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GENERALIZED MODEL OF STRAINS DURING BENDING OF METAL TUBES IN BENDING MACHINES

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According to the postulate concerning a local change of the "actual active radius" with a bending angle in the bent zone, a generalized model of strain during bending of ametal tube has been derived. The considered tubes should be subjected to bending in tube bending machines by the method of wrapping on a rotating template and with the use of a lubricated steel mandrel. The model is represented by three components of strain in the analytical form, including displacement of the neutral axis. Generalization of the model of metal tubes in the existing papers consists in including the displacement of the neutral axis and the possibility of determination of strains at each point along the thickness of the wall of the bent tube in the bending zone. The derived scheme of strain satisfies initial and boundary kinematic conditions of the bending process, conditions of continuity and inseparability of strains. The obtained analytical expressions can be classified as acceptable from the kinematic point of view.

Keywords: tubes bending, deformations, bending angle, thickness of the wall, neutral axis

1. Introduction

Tube bending (see e.g. Beskin, 1945; Boyle, 1971; Franz, 1961, 1969; Gruner, 1960; Grunow, 1985; Korzemski, 1968, 1971; Li *et al.*, 2006; Pesak, 1953; Śloderbach *et al.*, 1999, 2000, 2002, 2012; Tang, 2000; Wick *et al.*, 2001; Yang and Lin 2004; Zdankiewicz, 1970, 1998; Zhang *et al.*, 2011; Zhiqiang *et al.*, 2011) as a technological problem appeared in the end of the 19th century when production of tubes started to an industrial scale. Tubes were delivered mainly to industry of steam engines and boilers, gas engineering, power engineering, civil engineering. At present, tubes and elbows are purchased by almost all branches of the industry, and tube bending is a typical activity in many technological processes in the metal industry. Production of tubes and elbows is increasing more quickly than production of steel because tubes and elbows are made also of other materials, e.g. plastics.

At present, tube bending in tube bending machines using the method of wrapping on a rotating template with a lubricated mandrel is the most widely used. Such bending always leads to formation of thinner walls in the layers subjected to elongation, thickening and wrinkling in the layers subjected to compression, and deformation (ovalization) of the cross section. Such unfavorable phenomena should be included into the limits of tolerance given in European standards and recommendations (European Standard, 1993; Zdankiewicz, 1998), as well as regulations of UDT (Polish Office of Technical Inspection, UDT Conditions, 2003). The acceptable ovalization of the cross section according to the European Standard (EN 448, 1993) is up to 6%. In this paper, the author considers only cold bending of metal tubes of the assumed technological wall thickness $s^* \leqslant 0.10$ and maximum $d_{ext} = 160\,\mathrm{mm}$ (Śloderbach, 2012), where $s^* = g_0/d_{ext}$, g_0 and d_{ext} are the initial thickness and external diameter of the bent tube, respectively. In regulations of UDT (UDT Conditions, 2003), the pressure tubes are assumed as thin-walled, when $s^*_w \leqslant 0.05$, where $s^*_w = g_0/d_{int}$, $d_{int} = d_{ext} - 2g_0$, then $s^*_w = s^*/(1-2s^*)$. The above assumptions follow from the practical results obtained at the Research and Development Institute for Power Plant Maintenance in Wrocław, Poland (it stopped to exist in the 2006).

In literature, there are no analytical expressions for strains depending on an actual value of the bending angle, angles determining the position of each point (particle) in the bending zone, displacement of the neutral axis, bending radius, dimensions of the cross section of the bent tube and suitable technological-material coefficients k and λ_i dependent on technological parameters and tube materials. Tube bending always leads to reduction of the wall in elongated layers and an increase in tube thickness in layers subjected to compression, ovalization and formation of corrugation which distort the cross section.

In this paper, the author tries to describe the strain state, understanding the tube bending as a three-dimensional and heterogeneous aproblem including the influence of displacement of the neutral axis. Such influence is understood in the following way: the forming field of plastic strains generates displacement of the neutral axis (in this method, in the direction toward the layers subjected to compression), and the magnitude of the neutral axis displacement affectts the distribution of these strains.

From experiments, technological tests and industrial practice (Wick *et al.*, 2001), it appears that this displacement is only about 5% of the external diameter of the bent tube for thick-walled tubes $s* \geqslant 0.10$ bent at relatively large bending radii $R > 2d_{ext}$, and about 25% for very thin-walled tubes $s* \leqslant 0.01$ bent at very small bending radii $R \leqslant d_{ext}$. Thus, such a high displacement of the neutral axis of plastic bending should not be disregarded in analytical description of the strain state. Generalization of the strain description while bending metal tubes in bending machines related to the data from other papers (Śloderbach, 1999, 2000) consists in taking into account the neutral axis displacement.

It is assumed that strains in the tube bending process are identified with plastic strains. Thus, it appears that plastic strains are of the order of some tens of percent (even to 50%), and the maximum elastic strains are equal to decimal parts of the percent, and they are neglected in the description.

