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Numerical Modeling of Water Flow in Expansive Soils with Simplified Description of Soil Deformation

Sławomir Michalski, Adam Szymkiewicz*

*corresponding author, Gdańsk University of Technology, Faculty of Civil and Environmental Engineering, ul. Narutowicza 11, 80-233 Gdańsk, Poland, e-mail: adams@pg.edu.pl

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Abstract

In this paper we describe a numerical model of transient water flow in unsaturated expansive soils and the resulting soil volume change. The unsaturated flow equation is solved in a 2D domain using a finite-volume method and an explicit time discretization scheme. Strains in the soil mass are calculated by two simplified approaches, assuming that the strain state is either 1D (in the vertical direction only) or 2D with equal strains in horizontal and vertical directions. The model is applied to two cases described in the literature, in which the strains were computed from the solution of the stress equilibrium equation. It is shown that the simplified methods give results which are reasonably close to the more complex approach based on the equilibrium equations. The proposed model can be used to predict time-varying soil shrinkage and swelling caused by natural and anthropogenic factors.

Key words: expansive soils, unsaturated flow, hydro-mechanical coupling, root water uptake

1. Introduction

Expansive soils undergo significant volume changes during wetting (swelling) or drying (shrinkage). They are found in many parts of the world and pose considerable problems for civil engineers (e.g. Chen 2012, Nelson et al 2015). Expansive properties of soils are usually due to the presence of clay minerals from the smectite group (e.g. montmorillonite or beidellite). Many clays occurring in Poland show expansive behavior, especially the Pliocene clays from the Bydgoszcz and Poznań regions, where they are encountered at shallow depths. Volume changes in such soils can be triggered by a variety of factors related to both natural conditions and human activity, e.g. seasonal weather changes, planting or removing trees in the vicinity of buildings, excavations, leakage from pipelines or sewers(Kumor 2008). Reliable prediction of swell and shrink processes is an important task in geotechnical practice, it also constitutes a non-trivial scientific problem, due to the interaction between water flow and deformation, which must be considered in the framework of unsaturated soil mechanics (e.g. Fredlund et al 2012).

Over the years, many methods for estimating volume changes in expansive soils have been proposed in the literature (for an overview see e.g. works by Grabowska-Olszewska et al 1998, Morsi 2010, Vanapalli and Lu 2012, Fredlund et al 2012, Adem 2015, Adem and Vanapalli 2015). They can be broadly divided into three groups, differing in their complexity:

I. The first group of methods is based on the solution of stress equilibrium equations, with appropriate constitutive models describing stress-strain relationships.The equilibrium equations are coupled with an equation describing soil water flow.This is the most general, but also the most complex and computationally costly approach, which makes it possible to determine the time evolution of a 3D or 2D strain field, as influenced by the infiltration of rainfall, evaporation, transpiration by tree roots or loading by foundations (e.g. Zhang and Briaud 2015, Indraratna et al 2006).
II. The second group includes analytical methods, which are based on a number of simplifying assumptions, so that it is possible to compute soil strains without solving equilibrium equations. The most widely used assumption is that soil deforms only in the vertical direction (Vanapalli and Lu 2012). Another possibility is to consider 2D plane strain conditions with equal strains in vertical and horizontal directions (Navarro et al 2009). In both cases, strains and displacements can be calculated as a function of changes in soil suction (e.g. McKeen 1992) or water content (e.g. Garbulewski 2000, Briaud et al 2003) or on the basis of measurements of swelling pressure and swell under load in oedometers (e.g. Nelson et al 2015). The expected increments in suction or water content can be assumed a priori, or they can be obtained from the solution of the transient flow equation (Wray et al 2005).
III. The third group consists of empirical methods in the form of functional relationships between the anticipated vertical strain or displacement due to swell and some basic geotechnical properties, such as liquid limit, clay fraction content, or natural water content (e.g. Kumor 2008, Zumrawi 2013). While being a useful engineering tool, they lack an explicit physical basis and can be considered accurate only for a particular set of soils for which they have been developed. They are also not suited to include a variety of possible factors causing swelling and shrinkage.

