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HYDRAULIC FRACTURING OPTIMIZATION FRAMEWORK BASED ON PKN AND CINCO–LEY METHODS**

1. INTRODUCTION

In an era of decreasing number of discoveries of conventional hydrocarbon reservoirs, in the global oil and gas industry we can observe growing interest in unconventional resources. Conducting effective production from this type of reservoirs is associated with carrying out the intensification processes of production, among which hydraulic fracturing is the most popular. Several technological parameters are crucial in case of process effectiveness. The shape of the fracture is difficult to predict due to the local inhomogeneity of the reservoir. In the development projects of unconventional resources, characterized by the presence of significant contrasts of reservoir parameters, the geometry of the fracture can be successfully described by simplified mathematical models, and evaluate the effectiveness of fracturing can be based on analytical methods [13]. Production intensification treatments significantly effect on the physical properties of the near well zone, thereby increasing the productivity of the well. The effectiveness of such treatments can be assessed by analyzing the basic production parameters and compare the production capacity before and after treatment. Methods of determination of the effectiveness of the stimulation treatment include analytical and numerical methods [14]. Numerical methods required high quality and quantity data, therefore simple analytical methods like Cinco–Lay equivalent skin calculation can be successful used. Until the 90s of the twentieth century, 2D models were used in modeling of hydraulic

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fracturing. Currently, specialists are increasingly using 3D models, which, due to its complexity, are more difficult to use. Therefore, the 2D models are still used, while being simple and in many cases a sufficient approximation of the actual geometry of the fracture [2]. In order to describe the geometry of the fracture, PKN model was used [9, 10]. This model assumes a constant fracture height, regardless of the distance from the axis of the wellbore. The fracture width is a function of the distance from the horizontal axis, and adopts an elliptical outline. The maximum opening of the fracture occurs at the injection site fracturing fluid. The fracture width in the section plane is proportional to the net pressure which is defined as the pressure in the fracture decreased by the value of the stresses directed perpendicularly to the plane of the fracture. This model is used in situations where the length of the wing of the fracture is much higher than its height [6]. This paper concerns the various optimization methods, which allows the selection of appropriate parameters of fracturing technology in terms of maximizing the net value of the project. Optimization is a very important engineering tool that allows to achieve better results of hydraulic fracturing at relatively low cost. Optimization algorithm was developed based on two-dimensional PKN model and analytical method for assessing the effectiveness of Cinco–Lay.

2. MATHEMATICAL MODEL OF HYDRAULIC FRACTURING EFFECTIVENESS

In case of high contrast of vertical stress in reservoir, fracture geometry can be successfully described by simplifying Perkins and Kern model. PKN model like other 2D's assumes constant fracture height in reservoir. The net pressure in fracture defined as difference between pressure in fracture and reservoir stress in perpendicular fracture direction and is given by [1]:

$$p_{net} = \left(x_f \cdot \frac{32 \cdot \mu \cdot Q_i \cdot E'^3}{\pi \cdot h_f^{\,4}} \right)^{\frac{1}{4}} \ [\mathrm{MPa}] \tag{1}$$

where:

- p_{net} – net pressure [MPa],
- x_f – length of the wing of fracture [m],
- Q_i – injection rate corresponding to one side of the fracture [m^3/s],
- E' – plane strain modulus [MPa],
- h_f – height of the fracture [m],
- μ – viscosity of fracturing fluid [Pa·s].

Maximum fracture width can be calculated in coupling with equation (1) and can be expressed as [3]:

$$w_{f,\max} = \frac{p_{net} \cdot h_f \cdot (1-\nu)}{G} \ [\text{m}] \tag{2}$$

where:

$w_{f,\max}$ – maximum width of the fracture [m],
ν – Poisson ratio [–],
G – shear modulus [MPa].

Due to elliptical shape of fracture cross section, average width can be estimated with following formula [13]:

$$w_f = \frac{\pi}{4} \cdot w_{f,\max} \ [\text{m}] \tag{3}$$

where w_f – average width of the fracture [m].

Equations (1)–(3) provide description of fracture shape, but from engineering point of view, flow properties are curtailed for treatment effectiveness.

Average concentration of proppant in fracture is expressed by ratio of injected proppant mass per simplified fracture area [14]:

$$C = \frac{M}{2 \cdot x_f \cdot h_f} \left[\frac{\text{kg}}{\text{m}^2}\right] \tag{4}$$

where:

C – concentration of proppant in fracture [kg/m^2],
M – proppant mass [kg].