2. Geometric-analytical description of the bending process

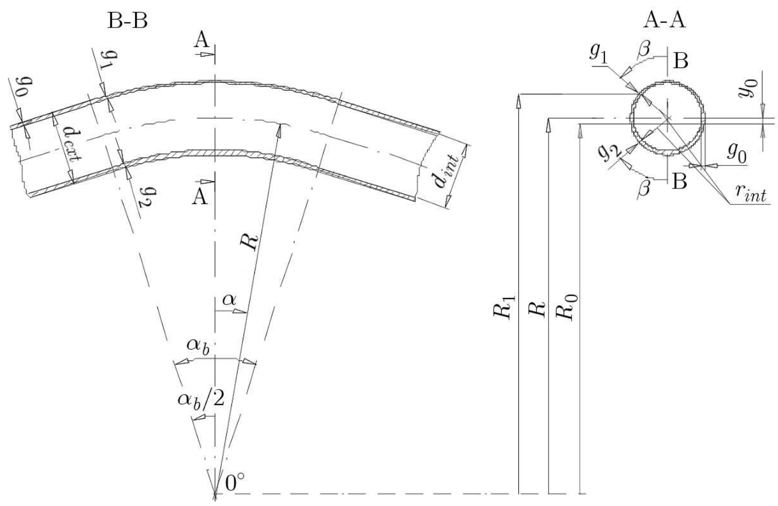

Fig. 1. Geometrical and dimensional quantities pertaining to tube-bending processes

d_{ext}, d_{int} – external and internal diameter of a bent tube,
g_0 – initial thickness of the bent tube,

g_i – actual thickness of the bend in the bending zone ($i = 1$ for elongated, $i = 2$ for compressed layers),
r_{ext}, r_{int} – external and internal radius of the bent pipe,
R – bending radius,
R_0 – radius of the neutral surface following the bending,
R_i – larger actual radius of the bend associated with longitudinal strain,
y_0 – displacement of the neutral surface with respect to the initial position,
α – actual angle of the bending zone determined at the principal bending plane and at planes parallel to it, $\alpha \in \langle 0°, \alpha_b/2\rangle$, where α_b is the active bending angle measured over the bending zone, $\alpha_b \in \langle 0°, 180°\rangle$,
α_0 – angle of bend (the angle by which a template or a former is rotated); in theory for spirals $\alpha_0 \in \langle 0°, \infty)$ but for the analyzed method $\alpha_0 \in \langle 0°, 180°\rangle$. Obviously, within the bending zone the two angles are equal $\alpha_0 = \alpha_b$. When the plateau zone is formed, then $\alpha_0 = \alpha_b + \alpha_{pl}$, where α_{pl} is the angle of the plateau zone (Franz, 1961; Śloderbach, 2002). Hence it follow that $\alpha_0 \leqslant \alpha_b$, becouse $\alpha_{pl} \leqslant 0$, see Franz (1961),
β – actual angle determined at the planes perpendicular to the bending plane, $\beta \in \langle 0°, 90°\rangle$.

3. The basic assumptions

It is assumed that the tube material is an incompressible rigid-plastic (with isothropic hardening) continuous medium satisfying the condition of plasticity (M-H-H) and the Levy-Mises flow laws. Its properties while bending are described by two technological-material coefficients k and λ_i. Thus, the constant plastic volume of the material is assumed before and after bending. The neutral axis of plastic bending is determined by the radius R_0. The axis separating the elongated and compressed layers in the bending zone is a line of stress discontinuity (Hill, 1986; Johnson and Mellor, 1975; Marciniak, 1971; Mendelson, 1988; Olszak *et al.*, 1985; Szczepinski, 1973; Śloderbach, 1999, 2002; Tang, 2000). It is also assumed that in the tip points of the elongated layers (environment of the mandrel), the tube bending process in bending machines (tube bending by wrapping on a rotational template and with the use of a lubricated mandrel) is a complex process of heterogeneous curvilinear elongation (biaxial stretch drawing) under a three dimensional stress state during bending of thick-walled tubes and a plane state for thin-walled tubes. In the case of the compressed layers, it is a composition of two processes: heterogeneous curvilinear compression and unfree upsetting. Introduction of the technological-material coefficient k allows one to include also the influence of mandrel friction with the tube wall on the strain distribution.

In real tube bending processes in bending machines, in the compressed layers, there are no such strain states as those occurring under free compression and upsetting. Tube bending is not a case of free (unbounded) bending but it is forced by the structure of working tools of the bending machine (matrix, mandrel, flatter, strip and its pressure force), and the lubrication is of great importance, too.

4. Procedure of strain component derivation

Derivation of the generalized expressions for three components of the strain state in the considered tube bending is going to be realized at three stages:

1. Derivation of the expressions for principal strains in relative and next in logarithmic measures while bending when the influence of the neutral axis displacement on the strain distribution is not taken into account.

2. Derivation of the expression for the neutral axis displacement in plastic bending y_0.
3. Formal derivation (according to Fig. 4) of the expressions for strain components in order to describe the deformation state for the problem including the neutral axis displacement by introduction of the derived relationship for y_0 into the equations.

4.1. Derivation of the expression with no neutral axis displacement

The basic aim is analytical derivation of the plastic strain components, i.e. the longitudinal (along the tube axis), circumferential and radial (along the thickness) components in measures of relative and logarithmic strains during tube bending in bending machines by wrapping on a rotating template using a mandrel or with no mandrel, on the assumption that $d_{int} \cong$ const. It concerns thin-walled and thick-walled metallic tubes subjected to cold bending in bending machines.

An important problem is to determine a form for a change of the "actual active radius" $r^{*(N)}$ in the bending zone (determined at each N-th point along thickness, and for total thickness of the bent tube, so as $r^* = r^{*(N)}$ for $g_0^{(N)} = g_0$), depending on a change of the bending angle α_b. That increase related to length of the bending radius R defines an increment of the relative longitudinal strain $d\varepsilon_1^{(N)}$. The local "actual active radii $r^{*(N)}$ and r^* depend on the actual value of the angles α and β determining positions of the points in the bending zone, and on the actual position of the external point in the considered N-th layer included into the wall of the bent tube (see Fig. 2), also on total dimensions of the cross section in the elongated and compressed layers.

