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Physico-mathematical model of complex heat exchange between an electric infrared radiant heating panel and the environment

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Abstract: One of the main elements of heating systems is a heating device, whose construction is constantly being improved upon on thanks to the development of the science surrounding the materials' physical and chemical properties of the working bodies, manufacturing and application technologies. This article is devoted to the development of a physical model of complex heat exchange between an electric infrared radiant heating panel based on an amorphous metallic radiating plate located in the middle of the panel and surrounding solids and air. The developed physical model is described by mathematical equations taking into account the boundary conditions of the third kind on the basis of complex heat exchange by radiation and convection both with surrounding solid bodies and passing airflow. The purpose of the solution is to determine the rational relation between the radiation and convective components of heat transfer, depending on the comfortable state of the person (observance of the technological process), and the operating modes of the heating system.

Keywords: electrical radiant heating panel, infrared heat transfer, heating system, physical model, lumped capacity method

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Introduction

Heating is one of the most expensive processes in engineering. Centralized heating systems of Former Soviet Union (USSR) countries are in a state of “deep crisis”. According to experts, energy loss for centralized heating systems can reach up to 40% and more than 70% of the heating networks require serious modernization. Therefore, today, there is an obvious transition from centralized to decentralized and individual heating systems. Considering large installed and developed systems of electrical power generation and distribution in most ex-USSR countries, one of the promising areas of development is electric heating. In confirming this trend we can note the fuel and energy balance in developed European countries which have a significant consumption of electricity for heating purposes. According to available data, in France, about 40% of all buildings are equipped with electrical heating, 30% in Spain and Finland, and more than 80% in Norway. Electrical heating systems have significant advantages that work mainly on the principle of heat transfer through radiation.

Medical-biological research have established that it is very important exactly how heat energy is transferred to the human body. Not all wavelengths of infrared radiation, which is used in radiant heating systems, can be applied for heating humans and animals. Infrared electromagnetic radiation contains a wavelength of 780 nm to 1 mm. It can also be divided into three regions, IR-A (wavelength from 780 to 1400 nm), IR-B (wavelength from 1400 to 3000 nm) and IR-C (wavelength from 3000 nm to 1 mm). It has been found that high-intensity IR-A radiation can cause thermal burns and in the long run can result in premature ageing to the skin. On the other hand, IR-C infrared waves from 3000 to 10000 nm do not cause negative impacts on humans and is more preferable for infrared heating in domestic environments (McGranaghan, 2014). Moreover, in some countries it is prohibited to use infrared heating in IR-A and IR-B range in areas that include the permanent presence of humans or animals.

Countless research has been made on the study of radiant heating and cooling systems (Cannistraro et al., 2015a; Cannistraro et al., 2015b; De Carli et al., 2012; Dudkiewicz & Jezowiecki, 2009; Miriel et al., 2002; Petras & Kalus, 2000; Zhelykh et al., 2016; Zhelykh et al., 2017), but of most interest is the theoretical and practical research of electrical radiation panels (Ferrarini et al., 2018) that was made with the use of the lumped capacity method which allows us to evaluate the heat transfer process in dynamics. Nevertheless, the mathematical model which was provided in the above mentioned paper, though suitable for certain experimental conditions, requires a more precise definition for its implementation in real life conditions. Such as, taking into account the influence of triatomic gases on radiant heat transfer and dividing convection and radiation components in the mathematical model formulation for the purpose of assessing the influence of each process separately from each other.

The purpose of this publication was to develop a mathematical model for infrared radiant heaters by using a lamp capacity method applied to the heat emitting layer which takes into account heat transfer between the heater surface, passing airflow and solid bodies.