Conductivity of selected proppant can be expressed as function of horizontal stress and proppant concentration, therefore [3]:

$$\omega = f(\sigma_h, C) \ [\text{mD} \cdot \text{m}] \tag{5}$$

where:

ω – conductivity [mD·m],
σ_h – effective horizontal stress [MPa].

Proppant conductivity is a proportional function of concentration. According to Sun J. et al. and Zhang J. et al [12, 15] proppant conductivity is given by:

$$\omega = \alpha(C) \cdot \varpi_b(\sigma_h, C_b) [\text{mD} \cdot \text{m}] \tag{6}$$

Coefficient of proportionality is equal to ratio between given concentration (C) and base concentration (C_b) which is used to determine proppant conductivity in horizontal stress range. Proppant conductivity curves for different concentration (C_b, $0.5C_b$ and $2C_b$) are presented in Figure 1.

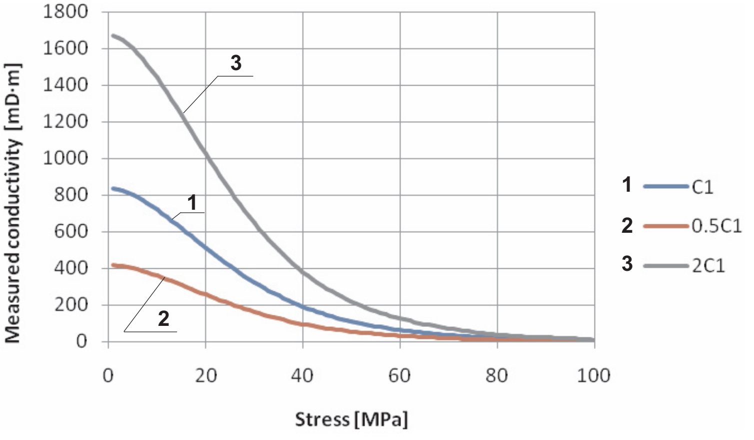


Fig. 1. Proppant conductivity vs. stress

The relationship between conductivity and permeability of proppant [3]:

$$k_f = \frac{\omega}{\overline{w}_f} \quad [\mathrm{mD}] \tag{7}$$

where k_f – permeability of the fracture [mD].

The permeability of the fracture allows to calculate the dimensionless fracture conductivity [3]:

$$c_{fD} = \frac{k_f \cdot \overline{w}_f}{k \cdot x_f} \quad [-] \tag{8}$$

where:

c_{fD} – dimensionless fracture conductivity [–],

k – permeability of the reservoir [mD].

Using the analytical methodology of Cinco–Lay, we can estimate the value of the equivalent skin effect after fracturing treatment using following formula [3]:

$$s_f = \ln\left(\frac{r_w}{x_f}\right) + \frac{1.65 - 0.328 \cdot u + 0.11 \cdot u^2}{1 + 0.18 \cdot u + 0.064 \cdot u^2 + 0.005 \cdot u^3} \quad [-] \tag{9}$$

where:

r_w – well radius [m],

s_f – equivalent skin effect [–].

$$u = \ln\left(c_{fD}\right) \; [-] \tag{10}$$

The forecast production after fracturing is calculated by [3]:

$$q_f = \frac{\left(p_z - p_{wf}\right) \cdot 2 \cdot \pi \cdot k \cdot h}{B \cdot \mu_R \cdot \left(\ln\left(\frac{0.472 \cdot r_e}{x_f}\right) + s_f + \ln\left(\frac{x_f}{r_w}\right)\right)} \left[\frac{\mathrm{m}^3}{\mathrm{s}}\right] \tag{11}$$

where:

q_f – production rate after fracturing treatment [m^3/s],

p_z – reservoir pressure [Pa],

p_{wf} – bottom hole pressure [Pa],

μ_R – oil viscosity [Pa·s],

h – reservoir thickness [m],

B – oil formation volume factor [–],

r_e – radius of the impact of the well [m].

In addition, before fracturing well performance is estimated using the equation for semi-steady flow [1]:

$$q_o = \frac{\left(p_z - p_{wf}\right) \cdot 2 \cdot \pi \cdot k \cdot h}{B \cdot \mu_R \cdot \left(\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right)} \left[\frac{\mathrm{m}^3}{\mathrm{s}}\right] \tag{12}$$

where q_o – production rate without fracturing treatment [m^3/s].