In terms of practical application, the methods from the second group are particularly interesting, because they are physically based and make it possible to calculate time-varying soil volume changes without the need to use complex, fully coupled hydro-mechanical models. These methods can be relatively easily coupled with existing numerical models for unsaturated water flow (1D, 2D or 3D, depending on the needs), which often include a detailed representation of soil-vegetation-atmosphere interactions, necessary to accurately capture soil swelling and shrinkage processes (e.g. Healy and Essaid 2012, Šimunek et al 2012, van Dam et al 2008) In this context, one of the issues that should be investigated in more detail is how the simplified methods of calculating soil deformation based on the assumption of 1D or 2D strain conditions compare to the solution of full equilibrium equations (i.e. the first group of methods). Such comparisons are presented in this paper.

Methods from both the first and the second groups require formulation of constitutive models describing soil volume changes as a function of stress variables. The choice of stress state variables for unsaturated soils remains an open scientific problem (Nuth and Laloui 2008, Sheng 2011). In unsaturated soils, water is under suction, i.e. it is bound to the soil skeleton by adsorptive, capillary and osmotic forces, resulting in negative (lower than atmospheric) values of measured pore water pressure. As the soil is dried, the water content decreases, while the suction increases (the water pressure becoming more negative), because the remaining amounts of water are more tightly bound to the skeleton by the surface forces. The largest values of suction, corresponding to oven-dry soil are commonly reported as $10^{5.25}$ to 10^6 kPa (Nelson et al 2015). The relationship between the soil water content and suction is called the soil water retention curve (SWRC) or the soil water characteristic curve (SWCC). It can be expressed with respect to the gravimetric water content w (mass of water / mass of soil skeleton), the volumetric water content θ (volume of water / volume of soil) or water saturation S_r (volume of water / volume of pores). In expansive and compressible soils, the shape of these functions is significantly affected by volume changes occurring during wetting and drying and by the external stress applied to the soil sample (Zhang and Briaud 2015).

Constitutive models for deformation of unsaturated soils can be formulated with respect to either one or two stress variables. In the first case, an effective stress is used, which combines the total stress and the negative pore water pressure, following the concept introduced by Bishop (1959). However, it is not clear to what extent the capillary, adsorptive and osmotic components of suction influence the mechanical stress state in soil (in dry soils, suction is very large, but does not influence the effective stress) (Baker and Frydman 2009, Yong 1999). In the second case, usually the total (net) stress σ and matric (i.e. capillary and adsorptive) suction ψ are chosen as the principal stress variables (Fredlund and Morgenstern 1976, Fredlund and Rahardjo 1993). The dependence of the gravimetric water content w, water saturation S_r and void ratio e on s and y is given in the form of constitutive surfaces – Fig. 1.

It is now generally accepted that unsaturated soil behavior can be accurately described using either a single stress variable or two variables, on the condition that the model is of elasto-plastic type (Jommi 2000, Alonso et al 2010). However, advanced elasto-plastic models usually include a large number of parameters, which can be difficult to estimate (e.g. Alonso et al 1990, 2010). Thus, simpler models based on constitutive surfaces, described above, remain a useful tool for prediction of soil deformations if we can assume either that the soil is elastic or that it undergoes plastic deformation along a specified monotonous path. Expansive soils can be considered elastic if the external loads are small (roads, light buildings), the soils are strongly over-consolidated (due to earlier swell-shrink cycles), and the deformation is

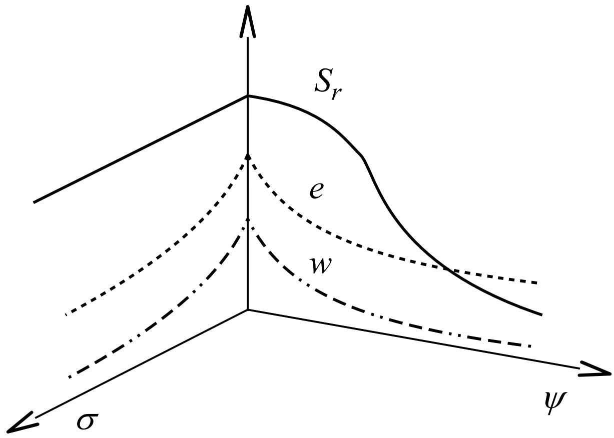

Fig. 1. Constitutive surfaces for unsaturated expansive soils (from Zhang 2004)

caused mainly by changes in water content and suction, as for example in the case of cyclic weather changes (Adem 2015, Bolzon and Schrefler 1995). Similarly, long-term shrinkage caused by planting a new tree or long-term swell caused by cutting down an existing tree are examples of monotonic loads, which can be described by elastic parameters (Navarro et al 2009, Indraratna et al 2006, Vu and Fredlund 2004).

