THE METHOD OF DETERMINING THE MAIN HARMONIC FREQUENCY COMPONENT

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Abstract

Analysis of vibroacoustic signals is one of the more frequently used diagnostic methods for mechanical devices occurring, among the others, in the car diagnostics. Often, it happens that the most important element of the recorded course is the fundamental harmonic frequency of vibrations. Fundamental frequency indicates the main process related to the operation of the device and allows to follow its course. In the article the author's method of determining the fundamental frequency in the signal, being the subject of a patent application, will be presented. Its theoretical basis and application examples were discussed comparing the accuracy of its use with the accuracy of other methods. The frequency range where the method finds application is shown. That is, where its accuracy turns out to be better than the accuracy of popular methods used to determine fundamental harmonic frequency component.

Keywords

fundamental harmonic frequency, Fourier transform

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1. Introduction

Analysis of vibroacoustic signals is one of the most frequently used diagnostic methods of mechanical devices. They are often encountered, among the others, in car diagnostics. It is possible to distinguish: indirect measurements, based on vibrations propagating in the air (sound measurements) and direct measurements of mechanical elements vibrating. The measured harmonic signal can be analysed for: amplitude (equated to the signal strength), frequency spectrum, phase shifts or other information contained in the signal [1, 3].

Among the examples of vibroacoustic analysis in the conditions of the vehicle inspection stations there are:
- analysis of sound coming from the car engine to determine the sound intensity [3],
- analysis of sound coming from the car engine to determine the current rotation speed [4],
- sound analysis from the bearings operation to identify possible damages [3],
- analysis of the current position of the plate forcing the suspension system vibrations to evaluate damping effectiveness (experimental work) [5].

Often, it happens that the most important element of the recorded course is the fundamental vibrations harmonic frequency component . The dominant frequency indicates the main process connected with the operation of the device and allows to follow its course. That is why the author's method will be presented in this article. It is used to determine the fundamental frequency in the signal and it is the subject of a patent application [7].

2. The proposed method of determining the fundamental harmonic frequency component

The discussed algorithm allows to determine the fundamental harmonic frequency component for a given function (course) of any physical or mathematical quantity. Due to the widespread use of digital signal processing systems, and what is connected with it, the calculations in the discrete field, the presented method will also be discussed in a discrete application for a given sampling frequency marked fpr [2, 6].

The principle of the method is based on the comparison of the amplitude of the signal with the amplitude of the derivative of the signal. For the harmonic signal the equation of motion may be in the form as follow:

$$F(t) = A \cdot \cos(2\pi f \cdot t + \varphi), \qquad (1)$$

where:

t – current time,
$F(t)$ – current location,
A – signal amplitude,
f – signal frequency,
φ – phase shift.

Then the derivative of the signal over time can be represented in the following form:

$$F(t) = -2A\pi f \cdot \sin(2\pi f \cdot t + \varphi). \qquad (2)$$

In this case, the amplitude of the signal derivative is equal to: . Thus, it can be seen that the frequency of the primary signal affects the amplitude of the signal derivative. So by comparing the amplitudes of the signal and its derivative over the time its frequency can be determined. For a discrete signal, this relationship can be represented by the following equation:

$$f = \frac{A_x}{2\pi A_x}, \qquad (3)$$

where:

f – frequency tested,
A_x – amplitude of the signal derivative,
A_x – signal amplitude.

To determine both amplitudes any method can be used. In practice, the maximum absolute value of the signal and its derivative can be used for this purpose. The presented algorithm is characterized by simplification in numerical terms which will be discussed in more detail below.

3. Frequency methods in common use [6]

Among the commonly used methods for determining the main harmonic component there are two types: those that analyse the signal in the time domain and those that analyse the signal in the field of frequency. Among the time-oriented methods, it is worth mentioning:

- methods based on the autocorrelation function and derivative methods,
- methods examining crossing the zero or other local changes in value over time.