Revenue after year, can be expressed as difference in oil production rate between treatment well and no treatment case:

$$\text{Revenue} = \left(q_f - q_o\right) \cdot R_o \cdot t \;\; [\text{PLN}] \tag{13}$$

where:

R_o – the price of oil [PLN/m^3],

t – production time [1 yr].

Objective function is expressed as simple NPV defined as:

$$J = \text{Revenue} - C_{tr} \;\; [\text{PLN}] \tag{14}$$

where C_{tr} – total cost of fracturing treatment (proppant, liquid, base price).

Optimization problem involves maximization function J by changing decision variable: fracture half length (x_f) and proppant mass (M) with respect of specific contrast:

$$J^* = \max(\text{Revenue (u)} - C_{tr}(u)),\ u \in \Re^2 \tag{15}$$

$$u = \begin{bmatrix} x_f & M \end{bmatrix}^T \tag{16}$$

With respect to constraints of solution space:

$$x_f^{\min} \le x_f \le x_f^{\max} \tag{17}$$

$$M^{\min} \le M \le M^{\max} \tag{18}$$

3. OPTIMIZATION METHODS

Optimization method for presented engineering problem based on two population type algorithms, which mimic nature mechanism principles and concepts. First one bases on wolf pack haunting mechanism – grey wolf optimizer, and the second one mimic swarm intelligence – particle swarm optimization. Due to analytical form of relation between variables and objective function, gradient optimization is used. Gradient methods are a directional search, where direction of updating variables depends on objection function gradient with respect to variables. The main disadvantage of gradient algorithm is its local search aspect. In case of non-smooth objective function, gradient methods converge to nearest start point local optimum. Therefore global optimization methods like GWO or PSO are used to evaluate optimal fracture design parameters. All of proposed algorithms were implemented in Matlab software.

Grey wolf optimizer – GWO

GWO algorithm was developed by Mirjalili [7]) inspiring by grey wolves pack. Algorithm based on leadership hierarchy and hunting mechanism of wolves in nature. Similar to nature equivalent algorithm base on four types of possible solution:

– alpha – which is the best solution.
– beta and delta – the second and third best solution,
– gamma – the rest of possible solutions.

Possible solutions (alpha, beta, delta and gamma) change position in search space with very strict haunting rules, detailed in Muro [8]. Firstly tracking, chasing and approaching the prey. Secondly pursuit, encircling and harassing the prey until it stops moving and the last step is attack the prey. Detailed mathematical description of GWO algorithm was presented in origin paper [7].

Particle swarm optimization – PSO

The PSO algorithm is a based stochastic optimization procedure developed by Kennedy and Eberhardt [4]. The algorithm mimics the social behaviors exhibited by swarms of animals. In the PSO algorithm, a point in the research space is called a particle. The collection of particles in a given iteration is referred as the swarm. PSO algorithm is used for problems where objective function is non-smooth, discontinuous and non-differentiable with non-linearity related parameters [5]. Change of particle position in space of optimization problem is updated with respect of inertia, cognitive and social component. The inertia component provides a degree of continuity in particle velocity from one iteration to the next, while the cognitive component causes the particle move towards its own previous best position. The social component moves the particle towards the best particle in its neighborhood. These tree component perform different role in optimization. The inertia components enable a broad exploration of search space, while cognitive and social components narrow search towards the promising solution found up to the current iteration. The main advantages of PSO are: intensiveness to scaling design variables, simple implementation, ease parallelism, derivative free and efficient global search. Authors tested possibility of implementation PSO algorithm to solve complex engineering problem related to well placement and control to maximize CO_2 trapping, where detailed description of algorithm can be found [11].

Gradient optimization

Gradient optimization using in this paper based on Newton method which include hessian of objective function to improve solution. Gradient of objective function can be calculated analytically or numerically depending on problem type. Subject of numerical calculations need an additional function evaluation.

4. CASE STUDY

For case study following engineering problem was stated: The wellbore W-1 pierced sandstone layer located at a depth of 3100 m. The thickness of the hydrocarbon-bearing rock is 15 m. These sandstones have Poisson ratio equal to 0.27, the Young's modulus is 8.5 GPa and permeability 5mD. Average formation pressure is 24 MPa, and a bottom hole pressure 22 MPa. Skin effect before the treatment is equal to 10. The viscosity of the oil is 5 cP, and the volume ratio of oil 1.101. The radius of the impact of well is 1000 m, and the wellbore radius is 0.2 m. The company performing hydraulic fracturing has pumped storage units with a capacity of 0.2 m^3/s. During the procedure, it is planned to create the fracture with a height of 15 m. Leak off coefficient is equal 0.0002 $m/s^{0.5}$.