This paper presents a numerical model developed by the first author as a part of his PhD thesis (Michalski 2016). The unsaturated flow equation is solved in a 2D domain using a finite-volume method and an explicit time discretization scheme. Strains in the soil mass are calculated using either a 1D or a 2D simplified approach. The model is applied to two cases described in the literature, in which strains were computed from the solution of a stress equilibrium equation.

2. Numerical Model

2.1. Main Assumptions

It is assumed that water flow in a two-dimensional unsaturated expansive soil domain can be described by the Richards' equation (Richards 1931):

$$\frac{\partial \theta(\psi)}{\partial t} - \frac{\partial}{\partial x}\left(k(\psi)\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial z}\left[k(\psi)\left(\frac{\partial u}{\partial z} - 1\right)\right] + R = 0, \tag{1}$$

where the water pressure potential u is the negative of soil suction $u = -\psi$, k is the permeability of soil, which,under unsaturated conditions, is variable and can be expressed as a function of the volumetric water content or suction, and R is the intensity of water uptake by tree roots. The form of $\theta(\psi)$ and $k(\psi)$ functions can be arbitrary, but their hysteresis is neglected. It is assumed that $\theta(\psi)$ includes both saturation changes and volumetric changes. The coupling between water flow and soil deformation is in one direction only, i.e. the values of suction obtained by solving Eq. (1) are used to calculate the volume change. The total stress in soil is considered constant. The strain is either one-dimensional (in the vertical direction only) or two-dimensional, with equal components in vertical and horizontal directions, as explained in more detail in section 2.3.

2.2. Solution of the Unsaturated Flow Equation

Eq. (1) is solved using finite-volume discretization in space, with a regular, structured grid consisting of rectangular cells (Fig. 2). For discretization in time, a first-order explicit scheme is used. While the Richards' equation is usually solved by implicit schemes (e.g. Szymkiewicz 2013), explicit schemes have the advantage of low complexity and ease of parallelization. Navarro et al (2009) used an explicit scheme to study soil shrinkage caused by root water uptake close to a line of trees. Following the approach described by Michalski (2016), the solution domain is covered by a rectangular grid of nodes, associated with the corresponding finite volumes (grid cells). The discrete form of Eq. (1) can be written as follows:

$$\frac{\theta_{i,j}^{m+1} - \theta_{i,j}^{m}}{\Delta t} + \frac{q_{i+1/2}^{m} - q_{i-1/2}^{m}}{\Delta x} + \frac{q_{j+1/2}^{m} - q_{j-1/2}^{m}}{\Delta z} + R_{i,j}^{m} = 0, \tag{2}$$

where i and j are node (cell) indices, m is a time-step index, Δt, Δx and Δz denote the discretization steps with respect to the independent variables. Water fluxes q between nodes are calculated as

$$\begin{aligned} q_{i+1/2}^{m} &= -\frac{k_{i+1,j}^{m} + k_{i,j}^{m}}{2}\left(\frac{u_{i+1,j}^{m} - u_{i,j}^{m}}{\Delta x}\right), \\ q_{j+1/2}^{m} &= -\frac{k_{i,j+1}^{m} + k_{i,j}^{m}}{2}\left(\frac{u_{i,j+1}^{m} - u_{i,j}^{m}}{\Delta z} - 1\right). \end{aligned} \tag{3}$$

As shown in Eqs. (3), in our study the nodal values of the permeability coefficient k are averaged arithmetically, although other averaging schemes can be also implemented (e.g. Szymkiewicz 2009). Eq. (2) makes it possible to calculate the value of water content at each node for a new time step, on the basis of values from the previous time step. The corresponding values of ψ and k are then obtained from the retention and permeability functions.

Explicit schemes are only conditionally stable, which means that the length of the time step is limited. The allowable time step can be calculated using the Courant criterion (Navarro et al 2009 and personal communication). Michalski (2016) proposed to use another approach, according to which the time-step size can be adjusted in calculations in such a way that the volumetric water content increment during a time step at any node is not larger than 0.0001. The algorithm has also been adapted to axi-symmetric geometries (e.g. moisture changes around a single tree). A more detailed description of the numerical procedure and its verification by comparison with an analytical solution can be found in Michalski (2016).