1. Analysis

To provide analytical research on heat transfer from infrared heating panels, it is necessary to take into account the place of installation. In this paper the three most commonly used heater constructions and mounting locations are mentioned (Motrechko et al., 2019). Heat fluxes from heaters of different construction and alignment are shown in Figure 1: 1a - displays a heating panel installed near an outer wall, 1b - displays a heating panel installed in the middle of the room, 1c - displays a heating panel installed in a false ceiling. In this study we will focus on the case shown in Figure 1a. Other cases can be described in a similar way. The equation of energy balance for a heating panel is shown in Figure 1a and will take following form:

$$Q_e = Q_{rad}^{f} + Q_{conv}^{f} + Q_{rad}^{t} + Q_{conv}^{t} + Q_{rad}^{b} + Q_{conv}^{b} + Q_{rad}^{r} + Q_{conv}^{r} + Q_{cond}^{r} + Q_{rad}^{h.e} \tag{1}$$

where:
Q_{rad}^{t} - radiant heat flux from the heating panel top surface, W;
Q_{rad}^{b} - radiant heat flux from the heating panel bottom surface, W;
Q_{rad}^{f} - radiant heat flux from the heating panel front surface, W;
Q_{rad}^{r} - radiant heat flux from the heating panel rear surface, W;
Q_{conv}^{t} - convective heat flux from the heating panel top surface, W;
Q_{conv}^{b} - convective heat flux from the heating panel bottom surface, W;
Q_{conv}^{f} - convective heat flux from the heating panel front surface, W;
Q_{conv}^{r} - convective heat flux from the heating panel rear surface, W;
Q_{cond}^{r} - conductive heat flux from the heating panel rear surface, W;
$Q_{rad}^{h.e}$ - radiant heat flux from the inner heating element, W;
Q_e - electrical flux supplied to inner heating element, W.

The radiant flux from the inner heating element can be written as follows:

$$Q_{rad}^{h.e} = Q_{rad}^{h.e,f} + Q_{rad}^{h.e,t} + Q_{rad}^{h.e,b} + Q_{rad}^{h.e,r} \tag{2}$$

where:
$Q_{rad}^{h.e,t}$ - radiant heat flux from the inner heating element directed to the top surface of the heating panel, W;
$Q_{rad}^{h.e,b}$ - radiant heat flux from the inner heating element directed to the bottom surface of the heating panel, W;
$Q_{rad}^{h.e,f}$ - radiant heat flux from the inner heating element directed to the front surface of the heating panel, W;
$Q_{rad}^{h.e,r}$ - radiant heat flux from the inner heating element directed to the rear surface of the heating panel, W.

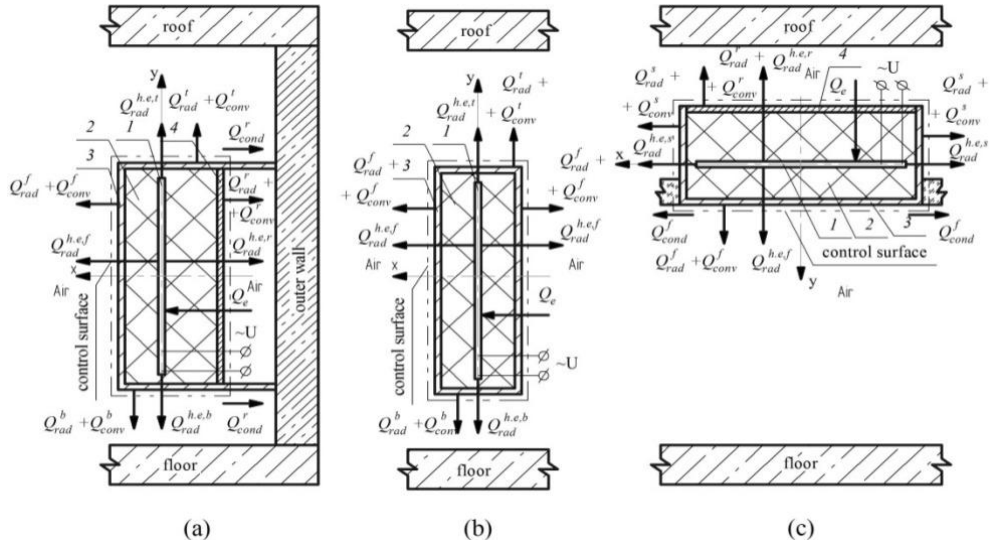

Fig. 1. The scheme of heat flows in radiant heating panels of different alignment: 1 - metallic heating element, 2 - thermal and electrical insulator, 3 - framework, 4 - shielding plate