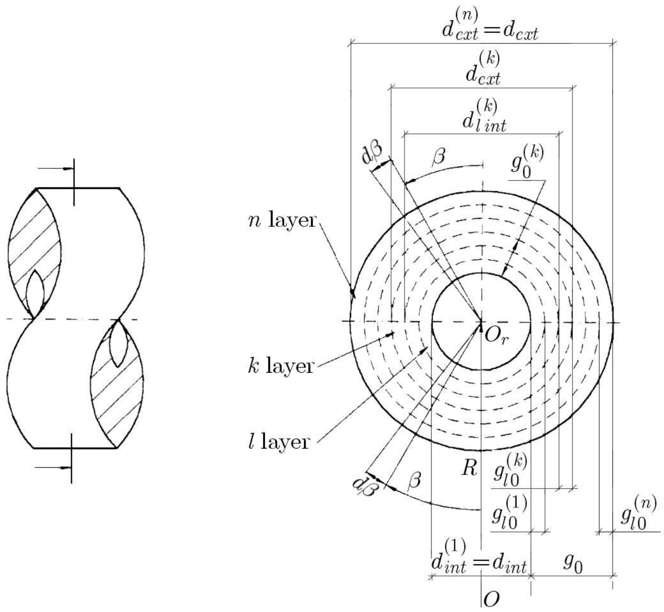

Fig. 2. A concept of the division of the transversal section of a thick walled pipe for the analytical and FEM method

In the analytical method, a possibile number of the analyzed points along the thickness of the bent tube wall is $N \in \langle 1, n)$, when $n \to \infty$. In this method, a division along the wall thickness is laminar into the N-th number of layers $g_{l0}^{(N)}$ in thickness, the thickness of which is measured from the internal tube surface to the external point of the N-th layer (see Figs. 1 and 2), where $g_0^{(k)} = (d_{ext}^{(k)} - d_{int})/2 = r_{ext}^{(k)} - r_{int}$.

Such division results from the fact that tube dimensions are given as $l \times d_{ext} \times g_0$, where l is the tube length. When $N = 1$, then $g_{l0}^{(1)} = g_0$, and when $N \to \infty$, then $g_{l0}^{(N)} \to 0$. In the case of the finite element method (FEM) applied for the thick walled tubes, the division is annular into a finite number of cylinders (rings) n of equal thickness. From Fig. 3 it also appears that when the wall thickness is considered as one layer for $N=1$, then $g_{l0}^{(1)}=g_0^{(1)}=g_0$, $d_{int}^{(1)}=d_{int}$ and $d_{ext}^{(1)}=d_{ext}$, where $d_{ext}=d_{int}+2g_0$ and $d_{l\,int}^{(k)}$, $d_{ext}^{(k)}$ are internal and external diameter of the k-th layer, respectively, d_{ext}, d_{int} are external and internal diameter of the tube for bending, respectively, $g_0^{(k)}$ – thickness of the k-th layer measured from d_{int}, $g_{l0}^{(1)}, g_{l0}^{(k)}, g_{i0}^{(n)}$ – thickness of the 1-st (first), k-th (currently) and n-th (last) layer.

The value of the relative strain ε_1, (i.e. the strain corresponding to the experimental results) obtained after previous integration of the expression for $d\varepsilon_1$ within the limits of the angle $\langle 2\alpha, \alpha_b\rangle$ depends on the actual value of the bending angle α_b, actual values of the point position angles α, β in the bending zone, actual dimensions of the bent tube, the bending radius R, and suitable technological-material coefficients k and λ_i. Let us describe the strain state assuming the following postulate.

Postulate A

The increment of the longitudinal component (along the tube axis) $d\varepsilon_1^{(N)}$ of the strain state, according to the notations in Figs. 1 and 2 (tube bending in bending machines by using a mandrel or with no mandrel but keeping almost the constant internal diameter $d_{int}\approx$ const), for the external point of every N-th layer included into the tube wall is directly connected with the increment of "the local active actual radius" $r^{*(N)}$, and the increments of circumferential and radial strains $d\varepsilon_2^{(N)}$ and $d\varepsilon_3^{(N)}$ which are presented in the following way

$$\begin{aligned} d\varepsilon_1^{(N)} &= \pm\frac{|d\mathbf{r}^{*(N)}|}{|\mathbf{R}|} \qquad & d\varepsilon_1 &= d\varepsilon_1^{(N)}\Big|_{g_0^{(N)}=g_0} \\ d\varepsilon_2^{(N)} &= \frac{|d\mathbf{r}_i^{(N)}|}{|\mathbf{r}_{ext}^{(N)}|} \qquad & d\varepsilon_3^{(N)} &= \frac{dg_i^{(N)}}{g_0^{(N)}} \end{aligned} \tag{4.1}$$

where $d\mathbf{r}^*=d\mathbf{r}^{*(N)}=(\mathbf{s}_{\alpha b}^{(N)}\times d\boldsymbol{\alpha}_b/2)\cos\beta$ for $g_0^{(N)}=g_0$, because when $N=\langle 1\rangle$, then $g_0^{(N)}=g_0$ and the sign $(+)$ is related to the elongated layers, and $(-)$ to the compressed ones.

For the division of a thick-walled tube into a finite number of cylinders (rings) for the FEM method the increment is

$$\begin{aligned} d\varepsilon_1^{(N)} &= \pm\frac{|d\mathbf{r}^{*(N)}|}{|\mathbf{R}|} \qquad & d\varepsilon_1 &= d\varepsilon_1^{(N)}\Big|_{g_0^{(N)}=g_0} \qquad & d\varepsilon_2^{(N)} &= \frac{|d\mathbf{r}_i^{(N)}|}{|\mathbf{r}_{ext}^{(N)}|} \\ d\varepsilon_3^{(N)} &= \frac{dg_{li}^{(N)}}{g_{l0}^{(N)}} \qquad & g_i &= \sum_{N=1}^{n} g_{li}^{(N)} \end{aligned}$$

where g_i is the running thickness of the bend within the bending and bend zone.

