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# PROFIT OPTIMALIZATION IN OPERATION SYSTEMS

**Leszek Knopik, Assoc. Prof.**
**Klaudiusz Migawa, Assoc. Prof.**
**Andrzej Wdzięczny, Ph. D.**
Bydgoszcz University of Science and Technology, Poland

## ABSTRACT

*The aim of this study is to analyse the effect of reducing the number of secondary damage to profit from the work of a technical object. Considering the criterion function that describes the average profit from the work of a technical object in the operation system. The study analyses a model of profit optimization, in which the design criterion function is based on the properties of Poisson branch process. Criterion function that describes the average gain is considered in the work at sufficiently general factors. Profit lifting model from the work of a technical facility is numerically exemplified. For the analysed electrical subsystem, intervals of time between the initial damage have exponential distribution, and between secondary damage - gamma distribution. In the presented example, the ability of profit optimization in operation systems of technical objects is demonstrated.*

**Keywords:** Branching Poisson process, primary failure, secondary failure, gamma distribution, mean profit, hazard function

## INTRODUCTION

In practice of the operational modes of transport, such as ships, issues regarding safety and reliability are very important. Issues of traffic safety of ships are considered in [6, 7]. An important issue in the operation of vessels is the reliability and readiness of the ship subsystems [8, 9]. In real operation systems, elimination of damage to a technical object is a desired action, which allows to provide the required level of reliability and increases the profits from the operation of a technical object. Reducing the number of defects can be achieved by improving the quality of diagnosis and improving the quality of repairs. Such actions require expenditures for equipment diagnostic subsystems and training of employees. It is known, that eliminating all or most of the damage is in practically impossible. This is related to high costs and expenditure on equipment diagnostic and repair subsystems. However, some measures for partial reduction of damage may be suitable. This work presents a model describing the change in profit while reducing some of the sudden or early damage. In practical reliability tests, a very important role is played by lifetime (time to failure) probability distributions of technical objects. The primary task in building operation process models is the identification of the available experimental data of lifespan time types distribution. These distributions describe the time to failure of an element, subsystem and operation system. Some damage arising in the operation process are the result of natural processes of wear and degradation of technical objects elements. However, other damage may result from inaccurate and ineffective repair of technical object parts that were damaged previously and also be the result of improper organization of repairs and insufficiently accurate diagnosis. Such damage in operation is called secondary. Improvement of this state often requires additional expenditure on diagnostic equipment and training of personnel. Analysis of operation systems work leads to the observation that damage to the technical object can be divided into two subsets. The first is the original damage, the other secondary damage. Secondary damage arises most frequently as a result of ineffective technical object items repair of previous damage. This fact makes the studied intervals population of time damage a statistically heterogeneous collection. In this case, for the description of life time, a mixture of probability distributions may be used. Mixtures of probability distributions were used as technical objects lifetime models in [15, 16, 17]. Analysis of the mechanism of damage generation leads to the conclusion, that most often original damage generates a series of secondary damage. In [18] an attempt to solve this problem has been undertaken based on the insight, that the time intervals between subsequent secondary damage in relation to the time interval between original damage are generally lower. Criterion function studied in [18] uses the properties of mixtures of probability distributions. Time to failure probability distributions are intensively used in the theory and practice of reliability. In practice, we encountered many situations of probability distributions conformance testing of time to failure with the known distributions. However, in operation practice, there are often situations where a the technical object is damaged suddenly and the recorded damage intervals are relatively small. The probability distribution of time to damage due to the heterogeneity is convenient to be described using mixtures of probability distributions. Time to failure in [16] is described by a distribution of exponential and Rayleigh mixture. Often in engineering practice, sudden damage are considered to be those, that occur on the same day after the repair. In this case, it is expedient to use mixtures of the known distributions such as exponential decay, gamma and Weibull distribution with a single point distribution. The problem of estimating parameters, when considering the mixture of single-point distribution with exponential distribution has been studied in [1, 12, 13, 14, 20, 21]. Parameter estimation of single-point distribution mixture with gamma distribution is presented in [22], while with the Weibull distribution in [23]. It is the motivation to use as a model the process of damage stationary branch Poisson process. The first time the Poisson branch process to describe the damage process was introduced by Bartlett [2] and Lewis [3, 19]. The study demonstrates, that this process can be an adequate model of the process of damage to the electrical subsystem for certain means of transport. The profits model uses the properties of Poisson branch process and the function describing effort to improve the diagnostics of a technical object state.