In turn, the analysis methods conducted in the frequency domain are based on the Fourier transform. The latter methods are the most popular. These include both advanced algorithms based on Dirac comb analysis or subspace analysis and the simple search for the maximum global function of the spectral density. However, in all such calculations, the key element is the Fourier transform or its modification in the form of a fast Fourier transform. Below, the basic problems related to the aforementioned most popular computational methods will be presented and their comparison with the presented algorithm.

The commonly used algorithms for determining the harmonic frequency components are associated with two constraints, the first of which is the numerical complexity of the algorithm. The basic solution, which is the Fourier transform, is characterised by numerical complexity increasing with the square of the number of samples of the signal being analysed. This in some cases excludes the use of this algorithm. The commonly used modification of the above solution, called the fast Fourier transform, reduces computational complexity and makes it dependent on the value: $N \cdot \log_2 N$, where N is the number of samples of the analyzed signal. However, this solution is only possible for the N values that are the power of the number 2, which is also cumbersome. Compared to the aforementioned solutions, the proposed algorithm is characterized by a linear dependence of computational complexity on the number N (analyzed signal samples), which is definitely a better solution than hitherto existing ones.

The second problem in the commonly used algorithms is the accuracy of calculations, which depends on the ratio of frequencies in the analysed signal to the sampling frequency and on the number of the analysed signal samples. Fourier transform (fast and normal) is characterized by the highest accuracy at frequencies close to one fourth of the sampling frequency of the signal. The frequency range for which the calculation accuracy is satisfactory depends on the number of the analysed signal samples. This is a big limitation, especially in the case of non-stationary signal analysis (in which the frequency may change over time). The possibility of analysis for as few samples as possible is the key but this is associated with a reduction in the accuracy of calculations, mainly at low frequencies. The proposed algorithm has a completely different characteristic of the accuracy of the calculations. It works best at low relative frequencies – close to one-tenth of the ratio of the measured frequency to the sampling frequency. Also it is not sensitive to reducing the number of the analysed samples as long as they cover at least three quarters of the period of the fundamental harmonic component. This makes the proposed algorithm the most accurate in those conditions in which common algorithms are no longer useful.

In turn, in the frequency range at which Fourier transform displays the highest accuracy, the proposed algorithm is characterized by significant deterioration of accuracy. Therefore, the presented algorithm should not be considered as more accurate in the universal sense, but as a method that works better in specific applications (at lower frequencies of the tested signal and with fewer signal samples).

With regard to the presented algorithm, there are also two limitations. The first is the fact that it does not determine the entire spectrum of harmonic components existing in the analysed signal but gives one value, corresponding to the frequency of the main harmonic component. The second limitation is the sensitivity of the algorithm to the existence of other harmonic components in the signal in addition to the main component. The fact of their existence in the signal in proportion to the main component exceeding ten percent limits the frequency range in which the main harmonic component is correctly detected. Still, however, this is a relatively low frequency range, in which the Fourier transform causes larger calculation errors than the presented method. Below will be presented results of simulation of the detection of the main harmonic component which were obtained using the presented method and the Fourier transform.

4. Simulation results of detecting the fundamental harmonic component

Fig. 1-6 shows the courses of the analysed signal x (t) generated at the frequency of 80 Hz, corresponding to different numbers of samples, and their spectra in the frequency domain calculated based on the Fourier transform.

Fig. 2 shows the frequency spectrum of the signal from Fig. 1. In the frequency domain, clear streaks are visible. First of them corresponds to the main harmonic component, and the second is its mirror image resulting from the duplication of the main component with respect to the sampling frequency (the graph of power spectral density is assumed to be symmetrical). A streak, based on which the signal frequency can be read, as expected, occurs slightly below value of 100 Hz, which corresponds to the set frequency of 80 Hz. However, there is a characteristic blur of the streak that prevents the precise reading of the frequency. The maximum value occurs exactly at 97 Hz, which means a frequency measurement error of 17 Hz.

