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# 45 YEARS OF DELIBERATIONS ON THE THERMO-MECHANICAL INTERPRETATION OF FRICTION AND WEAR. PART II – THE MICROSCOPIC INTERPRETATION OF THE TRIBOLOGICAL PROCESS

## 45 LAT ROZWAŻAŃ O CIEPLNO-MECHANICZNEJ INTERPRETACJI TARCIA I ZUŻYWANIA. CZĘŚĆ II – INTERPRETACJA MIKROKOPOWA PROCESU TRIBOLOGICZNEGO

**Key words:** friction heating, temperature, unit pressures, criterion of galling, intensive parameters, temperature gradient, disintegration of solids, switched nature of friction.

**Abstract:** In this second part, the pulse sources of heat, dependences among the intensive parameters of the friction process – temperature, unit pressures, friction velocity – and wear intensity, and some physical properties of the materials of solids in friction are presented. The flux densities are described in extensive quantities across the elementary surface dF of a nominal contact of solids. The thermodynamic criterion of galling is formulated, and the temperature characteristic of minimum wear and maximum resistance to wear is established. Minimum and maximum wear intensities and the specific work of wear are determined.
It is proven that a temperature measured in a selected point of friction couple element does not uniquely characterise thermal processes in a tribological system. However, it does characterise the maximum gradient of temperatures measured in at least two points situated as close to a friction contact as possible. A method of determining unit real pressures is proposed. The presence of a cooling effect in the process of tribological wear is disclosed. Wear is interpreted as a disintegration of a solid caused by volumetric and superficial mechanical work. A system of new wear measures is suggested.

**Słowa kluczowe:** nagrzewanie tarciowe, temperatura, naciski jednostkowe, kryterium zacierania, parametry intensywne, gradient temperatury, rozdrabnianie ciała stałego, impulsowy charakter tarcia.

**Streszczenie:** W drugiej części pracy przedstawiono impulsowe źródła ciepła, ustalono zależności między intensywnymi parametrami procesu tarcia – temperaturą, naciskami jednostkowymi, prędkością tarcia a intensywnością zużywania i niektórymi własnościami fizycznymi materiałów trących się ciał. Opisano gęstości strumieni wielkości ekstensywnych przepływających przez elementarną powierzchnię dF styku nominalnego ciał. Sformułowano termodynamiczne kryterium zacierania, ustalono temperaturę charakterystyczną dla minimalnego zużycia i maksymalnej odporności na zużywanie. Ustalono maksymalne i minimalne wartości intensywności zużywania oraz graniczne wartości pracy właściwej zużycia.
Wykazano, że temperatura zmierzona w wybranym punkcie elementu pary tarciowej nie charakteryzuje jednoznacznie procesów cieplnych zachodzących w systemie tribologicznym, lecz maksymalny gradient temperatur, zmierzonych w co najmniej dwóch punktach położonych możliwie blisko styku tarciowego. Zaproponowano sposób wyznaczania jednostkowych nacisków rzeczywistych. Ujawniono występowanie efektu chłodzenia w procesie zużywania tribologicznego. Zużywanie zinterpretowano jako rozdrabnianie ciała stałego, wywołane wykonaniem pracy mechanicznej objętościowej i powierzchniowej. Zaproponowano system nowych miar zużycia.

## INTRODUCTION

Besides the extensive thermodynamic quantities addressed before, some intensive magnitudes are analysed in this second part of friction and the wear's thermal and mechanical interpretation. They are attributed to some selected points of a tribological system and cannot flow inside and

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at its boundaries with the environment. They cause extensive quantities to flow, however. Analytical relationships between the two kinds of quantities are important to the macroscopic descriptions of friction-induced processes in actual mechanical equipment.

## NOMENCLATURE

$A_n$ – nominal contact surface of solids 1 and 2 [$m^2$],
$A_r$ – real contact surface of solids 1 and 2 [$m^2$],
$A_{ta}$ – friction work along path a [J],
$A_t$ – friction work at shift l [J],
$A_{t1-2}$ – friction work in the tribological system [J],
$A_{dyss}$ – work of mechanical dissipation [J],
$A_{dyssF}$ – superficial work of mechanical dissipation [J],
$A_{dyssV}$ – volumetric work of mechanical dissipation [J],
$\dot{A}$ – power of friction [W],
$\dot{A}_{dyss}$ – power of mechanical dissipation [W],
a – dimension of $A_n$ side, measured in the direction of friction path l [m],
$a_{dyss}$ – specific work of mechanical dissipation [J/kg],
b – width of friction path [m],
$c_p$ – specific heat [J/kgK],
dF – elementary area of solid contact [$m^2$],
$e_R^x$ – specific work of wear [J/kg],
H – hardness [MPa],
h – linear wear along friction path l [m],
$h^*$– apparent linear wear [m],
$h_o$ – linear wear along friction path a [m],
$I_h$ – linear intensity of wear,
i – mean specific enthalpy of wear products [J/kg],
$i_h$ – specific linear intensity of wear,
J – specific intensity of wear [$kg/m^2s$],
$J^*$– new interpretation of wear intensity
k – wear coefficient,
l – friction path [m],
$\Delta m$ – change of tribological system mass [kg],
m – mass wear [kg],
$\Delta m_a$ – mass wear along path a [kg],
$m_o$ – mass of momentary friction zone [kg],
$m_x$ – mass of separated wear particle [kg],
m – mass flux of wear products [kg/s],
$\dot{N}$ – normal force acting against surface $A_n$ [N],
$n_k$ – critical number of surface asperity contacts,
$n_o$ – number of surface asperity contacts on the nominal surface,
p – nominal unit pressure [MPa],
$p_r$ – real unit pressure [MPa],
q – unit power of friction [$W/m^2$],
$q_c$ – unit thermal flux [$W/m^2$],
$q_f$ – unit flux of dissipation heat [$W/m^2$],
$q_m$ – unit power of mechanical dissipation [$W/m^2$],
$q_i$ – density of a specific enthalpy flux of wear products [$W/m^2$],
$Q_{1-2}$ – heat [J],
$Q_{dyss}$ – dissipation heat [J],
$Q_i$ – heat pulse at a surface asperity contact [J],
t – friction time [s],
V – volumetric wear [$m^3$],
$V^*$– apparent volumetric wear [$m^3$],
v – velocity of friction [m/s],
x – distance from friction surface [m],
$\alpha$ – coefficient of heat absorption [$W/m^2K$],
$\alpha_p$ – coefficient of heat transfer to the environment [$W/m^2K$],
$\delta$ – thickness of friction zone [m],
$\lambda$ – thermal conductivity [W/mK],
$\eta$ – efficiency of the wear process,
$\eta_{A-E}$ – efficiency of T.A. Afanasjeva-Ehrenfest cycle,
$\mu$ – coefficient of friction,
$\mu_c$ – thermal friction coefficient,
$\mu_m$ – mechanical friction coefficient,
$\Theta$ – temperature of friction contact [K],
$\Theta^*$– temperature 0.5mm away from friction contact [K],
$\Theta_o$ – 'flash' temperature [K],
$\Theta_p$ – ambient temperature [K],
$\Theta_x$ – characteristic temperature [K],
1, 2 – indices attributed to elements 1 and 2 of the system, respectively.

## FRICTION HEATING AS HEAT ABSORPTION FROM A FRICTION CONTACT, INCLUDING WEAR

Thermal processes in solid friction manifest themselves on two scales of size and time – **Fig. 1 [L. 1, 2]**:

– On the microscopic scale – in the areas of surface asperity contacts, where they always appear momentary, dynamic;
– On the macroscopic scale – in the areas of contact of solids in friction, they can be stationary and non-stationary.