## BASIC DEFINITIONS AND CONCEPTS

The study assumes, that T is a positive random variable describing the life time of a technical object till it gets damaged. The non-negative random variable T has a finite mean value ET, distribution function F(t) satisfying the condition F(0) = 0, reliability function R(t) = 1– F(t), the probability density f(t). In this case, the risk function is defined as λ(t) = f(t) / R(t). In reliability theory and planning of preventive repair, a very large role has the distribution equivalent to the given distribution of the random variable T. The cumulative distribution of the equivalent distribution (see [4]) corresponding to the distribution with the cumulative distribution F(t) is defined as:

$$F_E(t) = \int_0^t R(u)du / ET$$

where ET is the mean value of a random variable T.

The variable probability density with equivalent distribution is expressed by the formula:

$$f_E(t) = R(t) / ET$$

the equivalent distribution of risk function is equal to:

$$\lambda_E(t) = R(t) / (ET\, R_E(t))$$

where RE(t) = 1– FE(t).

Many considerations in the theory of reliability use the function of the residual lifetime. If the condition ET < ∞ is satisfied, then the average residua lifetime function of the variable T is with R(t) > 0 defined as:

$$m(t) = E(X - t \mid X > t) = \int_t^\infty R(u)du / R(t)$$

whereas with condition R(t) = 0 it is assumed, that m(t) = 0.

The properties of m(t) function were studied in [5, 10, 11]. The give equality is true:

$$ET = m(0) = \int_0^\infty R(u)du$$

## THE DAMAGE PROCESS MODEL

In this chapter, a model of the damage process is presented. The damage process has been constructed as a superposition of two processes. It is assumed, that a sequence of independent random variables $Z_1, Z_2, \ldots$ is considered, where each of them determines the time to original failure. The original damage generates an additional series S of secondary damage. The intervals between secondary damage is determined by $Y_1$, $Y_2, \ldots, Y_S$. An assumption has been assumed, that random variables $Y_1, Y_2, \ldots, Y_S$ are independent and have the same distribution reliability function $R_Y(t)$. The random variable S can take the values 0, 1, 2,… The combined process is the superposition of the original damage process and secondary damage process. In the superposition of the processes, both types of damage are indistinguishable. The process is called the Poisson stationary branch process. The original damage resulting from the impact of the natural processes of wear and degradation of a technical object are separated by time intervals $Z_i$, i = 1,2,…. The first original damage occurs after the initial time Z1 and may generate secondary damage because of inefficient repair after time Y1. However, the repair of secondary damage may also be inefficient and after time Y2 another secondary damage may occur.

It is convenient to assume, that the random variables $Z_1, Z_2, \ldots$ are independent and have the same exponential distribution with density, then

$$f_Z(t) = \lambda \exp(-\lambda t),$$

where λ > 0 and t ≥ 0.

A sequence of random variables $Z_i$ i = 1, 2, … is generated by Poisson stationary process. Time between fallowing damage in Poisson stationary branch process will be marked with T. In [3, 19] it was shown, that the reliability function of the random variable T is given by:

$$R_T(t) = \frac{1 + ES \times R_Y(t)}{1 + ES} \exp\{-\lambda t - \lambda \times ES \int_0^t R_Y(u)du\} \quad (1)$$