Fig. 1. The course of the harmonic signal x(t) for frequency of 80 Hz and 32 registered samples

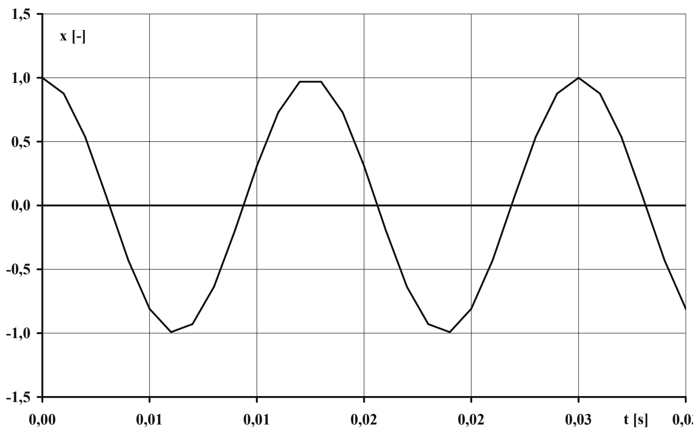
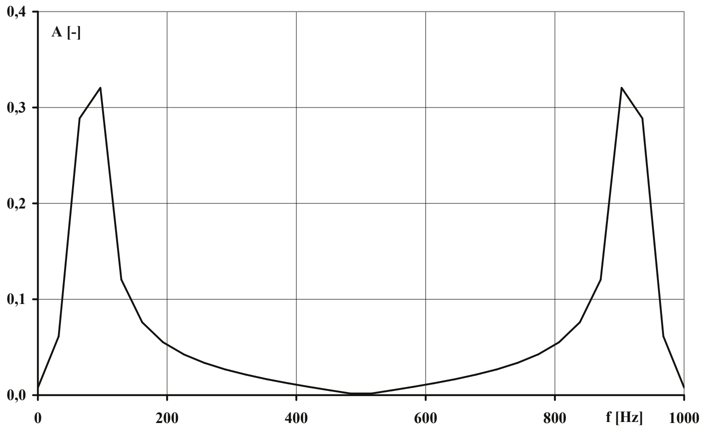


Fig. 2. Spectral power density for 80 Hz signal and 32 registered samples



Meanwhile, for the same signal analysed using the proposed algorithm, the calculated frequency value was 79 Hz. This results in a measurement error of 1 Hz.

This phenomenon is even more interesting after reducing the number of the analysed samples by half. The course of such a signal is shown in Fig. 3. As it can be seen, in the analyzed area there is a fragment of the harmonic wave that contains little more than one period of vibrations. This is a quite difficult situation for the frequency analysis using the Fourier transform.

The spectral power density calculated for this signal by the Fourier transform is shown in Figure 4. The streaks that occurred seem to be much more clear but the maximum of streak corresponding to the main harmonic component falls at the frequency of 67 Hz. It means an error of 13 Hz in relation to the set frequency of 80 Hz in the signal.

The frequency of the fundamental harmonic component calculated based on the presented method is again 79 Hz, which means an error within 1 Hz.

Fig. 3. Waveform of the harmonic signal x (t) for 80 Hz and 16 registered samples