The analysis of thermal processes in the areas of surface asperity contacts suggests the following conclusions:

- The friction heating of rough solid surfaces is a special case of heat absorption.
- The model of a friction source of heat based on Newton's law of heat absorption as supplemented with the energy balance equation referred to an elementary surface of macroscopic contacts of solids and to elementary times allows for consideration of:
  - The part of friction work associated with wear;
  - The presence of friction surface asperities and flash temperature;
  - The degree of friction contact's heating.

Heat transfer between a solid surface and a liquid (gas) is expressed especially as heat transfer from contact of rough solid surfaces. This is illustrated in a system solid-'gas'-solid (**Fig. 1, 2**), where a wall layer of liquid is absent. Heat $Q_i$ pulses, generated in an elementary contact of asperities, are all transmitted to solids in friction. This heating mechanism results in a density of heat flux $q_c$, in stationary processes equal to the density of dissipation heat flux; $q_f$, $q_c$ and $q_f$ are attributed to elementary contact surface dF at the macroscopic level of a friction couple.

The energy balance characterising friction in an elementary contact surface dF and the law of heat transfer becomes – **Fig. 1b [L. 1–3]**:

$$q = \mu pv = q_c + q_m = \alpha(\Theta_o - \Theta) + \eta\mu pv, \qquad (1)$$

where: $q_m$ – the unit power of mechanical dissipation, equal to the density of a specific enthalpy flux of wear products $q_i$, whereas $\eta = q_m/q$; if $\eta = 0$, then $\Theta = \Theta_x$, hence:

the coefficient of heat transfer from a friction contact:

$$\alpha = \frac{\mu pv}{\Theta_o - \Theta_x}, \qquad (2)$$

the unit thermal flux:

$$q_c = q_f = \frac{\mu pv}{\Theta_o - \Theta_x}(\Theta_o - \Theta), \qquad (3)$$

the unit power of mechanical dissipation:

$$q_m = q_i = \frac{\mu pv}{\Theta_o - \Theta_x}(\Theta - \Theta_x), \qquad (4)$$

the thermal coefficient of friction:

$$\mu_c = \frac{\Theta_o - \Theta}{\Theta_o - \Theta_x}\mu, \qquad (5)$$

the mechanical coefficient of friction:

$$\mu_m = \frac{\Theta - \Theta_x}{\Theta_o - \Theta_x}\mu = \eta\mu, \qquad (6)$$

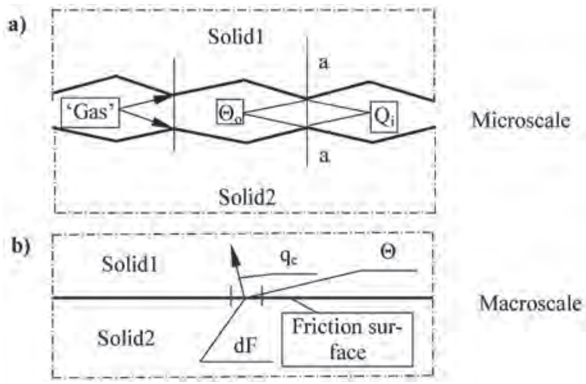


**Fig. 1. The friction heating of solids in a solid-'gas'-solid system: a) microscopic scale, b) macroscopic scale [L. 1–2]**

Rys. 1. Nagrzewanie tarciowe ciał w systemie ciało stałe – „gaz" – ciało stałe: a) skala mikroskopowa, b) skala makroskopowa **[L. 1–2]**

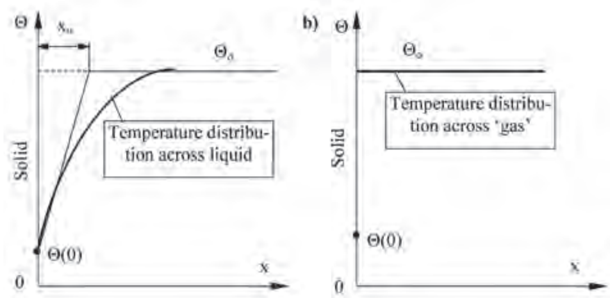


**Fig. 2. Heat transfer: a) between the solid surface and liquid (gas): b) across a solid-'gas'-solid system [L. 2]**

Rys. 2. Przejmowanie ciepła: a) między powierzchnią ciała stałego a płynem (gazem); b) w systemie ciało stałe – „gaz" – ciało stałe **[L. 2]**

specific intensity of wear:

$$J = \frac{1}{a_{dyss}} q_i = \frac{\mu pv}{a_{dyss}} \frac{\Theta - \Theta_x}{\Theta_o - \Theta_x}, \qquad (7)$$

specific work of wear:

$$e_R^x = a_{dyss} \frac{\Theta_o - \Theta_x}{\Theta - \Theta_x} = \frac{1}{-A + B\Theta}, \qquad (8)$$

characteristic temperature:

$$\Theta_x = \frac{A}{B}. \qquad (9)$$

Characteristic $\Theta_x$ and flash $\Theta_o$ temperatures are derived from calorimetric testing considering two values of $\eta$ at two values of $\Theta$ of calorimetric liquid, $\Theta_I$ and $\Theta_{II}$:

$$\Theta_x = \frac{\Theta_I \eta_{II} - \Theta_{II} \eta_I}{\eta_{II} - \eta_I}, \tag{10}$$

$$\Theta_o = \frac{\Theta_{II}(1-\eta_I) - \Theta_I(1-\eta_{II})}{\eta_{II} - \eta_I}. \tag{11}$$

The necessary and sufficient condition of galling is disturbing a dynamic equilibrium of a tribological system where stabilised wear takes place **[L. 4]**. In a system of quadrant coordinates p-v-Θ, **Fig. 3**, any point P of the shell $\delta$ is characterised by some critical values of friction parameters that condition the onset of galling **[L. 5]**:

$$p = p_{kr},\ v = v_{kr},\ \Theta = \Theta_{kr}. \tag{12}$$

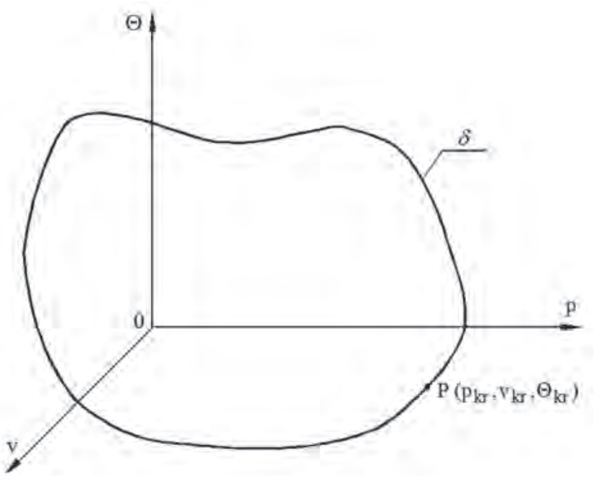


**Fig. 3. A geometric interpretation of the galling criterion [L. 5]**

Rys. 3. Interpretacja geometryczna kryterium zacierania **[L. 5]**

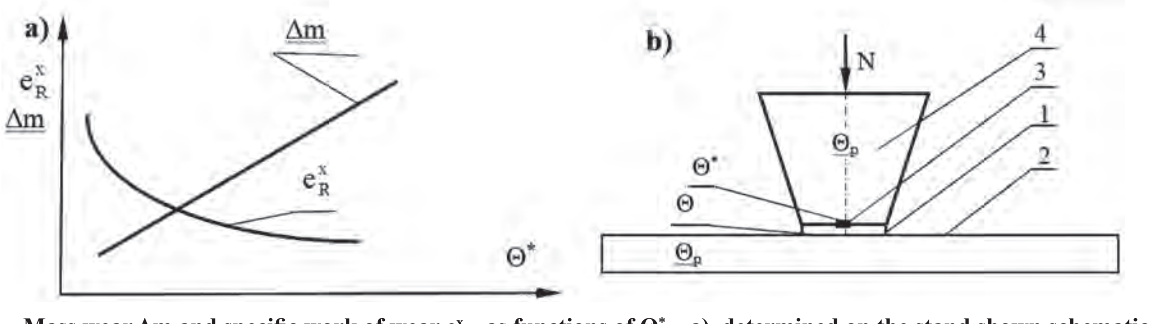


**Fig. 4. Mass wear Δm and specific work of wear $e^x_R$ as functions of $\Theta^*$ – a), determined on the stand shown schematically in b), assuming $\Theta^*$ is an independent parameter established with a thermostat. 1 – specimen, 2 – counters specimen, 3 – a thermoelemet placed 0.5 mm away from the friction contact, 4 – specimen handle – heat exchanger [L. 3]**

