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RECONSTRUCTION OF BOUNDARY CONDITION OF THE THIRD KIND BY APPLYING THE ANT COLONY OPTIMIZATION ALGORITHM

Summary. In this paper we present an application of the Ant Colony Optimization algorithm for solving the inverse heat conduction problem in which the state function and some of the boundary conditions should be determined. The ACO algorithm is a part of the swarm intelligence and it is inspired by the technique of searching for the shortest way connecting the ant-hill with the source of food. We propose to use this algorithm for minimizing the proper functional, which plays a crucial role in the method of solution and allows to reconstruct the value of heat transfer coefficient.

ODTWORZENIE WARUNKU BRZEGOWEGO TRZECIEGO RODZAJU PRZY ZASTOSOWANIU ALGORYTMU MRÓWKOWEGO

Streszczenie. W niniejszym artykule przedstawione zostało zastosowanie algorytmu mrówkowego do rozwiązania odwrotnego zagadnienia przewodnictwa ciepła, polegającego na wyznaczeniu funkcji stanu oraz rekonstrukcji jednego z warunków brzegowych. Algorytm mrówkowy należy do grupy algorytmów inteligencji roju i zainspirowany został techniką wyszukiwania najkrótszej drogi łączącej mrowisko ze źródłem pożywienia. W pro-

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ponowanym podejściu algorytm ten zostanie wykorzystany do wyznaczania minimum funkcjonału będącego istotnym elementem metody rozwiązania, umożliwiającym odtworzenie wartości współczynnika wnikania ciepła.

1. Introduction

In recent times, the new tools for solving optimization problems appeared, called the swarm intelligence algorithms. Algorithms of this kind belong to the group of artificial intelligence and they represent the special way of solving various problems in which the intelligent behavior results from the cooperation of many simple individuals. Single individuals are not aware of the complete problem, which should be solved, but big number of them and specific forms of their behavior cause the common success and, in result, lead to find the solution. In 1992, M.Dorigo introduced an algorithm, known as the Ant System (AS), useful for solving combinatorial optimization problems and imitating the behavior of ants, which evaluated in the algorithm known as the Ant Colony Optimization algorithm (ACO). The basic idea of this algorithm is inspired by the real ants which explore the environment for looking the best path leading to the source of food. During this search, they lay down a chemical substance, called as pheromone, directing each other ant to the best path. Artificial "ants" locate optimal solutions by moving through a parameter space, representing all the possible solutions, and in every iteration they record their positions and improve the quality of their solutions with the aid of special procedure imitating formation of the pheromone trail, so that in further iterations more ants can locate better solutions [4]. In this paper we want to present the idea of using the ACO algorithm for solving the inverse heat conduction problem. In order to minimize the proper functional we will use the ACO algorithm for finding the global minimum based on the approach proposed in [5].

2. Formulation of the problem

The inverse heat conduction is a problem with the incomplete mathematical description and it consists in determination of the function describing the distribution of temperature and some of the boundary conditions [1,2]. The inverse problem is much more difficult for solving than the direct heat conduction problem in which the initial and boundary conditions are known, only the temperature needs to be found. However, some methods for solving the inverse problem are proposed, like for example in papers [3,6,9–12]. In papers [7,8] the authors have applied the ACO and ABC algorithms, respectively, for minimizing a functional, being an important part of the proposed method of solving the inverse heat conduction problem with reconstructing the boundary condition of the second kind. In the current paper we consider the problem described by the following heat conduction equation:

$$c\,\rho\frac{\partial u}{\partial t}(x,t) = \lambda\frac{\partial^2 u}{\partial x^2}(x,t), \qquad x \in [0,d], \;\; t \in [0,T] \tag{1}$$

with the initial and boundary conditions of the form

$$u(x,0) = u_0, \qquad\qquad x \in [0,d], \tag{2}$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \qquad\qquad t \in [0,T], \tag{3}$$

where c is the specific heat, ρ denotes the mass density, λ is the thermal conductivity and u, t and x refer to the temperature, time and spatial location. On the boundary for $x = d$ we assume the boundary condition of the third kind

$$-\lambda\frac{\partial u}{\partial x}(d,t) = \alpha\,(u(d,t) + u_\infty), \qquad t \in [0,T], \tag{4}$$

where u_∞ describes the temperature of environment and α denotes the heat transfer coefficient, values of which is desired to be found. Another sought element is the distribution of temperature $u(x,t)$ in the considered region. If we set the value α of heat transfer coefficient as known, the problem, defined by equations (1)-(4), turns into the direct problem, solving of which gives the values of temperature $u(x_i,t_j)$ in the nodes of the mesh.