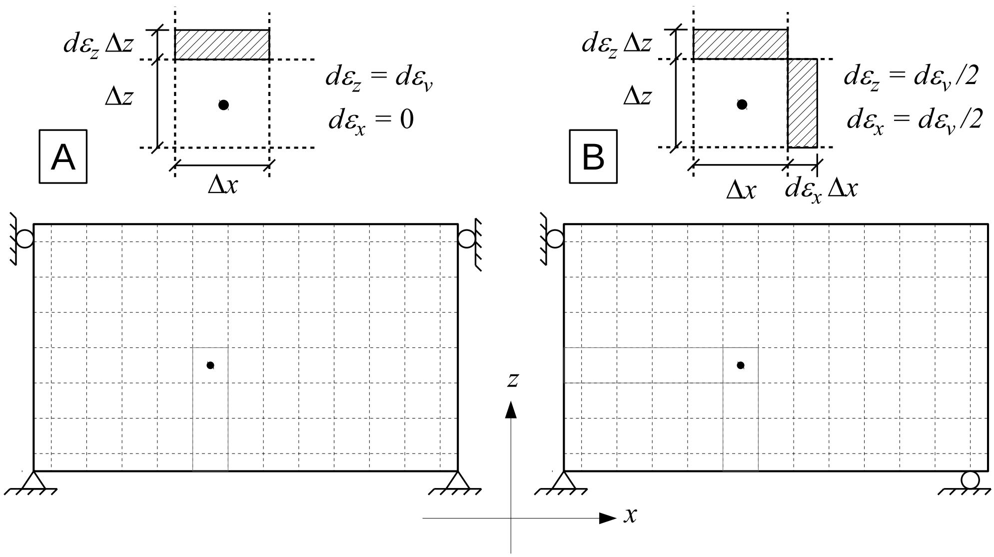

Fig. 2. Calculation of strains and displacements for 1D case (A) and 2D case (B) (Michalski 2016)

2.3. Calculation of Soil Deformation

Once the values of suction and water content have been obtained from the water flow equation for a new time step, the corresponding soil deformation is calculated. This can be carried out assuming either 1D vertical strain or 2D plane strain with equal strains in each direction. In the first case, the lower boundary of the solution domain is considered as a fixed boundary, and we calculate the vertical strain and the corresponding incremental displacement for each cell of the numerical grid. The displacements of the soil surface are obtained by summing incremental displacements in each column of the grid (Fig. 2A). In the 2D case, the incremental displacements are summed along columns and rows, assuming that the lower boundary and one of the vertical boundaries are fixed (Fig. 2B). While the assumption of 1D vertical strains is commonly used in models for expansive soils (Vanapalli and Lu 2012), the assumption of plane strain with $\varepsilon_x = \varepsilon_z$ seems to be considered only by Navarro et al (2009) in their study of the influence of a line of trees on a parallel row of buildings. In the following examples, it is shown that the results obtained from a full mechanical analysis (i.e. the solution of equilibrium equations for plane-strain conditions) partly fall in between these two cases.

In principle, the strains and displacements can be calculated using any formula which relates 1D or 2D strain to changes in soil suction or soil water content. The results shown below were obtained using the constitutive model described by Fredlund and Rahardjo (1993), in which the general formula for a small increment in the void ratio is:

$$\mathrm{d}e = \frac{\kappa_\sigma}{\sigma}\mathrm{d}\sigma + \frac{\kappa_\psi}{\psi}\mathrm{d}\psi, \tag{4}$$

where κ_σ and κ_ψ are constant coefficients. Integration of the above equation leads to a formula for finite increments in the void ratio, which is analogous to the well-known expression for soil compressibility under saturated conditions:

$$\Delta e = C_\sigma \log\left(\frac{\sigma}{\sigma_0}\right) + C_\psi \log\left(\frac{\psi}{\psi_0}\right), \tag{5}$$

where C_σ and C_ψ are compressibility indices. The volumetric strain can be expressed as:

$$\mathrm{d}\varepsilon_v = m_1^S \mathrm{d}\sigma + m_2^S \mathrm{d}\psi = \frac{3(1-2\nu)}{E}\mathrm{d}\sigma + \frac{3}{H}\mathrm{d}\psi, \tag{6}$$

where E and H are elastic moduli, ν is the Poisson ratio, and m_1^S and m_2^S are volume compressibility coefficients defined as

$$m_1^S = \frac{3\,(1-2\nu)}{E} = \frac{\partial \varepsilon_v}{\partial \sigma} = \frac{1}{1+e_0}\frac{\partial e}{\partial \sigma} = \frac{0.434}{(1+e_0)}\frac{C_\sigma}{\sigma}, \tag{7}$$

$$m_2^S = \frac{3}{H} = \frac{\partial \varepsilon_v}{\partial \psi} = \frac{1}{1+e_0}\frac{\partial e}{\partial \psi} = \frac{0.434}{(1+e_0)}\frac{C_\psi}{\psi}. \tag{8}$$

