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Mathematical Model of Particle Free Settling in a Vortex Apparatus

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1. Statement of the problem

In the production of thermal insulating materials, various heat and mass transfer apparatuses are widely used, in particular vortex ones, where the final stages of the technology are performed – drying or burning of fine particles (Pavlenko & Koshlak 2017, Pavlenko 2018). The scheme of such a device is shown in Fig. 1.

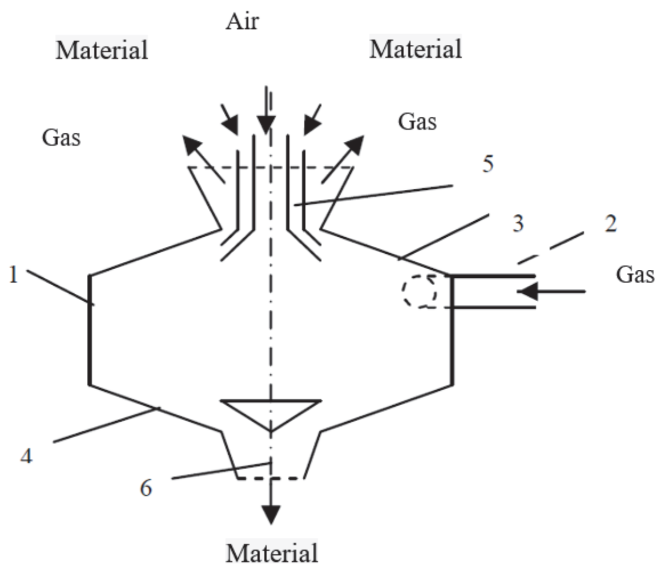

Fig. 1. Scheme of a vortex apparatus: 1 – body; 2 – lateral inlet pipe for gas supply; 3 – end upper wall; 4 – end bottom wall; 5 – feed hopper; 6 – discharge hopper

The vortex apparatus contains body 1 with lateral inlet pipe for heat transfer agent 2, connected tangentially to it, with end faces of the upper and lower walls 3 and 4, respectively, feed hopper 5 and discharge hopper 6.

Drying of materials in a vortex apparatus occurs during their free settling in the apparatus as a result of their interaction with air flows. Obviously, the longer the process, the more efficient it is.

2. Identification of previously unsettled parts of the general problem

A direct experimental study of the materials motion in vortex apparatuses is complicated by the process nonlinearity and non-stationarity. However, characteristics of the particle trajectories in the vortex apparatus can be determined by numerical modelling considering the available calculated velocity fields of the gas phase.

During drying, diameter of the particles changes, affecting the strength of their interaction with the gas phase. Available experimental data indicate an increase in the analysed particles diameter with temperature rise (Fig. 2). Thus, to determine trajectories of the introduced particles, it is necessary to calculate their temperature at the same time.

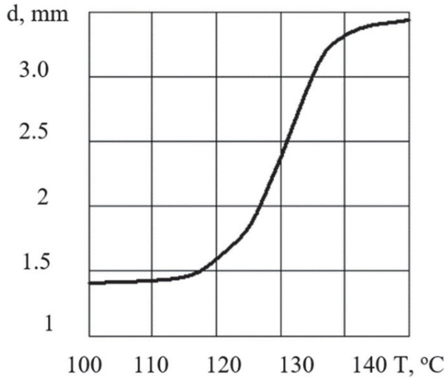

Fig. 2. Dependence of particle diameter d on temperature T

The efficiency of heat and mass transfer processes is largely determined by the ratio of geometric dimensions of this device, since hydrodynamic performance of the apparatus depends on them.

To rationalize technological parameters of the apparatus, as well as determine its efficiency, it is important to know the gas flow parameters (velocity and pressure components) in the entire volume of the apparatus.

Due to the specific apparatus design and gas supply, gas flows are substantially three-dimensional and largely perturbed. The experimental study of such flows is extremely difficult and requires large material costs.

On the other hand, an analytical solution to this problem is hardly possible without significant simplifications that can distort the entire process even at a qualitative level, therefore, in this paper we resorted to numerical modelling using the effective method of splitting by physical factors (Betyaev 2002). In this paper, this method, implemented in cylindrical coordinates in a three-dimensional formulation, is used to analyse the problem of gas dynamics study in a vortex apparatus.

3. Problem statement and methods of solution

In this paper, a complex mathematical model of the particle motion to be dried in a vortex apparatus with a simultaneous calculation of their temperature is developed.

Calculation procedure of the particle path under different conditions has been studied by many authors (see, for example: Betyaev 2002). In contrast to the specified works, in this paper the motion is calculated for the three-dimensional velocity field, obtained in the work (Mark 2011, Pavlenko & Koshlak 2019) for drying conditions in a vortex apparatus.