Rys. 4. Zużycie masowe Δm i praca właściwa zużycia $e^x_R$ jako funkcje temperatury $\Theta^*$ – a), ustalone na stanowisku według schematu b), przy założeniu, że temperatura $\Theta^*$ jest parametrem niezależnym ustalanym za pomocą termostatu. 1 – próbka, 2 – przeciwpróbka, 3 – termoelemet umieszczony w odległości 0,5 mm od styku tarciowego, 4 – uchwyt próbki – wymiennik ciepła **[L. 3]**

The direct measurement of Θ is difficult since the contact of elements in friction is inaccessible to a measurement sensor. In the circumstances, an approximate value of $\Theta^*$ is measured as close to that contact as possible. **Fig. 4a** shows mass wear and specific work of wear as functions of $\Theta^*$, determined experimentally by means of a head whose schematic is depicted in **Fig. 4b**. **Fig. 5** illustrates test results for the specific work of wear and characteristic temperature for eight selected metals.

As distinct from the above instance, where the point of temperature measurement is at a slight distance from the nominal surface and located on an artificially produced isothermal boundary, the temperature of the nominal surface itself Θ can be a parameter dependent on the conditions of friction as well. The ambient temperature $\Theta_p$, on the other hand, is stabilised by enforcing a heat exchange with the environment, and Θ is measured according to an original method described in **[L. 7]**.

All the parameters and conditions characterising friction, with the exception of sliding velocity, are assumed to remain stable. Thus, the sliding velocity v is the factor conditioning the variations of Θ at the test stand (**Fig. 6**). It is established assuming certain rotational speeds of the disc with an affixed counter specimen. A procedure is adopted similar to that for

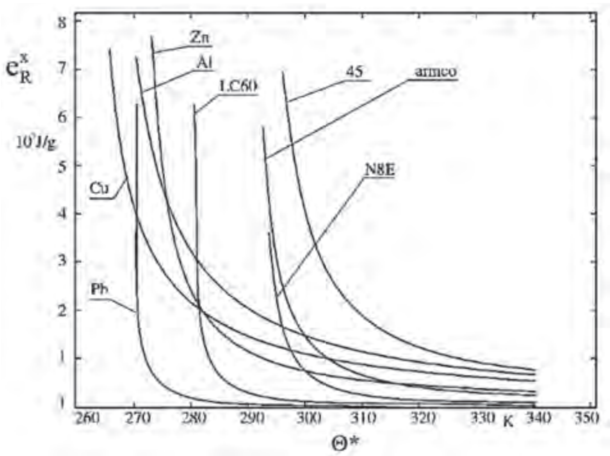


**Fig. 5. Specific work of wear $e^x_R$ tending towards a maximum close to characteristic temperature $\Theta^* \to \Theta_x$ [L. 6]**

Rys. 5. Dążenie pracy właściwej zużycia $e^x_R$ do wartości maksymalnych w pobliżu temperatury charakterystycznej $\Theta^* \to \Theta_x$ **[L. 6]**

the first example of tribological testing described above. The balance of energy flux densities in the process of stabilised friction is formulated as follows **[L. 8]**:

$$q = \mu pv = \alpha_p(\Theta - \Theta_p) + \eta\mu pv. \tag{13}$$

The density of dissipation heat flux $q_f$ equal to the density of heat flux $q_c$ released to the environment at a temperature $\Theta_p$, is described by:

$$q_c = q_f = \alpha_p(\Theta - \Theta_p) = \frac{\Theta - \Theta_p}{x}\lambda. \tag{14}$$

A coefficient expressing the capacity of a tribological system to deliver heat to the environment, $\alpha_p = \lambda/x$, characterises heat conduction across a sample material with a conductivity λ. At a distance x from a nominal surface with temperature Θ, the temperature of specimen material is $\Theta_p$. If $\eta = 0$, then $q_c = q_f = q$ and $\Theta = \Theta_o$. Therefore, (14) can be expressed as follows:

$$q_c = q_f = \frac{\mu pv}{\Theta_o - \Theta_p}(\Theta - \Theta_p), \tag{15}$$

where

$$\alpha_p = \frac{\mu pv}{\Theta_o - \Theta_p}. \tag{16}$$

The density of mechanical dissipation energy flux $q_m$ equal to the density of wear product specific enthalpy $q_i$ is described by:

$$q_i = q_m = \frac{\mu pv}{\Theta_o - \Theta_p}(\Theta_o - \Theta). \tag{17}$$

The intensity of stabilised wear:

$$J = \frac{1}{a_{dyss}} q_i = \frac{\mu pv}{a_{dyss}} \frac{\Theta_o - \Theta}{\Theta_o - \Theta_p}, \tag{18}$$

$$J = \mu pv(a - b\Theta), \tag{19}$$

Mass wear:

$$\Delta m = \mu Nl(a - b\Theta), \tag{20}$$

Specific work of wear:

$$e_R^x = \frac{\mu Nl}{\Delta m} = \frac{1}{a - b\Theta}. \tag{21}$$

Both Θ and $\Theta_p$ play major roles in (15). The greater Θ, the more dissipation heat is generated, the lower the wear and the greater the wear resistance – **Fig. 6a**. In the extreme (theoretical) case of $\Theta = \Theta_p$, dissipation heat is not released, and the entire work of friction causes wear. If Θ becomes maximum $\Theta_o$, on the other hand, there will be no wear.

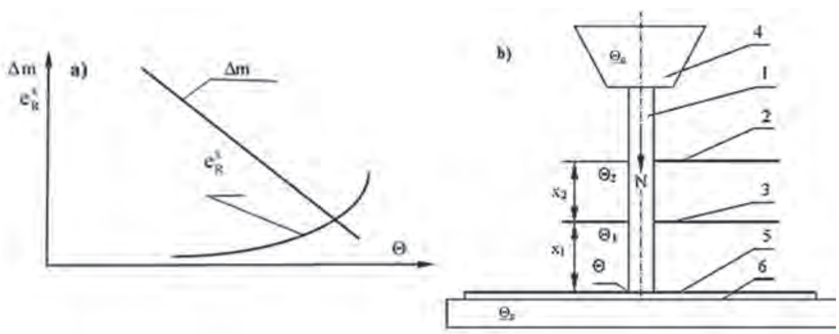


**Fig. 6. Mass wear Δm and specific work of wear $e^x_R$ as functions of Θ [L. 7]: a), determined on the stand as shown schematically in b) 1 – specimen, 2, 3 – thermoelements, 4 – specimen handle, 5 – counter specimen, 6 – disc – counter specimen handle**

Rys. 6. Zużycie masowe Δm i praca właściwa zużycia $e^x_R$ jako funkcje temperatury Θ **[L. 7]**: a) ustalone na stanowisku według schematu, b) 1 – próbka, 2, 3 – termoelementy, 4 – uchwyt próbki, 5 – przeciwpróbka, 6 – tarcza – uchwyt przeciwpróbki

The intensity of thermal processes can be controlled by varying the unit power of friction (**Fig. 6b**) and/or using a heat exchanger. Temperature, a result of heat generation and exchange, can be measured at a selected point of the tribological system. This analysis implies that the temperature, a system parameter, does not uniquely characterise either its thermal state or the process of wear. The discussion and experimental results presented in this chapter suggest an open issue of a dependence of the energy balance for a tribological system on the way of creating the temperature of a solid contact and on an adopted measurement location.

