{V_G} \tag{89}$$

and is described for BUCK in DCM in [17].

$$H_d(BUCK, DCM) = (V_G - V_O) \cdot \frac{(2 \cdot R_G \cdot M_V / D_A + R_P] \cdot (s \cdot C \cdot R_C + 1)}{s \cdot (C \cdot R_C \cdot R_{PG} + C_Z \cdot R_Y^2) + R_{PG} + G \cdot R_Y^2}, \tag{90}$$

$$Z_{out}(BUCK, DCM) = \frac{R_Y^2 (s \cdot C \cdot R_C + 1)}{s \cdot (C \cdot R_C \cdot R_{PG} + C_Z \cdot R_Y^2) + R_{PG} + G \cdot R_Y^2}. \tag{91}$$

All the above transmittances have the same single pole and the same single zero in left half-plane introduced by the parasitic resistance R_C of capacitor.

Quantities β_{d1}, β_{d2} and R_{wnT} for BOOST converter in DCM should be taken as β_1, β_2 and R_n in expressions (83)–(85) to obtain the respective transmittances. The result is:

$$H_g(BOOST, DCM) = \frac{G_A}{N} \cdot \frac{(2M_{VT} \cdot N - 1) \cdot (s \cdot C \cdot R_C + 1)}{d^*}, \tag{92}$$

where

$$d^* = s \cdot [C \cdot G_A \cdot R_C + C_Z \cdot (M_{VT} \cdot N - 1)^2] + (M_{VT} \cdot N - 1)^2 \cdot G + G_A,$$

where

$$N = 1 + D_A \cdot K, \tag{93}$$

$$G_A = D_A^2 \cdot G_Z, \tag{94}$$

M_{VT} is a DC voltage transfer function (see (89)) for BOOST converter.

$$H_d(BOOST, DCM) = \frac{D_A \cdot G_Z \cdot V_G}{N^2} \cdot \frac{(2M_{VT} \cdot N - N - 1) \cdot (s \cdot C \cdot R_C + 1)}{e^*}, \tag{95}$$

$$Z_{out}(BOOST, DCM) = \frac{(M_{VT} \cdot N - 1)^2 \cdot (s \cdot C \cdot R_C + 1)}{e^*}, \tag{96}$$

where

$$e^* = s \cdot [C \cdot G_A \cdot R_C + C_Z \cdot (M_{VT} \cdot N - 1)^2] + (M_{VT} \cdot N - 1)^2 \cdot G + G_A.$$

All above transmittances of BOOST converter, (similarly as for BUCK), have the same single pole and the same single zero in left half-plane, introduced by parasitic resistance R_C of capacitor. DC voltage transfer function M_{VT} for BOOST may be calculated from large signal model of Fig. 23. Taking the DC conditions we obtain:

$$M_{VT} = \frac{1 + \sqrt{1 + 4 \cdot G_A / G}}{2 \cdot (1 + D_A \cdot K)}, \tag{97}$$

where G_A and K are defined by Eqs. (94) and (69).

Other small-signal characteristics of BUCK and BOOST in DCM may be calculated from equivalent circuits shown in Fig. 24, with the use of expressions (70)–(81) for respective parameters. From Fig. 24 it is seen (without calculations) that input small-signal admittances of both converters in DCM are real numbers, therefore there is no phase shift between small-signal input current and voltage. The absence of second pole in transmittances H_g, H_d and Z_{out} is a consequence of zero value of average voltage v_{LS} on inductance L.

4. Examples of simulations and measurements

Models presented in Secs. 2 and 3 may be used in analysis and computer simulations of BUCK and BOOST converters and are applicable to use in popular circuit simulators, e.g. SPICE. Large-signal transients and DC characteristics for various operating conditions of converters may be calculated with the use of models shown in Figs. 6, 7, 15, 16, 20, 23. Small-signal analyses may be performed with application of models presented in Figs. 10, 11, 13, 14, 17, 18, 24. It should be pointed out, that models discussed in the paper are obtained by averaging the waveforms of currents and voltages over switching period T_S, therefore are valid only for sufficiently low frequencies. Small-signal models may be used directly to analysis in frequency domain or, after formal transformation, to analysis in time domain, for sufficiently small changes of variables. Simulations may be used to compare the results obtained for ideal and nonideal models or models presented in this paper and models described in other sources. Verification of models should be performed by measurements.