Based on this formula, one can determine most of the probabilistic properties of the process. Introducing designation

$$ET_T(t) = \int_0^t R_T(u)du$$

for $ET_T(t)$ occurs

$$ET_T(0) = 0$$

$$ET_T(\infty) = ET$$

Integrating equation (1) can be shown, that

$$ET_T(t) = (1- \exp(-\varphi(t)) / (\lambda\ (1+ ES))$$

where

$$\varphi(t) = \lambda\, t + ES \int_0^t R_Y(u)du \qquad (2)$$

Then, a formula for the mean value ET can be written

$$ET_T(\infty) = ET = 1/ (\ \lambda\ (1+ ES))$$

In [3, 19] formulas for the variance $D^2T$ have been derived from:

$$D^2T = (1 + 2\ ES \exp(-\lambda\ (1+ ES))\ EY) / (\lambda\ (1+ ES))^2$$

The variation coefficient V is now expressed by the formula:

$$V = 1 + 2\ ES \exp(-\lambda\ (1 + ES)\ EY)$$

The lower and upper limit for the variance and coefficient of variation based on [3, 19] can be written as:

$$1 / (\ \lambda^2\ (1 + ES)^2) \le D^2T \le (1 + 2\ ES\ ) / (\ \lambda^2\ (1 + ES)^2)$$

$$1 \le V \le 1{+}2\ ES$$

The last equality shows, that coefficient of variation for the time between damage is always greater than or equal to 1. From the reliability theory, it is known, that the coefficient of variation V is equal to 1 for the exponential distribution. The function of the average residual time of a random variable T can be written as:

$$m(t) = (ET - ET(t)) / R_Y(t)$$

and

$$m(t) = 1 / \varphi'(t)$$

Based on the above considerations, the following properties can be formulated for a random variable:

Property 1. Function m(t) of the average residual time of a random variable T is increasing

Property 2. For function φ(t) the inequalities φ'(t) > 0 and, φ''(t) ≤ 0 are true.

Property 3. Risk function of a random variable T has the form:

$$\lambda_T(t) = \varphi'(t) - \varphi''(t) / \varphi'(t)$$

Property 4. Cumulative distribution of equivalent distribution is expressed by the formula:

$$F_E(t) = 1-\exp(-\varphi(t)) \text{ for } t \ge 0$$

Property 5. The risk function of an equivalent distribution is expressed by the formula:

$$\lambda_E(t) = \varphi'(t)$$

Example 1. The object of the research in this example is a company holding 190 public transport buses. The company uses buses of different brands and types. In the example, it is considered that n = 1149 of time intervals between successive electrical bus subsystem damage. It is assumed, that the intervals between the secondary damage have gamma distribution with probability density in the form of:

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$$

where x > 0, α > 0, β > 0,

Γ(α) means the gamma function defined by the formula:

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

To evaluate the parameters of the distribution function of reliability (1) the method of maximum likelihood has been used. Parameters of the gamma distribution describe the time distribution between secondary damage rated by maximum likelihood method. The parameters of this distribution have values: α = 2,592, β = 0,3632. The parameter λ of exponential distribution describing the distribution of time between the original damage was assessed as: λ = 0,03804. The average value of a series of secondary damage has been rated as ES = 1,9997. On the basis of estimated values of the parameters α, β, λ and ES, calculated were the average value of time between original damage EZ = 26,88, the value of the average time between secondary damage EY = 0,9415. The value of the average time between any damage is equal to ET = 8,7630. The compliance of empirical distribution of time between damage with the distribution set by the reliability function RT(t) defined by the formula (1) was tested using two tests of conformity: λ – Kolmogorov and $\chi^2$ – Pearson test. The calculated statistics value λ = 0,647 is correspondent to the value of the level of significance p = 0,21, while for the test $\chi^2$ – Pearson calculated $\chi^2$ = 39,3, at p = 0,28. The applied compatibility tests confirm the high compatibility of the analysed data with the distribution resulting from the applying the Poisson stationary branch process to describe the process of damage with the function of reliability given by formula (1). The estimation of the damage parameters process model and testing its compatibility with data from the actual process of operation of the bus electric subsystem is illustrated in Figure 1.