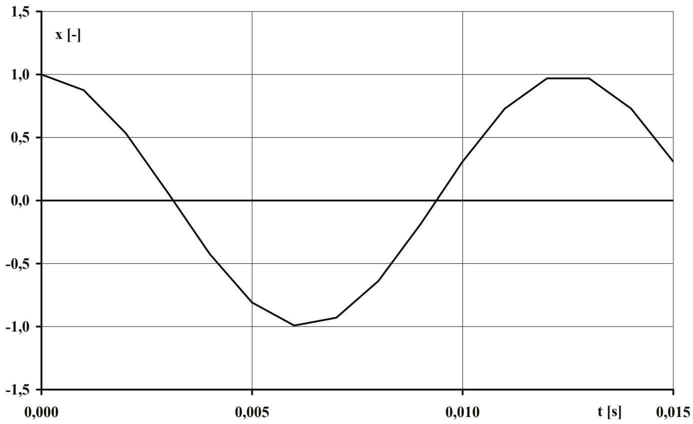


Fig. 5. Waveform of the harmonic signal x (t) for 80 Hz and 8 registered samples

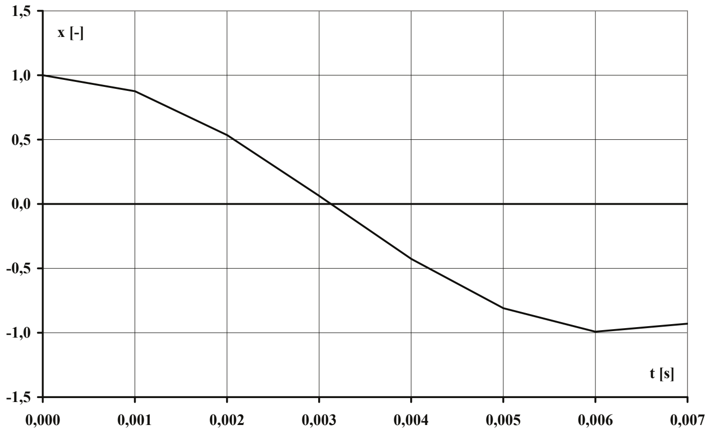


Fig. 4. Spectral power density for 80 Hz signal and 16 registered samples

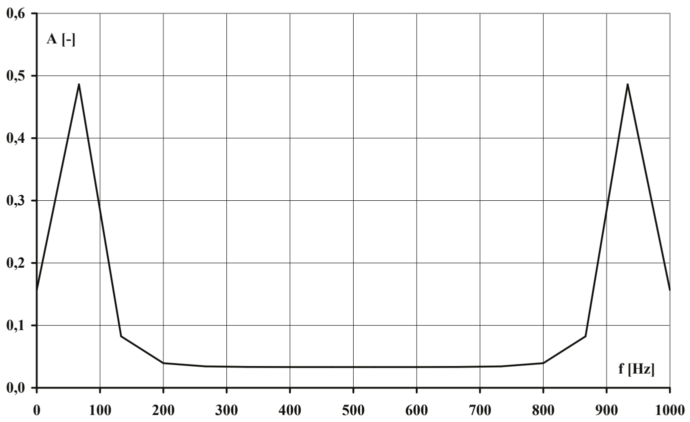


Fig. 6. Spectral power density for 80 Hz signal and 8 registered samples

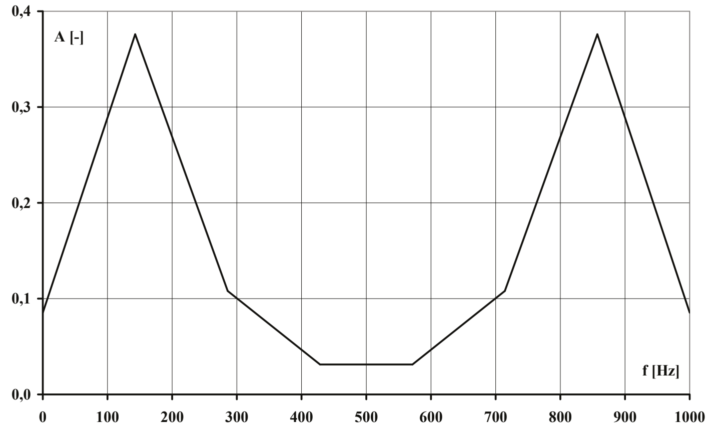


Fig. 5 shows the harmonic signal with a frequency of 80 Hz, represented this time by 8 samples. This means that within the analysed area even a single period of vibration described by the signal does not fit. The spectral power density for this signal is shown in Figure 6 and the main harmonic component read from the graph is 143 Hz, which causes a 63 Hz error (a relative error of almost 80%). Such a big error should not come as a surprise, considering the very small number of signal samples, which reduces the accuracy of calculations performed with the Fourier transform. For the same signal analysed using the presented algorithm, the read-out value of the main harmonic frequency was 78 Hz, which means an error within 2 Hz. This example illustrates the aforementioned fact that presented