In approach presented in this paper we propose to solve the direct heat conduction problem, described by equations (1)-(4), by taking the value of heat transfer coefficient as an unknown parameter α. Solution $u(d,t_j)$, received in this way, depends on the parameter α. Next, we determine the value of α by minimizing the following functional:

$$P(\alpha) = \sqrt{\sum_{j=1}^{m}\big(u(d,t_j) - \tilde{u}(d,t_j)\big)^2}, \tag{5}$$

representing the differences between the obtained results u and given values $\tilde{u}$ on the boundary for $x = d$, where the boundary condition is reconstructed. For minimizing the functional (5) we use the ACO algorithm.

3. Ant Colony Optimization algorithm

As we have mentioned before, answer to the questions how the almost blind ants can find the best way from the ant-hill to the source of food, how they communicate and what makes them to follow one after another gives the pheromone. Ants are not endowed with intelligence but they leave the pheromone trace in the ground which is the information for the other ants where to go and for the ant itself how to return to the ant-hill. The more ants traverse the trail, the stronger is the pheromone trace. The shorter is the way, the sooner can the ant reach the source of food and return to the ant-hill, which makes the pheromone trace stronger and force the other ants to choose this specific way. On the contrary, pheromone trails located far from the food source, not attended by the ants, evaporate.

Analogically like in nature, the role of ants is played by vectors $\mathbf{x}^k$, randomly dispersed in the searching region. In each step, one of the ants is selected as the best one, $\mathbf{x}^{best}$ - the one, for which the minimized function $F(\mathbf{x})$ takes the lowest value. In the next step, to each vector $\mathbf{x}^k$ is applied a modification, based on the pheromone trail. Vector of each ant is updated at the beginning of each iteration, by using the following formula: $\mathbf{x}^k = \mathbf{x}^{best} + \mathbf{dx}$, where $\mathbf{dx}$ is a vector determining the length of jump, elements of which are randomly generated from the interval $[-\beta, \beta]$ (where $\beta = \beta_0$ is a parameter of narrowing, defined in the initialization of the algorithm). At the end of each iteration the range of ants dislocations is decreasing, according to the formula $\beta_{t+1} = 0.1\beta_t$. This procedure simulates the evaporation of the pheromone trail in nature. The role of food source in our simulation is played by the point of the lowest function value, that is why the presence of ants – vectors is condensing around this point. The described procedure is iterated until the maximal number of iterations and, in general, runs as follows.

Initialization of the algorithm

1. Initial data:
   $F(\mathbf{x})$ – minimized function, $\mathbf{x} = (x_1, ..., x_n) \in D$;
   n^M – number of ants in one population;
   I – number of iterations;
   β – narrowing parameter.
2. Random selection of the initial ants localization: $\mathbf{x}^k = (x_1^k, ..., x_n^k)$, where $\mathbf{x}^k \in D$, $k = 1, 2, ..., n^M$.

3. Determination of the best located ant $\mathbf{x}^{best}$ in the initial ants population.

The main algorithm

1. Updating of the ants locations:
– random selection of the vector $\mathbf{dx}$ such that

$$-\beta_i \leqslant dx_j \leqslant \beta_i;$$

– generation of the new ants population:

$$\mathbf{x}^k = \mathbf{x}^{best} + \mathbf{dx}, \qquad k = 1, 2, ..., n^M.$$

2. Determination of the best located ant $\mathbf{x}^{best}$ in the current ant population.
3. Points 1 and 2 are repeated I^2 times.
4. Narrowing of the ants dislocations range: $\beta_{i+1} = 0.1\beta_i$.
5. Points 1 – 4 are repeated I times.

