Vasyl ANDRUNYK[1], **Petro MALACHIVSKYY**[2], **Yaropolk PIZYUR**[1], **Vasyl YATSUK**[1]
[1] "LVIV POLYTECHNIC" NATIONAL UNIVERSITY, L'VIV, UKRAINE
[2] CENTRE OF MATHEMATICAL MODELING OF IAPMM NAMED AFTER YA.S.PIDSTRYHACH, NAS UKRAINE, L'VIV

Continuous and smooth minimax spline-approximation of sensor temperature characteristic and its sensitivity

Master Vasyl ANDRUNYK

He graduated from Ukrainian Printing Academy. His master thesis theme is "Computer-Integration Technological Processes and Manufacture in Printing Industry". Since 2005 he has worked in „Lviv Polytechnic" National University at the department of Information Systems and Networks. He is involved in a scientific work and the dissertation theme is a new approach to spline – approximation.

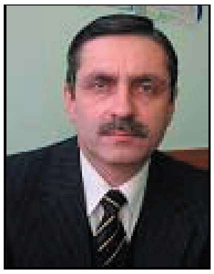
e-mail: aprox@complex.lviv.ua

PhD Petro MALACHIVSKYY

Senior research worker of the Centre of Mathematical Modeling of IAPMM named after Ya.S.Pidstryhach, Ukrainian National Academy of Sciences; compatibly the Associate Professor of the Automation and Computer Technology Department of Ukrainian Printing Academy where he lectures the following subjects: Mathematical Modeling and Instrumental Program Tools. His expertise area includes digital methods and applied software. The author of over 80 scientific works and several handbooks.

e-mail: psmal@cmm.lvi.ua

PhD Yaropolk PIZYUR



Scientific degree – PhD in physics and mathematics, title – Associated Professor, position - Associated Professor at the department of Applied Mathematics, "Lviv Polytechnic" National University. Scientific interests – Hermitian and Chebyshev spline approximation of functions. Near 40 scientific publications.

e-mail:pizyur@yahoo.com

Prof. Vasyl YATSUK

Full Professor of the Metrology, Standardization and Certification Department of "L'viv Polytechnic" National University. His expertise area is the measuring of electric and nonelectric values. The author of over 120 scientific works and the co-author of the textbook "Metrology and Measuring Technics". He has a row of inventions partly implemented in the serial digital tools A565, A566, ЦР7701 and ЦР7702 that are manufactured by the open joint-stock company "Mukachivprylad".

e-mail: vyatsuk@polynet.lviv.ua

Abstract

The problem under consideration is the construction of continuous and smooth spline-approximation whose any link is polynomial approximation according to a minimax criterion. The algorithm of such a spline-approximation with a priory given error is proposed. The practical sample of its application for the description of a temperature characteristic and sensitivity of cryogen thermodiode sensor is described.

Key words: spline-approximation, minimax (Chebyshev) polynom

Ciągła i gładka aproksymacja minimaksowa funkcjami sklejanymi charakterystyki czujnika temperatury

Streszczenie

Zbadano własności ciągłej i zarazem gładkiej aproksymacji funkcjami sklejanymi, w których wszystkie ogniwa są wielomianami. Zaproponowano algorytm konstrukcji takiego przybliżenia realizującego aproksymację z założoną dokładnością. Algorytm ten zastosowano do przybliżenia charakterystyki temperaturowej czujnika diodowego pracującego w obszarze temperatur kriogenicznych.

Słowa kluczowe: aproksymacja funkcjami sklejanymi

1. Introduction

To describe sensor static characteristics, rupture minimax polynomial splines are used frequently [1]. The approximations on distinct links are selected in such a way that a characteristic approximation error does not exceed an a'priori given value in any range point. The application expediency of such splines with dividing minimax approximation on the distinct parts of a measurement range could be justified by the possibility of necessary approximation accuracy attaining at the small values of parameters' quantity in polynomial approximation.