For population algorithm (GWO and PSO) 30 generation per 10 individual simulations were performed to converge. Stopping criteria for gradient algorithm were met when function gradient decreased to zero. Results of optimization in case of length of fracture wing and proppant mass are presented in table 1. The results of the optimization using GWO, PSO and gradient method are similar. The difference in the fracture wing length is about 3 cm. This value can be neglected, because the actual length of the fracture depends on the stress distribution in the reservoir and engineer is not able to estimate the fracture length with such precision. According to Table 1, GWO method yielded the greatest value of the fracture length: 175.5125. The values obtained by the PSO and gradient methods are almost identical (the difference below 1 cm). The similarity of these two methods is even more pronounced when we look at the values of the proppant weight. Both methods yielded results about 116 886 kg (difference below than 1 kg). The mass of proppant in the GWO method is equal to 11 7009.2 kg. The difference is noticeable – approximately 125 kg. Using the expert knowledge fracture would have 112 m of length and service would use 180 000 kg of proppant material. Changing of revenue, treatment cost and objective function value are presented in Figures 2–4, for GWO, PSO and gradient algorithm respectively.

Table 1

Optimal fracture design parameters and comparison with expert knowledge

	GWO	PSO	Gradient	Expert knowledge
x_f [m]	175.5125	175.4907	175.4895	112
M [kg]	117 009.2	116 885.4	116 886.2	180 000

During optimization using GWO and PSO methods 3000 calls were conducted. Referring to Figure 2 and 3, it can be observed, that after about 1500 calls revenue and treatment cost in PSO method are beginning to stabilize at a certain level. In the GWO method, substantially to the last calls fluctuations in values are observed – this is particularly evident looking at the first graph (Fig. 2). The PSO method is characterized by a smaller values dispersion during operation of the algorithm. Graphs for gradient method look differently. It should be note that in this method only 80 calls were used. After 10 calls value of the objective function started stabilization (Fig. 5). It is worth to mention, that on the graphs with treatment cost, value dispersion ranges from 0 to 25 million PLN (GWO and PSO optimization methods). Using gradient method we observed this dispersion from 1.5 to 6.1 million PLN.

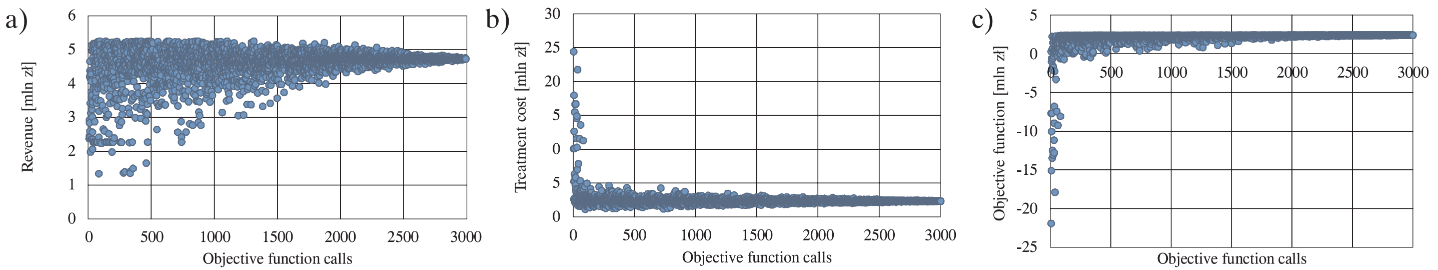


Fig. 2. Changing of revenue (a), treatment fracturing cost (b) and objective function for GWO algorithm (c)