4. Experimental results

The theoretical consideration will be illustrated by the example, in which we take $c = 1000\,[\mathrm{J/(kg\cdot\ K)}]$, $\rho = 2679\,[\mathrm{kg/m^3}]$, $\lambda = 240\ \ [\mathrm{W/(m\cdot\ K)}]$, $T = 1000\,\mathrm{s}$, $d = 1\,\mathrm{m}$, $u_0 = 1013\,\mathrm{K}$ and $u_\infty = 298\,\mathrm{K}$. Exact value of the sought parameter is the following: $\alpha = 28\,[\mathrm{W/(m^2\cdot\ K)}]$. For constructing the functional (5) we use the exact values of temperature, determined for the given α, and values noised by the random error of 1, 2 and 5%, located on the boundary for $x = 1$ with the step of 1 s. The initial population of ants seeking the best localization of the sought parameter is randomly selected from the range $[0, 500]$. We evaluate the experiment for number of ants $n^M = 10$ and number of iterations $I = 10$. The initial value of the narrowing parameter is $\beta_0 = 300$. The approximate values of reconstructed parameter is received by running the algorithm 20 times and by averaging the obtained results.

For the exact values of temperature, even for the small number of ants and iterations, we receive the exact solution which confirms correctness of the algorithm.

In Figure 1 the comparison between the exact values of temperature on boundary for $x = 1$, where the boundary condition of the third kind is reconstructed, and its approximated values calculated for input data perturbed by the error of 1% is presented. Distribution of the relative error for this reconstruction is displayed in Figure 2. Maximal relative error of the temperature distribution in the entire domain is equal to 0.02%, whereas the relative error of the heat transfer coefficient reconstructed for input data burdened by the error of 1% is equal to 0.745%.

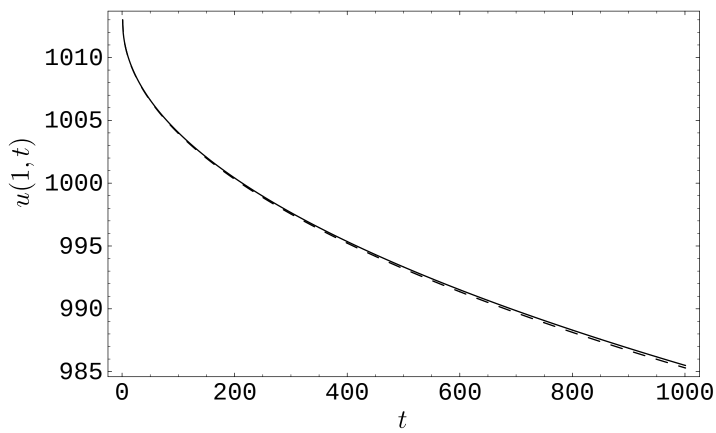

Fig. 1. Distribution of temperature $u(x, t)$ for $x = 1$ (solid line – exact solution, dashed line – approximated values obtained for input data noised by the error of 1%)
Rys. 1. Rozkład temperatury $u(x, t)$ na brzegu $x = 1$ (linia ciągła – wartości dokładne, linia przerywana – wartości odtworzone dla danych wejściowych zaburzonych 1% błędem)

Similarly, Figures 3 and 5 show the approximated values of temperature on boundary for $x = 1$ calculated for input data perturbed by the error of 2% and 5%, respectively, combined with the exact values. Distributions of the relative error for these reconstructions are presented in Figures 4 and 6, respectively. Relative error of the temperature reconstruction in the entire domain does not exceed the value of 0.021% for the 2% noise of input data and 0.03% for the case of 5% noise. Furthermore, the relative error of the heat transfer coefficient reconstructed for input data perturbed by the error of 2% is equal to 0.76%, whereas for the 5% perturbation this error does not exceed the value of 1.103%. Thus, we can notice that the errors of approximation increase as the error of input data grows, but much more slower.

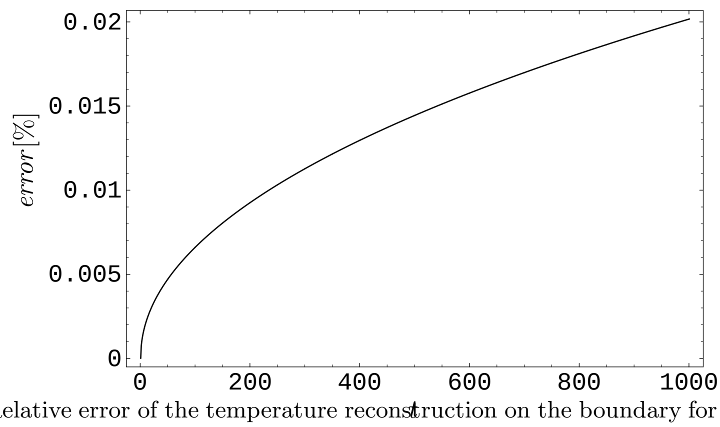

Fig. 2. Relative error of the temperature reconstruction on the boundary for input data noised by the error of 1%