Such approximations continuous on distinct links (the parts of the measurement range) could be used if a determining condition is the provision of function value reproducing with some error only. The example of such a task is the determination of a functional dependence for the description of thermotransducer static characteristics [2-4]. However, to explore sensor sensitivity, these rupture approximations could not be employed, since derivative values in rupture points have considerable discrepancies [5, 6]. To approximate a thermometric characteristics of silicon diode temperature sensors and their sensitivity, Chebyshev's polynomial approximations by the method of the least squares are used in the work [5]. Meanwhile, the satisfactory accuracy of sensor thermometric characteristic and its sensitivity approximation is being reached at hundredth and higher Chebyshev's polynomial degrees that complicates the practical implementations of such approximations due to pulsations relevant for high degrees.

The task of sensor static characteristics and its sensitivity reproducibility is narrowed to the construction of continuous and smooth spline-approximation. To reach an optimum accuracy, we should apply minimax spline-approximation that provides approximation with the smallest possible error at the given polynomial degree [1, 7].

The construction of smooth rupture spline-approximation by third degree polynomials with the usage of a minimax criterion is described in the work [7], although, the theoretical stipulation of such a construction is missing. The method of selection of such minimax polynomial approximations on distinct spline links that an approximation error is not over an a'priori given value within whole approximation range. Moreover, the spline constructed from them as well as its derivative as continuous functions is suggested in this article.

2. Determination and properties of continuous and smooth minimax spline-approximation with a given error

Let $f(x)$ be some continuous and differentiated function within the range $[\alpha, \beta]$ $\left(f(x) \in C^{(1)}[\alpha, \beta]\right)$, $P_m(a; x)$ – the polynomial of the degree m $(m \geq 3)$

$$P_m(a; x) = \sum_{i=0}^{m} a_i x^i \qquad (1)$$

and $w(x)$ – the weighting function continuous on $[\alpha, \beta]$, such that does not attain the zero value. It is expediently to approximate the function *f(x)* within the range $[\alpha, \beta]$ with the error G_0 by a continuous and smooth minimax spline.

$$S(x) = \sum_{j=1}^{q} P_m\left(a^{(j)};\, x\right) \Theta\left(\left(x - t_j\right)\left(t_{j+1} - x\right)\right), \qquad (2)$$

here $\Theta(u)$ – Hevisides function

$$\Theta(u) = \begin{cases} 1, & if \quad \mathrm{u} \geq 0 \\ 0, & if \quad \mathrm{u} < 0 \end{cases}.$$

The points t_j $\left(j = \overline{1,\, q+1}\right)$ of the spline (2) are spline knots, among which $t_1 = \alpha$, $t_{q+1} = \beta$ and $\left[t_{\mathrm{j}},\, t_{\mathrm{j+1}}\right]$, $j = \overline{1,\, q}$ – spline links where spline values are given by polynomials $P_m\left(a^{(j)};\, x\right)$, respectively.

In this spline anyone of the polynomials $P_m\left(a^{(j)};\, x\right)$, $j = \overline{1,\, q}$ is the minimax approximation of the function $f(x)$ on the interval $\left[t_{\mathrm{j}},\, t_{\mathrm{j+1}}\right]$ with the weight $w(x)$

$$\max_{t_j \leq x \leq t_{j+1}} \left| \frac{f(x) - P_m\left(a^{(j)}; x\right)}{w(x)} \right| = \min_{a} \max_{t_j \leq x \leq t_{j+1}} \left| \frac{f(x) - P_m(a; x)}{w(x)} \right|, \qquad (3)$$

and due to spline and its derivative continuity the value of these polynomials as well as their derivatives in the inner knots t_j, $j = \overline{2,\, q}$ coincide

$$P_m\left(a^{(j-1)};\, t_j\right) = P_m\left(a^{(j)};\, t_j\right), \qquad (4)$$

$$P'_m\left(a^{(j-1)};\, t_j\right) = P'_m\left(a^{(j)};\, t_j\right). \qquad (5)$$

In addition, if G_j – the error value of approximation of the function $f(x)$ with the weight $w(x)$ on the *j-th* spline link

$$G_j = \max_{t_j \leq x \leq t_{j+1}} \left| \left(f(x) - P_m\left(a^{(j)}; x\right)\right) / w(x) \right|, \qquad (6)$$

then

$$\max_{1 \leq j \leq q} G_j \leq G_0, \qquad (7)$$

here G_0 – the given error of spline-approximation.