Systematic investigation of presented models by simulations and measurements may be a very extensive task. In this paper, because of limited space, only a few examples are presented. BUCK converter working in CCM is selected for exemplary simulations. Nominal parameters of converter are: $V_G = 12$ V, $D_A = 0.5$, $G = 0.2$ S, $f_S = 100$ kHz, $L(\text{nom}) = 100$ μH, $C(\text{nom}) = 470$ μF. Parameters of components have been measured for conditions corresponding to converter operation. For a transistor and a diode it is obtained: $R_T = 28$ mΩ, $R_D = 300$ mΩ. A relatively large value of a diode resistance results from the assumed model (without DC offset voltage). Parameters of inductor and capacitor measured by Fluke PM 6306 meter depend on frequency. The values obtained for frequency $f = 700$ Hz (an approximate value of natural frequency of a power stage) are: $C = 487$ μF, $L = 92.2$ μH, $R_C = 42.8$ mΩ, $R_L = 40.1$ mΩ. The changes of measured values of C and L are below 4% for the frequency range 300 Hz to 3 kHz but changes of R_C and R_L are over 50%. The magnitude and phase of small-signal input-to-output transmittance H_g calculated from Eq. (29) are presented in Fig. 26a) and b).

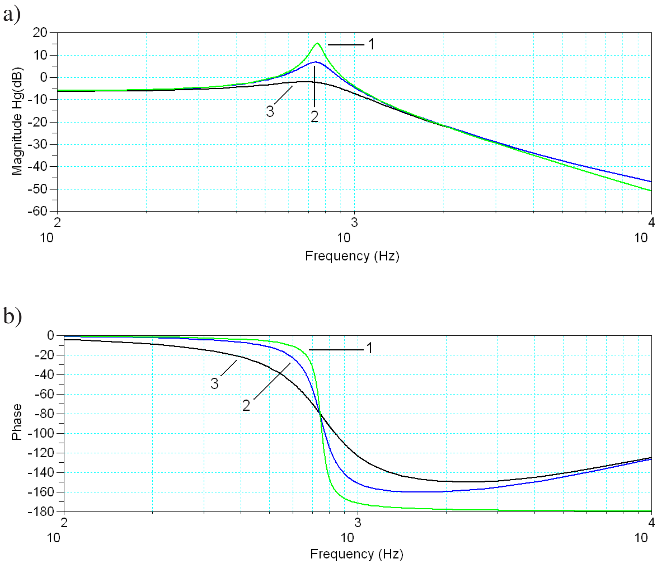

Fig. 26. Magnitude (a) and phase (b) of small-signal input-to-output transmittance H_g calculated for BUCK converter in CCM. Curve 1 – ideal converter; curve 2 – resistances of capacitor R_C and inductor R_L included; curve 3 – all parasitic resistances included

Curves 1 correspond to an ideal converter (no parasitic resistances), curves 2 – to including only R_C and R_L (i.e. assuming $R_D = R_T = 0$); curves 3 are obtained with all parasitic resistances included. Control-to-output transmittance H_d calculated for the same converter is presented in Fig. 27 (having similar structure as Fig. 26).

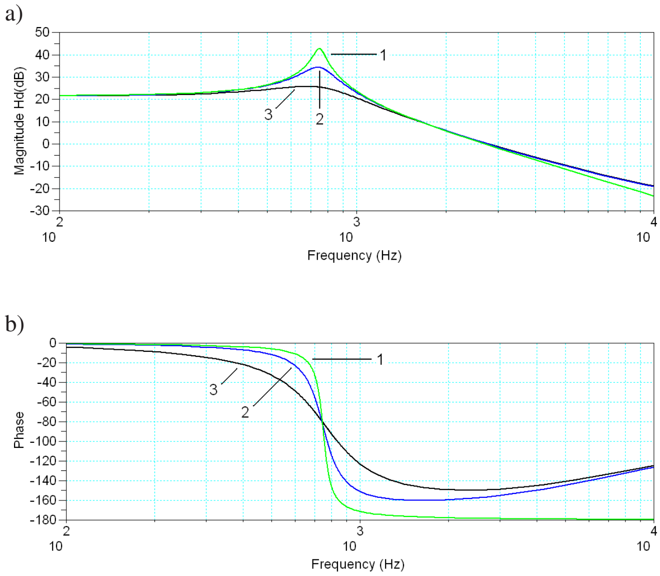

Fig. 27. Magnitude (a) and phase (b) of small-signal control-to-output transmittance H_d calculated for BUCK converter in CCM. Curve 1 – ideal converter; curve 2 – resistances of capacitor R_C and inductor R_L included; curve 3 – all parasitic resistances included

Magnitude of H_d is referenced to 1 V. Relatively high differences in magnitude of H_g and H_d between ideal and nonideal converter (nearly 20 dB at "resonant' frequency 750 Hz) are visible.