It is assumed that the particle falls vertically down with initial velocity v_o. When moving in the apparatus, a force acts on a particle:

$$\vec{F} = \vec{F_a} + \vec{F_s}, \tag{1}$$

consisting of buoyancy:

$$\vec{F_a} = (m - m^*)\vec{g}, \tag{2}$$

where *m* and *m** are particle mass, displaced air and resistance force:

$$\vec{F_s} = -CS\frac{\rho_r}{2}|v - v_r|(\vec{v} - \vec{v}_r), \tag{3}$$

where C is resistance coefficient, S is a unit cross section, ρ_r is its density, $\vec{v}, \vec{v}_r$ are particle and gas velocity respectively.

The equation of particle motion has the form:

$$\frac{d\vec{v}}{dt} = \vec{f}, \tag{4}$$

where $\vec{f} = \vec{F}/(m + km^*)$ and k are added-mass coefficients.

The process of a particle heating under the assumption of its sphericity is described by the one-dimensional equation of diffusive heat transfer:

$$\frac{\partial T}{\partial t} = a\frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{5}$$

where T is temperature, a is a particle thermal diffusivity, r is distance to its centre (radial coordinate).

Convective heat transfer takes place at the particle boundary:

$$q = \alpha\left(T_p - T_g\right), \tag{6}$$

determining the boundary conditions for equation (5), where q is heat flow density at the particle boundary, T_p and T_g are particle surface temperature and surrounding gas phase, respectively, and α is heat transfer coefficient.

Coefficient α depends on particle diameter d and it is convenient to express it in terms of the dimensionless Nusselt number Nu: $\alpha = Nu\,\lambda_e/d$, where λ_e is effective coefficient of gas thermal conductivity, considering the turbulent nature of motion. Its value is selected based on the ratio (Lobanov 2013, Koshlak & Pavlenko 2019): $\lambda_e = C\,\rho_g\,v_e$, where C and ρ_g are gas thermal conductivity and density, ν_e is effective kinematic viscosity coefficient, determined by a three-parameter algebraic turbulence model in gas-dynamic calculation.

We consider the particle motion in cylindrical coordinates ρ, φ, z. In this case, equation (4) takes the form:

$$\frac{\partial \nu_\rho}{\partial t} = \frac{\nu_\varphi^2}{\rho} + f_\rho, \tag{7}$$

$$\frac{\partial \nu_\varphi}{\partial t} = \frac{\nu_\rho\,\nu_\varphi}{\rho} + f_\varphi, \tag{8}$$

$$\frac{\partial \nu_z}{\partial t} = f_z, \tag{9}$$

where the index of velocity and specific gravity means their component in cylindrical coordinates. For velocity components we have:

$$\nu_\rho = \frac{d\rho}{dt}, \nu_\varphi = \rho\frac{d\varphi}{dt}, \nu_z = \frac{dz}{dt}. \tag{10}$$

Numerically, system of equations (7)-(10) was solved using Euler-Cromer method,

$$\nu_\rho^{n+1} = \nu_\rho^n + \Delta t\left[\frac{\left(\nu_\varphi^n\right)^2}{\rho^n} + f_\rho^n\right],$$

$$\nu_\varphi^{n+1} = \nu_\varphi^n + \Delta t\left[\frac{-\nu_\rho^n\nu_\varphi^n}{\rho^n} + f_\varphi^n\right],$$

$$\nu_z^{n+1} = \nu_z^n + \Delta t f_z^n,$$

$$\rho^{n+1} = \rho^n + \Delta t \nu_\rho^{n+1},$$

$$\varphi^{n+1} = \varphi^n + \Delta t\left(\nu_\varphi^{n+1}/\rho^{n+1}\right),$$

$$z^{n+1} = z^n + \Delta t \nu_z^{n+1},$$

where n is sacrificial layer number, and Δt is time increment.

The inner part of the cylinder was chosen as the calculation region. When determining resistance force (3) of the unit motion in a gas flow, causing its helical motion, its cross-sectional area is preliminarily calculated $S = \pi d^2/4$ using graphical dependency defined in Fig. 1. For this, the particle temperature is calculated using an explicit difference scheme:

$$T_i^{n+1} = T_i^n + \frac{\Delta t a[(i-1)T_{i+1} - 2(i-1.5)T_i + (i-2)T_{i-1}]}{\Delta r^2}, \quad (11)$$

where Δt and Δr are time and radius increments, respectively, n is sacrificial layer number, i is spatial cell number (temperatures are calculated in the centre of cells). As the temperature, determining the particle diameter, average temperature for all cells is selected.

4. Results and discussion

A series of test calculations of the model was performed, indicating its qualitative adequacy to the analysed process (Fig. 3-5).

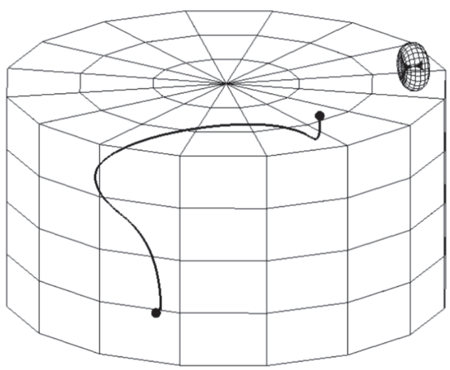

Fig. 3. Helicoidal particle path

The specific particle path depends considerably on the specific spot of its penetration into the vortex apparatus and is determined mainly (in addition to gravity) by vortex gas flows. So, being carried away by gas, a particle can make a helical motion (Fig. 3) until it leaves the apparatus.