## TEMPERATURE GRADIENT NEAR THE NOMINAL SURFACE OF THE FRICTION CONTACT [L. 9]

Temperature, an intensive quantity, cannot be averaged in superficial or spatial areas (with the exception of model cases, where these areas are sufficiently small or if the same temperature prevails in all of their points). In contrast, the thermal flux and temperature gradient can be averaged indirectly. Thus, the thermal flux's mean density corresponds to the mean temperature gradient in an area of the tribological system under analysis. The temperature gradient is the maximum of the nominal surface of solid friction.

To determine this, the temperature of a selected surface point and the temperature at another point, located at a known, possibly slight distance away from that surface, need to be established. The local density of stationary thermal flux can be calculated by having this information about the temperature gradient and thermal conductivity of friction element material. The required condition of thermal process stationarity allows for restricting the discussion to the cases of stabilised friction and wear, and this restriction provides for a physical interpretation of the processes and a physical exemplification of their model.

The field of temperature in the immediate environment of a solid nominal contact is illustrated in **Fig. 7**. **Fig. 7a** shows a schematic friction specimen consisting of 1 and 2 and friction parameters: normal load N, friction velocity v, and the thermal conductivities $\lambda_1$, $\lambda_2$ of the element materials. The friction's surface stationary state is characterised by $\Theta(0)$. Temperature changes in the particular elements correspond to straight lines, schematically illustrating temperature gradients. Friction surface $A_n$ is situated in a spatial area, a friction volume whose boundaries are defined by sections $\delta_1$and $\delta_2$. The temperature measurement location is at the distance of $x_1$ from that surface.

The method of realising the friction process depicted in **Fig. 7** implies that the friction source of heat is mobile. The smaller element 1 moves relative to the larger element 2 with a constant

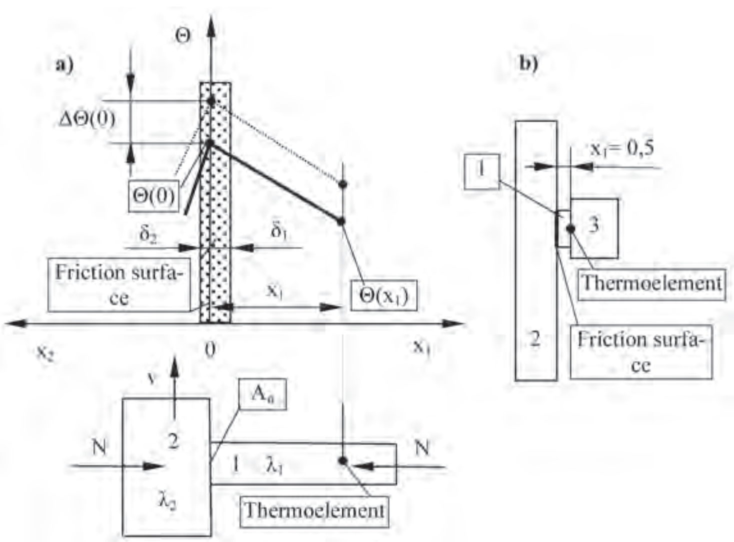


**Fig. 7. A schematic representation of a) temperature distribution around the nominal friction surface of the solids 1 and 2; $\delta_1$, $\delta_2$ – friction depth. b) elements 1 and 2 in friction and heat exchanger element 3 [L. 9]**

Rys. 7. Schematyczne przestawienie: a) rozkładu temperatury w otoczeniu powierzchni nominalnej tarcia ciał stałych 1 i 2; $\delta_1$, $\delta_2$ – głębokości tarcia. b) trące się elementy 1 i 2 i element wymiennika ciepła 3 **[L. 9]** velocity v. Thus, the temperature distribution in the friction contact's environment is shown in a moving system of coordinates in **Fig. 7**. a stationary temperature field prevails in element 1 and a quasi-stationary field in element 2. Therefore, the point of temperature measurement is tied to element 1.

A characteristic surface of nominal friction as a heat source is derived from the analysis of stationary thermal processes in the case of solid friction. It is constituted by a system of dependencies (22) – (25), which also addresses energy distribution between solids in friction. A volumetric source of friction heat is replaced with a superficial source of the same intensity. Once the densities of thermal fluxes to the friction specimen elements are established, maximum temperature gradients across these elements are generally described. On considering the information about the temperature of a fixed point of friction specimen element located at a known, possibly small distance away from a friction contact, the value of temperature Θ(0) is sought – (26).

$$q_{c1} = \frac{Q_{diss1}}{A_n t} = \frac{A_{t1\text{-}2}(1-k_1\frac{H_1}{p})\Delta m_1}{A_n t\Delta m} = \frac{\mu p v(1-k_1\frac{H_1}{p})\Delta m_1}{\Delta m}, \tag{22}$$

$$q_{c2} = \frac{Q_{diss2}}{A_n t} = \frac{A_{t1\text{-}2}(1-k_2\frac{H_2}{p})\Delta m_2}{A_n t\Delta m} = \frac{\mu p v(1-k_2\frac{H_2}{p})\Delta m_2}{\Delta m}, \tag{23}$$

$$[\mathrm{grad}\Theta(0)]_1 = \frac{-\mu p v(1-k_1\frac{H_1}{p})\Delta m_1}{\lambda_1\Delta m} = -\frac{\Theta(0)-\Theta(x_1)}{\delta_{x1}}, \tag{24}$$

$$[\mathrm{grad}\Theta(0)]_2 = \frac{-\mu p v(1-k_2\frac{H_2}{p})\Delta m_2}{\lambda_2\Delta m} = -\frac{\Theta(0)-\Theta(x_2)}{\delta_{x2}}, \tag{25}$$

$$\Theta(0) = \Theta(x_1) + \frac{\mu p v(1-k_1\frac{H_1}{p})\Delta m_1\delta_{x1}}{\lambda_1\Delta m}, \tag{26}$$

The following are the dependences that describe the wear intensity J of the system and the first element $J_1$ considering the wear coefficient $k_1$, the specific work of wear $e_R^x$, and wear intensity $J_2$ of the second system element.