The above equations describe the general case of 3D strain. For plain strain ($\varepsilon_{yy} = 0$), it is more convenient to write the constitutive equation in terms of the average stress $\sigma_{2D} = (\sigma_{xx} + \sigma_{zz})/2$ (Vu and Fredlund 2004):

$$\mathrm{d}\varepsilon_{v(2D)} = m_{1(2D)}^S \mathrm{d}\sigma_{2D} + m_{2(2D)}^S \mathrm{d}\psi = \frac{2(1+\nu)(1-2\nu)}{E}\mathrm{d}\sigma_{2D} + \frac{2(1+\nu)}{H}\mathrm{d}\psi. \tag{9}$$

In the 1D (vertical) strain case ($\varepsilon_v = \varepsilon_z$) the equation can be rewritten in terms of the vertical stress σ_z (Vu and Fredlund 2004):

$$\mathrm{d}\varepsilon_{v(1D)} = \mathrm{d}\varepsilon_z = m_{1(1D)}^S \mathrm{d}\sigma_z + m_{2(1D)}^S \mathrm{d}\psi = \frac{(1+\nu)(1-2\nu)}{E(1-\nu)}\mathrm{d}\sigma_z + \frac{(1+\nu)}{H(1-\nu)}\mathrm{d}\psi. \tag{10}$$

As already mentioned, only deformation due to suction changes is taken into account in the present model, so Eqs. (4)–(10) are simplified accordingly.

3. Examples of Calculations

3.1. Example 1: Infiltration

The first example, taken from Hung and Fredlund (2002), concerns 2D infiltration in a soil mass. The model domain and boundary conditions are shown in Fig. 3. Note that the boundary conditions for deformation refer to the 2D equilibrium equation as solved by Hung and Fredlund (2002), while in our simulations we computed the

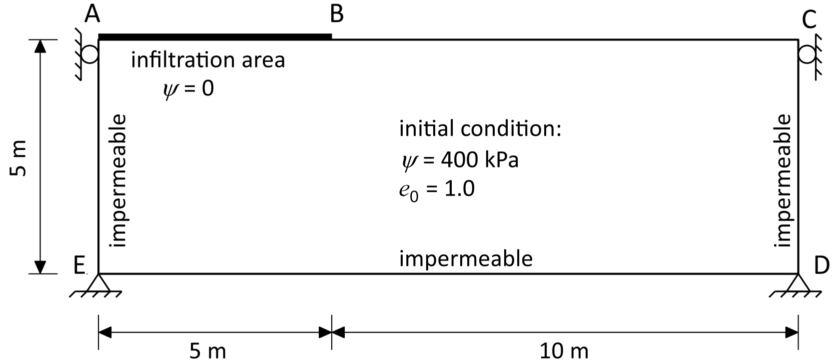

Fig. 3. Example 1: Geometry and boundary conditions (Hung and Fredlund 2002)

strains by simplified 1D and 2D approaches, described above. The initial void ratio was $e_0 = 1.0$, Poisson ratio $\nu = 0.35$, and the compressibility index $C_\psi = 0.1$. The retention function and the permeability function were given as

$$\theta = \frac{\theta_s}{\ln\left[e_n + \left(\frac{\psi}{a}\right)^n\right]}, \quad k = k_s\frac{\theta}{\theta_s}, \tag{11}$$

with e_n denoting the natural logarithm base, the volumetric water content at saturation $\theta_s = 0.45$, permeability at saturation $k_s = 1$ mm/day, and parameters $a = 100$ kPa, $n = 1.5$. For purposes of comparison, the parameters from Hung and Fredlund (2002) are used, although their physical consistency can be questioned (e.g. the volumetric water content at saturation θ_s is smaller than the porosity corresponding to the initial void ratio e_0). A uniform numerical grid with node spacing of 20 cm in each direction was used.