Each additive in equation (2) represents a radiation heat flux from the inner heating element to surrounding objects. Due to the availability of heat and the electrical insulation, as well as the framework, heat flux from the heating element is partially absorbed and dissipated before it exceed the limits of the radiant heater. As a result, the thermal process occurring in the radiation panel has a complicated nature of radiation-conductive heat transfer. Unfortunately, this task has a very complex analytical solution and can be solved only by means of a numerical technique. The general equation for describing the process of stationary radiation-conductive heat transfer in the middle of the panel can be written as follows (Jaeger, 1950; Siegel & Howell, 2001):

$$\nabla\left(\mathrm{k}_h \nabla T - q_{rad}^{h.e}\right) = 0 \tag{3}$$

where:
$q_{rad}^{h.e} = Q_{rad}^{h.e}/F_{h.s}$ - specific radiant flux from the inner heating element, W/m^2;
$F_{h.s}$ - heating surface of the radiant heating panel, m^2;
k_h - thermal conductivity of the radiant heater, W/(m^2·°C).

It should also be noted that for an opaque medium, the equation (3) takes the usual form of a conduction heat transfer equation in solids. Taking into account that most non-metallic heat and electrical insulators have a blackness degree and absorption capacity close to those of an absolutely black body, it is assumed that heat transfer in the middle of the radiation panel occurs only by means of conduction, and radiation heat transfer occurs only on the surface of the radiation panel. Consequently, equation (2) takes the form of:

$$Q_{rad}^{h.e} = 0 \tag{4}$$

Radiation heat flux from the surface of the radiant heater is supplied not only to solid bodies which have a mutual radiation surface with the heater, but also to triatomic gases which are present in the surrounding air. As standard, surrounding air contains carbon dioxide and water vapour triatomic gases, whose emission and absorption ranges are shown in Table 1. As mentioned previously, IR-C radiation with a wavelength of 3000 to 10000 nm is preferable for domestic heating purposes and by means of data provided by Table 1 it can be found that almost 64% of this wavelength range can be affected by the gases' emission and absorption. The latter means that it is very important to take into account the influence of triatomic gases by means of the selection of the radiation surface with the maximum emission in a stationary regime with wavelengths of 8500 nm. Otherwise most of the outgoing radiation from the radiant heating panel will be absorbed by the triatomic gases and thus there will be very little difference from ordinary convection type heaters (especially if the surrounding air consists of large amounts of CO_2 gases and has a high relative humidity value). Consequently, radiant flux from the surface of a radiant heater can be written as follows:

$$Q_{rad}^{h.s} = Q_{rad}^{h.s,s.b} + Q_{rad}^{h.s,g} \tag{5}$$

where:

$Q_{rad}^{h.s}$ - radiant heat flux from the heating surface of radiant panel, W;

$Q_{rad}^{h.s,s.b}$ - radiant heat flux from the surface of radiant heater supplied to solid bodies, W;

$Q_{rad}^{h.s,g}$ - radiant heat flux from the surface of radiant heater supplied to gases, W.

Table 1. Emission and absorption range for carbon dioxide and water vapor gases

CO_2		H_2O	
λ, nm	Δλ, nm	λ, nm	Δλ, nm
2400-3000	600	2200-3000	800
4000-4800	800	4800-8500	3700
12500-16500	4000	12000-30000	18000

This particular work concentrates on the situation where the surface emits the primary wavelength in the range of 8500 to 12000 nm, which is not affected by the triatomic gases, and thus it can be assumed that the radiation heat transfer only occurs between solid bodies.

Radiation transfer equations include the fourth degree temperature difference which can be linearized as follows:

$$\begin{aligned} T_{h.s}^4 - \overline{T}_{s.b}^4 &= \left(\overline{T}_{s.b} + \Delta T\right)^4 - \overline{T}_{s.b}^4 = \overline{T}_{s.b}^4\left[\left(1 + \frac{\Delta T}{\overline{T}_{s.b}}\right) - 1^4\right] = \\ &= 4\Delta T\overline{T}_{s.b}^3\left[1 + \frac{3}{2}\frac{\Delta T}{\overline{T}_{s.b}} + \left(\frac{\Delta T}{\overline{T}_{s.b}}\right)^2 + \frac{1}{4}\left(\frac{\Delta T}{\overline{T}_{s.b}}\right)^3\right] \approx 4a\overline{T}_{s.b}^3\Delta T \end{aligned} \tag{6}$$

where:
$T_{h.s}$ - temperature radiant heater surface, K;
$\overline{T}_{s.b}$ - average temperature of solid bodies, K;
$\Delta T = T_{h.s} - \overline{T}_{s.b}$ - temperature difference, K;
$a = [1 + 3/2 \cdot (\Delta T/\overline{T}_{s.b})]$.