From properties of the vector product, it results for $g_0^{(N)}=g_0$ that

$$d\varepsilon_1 = \pm\frac{cos\beta}{2|\mathbf{R}|}|\mathbf{s}_{\alpha b}^{(N)}\times d\boldsymbol{\alpha}_b| = \pm\frac{cos\beta}{2|\mathbf{R}|}\Big|\mathbf{r}_i^{(N)}\sin\frac{\alpha_b}{2}\times d\boldsymbol{\alpha}_b\Big| = \pm\frac{cos\beta}{2|\mathbf{R}|}\Big|\mathbf{r}_i^{(N)}\Big(\sin\frac{\alpha_b}{2}\times d\boldsymbol{\alpha}_b\Big)\Big| \tag{4.2}$$

for $g_0^{(N)}=g_0$ and

$$\mathbf{s}_{\alpha b}=\mathbf{s}_{\alpha b}^{(N)}=\mathbf{r}_i^{(N)}\sin\frac{\alpha_b}{2} \qquad \text{so} \qquad d\varepsilon_1=\frac{1}{2|\mathbf{R}|}|\mathbf{r}_i^{(N)}|\cos\beta\sin\frac{\alpha_b}{2}|d\boldsymbol{\alpha}_b|\Big|$$

Then for $g_0^{(N)}=g_0$

$$d\varepsilon_1=\pm\frac{1}{R}r_i^{(N)}\cos\beta\sin\Big(\frac{\alpha_b}{2}\Big)d\alpha_b \qquad d\varepsilon_2=\frac{dr_i^{(N)}}{r_{ext}^{(N)}} \qquad d\varepsilon_3=\frac{dg_i^{(N)}}{g_0^{(N)}} \tag{4.3}$$

where $r_i^{(N)}$ is the local actual radius related to the external point of the N-th layer included into the tube wall $r_i^{(N)} = r_{int} + g_i^{(N)}$ and $r_i^{(N)} \in \langle r_{int}, r_i \rangle$.

When $N = 1$ and when the averaged strains are related to the central layer, then $r_i^{(N)} \equiv r_{im} = r_{int} + g_i/2$, where r_{im} is the averaged small active bending radius related to a half of the actual thickness of the bent tube wall, $r_{ext}^{(N)}$ is the local external radius related to the external point of the N-th layer included into the tube wall, $r_{ext}^{(N)} \in \langle r_{int}, r_{ext} \rangle$ and $r_{ext}^{(N)} = r_{int} + g_0^{(N)}$. When $N = 1$, then $r_{ext}^{(N)} = r_{ext}$, where g_i is the local actual thickness of the wall of the whole section, $g_0^{(N)}$ and $g_i^{(N)}$ are calculation initial thicknesses of the considered N-th layer of the bent tube wall, such that $g_0^{(N)} \in \langle 0, g_0 \rangle$, and the actual local wall thickness for the external point of the N-th layer of the tube section such that $g_i^{(N)} \in \langle 0, g_i \rangle$.

After integration of expression $(4.3)_1$ in the range of change of the angle ξ from the actual value of the angle 2α up to α_b, i.e. in the range $\xi \in \langle 2\alpha, \alpha_b \rangle$, are obtains

$$\varepsilon_1 = \pm \Big\{ \Big[\frac{1}{R} r_i^{(N)} \cos\beta \Big(\cos\alpha - \cos\frac{\alpha_b}{2} \Big) \Big] \Big|_{g_0^{(N)} = g_0} + \frac{\cos\beta}{R} \Big[\int\limits_{2\alpha}^{\alpha_b} \cos\frac{\xi}{2} \frac{dr_i^{(N)}}{d\xi} \, d\xi \Big] \Big|_{g_0^{(N)} = g_0} \Big\} \tag{4.4}$$

As it results from expression (4.4), the longitudinal component of the plastic strain ε_1 (expressed in measures of relative strains) contains two terms. The second term estimated from calculations is negligibly low because its maximum value equals to only some percent as compared with the value of the first term. Thus, this term is disregarded in further considerations. Thus for $g_0^{(N)} = g_0$

$$\varepsilon_1 \cong \pm \frac{1}{R} r_i^{(N)} \cos\beta \Big(\cos\alpha - \cos\frac{\alpha_b}{2} \Big) \tag{4.5}$$

The expression determining the circumferential and radial components has been obtained on the assumption that during bending of the tube material particles are moving along the elbow radius to the center of the bent tube in the elongated layers and from the center in the compressed layers. The actual and averaged local components of the strain state take the following form for $g_0^{(N)} = g_0$

$$\begin{aligned} &\varepsilon_1 \cong \varepsilon_1^{(N)} = \pm \frac{1}{2R} d_i^{(N)} \cos\beta \Big(\cos\alpha - \cos\frac{\alpha_b}{2} \Big) \qquad \varepsilon_2 \cong \varepsilon_2^{(N)} = \frac{d_i^{(N)} - d_{ext}^{(N)}}{d_{ext}^{(N)}} \\ &\varepsilon_3 \cong \varepsilon_3^{(N)} = \frac{g_i^{(N)} - g_0^{(N)}}{g_0^{(N)}} \end{aligned} \tag{4.6}$$

The strains should be measurable during experiments performed in order to verify analytical or numerical calculations. Thus, transformation of Eqs. (4.6) for determination of strains for the total thickness of the elbow wall (it concerns especially thin-walled tubes) measured on the external surfaces (external measuring quantities after bending), or measurements of the initial thickness of the tube which is going to be bent requires the averaging and replacement of the values $d_{ext}^{(N)}$ from (4.6) by d_{ext}, and $g_0^{(N)} = g_0$, because when $N = 1$ then $g_0^{(N)} = g_0$, $d_{ext}^{(N)} = d_{ext}$, $d_i^{(N)} = d_i$. Now, it is necessary to derive measures of the logarithmic strains, useful in plastic work technologies in the case of large strains (Franz, 1961)

$$\begin{aligned} &\varphi_1 = \ln\Big[1 \pm \frac{1}{2R} d_i \cos\beta \Big(\cos\alpha - \cos\frac{\alpha_b}{2} \Big) \Big] \\ &\varphi_2 = \ln\frac{d_i}{d_{ext}} = \ln\frac{d_{int} + 2g_i}{d_{ext}} \qquad \varphi_3 = \ln\frac{g_i}{g_0} \end{aligned} \tag{4.7}$$