Fig. 4 shows the distribution of soil water suction after 25 days of infiltration. One can see that the results obtained with the model described in this paper are very close to the results reported by Hung and Fredlund (2002), which were obtained using implicit time discretization. This confirms the accuracy of the numerical solution method. The displacement (heave) of the soil surface obtained assuming 2D and 1D strain conditions is shown in Fig. 5. As could be expected, the heave values corresponding to the 2D strain assumption are significantly smaller than those for 1D vertical strain (a maximum heave of 144 mm vs. 222 mm). In the left-hand part of the domain, under the infiltration area, where the heave is largest, the results of Hung and Fredlund (2002) are in between the 1D and 2D cases (a maximum heave of 200 mm). In the part of the domain away from the infiltration area, the heave predicted by the simplified methods, especially for the 2D case, is smaller than the one obtained from the solution of equilibrium equations. This can be attributed to the influence of the boundary condition CD, which was set to zero horizontal strain in the solution of Hung and Fredlund (2002).

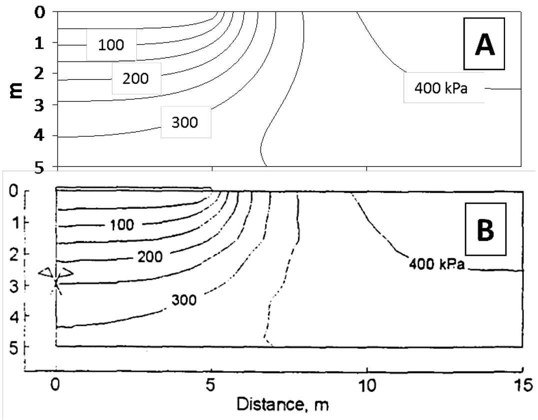

Fig. 4. Example 1: Distribution of soil suction after 25 days of infiltration A – obtained with the model described in this paper, B – reported by Hung and Fredlund (2002)

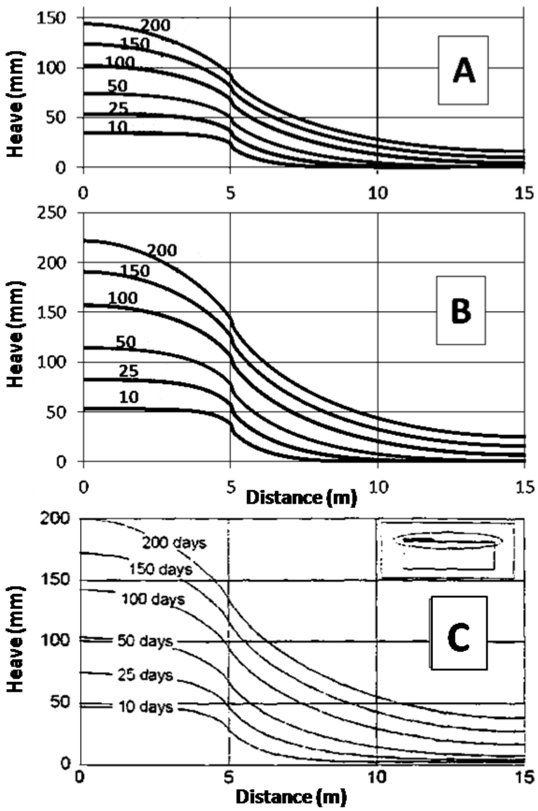

Fig. 5. Example 1: Comparison of soil surface heave. A – simplified method, 2D strain, B – simplified method, 1D strain, C – solution of the equilibrium equations reported by Hung and Fredlund (2002)

3.2. Example 2: Water Uptake by a Line of Trees

The second example, taken from Fredlund and Hung (2001) and Vu (2002), concerns water uptake by tree roots and the associated settlement caused by soil shrinkage. The aim is to obtain the flow and deformation field in a steady state in the vicinity of a line of trees (Fig. 6). It is assumed that the distance between the trees is equal to 5 m and each tree transpires 0.3 m^3 of water per day, with the transpiration rate varying linearly between depths of 1 m and 3 m, as shown in Fig. 6. The initial void ratio is $e_0 = 1.0$, the Poisson ratio $\nu = 0.3$, and the compressibility index $C_\psi = 0.2$. The steady state solution of Eq. (1) does not depend on the retention function - the same retention function as in Example 1 was used to reach a steady state in transient simulation. The permeability function is given by the formula

$$k = k_s \frac{1}{1 + \alpha \left(\frac{\psi}{\gamma_w} \right)^\beta} \tag{12}$$

with permeability at saturation $k_s = 5$ mm/day, the volumetric weight of water $\gamma_w = 10\,\text{kN/m}^3$, ψ given in kPa, and parameters $\alpha = 0.001\ \text{m}^{-1}$ and $\beta = 2$. The node spacing was equal to 25 cm in each direction.