The determination task of spline-approximation $S(x)$ (2) with the mentioned above properties on the interval $[\alpha, \beta]$ for the function $f(x)$ roots in the provision of approximation given error G_0 reaching at the least possible link quantity. This task solution is restricted to such a choice of link limits – the knots t_j, $j = \overline{2,\, q}$ of spline-approximation (2) when all links' lengths maybe except the last one are maximally admissible for the given error of approximation G_0.

Spline-approximation that provides some approximation error at the smallest possible link quantity according to the mentioned conditions is called balance spline-approximation with the error G_0 [1]. In practice, balance spline-approximations with the weighting functions $w(x) = 1$ and $w(x) = f(x)$ are used most frequently. In case, when the weighting function is equal to 1 we gain approximation with an absolute error, and in case, $w(x) = f(x)$ – with a relative error.

To construct spline-approximation (2), we could employ the algorithms of balance minimax spline-approximation construction, which is described in the works [1, 10], with the prescription of following of mentioned below conditions (3)-(5) concerning the smoothness and continuity of a spline.

3. Algorithm of the construction of continuous and smooth minimax spline-approximation with the given error

Let us take the algorithm described in the work [8] as the basis of algorithm of continuous and smooth minimax spline-approximation (2) determination. This algorithm is based on the specified definition of approximation to the intermediate knots of the searched spline. According to this algorithm, the construction of balance spline-approximation to some continually differentiated function $f(x)$ with the given error G_0 roots in the gradual determining of link limits, i.e. the inner spline knots t_j, $j = \overline{2,\, q}$. The uttermost spline knots t_1 and t_{q+1} coincide with the limits of the interval $[\alpha, \beta]$ – $t_1 = \alpha$, $t_{q+1} = \beta$, where spline-approximation is being constructed. In the beginning, while an approximation accuracy is being provided and spline continuity and smoothness conditions are being followed, the right utmost limit t_2 of the first link $[t_1,\, t_2]$, subsequently, of the second and so on, could be determined. The optimum limits of spline links are determined by an iteration method, when the specification of the sample knot t_j value is being conducted depending on the error of corresponding current link approximation within the range $\left[t_{j-1},\, t_j\right]$.

The parameters of spline-approximation (2) polynomials on every link are determined according to the criterion of minimax approximation with the precise reproduction of the function value and the derivative in the given points [10]. To gain the continuous and smooth spline according to the conditions mentioned below (4)-(5), it is necessary to provide the coincidence of spline link values and their derivatives in the inner spline knots t_j, $j = \overline{2,\, q}$. Minimax approximation with the precise reproduction of the function value and its derivative in the utmost right link point t_2 is used on the first spline link

$$\begin{cases} \left(f\left(z_i^{(1)}\right) - P_m\left(a^{(1)}; z_i^{(1)}\right)\right) / w\left(z_i^{(1)}\right) = (-1)^i \mu_1, & i = \overline{1,\, m} \\ P_m\left(a^{(1)};\, t_2\right) = f(t_2) & \\ P'_m\left(a^{(1)};\, t_2\right) = f'(t_2) & \end{cases}, \qquad (8)$$

here $z_i^{(1)}$, $i = \overline{1,\, m}$ – the ordered by rising $z_i^{(1)} < z_{i+1}^{(1)}$ $\left(i = \overline{1,\, m-1}\right)$ points of alternance within the range $[t_1,\, t_2]$. To determine these alternance points, Remez's scheme with the one-point substitution of approximation to alternance points by Valle-Pussens algorithm [10] could be used. The quantity of alternance points that determines such a minimax approximation makes – m. During a choice of preliminary approximation to these points, we should remember that the utter right link point could not be localised inside alternance. Moreover, the approximation error on the first link is equal to the module of μ_1 $\left(G_1 = |\mu_1|\right)$.