The expressions for component strains (4.1)-(4.7) describe deformation in bent tubes made of plastic and incompressible continuous media (Śloderbach, 1999; 2002). Real materials undergo deformation in another way (especially in the compressed layers), and tube bending in bending machines is not unbounded upsetting in those layers. There are boundary limitations and forcing for displacements (especially in the compressed layers of the bent tube) resulting from the structure of the bending machine and its rigidity. There are also forces of external friction of the tube with the bending machine tools, and internal friction in the bent tube materials, and others. From the tests described in literature (Franz, 1961; Gruner, 1960; Grunow, 1985; Korzemski, 1968, 1971; Wick *et al.*, 2001; Yang and Lin, 2004; Zhang *et al.*, 2011; Zhiqiang *et al.*, 2011) and the tests performed by the author (Śloderbach, 1999, 2000, 2002) it appears that expressions (4.1)-(4.7) should be modified. Only the longitudinal component (along the tube axis) should be modified because values of the circumferential and radial components (along thickness) result directly from the longitudinal component and the condition of plastic incompressibility of the material. The longitudinal component expressed in logarithmic measures of strains (4.7) is modified because the results of experimental measurements are defined in such measures and determined on the external surfaces of the bent tube (Franz, 1961; Gruner, 1960; Grunow, 1985; Korzemski, 1968, 1971; Śloderbach, 1999, 2000, 2002).

The modification of expressions (4.7) consists in introduction – according to the experimental data – of two parameters k and λ_i (technological-material parameters of the tube bending process). Thus

$$\begin{aligned} &\varphi_1 = \lambda_i \ln\Big[1 \pm \frac{1}{2R} d_i \cos\beta\Big(\cos(k\alpha) - \cos\frac{k\alpha_b}{2}\Big)\Big] \\ &\varphi_2 = \ln\frac{d_i}{d_{ext}} = \ln\frac{d_{int}+2g_i}{d_{ext}} \qquad\qquad \varphi_3 = \ln\frac{g_i}{g_0} \end{aligned} \tag{4.8}$$

where g_i is the local actual thickness of the bent elbow wall, d_i – local actual external diameter of the elbow in the bending zone: $d_i = 2r_i$, $d_i = d_{int} + 2g_i$, λ_i – technological-material coefficient of strain distribution in the elongated layers $i = 1$ and compressed layers $i = 2$, defined from experiments, so as $\lambda_1 \cong 1$ and $\lambda_2 \in \langle 0, 1\rangle$. In the most cases of known tests on the tube bending process using the method considered in this paper it can be assumed that $\lambda_2 \approx 0.5$, k – technological-material coefficient dependent on the bent tube material and the applied bending technology determining the bending zone range in the bent zone. This coefficient is defined during experiments, theoretically $k \in \langle 1, \infty)$. It seems that in the case of most of metallic materials it is sufficient when $k \in \langle 1, 6\rangle$. From the recognized tests and calculations it even appears that $k \in \langle 1, 3\rangle$ (see e.g., Franz, 1961; Gruner, 1960; Grunow, 1985; Korzemski, 1968, 1971; Śloderbach, 1999, 2000, 2002). In the case of more ductile, soft, plastic materials bent at elevated temperatures (hot, semi-hot or preheated bending) or bent with a greater radius R and with a more fitted expanding mandrel (segment with an adjusted external diameter) with rich lubrication of the mandrel and the tube interior the coefficient k is lower (tends to unity, $k \to 1$). Thus, it appears that the coefficient k allows one to include (indirectly and in part) some effects of friction between the mandrel and the bent tube wall. For elbows bent to $180°$, the coefficient k expresses the ratio of the bending angle α_0 to the real value of the bending angle α_b, i.e. $k = \alpha_0/\alpha_b$. When the bending angle is $\alpha_0 = k\alpha_b = 180°$, for example as in Franz (1961), Korzemski (1971), Śloderbach (1999, 2002), then $k = 180°/\alpha_b$. If $\alpha_0 = 90°$, then $2\alpha_0 = k\alpha_b = 180°$, when $\alpha_0 = 60°$, then $3\alpha_0 = k\alpha_b = 180°$, etc.

From the known tests on bending using the method of wrapping and the template and mandrel, it appears that the coefficient k decreases when the mandrel is well chosen, put forward and lubricated, and if the applied metals and their alloys are soft and very soft.

The transformed and adapted for analytical calculations of strains in external points of every N-th layer included into the bent tube wall (especially, it concerns the division into a finite number of cylinders (rings) for thick-walled tubes and the FEM method) in the elongated and compressed layers expressions (4.8) take the form

$$\begin{aligned}
&\varphi_1^{(N)} = \lambda_i \ln\Big[1 \pm \frac{1}{2R} d_i^{(N)} \cos\beta \Big(\cos(k\alpha) - \cos\frac{k\alpha_b}{2}\Big)\Big] \\
&\varphi_2^{(N)} = \ln\frac{d_i}{d_{ext}^{(N)}} = \ln\frac{d_{int}+2g_i}{d_{ext}} \qquad \varphi_3^{(N)} = \ln\frac{g_i}{g_0} \qquad g_i = \sum_{N=1}^{n} g_{li}^{(N)}
\end{aligned} \tag{4.9}$$

From the assumptions and derived expressions (4.1)-(4.9), it appears that the bending angle α_b is the basic parameter determining the advancing of the bending process.