computational method at the low relative frequencies is characterized by very low sensitivity of the calculations accuracy to the number of samples in the signal as long as the signal fragment contains at least three quarters of the period of the main harmonic component.

Simulations of the use of two compared computational methods to determine the main harmonic component were conducted for different signal frequencies. In the following figures (Fig. 7 – Fig. 9), the relative error values in the domain of the frequency were presented. The solid line represents the results of the Fourier transform and the dashed line the presented method.

Fig. 7 shows results for the number of samples being 32, in Fig. 8. for the number of samples equal to 16 and in Fig. 9. for the number of samples equal to 8.

Fig. 7. Relation of the relative error in the domain of the fundamental signal frequency for 32 samples

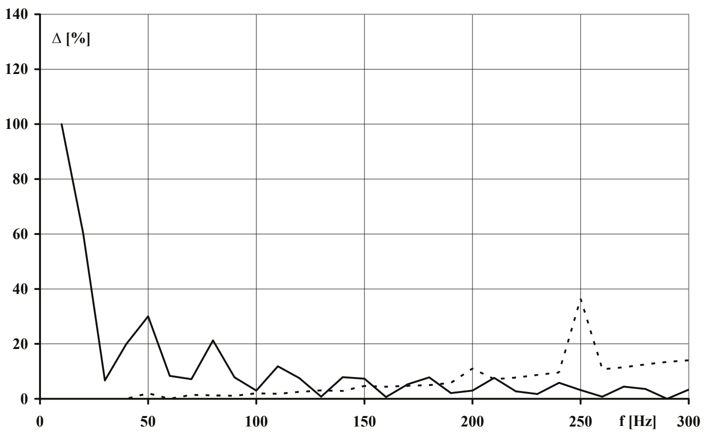


Fig. 9. Relation of the relative error in the domain of the fundamental signal frequency for 8 samples

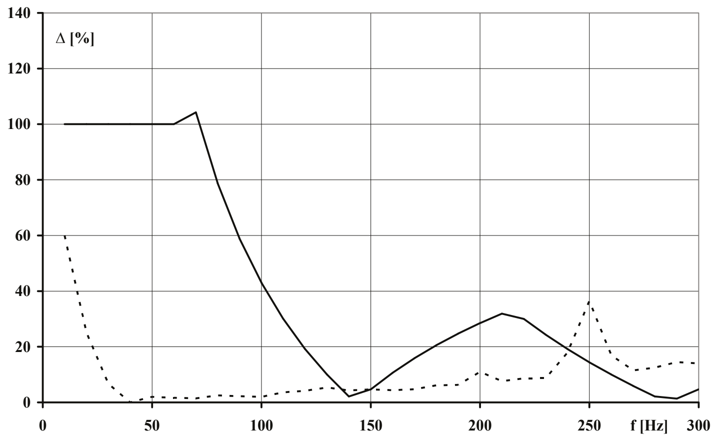


Fig. 8. Relation of the relative error in the domain of the fundamental signal frequency for 16 samples