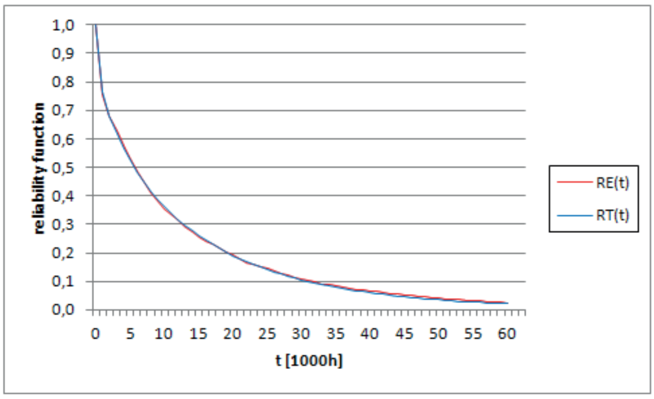


*Fig. 1. Theoretical $R_T(t)$ and empirical $R_E(t)$ function reliability chart*

Figure 1 shows the course of empirical reliability function $R_E(t)$ and the reliability function of the test model $R_T(t)$.

## PROFIT OPTIMIZATION MODEL BASED ON POISSON BRANCH PROCESS

It was assumed, that the average profit from the work of a technical object is proportional to the mean time till damage ET. This profit was determined by $g_1(t)$ and assumes that

$$g_1(t) = e / ( \lambda (1+ t ))$$

where $e > 0$, $0 \le t \le ES$.

It is obvious, that $g_1(0) = e/\lambda$ means profit from the work of a technical object, in which there are no secondary damage, while $g_1(ES) = e /( \lambda (1+ ES ))$ is the profit from the work before trying to invest in reducing the number of secondary damage. The reduction of secondary damage amount is realized in the model by reducing the average value of ES, whereas other parameters of the reliability function (1) remain unchanged.

By $h_2(t) = - g_2(t)$ effort to eliminate secondary damage has been marked. It has been assumed, that the function $g_2(t)$ is twice differentiable on the interval (0, ES) and meets the following assumptions:

$$g_2(t) < 0,\ g_2(0+) = - \infty,\ g_2(ES-) = 0, \quad (3)$$

$$g_2'(t) > 0,\ g_2'(0+) = + \infty,\ g_2'(ES-) = 0, \quad (4)$$

$$g_2''(t) < 0, \quad (5)$$

Criterion function expressing the average profit from the work of a technical object has the form;

$$g(t) = g_1(t) + g_2(t)$$

At this point of work, it was assumed, that the function $g_2(t)$ has the form:

$$g_2(t) = \frac{b}{t^2} + \frac{c}{t} + d$$

Criterion function for the optimization problem is now as follows:

$$g(t) = \frac{a}{1+t} + \frac{b}{t^2} + \frac{c}{t} + d$$

where $a = e / \lambda$.

From the form of function $g_2(t)$ is clear, that $g_2(t) = - \infty$ if and only if $b < 0$, and $g_2(ES-) = 0$ if and only if

$$b + c\,ES + d\,(ES)^2 = 0 \quad (6)$$

Derivative $g_2'(t)$ is:

$$g_2'(t) = -\frac{2b}{t^3} - \frac{c}{t^2}$$

Equality $g'_2(0^+) = + \infty$ occurs because $b < 0$. Equality $g'_2(ET) = 0$ is true if and only if

$$b = - 0{,}5c\,ES \quad (7)$$

One can check, that if $b = - 0{,}5c\,ES$, than $g'_2(t) \ge 0$, for $t \in (0, ES)$.

The second order derivative is given by formula:

$$g''_2(t) = \frac{6b}{t^4} + \frac{2c}{t^3} \quad (8)$$

Equality $g''_2(0^+) = - \infty$ is true because $b < 0$.

From inserting (7) into equation (6) we have

$$g''_2(t) = \frac{-3cES}{t^4} + \frac{2c}{t^3}$$

From the form of the last formula, it is concluded, that if $t \in (0, ES)$, then $g_2''(x) < 0$.

From inserting (8) into (9), one gets

$$d = - e/(2\,ES)$$

The function g2(t) can be written as:

$$g_2(t) = -\frac{1}{2}\frac{cES}{t^2} + \frac{c}{t} - \frac{c}{2ES}$$

From the embodiment of the above written formula, one can see, that function $g_2(t)$ depends only on parameter c, whereas the criterion function depends on a and c.

Below, the criterion function g(t) is tested by formula (6).