The parameters of spline - approximation (2) polynomials for the function $f(x)$ on inner links, from the second one to one before the last, are determined according to the criterion of minimax approximation under the condition of the precise reproduction of the function and the derivative values in the utmost points of any link $[t_j,\ t_{j+1}]$, $j=\overline{2,\ q-1}$.

According to the characteristic property of minimax approximation with both the function and the derivative value precisely reproducing in two points [10], the coefficient values of these polynomials fit the equation system:

$$\begin{cases} P_m\left(a^{(j)};t_j\right)=f\left(t_j\right) \\ P'_m\left(a^{(j)};t_j\right)=f'\left(t_j\right) \\ \left(f\left(z_i^{(j)}\right)-P_m\left(a^{(j)};z_i^{(j)}\right)\right)/w\left(z_i^{(j)}\right)=(-1)^i\mu_j,\ i=\overline{1,\ m-2} \\ P_m\left(a^{(j)};t_{j+1}\right)=f\left(t_{j+1}\right) \\ P'_m\left(a^{(j)};t_{j+1}\right)=f'\left(t_{j+1}\right) \end{cases}, \qquad (9)$$

here $z_i^{(j)}$, $i=\overline{1,\ m-2}$, $j=\overline{2,\ q-1}$ – the ordered by rising $z_i^{(j)}<z_{i+1}^{(j)}$, $\left(i=\overline{1,\ m-3}\right)$ points of the j-th link alternance, i.e. on the interval $[t_j,\ t_{j+1}]$. In this case, there are only $(m-2)$ alternance points. During the choice of preliminary approximation to alternance points, we should remember that the utmost points of any link could not be included into alternance. The same as for the first link, an approximation error on any link is equal to the module of the corresponding value μ_j $\left(G_j=|\mu_j|\right)$, $j=\overline{2,\ q-1}$.

To determine polynomial approximation on the last spline link, minimax approximation with the function and its derivative value precisely reproducing in the utter left link point – t_q is used. The coefficient values of this polynomial approximation fit the equation system:

$$\begin{cases} P_m\left(a^{(q)};t_q\right)=f\left(t_q\right) \\ P'_m\left(a^{(q)};t_q\right)=f'\left(t_q\right) \\ \left(f\left(z_i^{(q)}\right)-P_m\left(a^{(q)};z_i^{(q)}\right)\right)/w\left(z_i^{(q)}\right)=(-1)^i\mu_q,\quad i=\overline{1,\ m} \end{cases}, \qquad (10)$$

here $z_i^{(q)}$, $i=\overline{1,\ m}$ – ordered by rising $z_i^{(q)}<z_{i+1}^{(q)}$, $\left(i=\overline{1,\ m-1}\right)$ points of alternance on the interval $[t_q,\ t_{q+1}]$. Similarly to the first link, this approximation has m alternance points and while choosing preliminary approximation to these points, we should remember that the utter left link point could not be included into alternance. An approximation error on the last link is equal to the module of μ_q $\left(G_q=|\mu_q|\right)$.

The task of balance spline-approximation with the given error in case of the approximation of table-represented functions is not always soluble. It has no solution if the error G exceeding the given G_0 $(G>G_0)$ is gained on some of subintervals $\left[x_{i_j},\ x_{i_{j+1,r}}\right]$, $j=1,2,\ldots,q$ of a minimally admissible link length. For spline-approximation by the m degree polynomial, the minimally necessary point quantity for the determination of the first and the last links makes $(m+1)$, for the rest of links – m. If during the construction of balance spline-approximation with the given error, the error larger than this is gained, then the polynomial of a higher degree could be chosen or an approximation error could be increased, according to a concrete task. In this case, other solutions are also possible. For instance, on subintervals where minimax approximation with the needed error was not found, we could use ermit interpolation [11] with function and derivative values' reproducing in corresponding utter link points: on the first link in the utter right point, on the last – in the utter left point, and for inner links – in both utter points.