The time-domain response of output voltage v_O(t) to input step change $v_G(t) = 12$ V$\cdot\mathbf{1}(t)$ are calculated from large-signal model shown in Fig. 6, with the use of SPICE simulator. Results of simulations for ideal converter and for converter including resistances R_C and R_L (with $R_T = R_D = 0$) are shown in Fig. 28.

Simulation results for all parasitic resistances included are compared with results of experiment in Fig. 29.

It is observed, that the existence of parasitic resistances strongly influences transient states in converter power stage. Results of calculations in Figs. 28 and 29 (curve a) differ substantially. On the other hand, results of measurements are similar to results of calculations including all parasitic resistances. The consistence of simulation and measurement results in Fig. 29 is not ideal because the models of real capacitor and inductor are approximate (that results in relatively large changes of measured parasitic resistances with frequency).

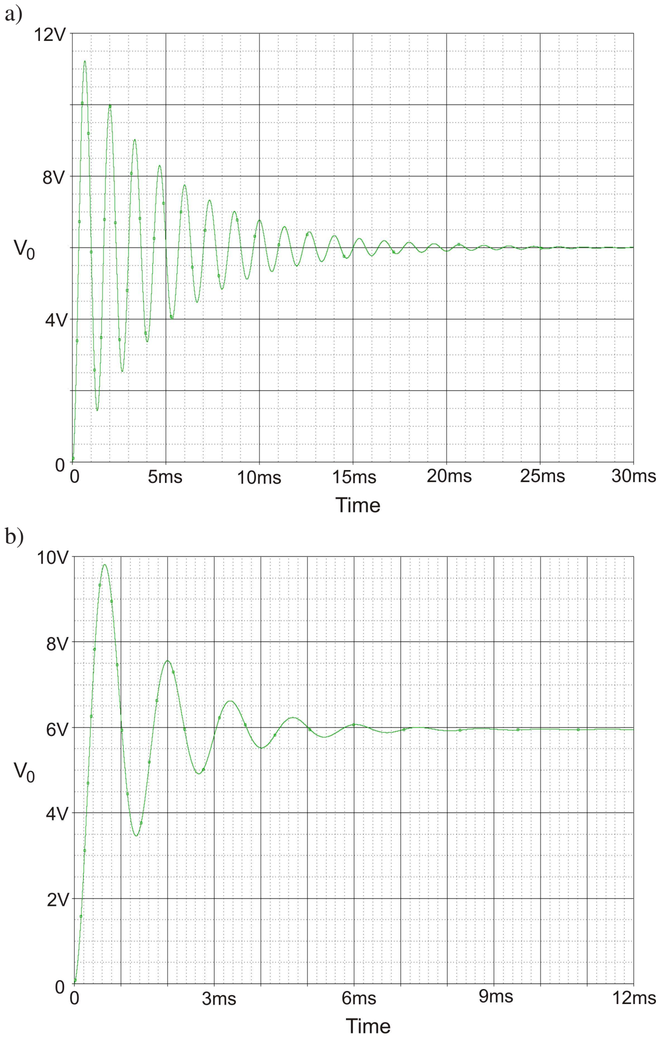

Fig. 28. Large-signal, time-domain response of output voltage v_O(t) to step change of input voltage v_G in CCM, obtained by SPICE simulation for ideal BUCK (a) and for BUCK with resistances R_C and R_L included (b)