4.2. Derivation of the expression for displacement of the neutral axis

The aim is to derive an extended (for transient zones and for unfree bending) expression determining displacement of the neutral axis in plastic bending. In the paper by Tang (2000), the autor derived the following approximate expression for the displacement of the neutral axis, see Fig. 3

$$y_0 = \frac{0.42}{\widetilde{r}} r_m \tag{4.10}$$

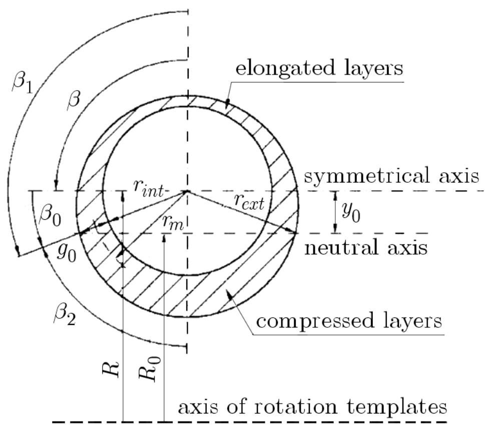

Fig. 3. Schematic picture of the elbow cross-section and its characteristic parameters

The extended expression determining the displacement of the neutral axis resulting from Section 4.1 in this paper (with no derivation), valid for transient zones and not unbounded bending, is

$$y_0 \cong \lambda_0 \frac{0.42}{\widetilde{r}} r_m \Big(\cos(k\alpha) - \cos\frac{k\alpha_b}{2}\Big) \tag{4.11}$$

where λ_0 is the correction coefficient of displacement of the neutral layer, $\lambda_0 \in \langle 0,1 \rangle$, $\widetilde{r}$ – relative radius of bending, $\widetilde{r} = R/d_{ext}$, r_m – mean radius of the bent tube, $r_m = r_{int} + g_0/2$.

The coefficient λ_0 determines characteristic technological-material parameters of the tube bending process, such as kind of the mandrel, tube material, shape of the template and the flatter, strip pressure, clearances, forces of friction between the bent tube and the bending machine rigidity of the bending machine, kind of bending (cold, hot, self-hot, with preheating). From Eqs. (4.10) and (4.11), it appears that for very small bending radii $R \in \langle 0.5d_{ext}, d_{ext} \rangle$ and more thin-walled tubes $s_w^* \ll 0.05$, the maximum displacement of the neutral axis can be equal to $\sim 25\%$ of the diameter of the tube which is going to be bent. Greater displacements of the neutral axis can be caused by another bending technology because in the case of the considered ranges $\widetilde{r}$ and s_w^* tubes are often bent with the use of a force which is opposite to the force rotating the template so as to obtain a suitable stress distribution in the cross section. From extended Eq. (4.11), it also appears that the displacement of the neutral axis is influenced not only by the bending radius and the tube thickness (thin-walled) (see Franz, 1961; Hill, 1986; Korzemski, 1968, 1971; Śloderbach, 1999, 2002; Wick *et al.*, 2001, Yang and Lin, 2004), but by suitable technology, bending parameters and the tube material as well. From Eq. (4.11), it also appears that there are three additional parameters determining the displacement of the neutral axis and its position in the bending zone: the bending angle and the angle determining the position of the point in the bending zone and the coefficient k. Thus, if $\cos(k\alpha) = 1$ and $\cos(k\alpha_b/2) = 0$ then $y_0 = y_{0\,max} \cong \lambda_0(0.42/\widetilde{r})r_m$, see Eq. (4.11).

The introduced limitations concerning the tube bending parameters cause that, for example, the maximum displacement (for instance for $R = d_{ext}$, $s_w^* = 0.03$ and $\lambda_0 = 0.5$) relative to the external diameter of the bent tube of the neutral axis is $y_0/d_{ext} \approx 10\%$. However, for some ranges R and s_w^* and bending technologies and tube materials, relationships (4.8) or (4.9) which do not include the displacement of the neutral axis y_0 can be applied in the strain description. Thus, they are used for precise description of fundamental experiments presented by Franz (1961). The estimated maximum value y_0 can be in practice even lower owing to a suitable selection and set-up of tooling of the bending machine, removal of clearances, a more plastic material for the bent tube, application of bending at elevated temperatures, increase of rigidity of the bending machine and so on. In the compressed layers, the effects resulting from not unbounded upsetting may be less, they will be more intense along the perimeter of the displacement of the bent tube material to the sides, upward and along the bent axis. These can cause lower values of the coefficient λ_0.

4.3. Relationshisps including displacement of the neutral axis

The derived relationships for the strain state determination which describe the problem of the displacement of the neutral axis in plastic bending (according to Figs. 2-4) and according to Sections 4.1 and 4.2 by suitable substitution of expression (4.11) determining y_0 to modified Eqs. (4.8) and (4.9) are as follows:

— for elongated layers

$$\begin{aligned} &\varphi_1 \cong \lambda_i \ln\Big[1 + \frac{d_i \cos\beta + 2y_0}{2(R - y_0)}\Big(\cos(k\alpha) - \cos\frac{k\alpha_b}{2}\Big)\Big] \\ &\varphi_2 \cong \ln\frac{d_i}{d_{ext}} \qquad\qquad \varphi_3 \cong \ln\frac{g_i}{g_0} \end{aligned} \tag{4.12}$$

— for compressed layers

$$\begin{aligned} &\varphi_1 \cong \lambda_i \ln\Big[1 - \frac{d_i \cos\beta - 2y_0}{2(R - y_0)}\Big(\cos(k\alpha) - \cos\frac{k\alpha_b}{2}\Big)\Big] \\ &\varphi_2 \cong \ln\frac{d_i}{d_{ext}} \qquad\qquad \varphi_3 \cong \ln\frac{g_i}{g_0} \end{aligned} \tag{4.13}$$