4. Continuous approximation of thermodiode sensor temperature characteristic and sensitivity

Let us consider the approximation of the temperature characteristic of the DT-471 type thermodiode sensor manufactured by the "Lake Shore" firm, presented on the site [12] (Curve 10). The temperature characteristic of this sensor is given by 120 values within the range from 1.4 K to 475 K. The graph of this temperature characteristic is shown in the fig. 1. In practice, temperature dependence is described by a quite complex mathematical dependence. The voltage drop on a silicon (p-n) junction is described by the following equation [13]

$$U=IR_{BL}+f_T\ln\left\{\frac{I}{I_{SD}}\left[\sqrt{\frac{I_R^2}{4I_{SD}I}+\frac{I_{SD}+I_R}{I}+1}-\sqrt{\frac{I_R^2}{4I_{SD}I}}\right]^2\right\}, \qquad (11)$$

where $U_{BL}=IR_{BL}$ – voltage drop on equivalent base resistance; T_x – p-n junction temperature; $R_{BL}=R_B+R_{BP}+R_{EP}+R_{CW}$ – equivalent base resistance; R_B, R_{BP}, R_{EP}, R_{CW} – accordingly resistances of a base, base and emitter pins and connection wires; $f_T=kT_x/q$ - temperature potential; I_{SD}, I_R – equivalent heat current and recombination current, respectively.

The analysis of dependence (11) with taking into consideration the methodical errors points out the expediency of experimental research result usage.

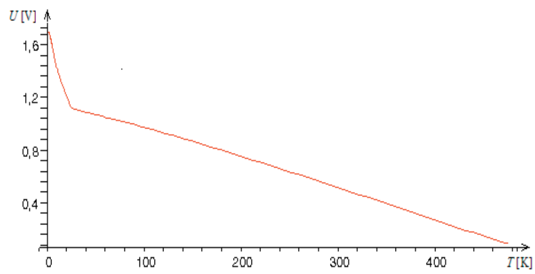


Fig. 1. Graph of the DT-471 type sensor temperature characteristic.

The graph of this sensor sensitivity is shown in the fig.2.

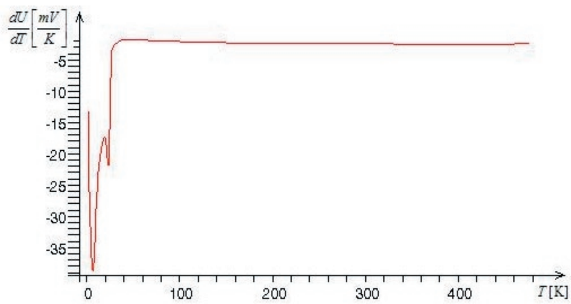


Fig. 2. Graph of the DT-471 type sensor sensitivity.

To describe this temperature characteristic, the continuous and smooth balance spline whose relative error is not over 0.03% is used. This spline consists of 6 links determined according to the minimax criterion, anyone of them is represented by the fifth degree polynomial

$$U_j(T) = \sum_{i=0}^{5} a_i^{(j)} T^i, \qquad (12)$$

here j – the link number.

This polynomial coefficient values for any link of this spline as well as link limits are notified in the table 1. The relative error value of the approximation by the balance spline of sensor temperature characteristics and sensitivity on any link is indicated in percents.

Tab. 1. Approximation results of the DT-471 type sensor temperature characteristic.