Consequently, the radiant heat transfer coefficient from the surface of the radiant heat supplied to the solid bodies can be written as follows:

$$h_{rad}^{h.s,s.b} = \frac{Q_{rad}^{h.s,s.b}}{\left(T_{h.s} - \overline{T}_{s.b}\right)F_{h.s}} = 4aA_{h.s,s.b}\sigma\overline{T}_{s.b}^{3} \tag{7}$$

where:
$h_{rad}^{h.s,s.b}$ - radiant heat transfer coefficient from the surface of radiant heater supplied to solid bodies, W/(m^2·K);
$A_{h.s,s.b}$ - total absorption capacity for radiant heat transfer between radiant heater and solid body;
$\sigma = 5.67 \cdot 10^{-8}$ - Stefan-Boltzmann constant, W/(m^2·K^4).

The total absorption capacity for the radiant heat transfer between the radiant heater and a solid body can be written as follows:

$$A_{h.s,s.b} = \frac{1}{\frac{1}{A_{h.s}} + \left(\frac{1}{A_{s.b}} - 1\right)\frac{F_{h.s}}{F_{s.b}}} \tag{8}$$

where:
$A_{h.s}$ - absorption capacity of radiant heating panel;
$A_{s.b}$ - absorption capacity of the solid body;
$F_{s.b}$ - surface of the solid body that exchanges heat with the radiant heater, m^2.

The convection heat transfer coefficient in the case of airflow which passes along a flat radiant heater surface can be determined by the following equation:

$$h_{conv} = \frac{k_{air}C}{l_0}(Gr \cdot Pr)^n \tag{9}$$

where:
h_{conv} - convective heat transfer coefficient from the surface of the radiant heater supplied to the airflow, W/(m^2·K);
k_{air} - thermal conductivity of the airflow, W/(m^2·°C);
C, n - coefficients according to Table 2;
l_0 - specified dimension, m;
Gr - Grashof number;
Pr - Prandtl number.

The specified temperature for the air thermal properties determined can be obtained by the following equation:

$$T_0 = 0.5\left(T_{h.s} + \overline{T}_{air}\right) \tag{10}$$

where $\overline{T}_{air}$ - average temperature of airflow, K.

Table 2. Values for C and n coefficients

Gr·Pr	C	n
$< 10^{-3}$	0.5	0
10^{-3}-$5\cdot10^{2}$	1.18	0.125
$5\cdot10^{2}$-$2\cdot10^{7}$	0.54	0.25
$2\cdot10^{7}$-10^{13}	0.135	0.33

For heaters installed in false ceilings or in floors (radiant floor systems) it should also be noted that in the case where the hot surface of the horizontal heater plate is orientated upwards, the value of the heat transfer coefficient found by formula (9) should be increased by 25%. In the case where the heating surface of the horizontal heater plate is oriented downwards, the value of the heat transfer coefficient found by formula (9) should be reduced by 25%. The specified dimensions for heaters with horizontal mountings should be taken from the shorter linear dimension. For vertical orientated heaters, the specified dimension should be taken from the height.