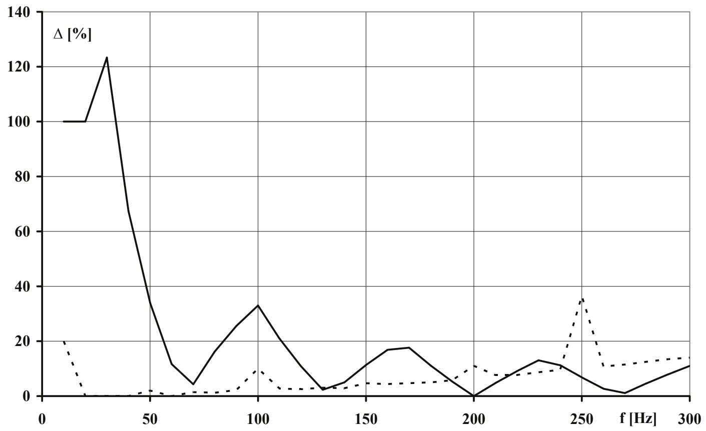
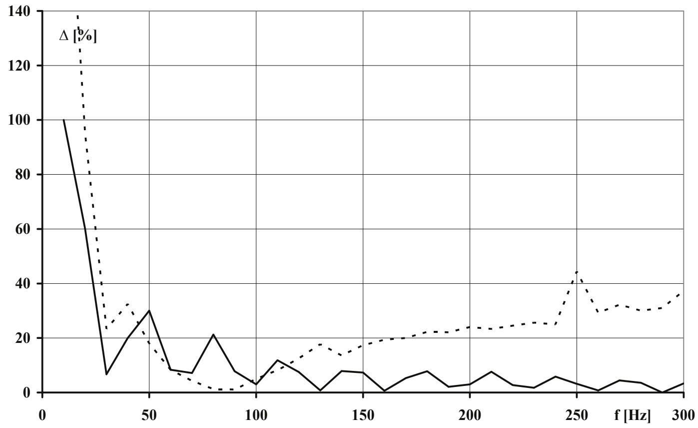


Fig. 10. Relation of the relative error in the field of the fundamental signal frequency for 32 samples with additional harmonic components



As it can be seen on the graphs above, in the lower frequency ranges the relative error values are lower in the case of the presented method, however, each time there is a certain limit frequency above which the Fourier analysis gives better results. It is worth noting that this limit frequency increases with the decreasing number of samples. This is due to the fact, which was already underlined in this article, that the presented method is only slightly sensitive to the decreasing the number of signal samples. This means that in the case of a small number of samples, its use is the most reasonable.

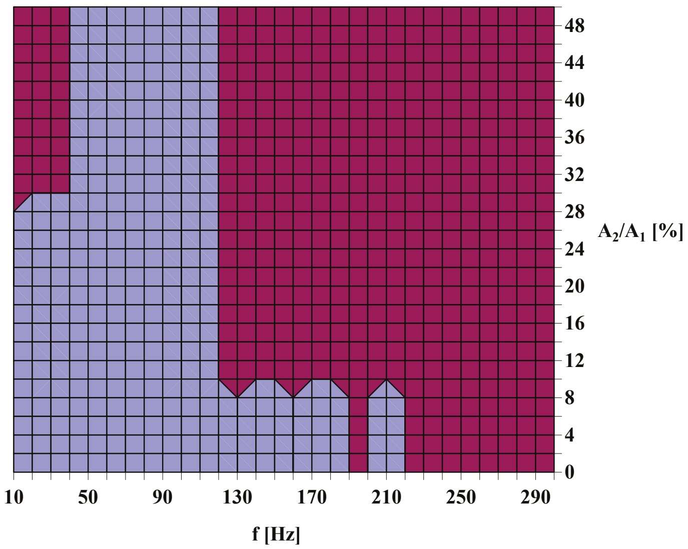
There remains the problem of the sensitivity of the presented method to the „pollution" of the analysed signal with the other harmonic components. As mentioned in chapter 3, the method presented performs best in the situation in which additional components do not exceed ten percent of the main component. This is the case when the signal comes from one dominant source – for example from a combustion engine in a holistic approach [4].