$$J = \frac{\Delta m}{tA_n}, \tag{27}$$

$$k_1 = \frac{V_1 H_1}{lN} = \frac{V_1 H_1 \rho_1}{vtA_n p\rho_1} = \frac{J_1 H_1}{vp\rho_1}, \tag{28}$$

and on considering (26)

$$k_1 = \frac{p}{H_1} - \frac{[\Theta(0)-\Theta(x_1)]\lambda_1\Delta mp}{\delta_{x1}\mu vp\Delta m_1 H_1}, \tag{29}$$

$$J_1 = v\rho_1 p\frac{1}{H_1}\{\frac{p}{H_1} - \frac{[\Theta(0)-\Theta(x_1)]\lambda_1\Delta mp}{\delta_{x1}\mu vp\Delta m_1 H_1}\}, \tag{30}$$

$$J_1 = v\rho_1 p\frac{1}{H_1}\{\frac{p}{H_1} + [\mathrm{grad}\Theta(0)]_1\frac{\lambda_1\Delta m}{\mu v\Delta m_1 H_1}\}, \tag{31}$$

$$e_R^x = \frac{\Delta m_1\mu H_1}{\Delta mk\rho_1} = \frac{\mu^2 H_1^2\delta_{x1}v\Delta m_1^2}{\Delta m\rho_1\{p\delta_{x1}\mu v\Delta m_1 - \lambda_1\Delta m[\Theta(0)-\Theta(x)]\}}, \tag{32}$$

$$e_R^x = \frac{\mu^2 H_1^2 v\Delta m_1^2}{\Delta m\rho_1\{p\mu v\Delta m_1 + \lambda_1\Delta m[\mathrm{grad}\Theta(0)]_1\}}, \tag{33}$$

$$J = \frac{\mu p v}{e_R^x} = v\rho_1 p\frac{\Delta m}{H_1\Delta m_1}\{\frac{p}{H_1} - \frac{[\Theta(0)-\Theta(x_1)]\lambda_1\Delta m}{\delta_{x1}\mu v\Delta m_1 H_1}\}, \tag{34}$$

$$J_1 = J\frac{\Delta m_1}{\Delta m}, \tag{35}$$

$$J_2 = J - J_1 = J(1-\frac{\Delta m_1}{\Delta m}) = J\frac{\Delta m_2}{\Delta m}. \tag{36}$$

The greater temperature Θ(0), with a constant $\Theta(x_1)$, corresponds to a lower wear intensity. In turn, the higher temperature $\Theta(x_1)$, at a constant Θ(0), is associated with more intensive wear. These regularities suggest not the particular temperatures Θ(0) and $\Theta(x_1)$, but the difference between them conditions the process of wear. The wear intensity will not change while an identical value reduces Θ(0) and Θ(x1), e.g. ΔΘ(0), at the same time (**Fig. 7a**) and where a constant wear mechanism is maintained. The method described above describes the mobile element's thermal processes based on friction work information, wear in the tribological system and its elements, and thermal processes in the fixed element.

## REAL UNIT PRESSURES AND REAL CONTACT SURFACE OF SOLIDS IN FRICTION [L. 10]

The analytical descriptions of tribological processes provisionally assume real unit pressures are equal to the hardness of a softer material of friction solids or the unit pressure of the plastic flow. This assumption stems from the fact that the direct contact of surface asperities is plastic. These material characteristics are static and carry no information about friction and wear conditions.

An attempt is made below to determine unit pressures considering this information and relying on the laws of mass and energy conservation. The momentary real surface of solid contact is then described in analytical terms. The discussion is restricted to the model case of friction of a specimen material whose hardness is many times lower than that of the counter specimen material, and this allows for neglecting energy dissipation and wear in the counter specimen. This also denotes meeting J.F. Archard's postulate **[L. 11]** that the more plastic material of a friction specimen element conditions the values of real unit pressures.

A superficial friction heat source will be replaced with a volumetric source of the same unit power.

$$q_c = m_o c_p(\Theta_o - \Theta)\frac{v}{aA_n} = \alpha(\Theta_o - \Theta), \tag{37}$$

where: $m_o = \rho\frac{aN}{p_r} = \rho\frac{A_n\delta}{p_r}$ – the heated mass of energy dissipation zone, $p_r$ – sought real unit pressures. The coefficient of heat transfer from the friction contact results:

$$\alpha = \rho c_p v\frac{p}{p_r} = \frac{\mu p v}{\Theta_o - \Theta_x} \tag{38}$$

and

$$\Theta_o - \Theta_x = \frac{\mu p_r}{\rho c_p}, \tag{39}$$

$$q_c = \rho c_p v\frac{p}{p_r}(\Theta_o - \Theta), \tag{40}$$

$$q_m = \rho c_p v\frac{p}{p_r}(\Theta - \Theta_x), \tag{41}$$

$$\eta = \frac{\rho c_p}{\mu p_r}(\Theta - \Theta_x) = k\frac{p_r}{p}. \tag{42}$$

The mass of worn material $\Delta m_a$ along a path equal to a:

$$\Delta m_a = \rho V_a = \rho k\frac{a\mu N}{\mu p_r} = \rho k\frac{A_{ta}}{\mu p_r} \tag{43}$$

Specific work of wear

$$e_R^x = \frac{A_{ta}}{\Delta m_a} = \frac{A_t}{\Delta m}, \tag{44}$$

where: $A_t$ – work of friction along any displacement l, $\Delta m$ – its corresponding stationary mass wear.

Wear coefficient

$$k = \frac{\mu p_r}{\rho e_R^x} \tag{45}$$

and

$$\frac{\rho c_p}{\mu p_r}(\Theta - \Theta_x) = \frac{\mu p_r^2}{\rho e_R^x p}. \tag{46}$$

The above equation implies the sought-after real unit pressure:

$$p_r = \sqrt[3]{\left(\frac{\rho}{\mu}\right)^2 e_R^x p c_p (\Theta - \Theta_x)} \tag{47}$$

and the real surface of friction contact:

$$A_r = \frac{N}{p_r}. \tag{48}$$

As suggested by (47), real unit pressures depend on friction and wear conditions. Soft materials (e.g. Cu, Zn, Pb) are several times greater than their hardnesses **[L. 10]**. The surface $A_r$ is likewise dependent on these conditions.

## MATERIAL DISINTEGRATION IN AN ENVIRONMENT OF SOLID SURFACE ASPERITY CONTACTS

Seeing tribological wear as a waste of solid mass or volume does not fully reflect its nature since the disintegration of a substance in the friction zone is ignored as part of this approach. Wear product particles are separated from top layers in the contact areas of surface asperities of solids in friction, that is, on the real friction surface $A_r$. A steady wastage of mass or volume of an element in friction can be observed on the nominal friction surface $A_n$ – **Fig. 8**.

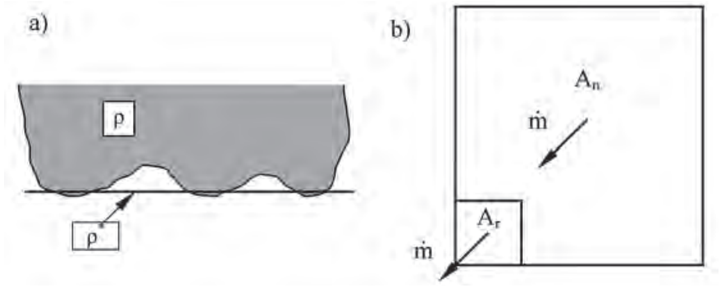


**Fig. 8. Determination of worn material density ρ and apparent density ρ*on a nominal surface $A_n$ as wear particles are formed – a); determination of wear product mass flux ṁ on the nominal $A_n$ and real $A_r$ solid contact surfaces – b) [L. 12, 13]**

Rys. 8. Oznaczenie gęstości materiału zużywanego ρ i jego gęstości pozornej ρ* na powierzchni nominalnej $A_n$ w trakcie tworzenia się cząstek zużycia – a); oznaczenie strumienia masy produktów zużycia ṁ na powierzchniach nominalnej $A_n$ i rzeczywistej $A_r$ styku ciał stałych – b) **[L. 12**, **13]**

**Fig. 9a** presents a schematic nominal surface $A_n$ divided into $n^2$ identical fields, which make up the real surface $A_r$ **[L. 13]**. According to J.F. Archard, a wear particle of mass $m_x$ is separated following $n_k$ impacts against the surface asperities of the other solid in friction – **Fig. 9b**. Volumetric wear on the macroscopic scale is described with (49) **[L. 11]**:

$$V = klN/H. \tag{49}$$

Another possible mechanism of material dispersion is illustrated in **Figs. 10a** and **10b**. The author bases it on a thermodynamic analysis of the energy dissipation process in solid friction. The limitation found on a possible total conversion of mechanical energy into heat suggests wear takes place in each elementary contact of surface asperities. Therefore, the particles generated will be more numerous and smaller than in the model in **Fig. 9**.