The system of equations (4.12), (4.13) is the searched set of expressions for description of the strain state in the bending process in bending machines (thin-walled metal tubes) with the method of wrapping on a rotating template and using the mandrel, or without a mandrel, but when $d_{int} \approx$ const is kept while bending. The derived equations include the effect of the displacement of the neutral axis in plastic bending on the deformation field. If in equations (4.12) and (4.13) we substitute $y_0 = 0$, then the considered problem does not include the influence of the displacement of the neutral axis in plastic bending on the strain distribution. When, for example $R = d_{ext}$, $s_w^* = 0.03$ and $\lambda_0 = 0.5$, then $y_{0\,max}/d_{ext} \approx 10\%$, and the calculated increments of longitudinal and equivalent strains including the effect of the displacement of the axis are by $\sim 20\%$ greater as compared with the values obtained for the case which does not include the axis displacement y_0. It means that for some certain values of the bending radius $R < 1.5d_{ext}$ and for thin-walled tubes $s_w^* \leqslant 0.03$, the displacements of the neutral axis should not be neglected in the analytical description of the strain state. The estimated maximum value of y_0/d_{ext} can be lower when, for example, $R \geqslant 1.5d_{ext}$ and $0.03 \leqslant s_w^* < 0.125$, then $y_{0\,max}/d_{ext} \approx 6\%$ and less. This is an additional reason why relationship (4.8) can be used for analysis and description of experimental results given in the fundamental paper by Franz (1961), not including the displacement of the neutral axis. Relationships (4.8) used in Śloderbach (1999, 2000) do not include the displacement of the neutral axis y_0, because the bent tube tested by Franz (1961) was thick-walled, in which $s_w^* \approx 0.127$ (then $s^* \approx 0.1$), and it was bent at the radius $R \cong 1.73d_{ext}$ (so $R > 1.5d_{ext}$). Then for $\lambda_0 = 0.5$, $y_{0\,max} \approx 2.43\,\mathrm{mm}$ and $y_{0\,max}/d_{ext} \approx 5.5\%$.

Expressions (4.12) and (4.13) can be written in a more compact form as

$$\begin{aligned} &\varphi_1 \cong \lambda_i \ln\Big[1 \pm \frac{d_i \cos\beta \pm 2y_0}{2(R-y_0)}\Big(\cos(k\alpha) - \cos\frac{k\alpha_b}{2}\Big)\Big] \\ &\varphi_2 \cong \ln\frac{d_i}{d_{ext}} \qquad \varphi_3 \cong \ln\frac{g_i}{g_0} \end{aligned} \tag{4.14}$$

According to the assumptions that the derived expressions for strain components in tube bending processes are identified with plastic strains, we obtain

$$\begin{aligned} &\varphi_1 = \varphi_1^p \qquad \varphi_2 = \varphi_2^p \qquad \varphi_3 = \varphi_3^p \qquad \varphi_{(i)} = \varphi_{(i)}^p \\ &\varepsilon_1 = \varepsilon_1^p \qquad \varepsilon_2 = \varepsilon_2^p \qquad \varepsilon_3 = \varepsilon_3^p \qquad \varepsilon_{(i)} = \varepsilon_{(i)}^p \end{aligned} \tag{4.15}$$

5. Initial and boundary conditions for the model of strains

Expressions (4.12)-(4.14) satisfy the following boundary and initial conditions

a) when $\alpha = \alpha_b/2 = 0$, we have the beginning of the bending process (no bending),

b) when $\alpha = \alpha_b/2 \neq 0$ – beginning and end of the bending zone,

c) when $\beta_1 = 90° + \beta_0$ and $\beta_2 = 90° - \beta_0$ – position of the layer of zero elongations (the neutral axis of plastic bending) defined by the radius R_0 in the bent zone. Then

$$R_i = R_0 \qquad r_i = r_{ext} \qquad g_i = g_0$$

and

$$\varphi_1 = \varphi_2 = \varphi_3 = \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0 \qquad \text{and also} \qquad \varphi_{(i)} = \varepsilon_{(i)} = 0$$

d) when $k\alpha = \beta_1 = \beta_2 = 0°$ – tip points of the bending zone, and $k\alpha_b \in (0°, 180°)$, then

$$R_i = R - y_0 \pm (r_{int} + g_i \pm y_0)\Big(1 - \cos\frac{k\alpha_b}{2}\Big) \qquad r_i = r_{int} + g_i$$

e) when $k\alpha = \beta_1 = \beta_2 = 0°$ and $k\alpha_b = 180°$, then R_i, and r_i, g_i reach their extreme values, i.e. the maximum and minimum (in elongated layers) or the minimum and maximum (in compressed layers), respectively. It is the condition of initiation of the maximum strains at that point, and formation of the plateau zone (see, Franz, 1961; Korzemski 1971; Śloderbach, 1999, 2000, 2002).

– Zone of elongated layers

$$g_1 = g_{1\,min} \qquad R_1 = R - y_0 + (r_{int} + g_{min} + y_0) \qquad r_1 = r_{int} + g_{1\,min}$$

– Zone of compressed layers

$$g_2 = g_{2\,max} \qquad R_2 = R - y_0 - (r_{int} + g_{max} - y_0) \qquad r_2 = r_{int} + g_{2\,max}$$

In this case, the main components of logarithmic and relative strains and their intensities also reach the extreme values which differ in the zones of elongated and compressed layers, respectively.

f) when $d_i = d_{int} = d_{ext}$ (internal surface of the bent tube), then – according to expressions (4.12)-(4.14) – it appears that $\varphi_2 = 0 \Rightarrow \varepsilon_2 = 0$.

Note: Derived expressions (4.12)-(4.14) have a physical sense when the conditions for $R > y_0$ or for $R > y_{0\,max}$ are satisfied, and in practice they are always satisfied.