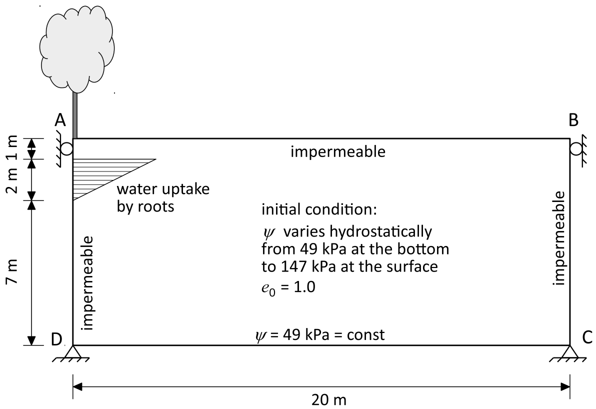

Fig. 6. Example 2: Geometry, initial and boundary conditions (Fredlund and Hung (2001) and Vu (2002))

Similarly to Example 1, the distribution of soil suction obtained by the model described in this paper was very close to the one given by Fredlund and Hung (2001) (not shown here). However, differences were observed in terms of the calculated settlements of the soil surface (Fig. 7). The relation between various solutions is consistent with Example 1. The maximum settlement occurs under the trees and is equal to 110 mm for the 1D case and 77 mm for the 2D case, while the solution of equilibrium equations yields 85 mm. In the zone of large settlements the results of Fredlund and

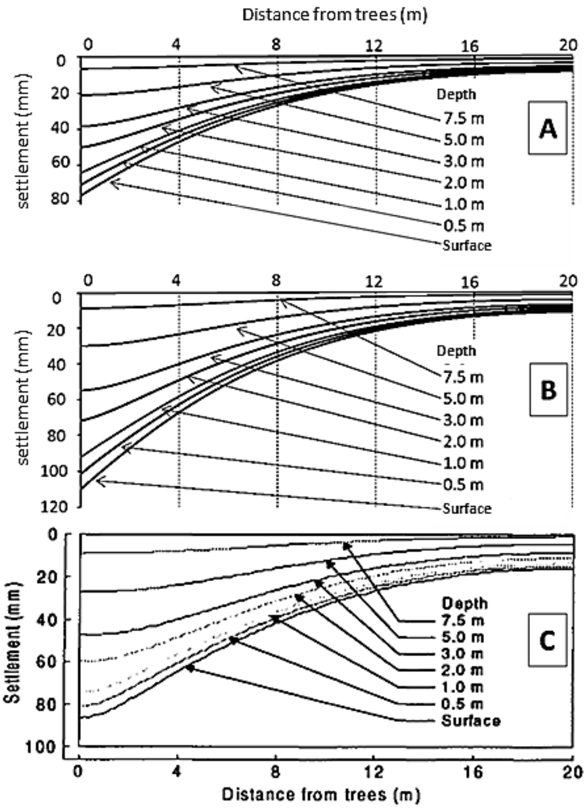

Fig. 7. Example 2: Comparison of soil surface settlements. A – simplified method, 2D strain, B – simplified method, 1D strain, C – solution of the equilibrium equations reported by Fredlund and Hung (2001)

Hung (2001) are between the results obtained by the simplified 1D and 2D approaches. Close to the right-hand boundary, the settlement predicted by the equilibrium equation is larger than that obtained by the simplified methods and the difference is larger for the 2D method.

4. Conclusions

The model proposed in this paper can be a useful tool for estimation of time-varying volume change in expansive soils. The solution of the unsaturated flow equation by the explicit scheme led to results very similar to the widely used implicit time discretization. Using both 1D and 2D simplified methods of strain calculation one obtains a range of heave or settlement values which are in reasonable agreement with values calculated from the equilibrium equations. The same procedure for calculating strains and displacements can be coupled with other numerical codes solving the unsaturated flow equation.

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