Fig. 10 presents the relation of the relative error of the considered methods in the frequency domain of the main signal disturbed by other frequency components at the level of forty percent of the main signal. This means very unfavourable conditions for the presented method.

As it can be seen in the above case, the frequency range at which the proposed algorithm causes a smaller error than the Fourier transform, falls within a small range (40 - 130 Hz).

Therefore, the usefulness of the discussed method (compared to the Fourier transform analysis) can be presented in the two-dimensional field: frequency of the analysed signal and the ratio of additional components amplitudes to the amplitude of the main signal component. Such dependence is shown in Fig. 11. The bright area corresponds to the situation in which the presented method gives results more accurately than the Fourier transform.

Fig. 11. The range of usefulness of the presented method in the field of the main signal frequency and the ratio of the additional harmonic components amplitude to the main component amplitude



As it is shown, in the case of amplitude ratios below 10%, the range of useful frequencies reaches up to around 200 Hz but when this ratio increases, the range of useful frequencies decreases to 120 Hz. This gives a certain framework for the applicability of the presented method. It should be emphasized, however, that in the case of reducing the number of samples of the analysed signal, the applicability of the method will increase as a result of the deterioration of the properties of the Fourier analysis. The above considerations are presented for a number of samples equal to 32.

5. Summary

The presented method of determining the fundamental harmonic signal component can be used in all kinds of detection systems operating in real time, that require frequency determination with which moving parts of devices move. In such solutions, devices generate a clear picture of the main harmonic component against the background of other components. The advantages of the discussed algorithm will then be:
-the possibility of using relatively short signal waveforms in the analysis (consisting of only a few samples),
-relatively high accuracy of calculations with a small number of samples,
-relatively low numerical complexity (even compared to the fast Fourier transform),
-thanks to the above features - the ability to monitor frequency changes even in real time.

With regard to the above applications, the presented algorithm can analyse signals of various origins like sound, vibration and others. It should be noted, however, that the optimal frequency for which the use of this method is the most beneficial oscillates around one-tenth of the sampling frequency. For the examples considered, the sampling rate was 1000 Hz, so that near 100 Hz the calculation results were the most accurate with the proposed method.

Bibliography

[1] A. Gajek, Z. Juda: Czujniki. Mechatronika samochodowa, Wydawnictwa Komunikacji i Łączności, Warszawa 2011.
[2] B. Pańczyk, E. Łukasik, J. Sikora, T. Guziak: Metody numeryczne, Politechnika Lubelska, Lublin 2012.
[3] K. Sitek, S. Syta: Badania stanowiskowe i diagnostyka. Pojazdy samochodowe, Wydawnictwa Komunikacji i Łączności, Warszawa 2011.
[4] T. Szczepański: Metoda wyznaczania charakterystyki prędkościowej momentu obrotowego i mocy silnika spalinowego za pomocą analizy sygnału dźwiękowego, praca magisterska pod opieką prof. Z. Chłopka, Politechnika Warszawska, Wydział Samochodów i Maszyn Roboczych, Warszawa 2008.
[5] S. Traczyk, A. Damm, R. Czajka, M. Gołaszewski. T. Szczepański: Opracowanie konstrukcji i wykonanie stanowiska badawczego do oceny skuteczności tłumienia drgań zawieszenia pojazdu, metodą EUSAMA, umożliwiającego rejestrację przebiegu drgań, w trakcie swobodnego ich wygaszania, przy różnych częstotliwościach wymuszania drgań w zakresie od 2 do 30 Hz, praca statutowa nr 6201/ZDO realizowana w Instytucie Transportu Samochodowego w latach 2012-2015.
[6] T. Zieliński: Cyfrowe przetwarzanie sygnałów, Wydawnictwa Komunikacji i Łączności, Warszawa 2005.
[7] Zgłoszenie patentowe nr P.423382, pod tytułem: Sposób pomiaru częstotliwości drgań, 07.11.2017.

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