№ link	Link limits	Polynomial (12) coefficient values in the order of degree rising	Approximation error [%]	
			characteristics	sensitivity
1	1.4; 12	1.7103; $1.9989\cdot10^{-3}$; $-4.8893\cdot10^{-3}$; $1.9990\cdot10^{-5}$; $3.4349\cdot10^{-5}$; $-1.4507\cdot10^{-6}$	0.025	16.2
2	12; 22	2.4487; -0.25724; $2.7377\cdot10^{-2}$; $-1.6298\cdot10^{-3}$; $4.9900\cdot10^{-5}$; $-6.2051\cdot10^{-7}$	0.008	0.73
3	22; 26	81.1532; -19.9594; 1.9429; $-9.2568\cdot10^{-2}$; $2.1651\cdot10^{-3}$; $-1.9942\cdot10^{-5}$	0.022	2.57
4	26; 46	3.5701; -0.32433; $1.7194\cdot10^{-2}$; $-4.5597\cdot10^{-4}$; $6.0103\cdot10^{-6}$; $-3.1483\cdot10^{-8}$	0.026	7.1
5	46; 390	1.1489; $-1.3559\cdot10^{-3}$; $-4.7383\cdot10^{-6}$; $1.0344\cdot10^{-8}$; $-9.9551\cdot10^{-12}$; $1.8461\cdot10^{-15}$	0.025	0.63
6	390; 475	-34.0714; 0.42745; $-2.09407\cdot10^{-3}$; $5.1036\cdot10^{-6}$; $-6.2237\cdot10^{-9}$; $3.0375\cdot10^{-12}$	0.004	0.23

The relative error, 16%, of reproducing of the sensitivity value by a spline on the first link is caused by the sensitivity reproducing error at the temperature 1,4 K, it makes 6,77% at the temperature 1,6 K, and the sensor sensitivity reproducing error does not exceed 4% in the rest of observation points of this link. The high error, 7,1%, of sensor sensitivity spline reproducing on the fourth link could be explained by the local function minimum describing sensor sensitivity that is available on this interval (see fig. 2). However, the qualitative nature of sensor sensitivity altering within the range of the first four links, i.e. for temperature from 1,4 K to 4,6 K is reproduced satisfactory by the spline derivative. The results of the continuous approximation of temperature characteristics and the sensitivity of a thermodiode sensor notified in the table prove the usage expediency of spline-approximation with the links determined by the minimax criterion. The gained spline reproduces the temperature characteristics of the thermodiode sensor within the temperature range from 1,4 K to 475 K with the relative error less than 0,026%. The comparison of this spline parameter quantity with high degree Chebyshev's polynomial approximation proposed in the work [5] looks especially efficient. The approximation parameter total quantity in this case is much less, since the minimax criterion provides the attaining of the smallest approximation error for the given parameter quantity. According to the definition of balance spline-approximation, we could declare that spline-approximation with the (12) – looked polynomial links' quantity less than six which with the relative error, 0,03%, approximates the dependence of voltage drop on temperature for the static temperature characteristics of the DT-471 type thermodiode sensor within the range [1,4 K; 475 K] does not exist. Any link of this spline-approximation is determined from the condition of attaining of the largest possible length at which the given approximation accuracy is provided. Since approximation on any link is conducted according to the minimax criterion, the quantity of these links is the smallest possible. However, it is sometimes possible to reduce the spline-approximation error for the same link quantity at the expense of some increase of the approximation error on the last link. In this case, we should fix the gained link quantity and, cutting the lengths of some links, try to reach the certain reducing of spline-approximation error. This task is called the task of construction of spline-approximation with the given link quantity. The construction algorithms of nonsmooth spline-approximation with the given link quantity is described in the work [1].

5. Conclusions

The construction of continuous and smooth balance spline-approximation with polynomial links where approximation on any link is determined by the minimax criterion is based on the application of minimax approximation that reproduces the precise values of the function and its derivative in the given points. The approximation parameter value on the first link of this spline is determined by the characteristic property of minimax approximation which in the utter right link point reproduces the function and its derivative values. Polynomial approximations on the inner spline links, from the second one to the one before the last, are determined according to the characteristic property of minimax approximation which reproduces the function and derivative values in the utter points of these links. Minimax approximation on the last spline link reproduces precisely the values of the function and its derivative in the utmost left link point. Moreover, the lengths of all spline links except the last one are chosen as maximally possible for the given spline-approximation error.

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