Rys. 2. Błąd względny odtworzenia temperatury na brzegu obszaru dla danych wejściowych zaburzonych 1% błędem

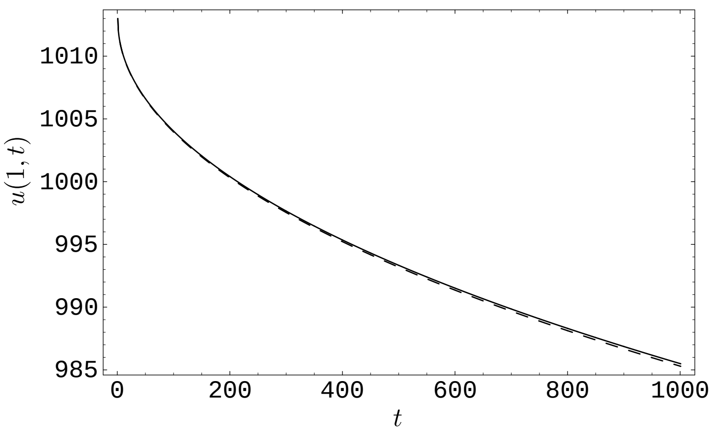

Fig. 3. Distribution of temperature $u(x,t)$ for $x = 1$ (solid line – exact solution, dashed line – approximated values obtained for input data noised by the error of 2%)

Rys. 3. Rozkład temperatury $u(x,t)$ na brzegu $x = 1$ (linia ciągła – wartości dokładne, linia przerywana – wartości odtworzone dla danych wejściowych zaburzonych 2% błędem)

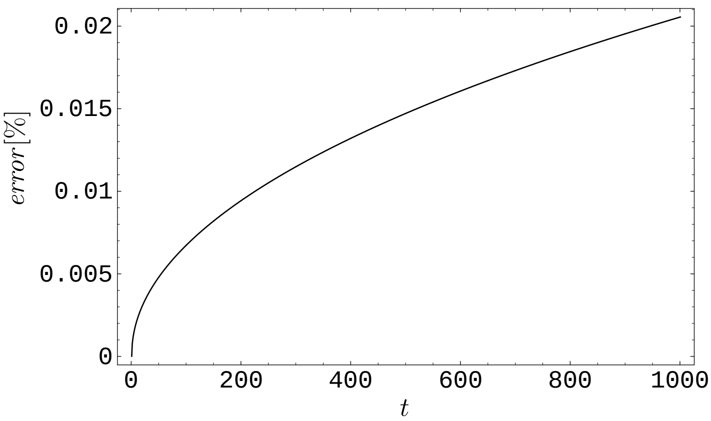

Fig. 4. Relative error of the temperature reconstruction on the boundary for input data noised by the error of 2%

Rys. 4. Błąd względny odtworzenia temperatury na brzegu obszaru dla danych wejściowych zaburzonych 2% błędem

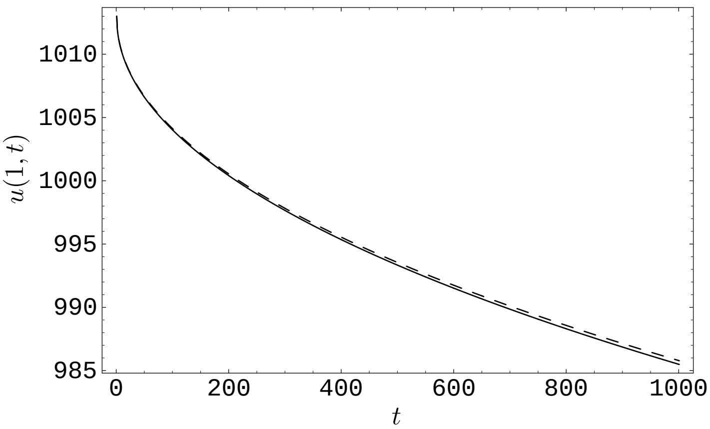

Fig. 5. Distribution of temperature $u(x,t)$ for $x = 1$ (solid line – exact solution, dashed line – approximated values obtained for input data noised by the error of 5%)