It is known, that $g(0^+) = 0$ if and only if $b < 0$. The derivative of the criterion function has the form

$$g'(t) = \frac{-a}{(1+t)^2} - \frac{2b}{t^3} - \frac{c}{t^2}$$

It is easy to note, that $g'(0^+) = +\infty$ and that

$$g'(ES) = -\frac{a}{(1+ES)^2} < 0$$

Thus, the derivative g'(t) changes the sign from + to – in the interval (0, ES).

The following shows, that the sign change occurs exactly once. The second order derivative of the criterion function is given by formula:

$$g''(t) = \frac{2a}{(1+t)^3} + \frac{6b}{t^4} + \frac{2c}{t^3}$$

Taking into account (7) one can store the following inequality:

$$g''(t) < \frac{2a}{t^3} + \frac{2c}{t^3} - \frac{3cES}{t^4}$$

Inequality g''(t) < 0 comes from inequality

$$2(a + c)t - 3cES < 0$$

If 2a < c, then for $t \in (0, ES)$ occurs g''(t) < 0. This means, that the derivative g'(t) changes sign from „+" to „–" exactly once. One, one can formulate:

**Conclusion:** If 2a < c and relations (3), (4) and (5) are true, then the criterion function g(t) reaches a clear maximum in the interval (0, ES).

The following example contains the numerical illustration of this proposal.

Example 2. In the analysed example, the base is the process model of the electrical system failures considered in example 1 of this work. The intervals between the original damage have the exponential distribution with parameter $\lambda = 0{,}03804$, whereas intervals between secondary damage have the gamma distribution with parameters $\alpha = 2{,}592$, $\beta = 0{,}3632$. The average value of the secondary damage value length has been assessed as ES = 1,9997.

In the discussed example, it is assumed, that the ratio of the criterion function is a = 0,0833. The calculations were performed for three different values of the parameter c, $c \in \{0{,}8; 1; 1{,}2\}$. Figure 2 shows the waveform of the criterion function for three different values of the parameter c. Each analysed realization of the criterion function is achieves a maximum value at a certain point.

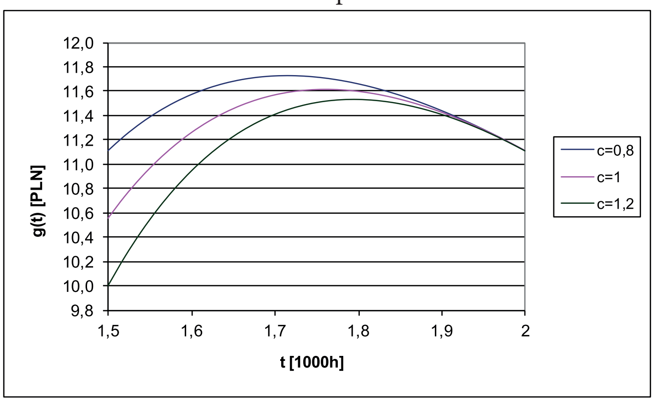


*Fig. 2. The dependency of the average profit g(t) from time t [1000h]*

## CONCLUSIONS

The study shows, that if the general conditions of the process of damage are met, it is possible to raise the profits of the operation of a technical object in the operation system. The basic assumption in this work is the presence of two types of damage that are called original and secondary damage in this work. The model of profit function has been based on the properties of the Poisson branch process. The numerical example presented in the second part of the work shows, that it is possible, under certain expenditures to eliminate secondary damage and obtain higher profits in the operation system.

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## CONTACT WITH THE AUTHOR

Leszek Knopik

Faculty of Management
Bydgoszcz University of Science and Technology
Fordońska 430 St.
85-790 Bydgoszcz
Poland

e-mail: knopikl@utp.edu.pl

Klaudiusz Migawa

Faculty of Mechanical Engineering,
Bydgoszcz University of Science and Technology
Prof. S. Kaliskiego 7 St.
85-763 Bydgoszcz
Poland

e-mail: klaudiusz.migawa@utp.edu.pl

Andrzej Wdzięczny

Faculty of Mechanical Engineering
Bydgoszcz University of Science and Technology
Prof. S. Kaliskiego 7 St.
85-763 Bydgoszcz
Poland

e-mail: andy@utp.edu.pl