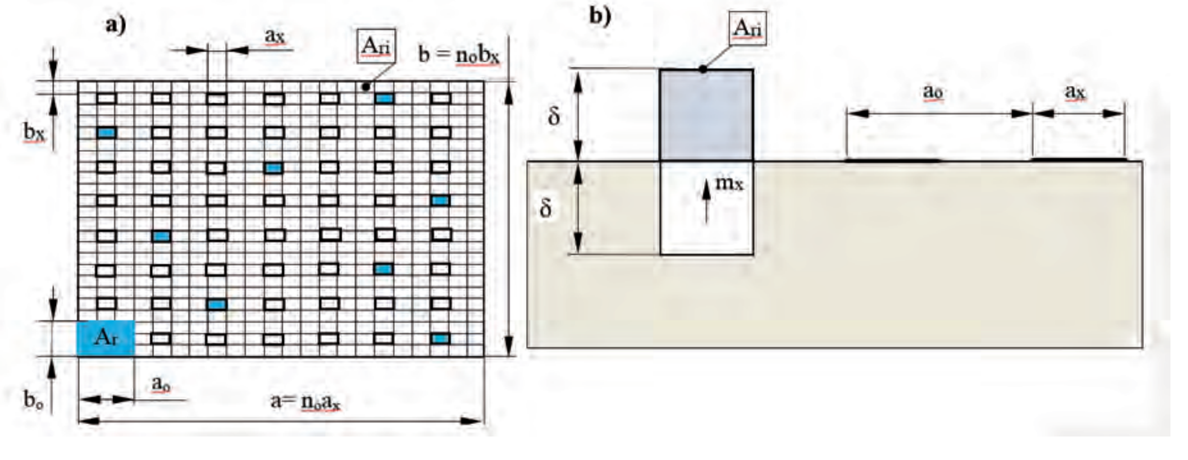


**Fig. 9. Tribological wear according to J.F. Archard: a) on the nominal surface $A_n$ = a·b, real areas $A_{ri}$ of asperity contacts are shown in bold frames; only those additionally darkened are subject to wear, b) a schematic representation of a wear particle formation [L. 14, 15]**

Rys. 9. Zużywanie tribologiczne według J.F. Archarda: a) na powierzchni nominalnej $A_n$ = a·b rzeczywiste pola $A_{ri}$ styku nierówności oznaczone są pogrubionymi ramkami; tylko te z nich podlegają zużywaniu, które dodatkowo zaciemniono, b) schematyczne przedstawienie tworzenia się cząstki zużycia **[L. 14**, **15]**

The flux of wear product mass across both surfaces is identical and equal ṁ. If the density of the element material subject to stationary wear is ρ and the element's linear dimension reduces by h after a time t, mass wear with reference to the real surface $A_r$ will be defined as follows:

$$m = A_r h\rho = V\rho. \tag{50}$$

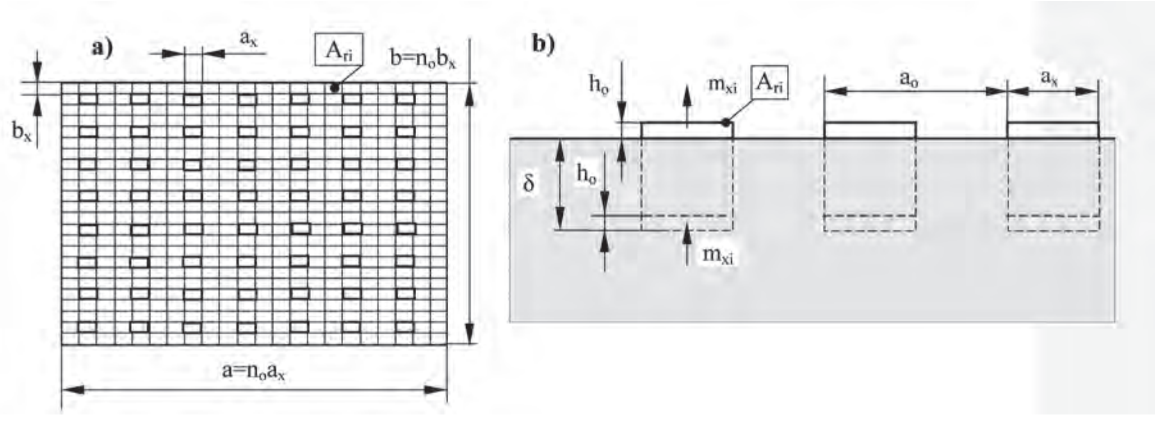


**Fig. 10. Tribological wear according to the new concept: a) all the areas shown in bold frames are subject to wear, b) generation of a particle with a mass $m_{xi}$ [L. 15]**

Rys. 10. Zużywanie tribologiczne według nowej koncepcji: a) wszystkie pola oznaczone pogrubionymi ramkami podlegają zużywaniu, b) powstanie cząstki o masie $m_{xi}$ **[L. 15]**

It can also be defined for the nominal contact surface, considering material discontinuities around asperity contacts and the consequent reduction of its density from the primary, real ρ to apparent density $\rho^*$ – **Fig. 8a**:

$$m = A_n h \rho^*. \quad (51)$$

Thus, the apparent density of wear product mass flux is:

$$\rho^* = \frac{A_r}{A_n}\rho = \frac{p}{H}\rho. \quad (52)$$

**Fig. 11a** shows the relationship between the volume of worn material $V = h \cdot A_r$ of density ρ and the apparent volume of tribological wear $V^* = h \cdot A_n$ of a material with density $\rho^*$. This illustrates the rise of material volume from V to $V^*$, associated with the tribological wear of rough contact surfaces, interpreted as the formation of wear products.

Mass wear: $m = V^*\rho^* = V\rho$, hence the apparent volume $V^*$:

$$V^* = V\frac{\rho}{\rho^*} = V\frac{H}{p}. \quad (53)$$

It corresponds to the apparent linear wear $h^*$ – **Fig. 11b**:

$$h^* = h\frac{\rho}{\rho^*} = h\frac{H}{p}. \quad (54)$$

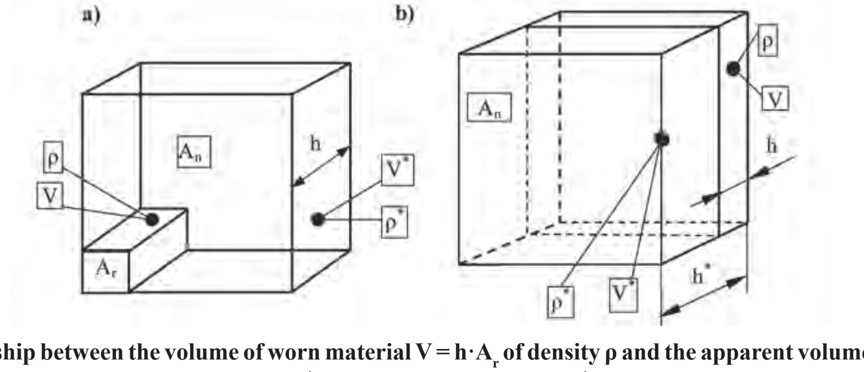


**Fig. 11. The relationship between the volume of worn material $V = h \cdot A_r$ of density ρ and the apparent volume of tribological wear $V^* = h \cdot A_n$ of a material with density $\rho^*$: a) the case of $V = h \cdot A_r$, $V^* = h \cdot A_n$, b) the case of $V = h \cdot A_n$, $V^* = h^* \cdot A_n$ [L. 12, 13]**