6. Exemplary calculations

This Section presents exemplary calculations of variation of longitudinal strains and wall thickness, including the displacement of the neutral axis of in plastic bending related to the external diameter of the bent tube from the range from 0% up to the maximum value of 25%. Simulation calculations have been performed for a metallic (steel) tube of dimensions ∅44.5 mm × 4.5 mm and $s_w^* \approx 0.127$ in the main bending plane in elongated layers $\alpha = \beta_1 = 0°$ for the bending angle $\alpha_0 = k\alpha_b = 180°$. For calculations, the following values of the technological-material coefficients have been assumed: $k \cong 3$ and $\lambda_1 \cong 1$ and the bending radius $R \approx 77$ mm ($R \approx 1.73 d_{ext}$). The data for calculations have been taken of Franz (1961). The calculations have been performed using expressions (4.14) and the condition of plastic incompressibility of the bent tube material. The calculation results are presented in Figs. 4 and 5.

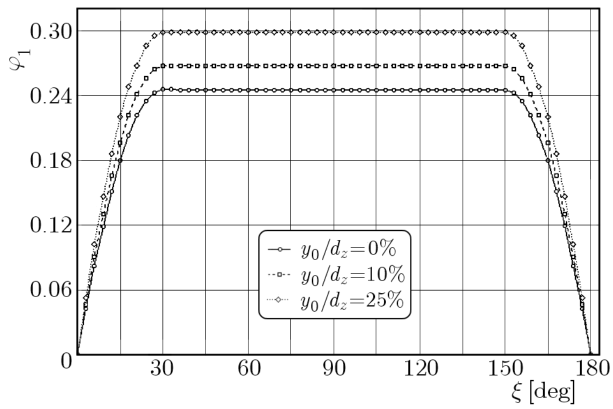

Fig. 4. Exemplary calculations of longitudinal deformations, depending on the neutral axis displacement y_0, where $dz \equiv d_{ext}$

7. Conclusions

- The paper contains derivation of the generalized relationships for logarithmic and relative measures of strains: longitudinal (along the tube axis), circumferential and along thickness (radial) during bending of thin- and thick-walled metallic tubes in bending machines. Generalization of the strain description as compared to the previous papers by the author

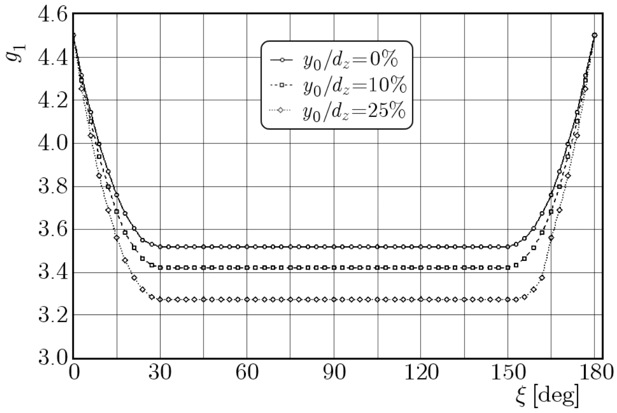

Fig. 5. Exemplary calculations of the wall thickness distribution as depending on the neutral axis displacement y_0, where $dz \equiv d_{ext}$

(Śloderbach, 1999; Śloderbach and Rechul, 2000) consists in including the displacement of the neutral axis. The strains can be defined in the main bending plane and each parallel or perpendicular plane, i.e. in all points of the bending zone. The derived relationships describing the measures of logarithmic and relative strains depend on the bending radius R, geometrical dimensions of the tube, bending angle α_b, angular coordinates α and β, which describe the bending zone in the range of the bending angle $k\alpha_b \in \langle 0°, 180° \rangle$, displacement of the neutral axis y_0 and two technological-material coefficients k and λ_i. The results of exemplary calculations of longitudinal strains and wall thickness distribution for the elongated layers performed for the bending angle $\alpha_0 = k\alpha_b = 180°$ including the displacement of the neutral axis related to the external diameter from the range from 0% up to the maximum value of 25% are shown in form of suitable curves. The calculations include the value of the coefficient of the bending zone range $k = 3$ and the coefficient of strain distribution in the elongated layers $\lambda_1 = 1$. From the obtained graphs, it appears that there is a certain proportionality between the values of displacement of the neutral axis expressed in % and the relative increment of the longitudinal strain as well as reduction of the elbow wall thickness.

- In future tests an explicit (analytical) form of k depending on the coefficient of friction and suitable technological-material parameters of bending can be searched. When the coefficient of friction between the mandrel and the internal wall of the bent tube tends to infinity, then the coefficient k tends to infinity, too. It means no bending because the angular range of the bending zone tends to zero $\alpha_b \to 0°$. Then, also the angle of bend zone tends to zero $\alpha_0 \to 0°$.

- In the considerations on the problem of taking into accout the displacement of the neutral axis in plastic bending, some simplifications can be introduced, for example the neglecting of y_0 in the first or second term in the numerator in expressions $(4.12)_1$ and $(4.13)_1$ or the numerators or denominators of expressions $(4.12)_1$ and $(4.13)_1$ in order to obtain the best conformity of the calculated quantities with the experimental data or those taken from literature. Such simplifications should be dependent on the bending parameters occurring in Eqs. (4.10) and (4.11).

- Śloderbach (1999) presented the results of the analytical and numerical calculations based on derived relationships (4.12)-(4.14). The results of calculations coincide with the experimental data of Franz (1961) for a tube of dimensions $\emptyset 44.5\,\mathrm{mm} \times 4.5\,\mathrm{mm}$ and $s_w^* \approx 0.127$, the bending radius $R \approx 77\,\mathrm{mm}$ $(R \approx 1.73 d_{ext})$. Corresponding calculations results have been shown in form of appropriate graphs.

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