As it was shown in equation (1) that the infrared heater thermal balance in most common cases consist of radiation and convection fluxes from its surfaces and additional heat fluxes (radiation from the inner layers, conduction heat transfer). For this study we will ignore additional heat fluxes, as they have rather small values compared to convection and radiation fluxes from the heating surfaces. For further research we will use a lumped capacity method for defining physical processes on the front surface of the infrared heater shown in Figure 1a. According to the first law of thermodynamics, the following equation can be written for the emitting surface of the radiant heater:

$$\begin{aligned}\rho_{h.s}C_{h.s}\delta_{h.s}\frac{dT}{d\tau} &= q_e - \left(q_{conv} + q_{rad}\right) = \\ &= q_e - h_{conv}\left(T_{h.s} - \overline{T}_{air}\right) - h_{rad}\left(T_{h.s} - \overline{T}_{s.b}\right)\end{aligned} \tag{11}$$

where:

$\rho_{h.s}$ - density of the radiant heating panel emitting surface, kg/m^3;
$C_{h.s}$ - thermal capacity of the radiant heating panel emitting surface, kJ/(kg·K);
$\delta_{h.s}$ - thickness of the radiant heating panel emitting surface, m;
T - temperature of the emitting surface of the radiant heater, K;
τ - time, s;

$q_e = Q_e/F_{h.s}$ - specific electrical flux supplied to the inner heating element, W/m^2;
$q_{rad} = Q_{rad}/F_{h.s}$ - specific radiant heat flux from the heating panel, W/m^2;
$q_{conv} = Q_{conv}/F_{h.s}$ - specific convection heat flux from the heating panel, W/m^2.

Initial conditions for the above mentioned equation can be written as follows:

$$\tau = 0, \quad \mathrm{T}(\tau) = \mathrm{T}(0) = T_{int} \tag{12}$$

where T_{int} - initial temperature of the infrared heater emitting layer, K.

The specific electrical flux supplied to the inner heating element can be written as follows:

$$q_e = \frac{I^2 R}{l_{h.s} h_{h.s}} \tag{13}$$

where:
I - electrical current supplied to the inner heating element, A;
R - electrical resistance of the inner heating element, Ω;
$l_{h.s}$ - length of the inner heating element, m;
$h_{h.s}$ - height of the inner heating element, m.

According to all of the above, the solution can be written as follows:

$$\mathrm{T}(\tau) = \psi\Upsilon + 1 + (T_{int} - \psi\Upsilon - 1)e^{-\frac{h_{rad}}{\rho C_p \delta \psi}\tau} \tag{14}$$

$$\psi = \frac{h_{rad}}{h_{rad} + h_{conv}}; \quad \Upsilon = \frac{q_e}{h_{rad}} + \overline{T}_{s.b} - 1;$$

Conclusions

This paper developed a physical model for an electrical infrared radiating panel, which is described in the form of mathematical equations, taking into account the boundary conditions of the third kind on the basis of complex heat exchange by radiation and convection, both with surrounding solid bodies and with passing airflow. The mathematical model for an infrared radiant heater was developed by using a lamp capacity method applied to a heater emitting layer, which takes into account the heat transfer between the heater surface, passing airflow and solid bodies.

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Model fizyko-matematyczny złożonej wymiany ciepła między elektrycznym promiennikiem podczerwieni a środowiskiem

Streszczenie: Jednym z głównych elementów systemów grzewczych jest urządzenie grzewcze, którego konstrukcja jest stale udoskonalana dzięki rozwojowi wiedzy w zakresie fizycznych i chemicznych właściwości materiałów, stosowanych technologii i sposobów produkcji. Artykuł poświęcony jest opracowaniu fizycznego modelu złożonej wymiany ciepła między elektrycznym promiennikiem podczerwieni, w którym zastosowano amorficzną metalową płytę promieniującą, umieszczoną na środku panelu. Opracowany model fizyczny opisano równaniami matematycznymi uwzględniającymi warunki brzegowe trzeciego stopnia przy założeniu złożonej wymiany ciepła przez promieniowanie i konwekcję zarówno z otaczającymi ciałami stałymi, jak i przepływającym powietrzem. Celem analizy jest określenie udziału promieniowania i konwekcji w wymianie ciepła, a także zależności między wymianą ciepła a warunkami komfortu cieplnego poszczególnych osób (uwzględniając przestrzeganie procesu technologicznego) oraz trybów pracy systemu grzewczego.

Słowa kluczowe: elektryczny radiacyjny panel grzewczy, przekazywanie ciepła w podczerwieni, system grzewczy, model fizyczny, metoda skupionej wydajności