Rys. 11. Relacja między objętością zużytego materiału $V = h \cdot A_r$ o gęstości ρ i pozorną objętością zużycia tribologicznego $V^* = h \cdot A_n$ materiału o gęstości $\rho^*$: a) przypadek $V = h \cdot A_r$, $V^* = h \cdot A_n$, b) przypadek $V = h \cdot A_n$, $V^* = h^* \cdot A_n$ **[L. 12, 13]**

Wear generates an increased volume. This results in a new measure of volumetric wear, described as:

$$\Delta V = V^* - V = A_n(h^* - h). \tag{55}$$

In parallel, the following difference should be accepted as the measure of linear wear:

$$\Delta h = h^* - h. \tag{56}$$

(55) and (56) can be expressed differently:

$$\Delta V = V(\frac{\rho}{\rho^*} - 1) = V(\frac{A_n}{A_r} - 1) = V(\frac{H}{p} - 1), \tag{57}$$

$$\Delta h = h(\frac{\rho}{\rho^*} - 1) = h(\frac{A_n}{A_r} - 1) = h(\frac{H}{p} - 1). \tag{58}$$

When the new measure of mass wear is defined, the difference between the masses of worn material and the wear of products is absent. Considering the reduced density of material in friction and introducing a new measure of mass wear are suggested, therefore:

$$\Delta\rho = \rho - \rho^*, \tag{59}$$

$$\Delta\rho = \rho(1 - \frac{A_r}{A_n}) = \rho(1 - \frac{p}{H}). \tag{60}$$

The increases of material volume ΔV, of the linear dimension Δh, and of Δρ would have no effect were the real surface equal to a nominal surface or if the unit pressures on the real and nominal surfaces were equal, which is not the case in the contacts of solids in friction.

**Table 1. The new definitions of wear intensity and their derivatives [L. 13]**
Tabela 1. Nowe definicje intensywności zużywania i ich pochodne **[L. 13]**

| Measure of wear | Wear intensity, quantity, unit | Wear resistance, quantity, unit | Density of wear intensity, quantity, unit | Wear resistance, quantity, unit |
|---|---|---|---|---|
| Volumetric | $J^* = \frac{\Delta V}{t}$; $[m^3 \cdot s^{-1}]$ | $1/J^*_{Vt}$; $[s \cdot m^{-3}]$ | $J^*_{VtAn} = \frac{\Delta V}{t \cdot A_n}$; $[m^3 \cdot s^{-1} \cdot m^{-2}]$ | $1/J^*_{VtAn}$; $[s \cdot m^{-1}]$ |
| | $J^*_{Vl} = \frac{\Delta V}{l}$; $[m^3 \cdot m^{-1}]$ | $1/J^*_{Vl}$; $[m^{-2}]$ | $J^*_{VlAn} = \frac{\Delta V}{l \cdot A_n}$; $[m^3 \cdot m^{-3}]$ | $1/J^*_{VlAn}$; $[m^3 \cdot m^{-3}]$ |
| | $J^*_{VAt} = \frac{\Delta V}{A_t}$; $[m^3 \cdot J^{-1}]$ | $e^*_R = 1/J^*_{VAt}$; $[J \cdot m^{-3}]$ | $J^*_{VAtAn} = \frac{\Delta V}{A_t \cdot A_n}$; $[m^3 \cdot J^{-1} \cdot m^{-2}]$ | $1/J^*_{VAtAn}$; $[J \cdot m^{-1}]$ |
| Linear | $J^*_{ht} = \dot{h} = \frac{\Delta h}{t}$; $[m \cdot s^{-1}]$ | $1/J^*_{ht}$; $[s \cdot m^{-1}]$ | $J^*_{htAn} = \frac{\Delta h}{t \cdot A_n}$; $[m \cdot s^{-1} \cdot m^{-2}]$ | $1/J^*_{htAn}$; $[s \cdot m]$ |
| | $J^*_{hl} = \frac{\Delta h}{l}$; $[m \cdot m^{-1}]$ | $1/J^*_{hl}$; $[m \cdot m^{-1}]$ | $J^*_{hlAn} = \frac{\Delta h}{l \cdot A_n}$; $[m \cdot m^{-3}]$ | $1/J^*_{hlAn}$; $[m^2]$ |
| | $J^*_{hAt} = \frac{\Delta h}{A_t}$; $[m \cdot J^{-1}]$ | $1/J^*_{hAt}$; $[J \cdot m^{-1}]$ | $J^*_{hAtAn} = \frac{\Delta h}{A_t \cdot A_n}$; $[m \cdot J^{-1} \cdot m^{-2}]$ | $1/J^*_{hAtAn}$; $[J \cdot m]$ |

The new measures of mass Δρ, volumetric ΔV, and linear Δh wear are absolute. Relative measures are created by referring those absolute ones to some selected parameters, typically: friction time t, friction path l, and friction work $A_t$ of the friction process, or possibly to the nominal surface (**Table 1**).

Wear intensity and its derivatives are not based on mass wear Δρ, since the latter is independent of the time, path, and friction work in stabilised wear. Its change over time, meanwhile, is characteristic of a changing wear mechanism. Δρ also adds some information about wear intensity and resistance, as it is a variation of density, a quantity referred to as a unit of volume (and, indirectly, to the units of surface and length). Therefore, the proposed new measure of mass wear Δρ seems to be a versatile quantitative characteristic of any wear mechanism and a basis for their classification.

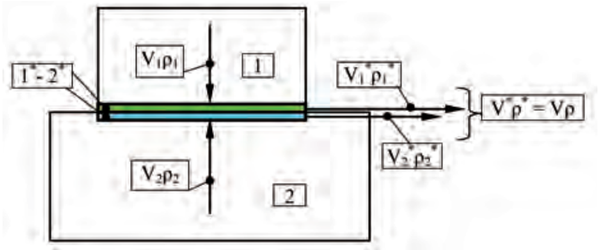


**Fig. 12. Tribological wear as a two-stage process: indirect wear – the passage of material masses $m_1 = V_1 \cdot \rho_1$ and $m_2 = V_2 \cdot \rho_2$ to an intermediating solid $1^*$- $2^*$ and the removal of $V_1^* \cdot \rho_1^*$ and $V_2^* \cdot \rho_2^*$ of the solid $1^*$- $2^*$ out of a friction couple; the mass wear of the system is described with the equality $m = V^* \cdot \rho^* = V \cdot \rho$ [L. 13]**

Rys. 12. Zużywania tribologiczne jako proces dwuetapowy: zużywanie pośrednie – przechodzenie masy materiału $m_1 = V_1 \cdot \rho_1$ i $m_2 = V_2 \cdot \rho_2$ do ciała pośredniczącego $1^* – 2^*$ oraz usuwanie materiału $V_1^* \cdot \rho_1^*$ i $V_2^* \cdot \rho_2^*$ ciała $1^* – 2^*$ poza parę tarciową; zużycie masowe systemu opisuje równość $m = V^* \cdot \rho^* = V \cdot \rho$ **[L. 13]**

Wear, interpreted as a material disintegration, suggests it is a two-stage process. This means the material of friction pair elements is not subject to wear directly but passes gradually to the friction zone, forming a new intermediate element of the tribological system, 1* – 2*. This is only this element, whose material is of a mean density $\rho^*$ lower than the mean density of element 1 and 2 material ρ, is the source of wear products. This situation can be illustrated schematically, like in **Fig. 12**.