Rys. 5. Rozkład temperatury $u(x,t)$ na brzegu $x = 1$ (linia ciągła – wartości dokładne, linia przerywana – wartości odtworzone dla danych wejściowych zaburzonych 5% błędem)

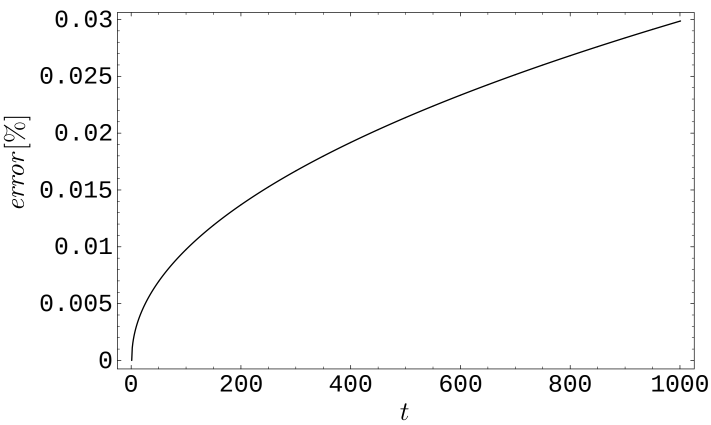

Fig. 6. Relative error of the temperature reconstruction on the boundary for input data noised by the error of 5%

Rys. 6. Błąd względny odtworzenia temperatury na brzegu obszaru dla danych wejściowych zaburzonych 5% błędem

5. Conclusions

The paper contains a proposal of the method for solving the inverse heat conduction problem with the boundary condition of the third kind. Important part of this procedure is the minimization of the appropriate functional, realized with the aid of Ant Colony Optimization algorithm. Results received for the example in which we reconstructed the value of heat transfer coefficient appearing in the boundary condition of the third kind are satisfying for the input data noised by the error of different values, for relatively small number of ants in the algorithm and for relatively small number of iterations. For the exact data the methods gives the exact results, whereas the errors of approximations obtained by means the proposed procedure increase significantly more slowly than the error of input data which shows the stability of presented method.

The advantages of using approach with the ACO algorithm are at least two – the time needed for finding the global solution is respectively short and the only assumption needed by the algorithm is the existence of solution. If the solution of the optimized problem exists, it will be found, with some given precision of course. Additionally, it is worth to mention that solution received by using the algorithm should be treated as the best solution in the given moment. Running the algorithm again can give different solution, even better. But it does not decrease its effectiveness.

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Omówienie

Niniejszy artykuł przedstawia propozycję metody rozwiązywania odwrotnego zagadnienia przewodnictwa ciepła z warunkiem brzegowym trzeciego rodzaju, wykorzystującej algorytm mrówkowy. Algorytm ten należy do grupy algorytmów optymalizacyjnych, inspirowanych inteligencją roju i naśladujących zachowania rzeczywistych kolonii, w tym przypadku mrówek. Wyniki otrzymane dla rozważanego zadania odtworzenia współczynnika wnikania ciepła, występującego w warunku brzegowym trzeciego rodzaju, okazały się zadowalające dla różnych wartości zakłóceń danych wejściowych, dla stosunkowo małej liczby mrówek oraz niewielkiej liczby iteracji. Dla niezakłóconych danych wejściowych otrzymano rozwiązanie dokładne, natomiast błędy uzyskanych przybliżeń rosną znacznie wolniej niż błędy danych wejściowych, co świadczy o stabilności metody. Korzyści wynikające z zastosowania algorytmu mrówkowego są co najmniej dwie – czas działania algorytmu jest stosunkowo krótki, a jedynym założeniem wymaganym przez algorytm jest istnienie rozwiązania. Niezależnie od rodzaju optymalizowanego problemu, rozwiązanie, jeśli tylko istnieje, zostanie znalezione, z większą lub mniejszą precyzją. Warto przy tym wspomnieć, że otrzymane rozwiązanie jest najlepsze w danym momencie, co jednak nie umniejsza skuteczności metody.