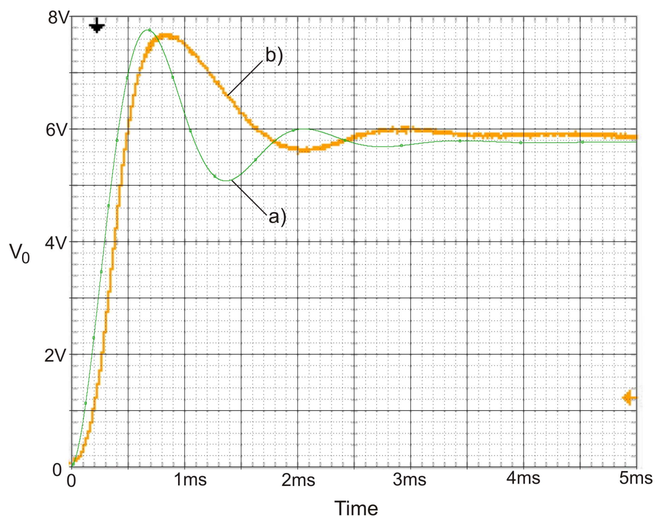

Fig. 29. Large-signal, time-domain response of output voltage $v_O(t)$ to step change of input voltage v_G for BUCK in CCM, obtained by simulation for all parasitic included (curve a) and observed experimentally (curve b)

5. Conclusions

In the paper, averaged models of basic switch-mode DC-DC power converters (BUCK and BOOST) in the form of large-signal and small-signal equivalent circuits are presented. These equivalent circuits are derived with the use of separation of variables approach introduced in [17]. The paper is therefore a continuation of considerations presented in [17]. Some equations describing averaged models, presented in [17], are repeated here for convenience; in addition some new formulas are derived, for example formulas for BOOST converter in DCM, and expressions for output impedances and some other small-signal transmittances for BUCK and BOOST in CCM and DCM mode. The main advantage of models in the form of equivalent circuits over models in the form of mathematical equations is the simplicity of application in popular circuit simulators. Models obtained with the use of the switch averaging approach are also presented in some references in the form of equivalent circuits but some informalities in their derivation for converters with parasitic resistances or working in DCM mode leads to difficulties in obtaining proper description of converters [9]. As a result, models based on switch averaging are usually presented in the literature with neglecting all or a part of parasitic resistances [1, 7, 20–22] or some inaccuracies in models may be observed [2, 23, 24].

Theoretical considerations of the paper are accompanied by some examples of simulations and measurement results shown in Sec. 4. It may be observed, that parasitic resistances of converter components strongly influence converter characteristics, therefore models neglecting (fully or partially) these parasitics, often met in the literature, may lead to inaccurate results of simulations and their usefulness in designing process is problematic. On the other hand, a simple description of the component parasitics in the form of additional series resistances, used in this paper, may be insufficient for a more precise analysis, and more involved models should be used.

The proposed method of deriving averaged models is applied here only to the most popular switch-mode converters: BUCK and BOOST, but may be used in the description of other power converters as well.

Acknowledgements. The author would like to thank Mr. Maciej Bączek and Mr. Marcin Walczak for performing exemplary simulations and measurements.

REFERENCES

[1] R.W. Erickson and D. Maksimovic, *Fundamentals of Power Electronics*, Kluwer, Amsterdam, 2002.
[2] M.K. Kazimierczuk, *Pulse-Width Modulated DC–DC Power Converters*, J. Wiley, London, 2008.
[3] P. Basso, *Switch-Mode Power Supply Spice Cookbook*, McGraw-Hill, London, 2001.
[4] K.C. Wu, *Switch-Mode Power Converters,* Elsevier Academic Press, New York, 2006.

[5] N. Mohan, T.M. Undeland, and W.P. Robbins, *Power Electronics: Converters, Applications and Design*, J. Wiley, London, 2003.

[6] D. Maksimowic, A.M. Stankovic, V.J. Thottuvelil, and G.C. Verghese, "Modeling and simulation of power electronic converters", *Proc. IEEE* 89 (6), 898–912 (2001).

[7] D. Maksimovic and R.W. Erickson, "Advances in averaged switch modeling and simulation", *Power Electronics Specialists Conf.* 1, CD-ROM (1999).

[8] E. van Dijk, H. Spruijt, D. O'Sullivan, and J. Klaassens, "PWM-switch modeling of DC – DC converters", *IEEE Trans. on Power Electronics* 10 (6), 659–664 (1995).