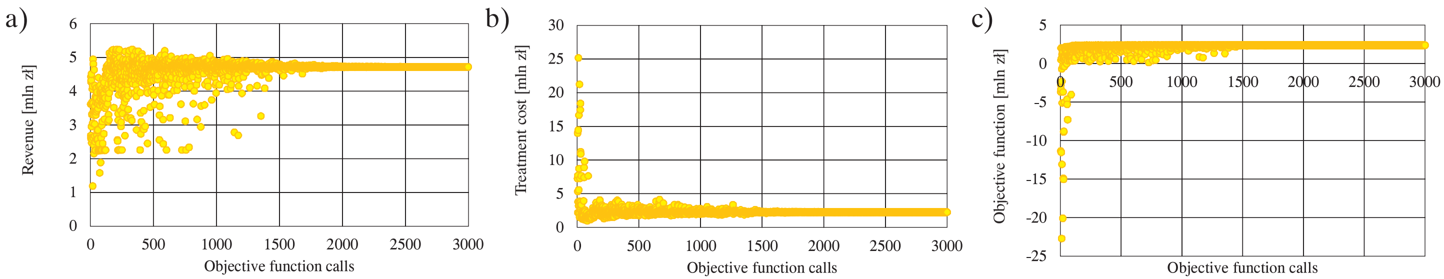


Fig. 3. Changing of revenue (a), treatment fracturing cost (b) and objective function for PSO algorithm (c)

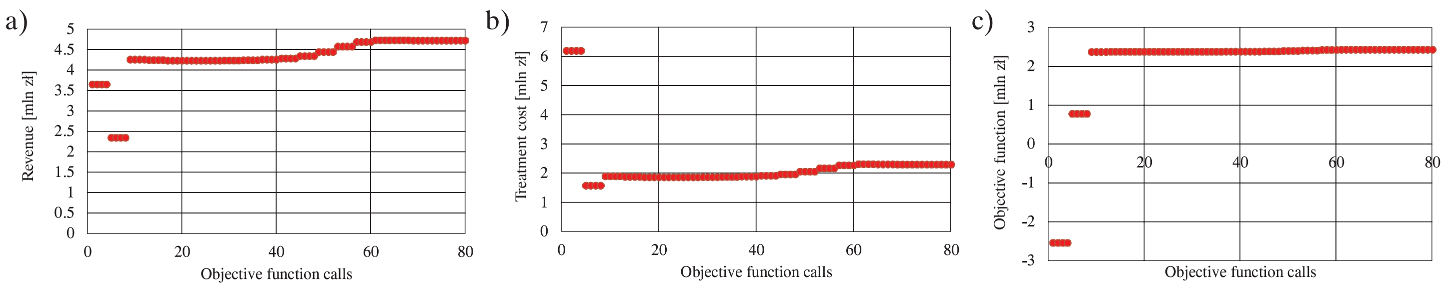
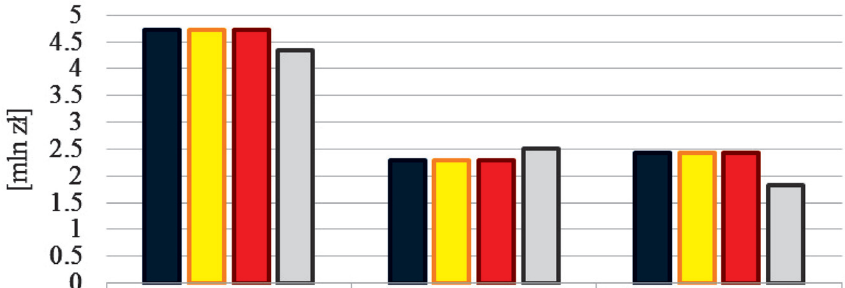


Fig. 4. Changing of revenue (a), treatment fracturing cost (b) and objective function for gradient algorithm (c)



	Revenue	Treatement cost	Objective function
GWO	4.7223	2.2965	2.4258
PSO	4.7213	2.2955	2.4258
Gradient	4.7212	2.2955	2.4258
Expert knowledge	4.3445	2.516	1.8285

Fig. 5. Results of revenue, treatment cost and objective function value for optimal fracturing design

5. CONCLUSIONS

Comparing the obtained results it can be stated with certainty, that used optimization algorithms are useful engineering tool with which company can quickly get additional profit. The results obtained with all of the three algorithms are very similar. The biggest revenue is observed using GWO method, which is related to the selection of the largest proppant weight by this algorithm (better fracture permeability). As is known, the greater weight of the proppant is, the greater is cost – and therefore the costs of the treatment are also the greatest in this method. The objective function for each algorithm has the final value 2.4258 million PLN. Profit obtained through the use of expert knowledge is almost 25% smaller and is equal 1.8285 million PLN. Using optimal fracture design parameters proposed by expert we obtain greater losses and lower profits which naturally results in the lower objective function value. The use of optimization methods allow to earn extra money with very little risk of failure.

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