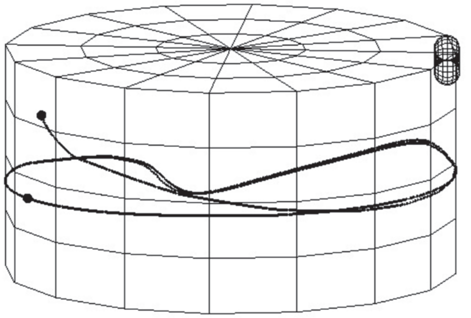

Fig. 4. Part of the quasi-stationary particle free settling path

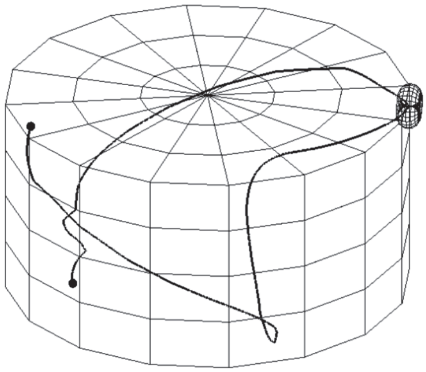

Fig. 5. Complex irregular particle path

However, numerical experiments show a rather irregular nature of particle motion. So, in the calculations, we observed both paths with stationary free settling sections (Fig. 4), as well as quite interesting variants of possible particle motions, such as, for example, as the variant, presented in Fig. 5. A particle under the action of gravity and gas flows can generally move down, and then, falling into ascending flows, again move upwards for a certain time. Finally, it leaves the apparatus.

The study of a large number of calculation results allows, still, (despite the irregularity of particle paths) to draw a qualitative conclusion that, in general, particles that fall into the vortex apparatus closer to the side wall are free settling for a longer period, and, consequently, they are dried longer, which is preferable from a technological viewpoint.

5. Conclusions

A complex mathematical model of heating and three-dimensional particle flow in a vortex apparatus was developed.

Analysing the obtained results, we can conclude that duration of the material particle heat treatment until it is completed, as shown in the figures above, can be different and depends, mainly, on the intensity of particle sweep with the heat transfer agent flow. In general, the obtained calculated information may serve as the basis for construction and optimization of the apparatus design from the viewpoint of energy usage reduction. The path configuration and its length determine overall dimensions of the apparatus and flow characteristics of the heat transfer agent. Depending on the required heat treatment intensity (and heat treatment intensity, as shown in the second section, determines the thermophysical properties of the finished product), the point of particle introduction and heat transfer agent velocity may vary.

Test calculations carried, performed according to the presented model, testify to its qualitative adequacy and possible use of this model to calculate various modes of material drying in vortex apparatuses.

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Abstract

A mathematical model of gas dynamics in a vortex apparatus during heat treatment is presented in the paper. The parameters of gas flows in the vortex apparatus, optimal ratios of the vortex apparatus geometric dimensions, as well as hydrodynamic parameters are determined, making it possible to develop effective design solutions of this equipment.

The mathematical model allows one to carry out computational experiments and determine particle trajectories, their temperature, particle size and humidity at various points in time and evaluate the dynamics of these parameters.

Using the calculation experiment method makes it possible to quickly and without financial costs determine the technological modes of heat treatment of dispersed material in vortex devices.

The obtained data can be used in calculation methods of heat and mass transfer vortex apparatuses

Keywords:
mathematical modelling, drying of particulates, vortex layer

Model matematyczny procesu swobodnego osadzania cząstek w aparacie wirowym

Streszczenie

W artykule przedstawiono matematyczny model dynamiki gazu w aparacie wirowym podczas obróbki cieplnej zdyspergowanych materiałów. Określane są parametry przepływów gazu w aparacie wirowym, identyfikowane są optymalne proporcje wymiarów geometrycznych aparatu wirowego, a także parametry hydrodynamiczne, które pozwalają opracowywać skuteczne rozwiązania konstrukcyjne tego sprzętu.

Model matematyczny umożliwia przeprowadzanie eksperymentów obliczeniowych i określanie trajektorii cząstek, ich temperatury, wielkości cząstek i wilgotności w różnych punktach czasowych oraz ocenić dynamikę zmian tych parametrów.

Zastosowanie metody eksperymentu obliczeniowego pozwala szybko i bez kosztów finansowych określić warunki technologiczne do obróbki termicznej rozproszonego materiału w urządzeniach wirowych o różnych konstrukcjach.

Uzyskane dane można wykorzystać w metodach obliczania urządzeń wirowych z wymianą ciepła i masy.

Słowa kluczowe:
modelowanie matematyczne, suszenie cząstek, warstwa wirowa