The mean material densities are defined by:

$$\rho = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}, \tag{61}$$

$$\rho^* = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1^* + V_2^*}. \tag{62}$$


**Fig. 13. The work of mechanical dissipation $A_{dyssV}$ in the system shear stress – volume[L. 13]**

Rys. 13. Praca dyssypacji mechanicznej $A_{dyssV}$ w układzie naprężenie styczne – objętość **[L. 13]**

The work of mechanical volumetric dissipation in the system shear stress – volume is illustrated in **Fig. 13** as a dark rectangle whose sides are $\mu \cdot H – \mu \cdot p$ and $V^*$-V. This work conditions the process of solid disintegration with the effect of friction. It becomes 0 for H = p. In the event J.F. Archard's dependence (49) is simplified into $V = klA_n$, hence the linear wear intensity meets the following equation:

$$I_h = h/l = k = i_h, \tag{63}$$

Thus, in this special case, the linear wear intensity $I_h$ equals the linear specific wear intensity $i_h$.

The total work of mechanical dissipation $A_{dyss}$ can be determined if the value of η is known. It results from the following system of dependences **[L. 16, 17]**:

$$H = n_o p = n_k \eta p, \tag{64}$$

$$\eta = \frac{n_o}{n_k} = kn_o = k\frac{H}{p}. \tag{65}$$

$A_{dyss} = A_t\eta = A_t k\frac{H}{p}$ has two components: volumetric

$$A_{dyssV} = V\frac{\mu(H-p)^2}{p} = k\frac{H}{p}\frac{pA_t}{\mu HH}\frac{\mu(H-p)^2}{p} = A_{dyss}\left(\frac{H-p}{H}\right)^2 \tag{66}$$

and superficial $A_{dyssF}$, derived from the balance $A_{dyss} = A_{dyssV} + A_{dyssF}$:

$$A_{dyssF} = A_t\eta - A_{dyssV} = A_{dyss}[1 - (1 - \frac{p}{H})^2]. \tag{67}$$

It is a small part of the superficial component since H»p is most commonly:

$$\frac{A_{dyssF}}{A_{dyssV}} = \frac{p}{H-p}. \tag{68}$$

The question of the existence of any limits to wear intensity relates to the limited possibility of a total conversion of mechanical energy into dissipation heat in the process of solid friction. In the thermodynamic system of pressure-unsteady solids, the maximum efficiency of the heat generation process is defined by T.A. Afanasjeva-Ehrenfest principle **[L. 18]**: $\eta_{A-E} = 1-p_1/p_2$, where

$p_1$ – the lower pressure, $p_2$ – the greater pressure. As a system of solids with rough surfaces in friction is characterised by unit pressures p on the nominal surface $A_n$ and hardness H on the real surface $A_r$, the minimum wear efficiency $\eta_{min} = 1 - \eta_{A-E} = 1 - 1 + p/H = p/H$ **[L. 19]**. Based on (49), the minimum specific linear wear intensity $i_{hmin}$ is described with the following dependence**[L. 13, 14]**:

$$i_{hmin} = \left(\frac{p}{H}\right)^2, \tag{69}$$

while the maximum intensity results from:

$$i_{hmax} = \frac{p}{H}. \tag{70}$$

The range of intensity $i_h$ variations means the impossibility of completely avoiding wear. This conclusion from the microscopic analysis of the process of pulse separation of wear particles from the top layer limits the assumption of zero wear, adopted in the macroscopic model up to its certain minimum value (49) – depending on the ratio of unit pressures p/H. Wear particles are not formed if p = H, then the surfaces $A_n$ and $A_r$ are identical, and the material of one or both the solids in friction is plastically deformed as part of fluid friction.

The specific linear wear intensity $i_h$ is associated with the linear wear intensity $I_h$. Its boundary values are described as follows:

$$I_{hmin} = \left(\frac{p}{H}\right)^3, \tag{71}$$

$$I_{hmax} = \left(\frac{p}{H}\right)^2. \tag{72}$$

J.F. Archard's dependence can also be referred to as mass wear: $m = V\rho = klN\rho/H = kA_t\rho/\mu H$, hence the relation of the specific work of wear and wear coefficient $k = i_h$: $e_R^x = \frac{\mu H}{k\rho}$. When considering (69) and (70), the minimum and maximum specific works of wear are defined, namely:

$$e_{Rmin}^x = \frac{\mu H^2}{p\rho}, \tag{73}$$

$$e_{Rmax}^x = \frac{\mu H^3}{p^2\rho}. \tag{74}$$

The process of mechanical energy dissipation, conditioned by the execution of mechanical work $A_{dyss} = A_{dyssV} + A_{dyssF}$, takes place in a zone where wear particles are generated. The zone is numbered 1 in **Fig. 14**. In zone 2, the material is not disintegrated, only subject to an elastoplastic strain, and dissipation heat is generated. Thus, the formation of wear particles contributes to limiting the quantity of friction heat, that is, to the cooling effect in the process of solid friction.

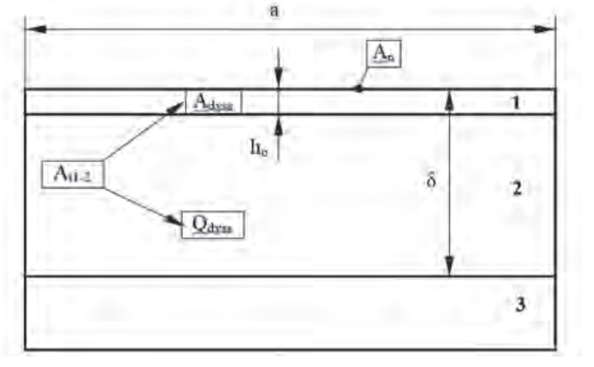


**Fig. 14. A schematic representation of the energy dissipation areas as the location of the cooling effect in solid friction: 1 – the zone of mechanical dissipation and material disintegration, 2 – a friction source of heat, 3 – the zone of secondary effects caused by the energy dissipation in zones 1 and 2 [L. 20]**

Rys. 14. Schematyczne przedstawienie obszaru dyssypacji energii jako miejsca występowania efektu chłodzenia podczas tarcia ciał stałych: 1 – strefa dyssypacji mechanicznej i rozdrabniania materiału, 2 – tarciowe źródło ciepła, 3 – strefa oddziaływań wtórnych wywołanych dyssypacją energii w strefach 1 i 2 **[L. 20]**

## CONCLUSIONS

A synthesis is presented here of some major results of the author's studies during the last 45 years that better explicate the physical nature of solid friction. The approach proposed differs considerably from other models known from engineering literature. The equations formulated are direct results of the laws of physics, particularly thermodynamics, and express analytical dependences between friction and wear parameters, considering the various types of energetic effects.

Some descriptions address the macroscopic interpretation of phenomena, which is important to machine operation. Thermodynamic notions and quantities need to be used, absent from well-known mechanistic interpretations. A systemic approach is key to building a test object model and determining energetic transformations inside the object and at its boundaries with the environment. An unequivocal interpretation of extensive parameters is thus arrived at, of paramount importance to tribology. The macroscopic analysis of friction and its effects, despite its complexity and extent, is limited as it ignores some elementary phenomena at a lower level in the hierarchy of matter organisation.

A description of the pulse nature of the energy dissipation process supplements the system of macroscopic relationships with intensive parameters. The microscopic interpretation also places some restrictions on the applicability of some conclusions arising from the phenomenological approach. The mathematical dependences are presented to form an outline of the thermodynamic theory of friction and wear of solids. It can provide a source for guidelines conditioning test methods to establish the values of structural parameters in these models.

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