[9] W. Janke, "Averaged models of pulse-modulated DC-DC converters. Part I. *Discussion of standard methods*", *Archives of Electrical Engineering* 61 (4), 609–631 (2012).

[10] V. Vorperian, "Simplified analysis of PWM converters using model of PWM switch. Part I. Continuous conduction mode", *IEEE Trans. on Aerospace and Electronic Systems* 26 (3), 490–496 (1990).

[11] V. Vorperian, "Simplified analysis of PWM converters using model of PWM switch. Part II: Discontinuous conduction mode", *IEEE Trans. on Aerospace and Electronic Systems* 26 (3), 497–505 (1990).

[12] J. Sun, D. Mitchell, M. Greuel, P. Krein, and R. Bass, "Averaged modeling of PWM converters operating in discontinuous conduction mode", *IEEE Trans. on Power Electronics* 16 (4), 482–492 (2001).

[13] J. Xu and J. Wang, "Bifrequency pulse-train control technique for switching DC-DC converters operating in DCM", *IEEE Trans. on Industrial Electronics* 58 (8), 3658–3667 (2011).

[14] D. Biolek, V. Biolkova, and Z. Kolka, "Averaged modeling of switched DC-DC converters based on SPICE models of semiconductor switches", *Proc. 7-th WSEAS Conf. Circuits, Systems, Electronics, Control and Signal Proc.* 1, 162–167 (2008).

[15] E. Niculescu, D. Purcaru, and M. Niculescu, "Small-signal models of some basic PWM converters", *12-th WSEAS Int. Conf. on Circuits, Heraklion* 1, 321–326, (2008).

[16] S. Gong, L. Chen, C. Yu, and G. Xie, "Modeling and simulation of non-ideal buck converter in DCM", *Int. J. Computer Technology and Electronics Engineering* 2 (2), 72–75 (2012).

[17] W. Janke, "Averaged models of pulse-modulated DC-DC converters. Part II. Models based on the separation of variables", *Archives of Electrical Engineering* 61 (4), 633–654 (2012).

[18] W. Janke, M. Walczak, and M. Bączek, "Input and output characteristics of BUCK and BOOST voltage converters with parasitic resistances taken into account", *Electrical Review* (2012), (to be published), (in Polish).

[19] W. Janke, M. Walczak, and M. Bączek, "Output characteristics of step-down (Buck) power converter", *Bull. Pol. Ac.:Tech.* 60 (4), 751–756 (2012).

[20] M. Qiao, P. Parto, and R. Amirani, "Stabilize the buck converter with transconductance amplifier", *Int. Rectifier, Appl. Note* AN-1043, CD-ROM (2002).

[21] S. Choudhury, "Designing a TMS320F280x based digitally controlled DC-DC switching power supply", *Texas Instruments, Appl. Report* SPRAAB3, CD-ROM (2005).

[22] R. Zaitsu, "Voltage mode boost converter small signal control loop analysis using the TPS61030", *Texas Instruments, Appl. Report* SLVA274A, CD-ROM (2009).

[23] D. Czarkowski and M.K. Kazimierczuk, "Energy-conservation approach to modeling PWM DC – DC converters", *IEEE Trans. on Aerospace and Electronic Systems* 29 (3), 1059–1063 (1993).

[24] M.K. Kazimierczuk and D. Czarkowski, "Application of the principle of energy conservation to modeling the PWM converter", *2-nd IEEE Conf. on Control Applications* 1, 291–296 (1993).

[25] "Voltage Regulator Module (VRM) and Enterprise Voltage Regulator-Down (EVRD)", *11.1 Design Guidelines, Intel Corp.* 1, CD-ROM (2009).

[26] R. Ahmadi, D. Paschedag, and M. Ferdowsi, "Closed loop input and output impedances of DC-DC switching converters operating in voltage and current mode control", *IECON 36-th Ann. Conf. IEEE Industrial Electronic Society* 1, 2311–2316 (2010).

[27] J. A. Qahouq and V.P. Arikatla, "Power converter with digital sensorless adaptive voltage positioning control scheme", *IEEE Trans. on Industrial Electronics* 58 (9), 4105–4116 (2011).

[28] V. Švikovič, J. A. Oliver, P. Alou, O. Garcia, and J.A. Cobos, "Synchronous buck converter with output impedance correction circuit", *APEC 27-th Ann. Conf.* 1, 727–734 (2012).