

## Remaining useful life prediction of cylinder liner based on nonlinear degradation model

Indexed by:



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### Highlights

- A nonlinear degradation model is established to predict the RUL.
- The off-line and on-liner data are used to estimate the unknown parameters adaptively.
- The proposed model has a high evaluation accuracy

### Abstract

In order to effectively monitor the wear and predict the life of cylinder liner, a nonlinear degradation model with multi-source uncertainty based on Wiener process is established to evaluate the remaining useful life (RUL) of cylinder liner wear. Due to complex service performance of cylinder liner, the uncertainty of operational environment and working conditions of cylinder liner wear are considered into the model by a random function. The probability density function (PDF) formula of RUL is derived, and the maximum likelihood estimation method is adopted to estimate the unknown parameters of PDF. Considering the evaluated parameters as the initial values, the model parameters are updated adaptively, and an adaptive PDF is obtained. Furthermore, the proposed model is compared with two classical degradation models. The results show that the proposed model has a good performance for predicting the life, and the error is within 5%. The method can provide a reference for condition monitoring of cylinder liner wear.

### Keywords

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remaining useful life, cylinder liner wear, nonlinear degradation, maximum likelihood estimation, adaptive PDF.

## 1. Introduction

Failure analysis methods are widely used in aviation, navigation, wind power and other industries. To prevent sudden failure of components, a hybrid approach with the fusion of model-based and data-driven approaches was proposed, and the nonlinear degradations of dynamical system components was analyzed [17]. In order to investigate the effects of stiffness degradation of fiber reinforced polymer (FRP) on reliability and failure, the relationship between stiffness degradation and strength degradation was established, and a probability model of stiffness degradation of FRP was presented [6]. Aim to control the operation risk of the internal combustion engines (ICEs), the diagnosis [20, 29], repair and replacement [23] technologies were used to assess service status. Piston ring-cylinder liner (PRCL) system is the core component of ICEs, and it determines the power conversion efficiency of the system. The cylinder liner plays an important role in PRCL, and the wear of cylinder liner greatly impacts on the safe operation and long life of the system. Prognostics and health management (PHM) is an important method to monitor operational condition of cylinder liner wear. Therefore, to ensure high-efficiency and safe operation of the system, the degradation performance of

cylinder liner needs to be evaluated, and RUL needs to be predicted based on degradation data.

Cylinder liner wear occurs during the operational process inevitably, and it is related to the tribological characteristics [5], dynamic characteristics [1] and operation conditions of PRCL system [2]. It is difficult to acquire a large number of data sets because that the wear changes randomly and slowly. It increases the difficulty of life assessment of cylinder liner. In recent years, many works have been done to reveal the mechanism of cylinder liner wear. The friction and wear characteristics of PRCL at top dead center (TDC) were investigated by numerical method, and the oil and lubrication types, surface coatings were considered into the testing [32]. The wear behaviors of cylinder liners and piston rings were investigated in a linear reciprocating tribometer, and the wear of cylinder liners of different material was compared in boundary lubrication [26]. Synchronously, the friction and wear of honing cylinder liner were discussed based on experiment analysis [12]. The above literatures analyzed the cylinder liner wear from different factors, and the results provide a reference for wear evaluation. However, the models cannot be used in the PHM. Currently, the vibration, acoustic and wear signal are used to diagnose, monitor and predict the condition of ICEs. To monitor the scuffing fault of

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cylinder liner, the vibration and acoustic emission analysis were applied, the influence of emissions on scuffing fault was analyzed [19]. The failures of timing device piston and supply pump in transport utility vehicles were evaluated, and the remedial actions to avoid the failure of the system were suggested [3]. The wavelet transform and neural network were applied to extract feature and recognize the fault automatically, and the method was applied to diagnose the wear of the hydraulic cylinder [11]. Based on evidential reasoning (ER) rule, a multi-model fusion system was proposed to diagnose the wear faults of diesel engine [29]. RUL prediction is an effective way to prevent accidents, and it is conducive to monitor the operational condition of the system. A method was presented to predict the wear of piston/cylinder pair, the load-bearing and lubrication parameters were considered into the model, and the model was validated by the experiments [14]. RUL of an aviation hydraulic pump was predicted by using the numerical approach [15], and the Monte Carlo method was used to simulate the wear debris features. Considering the effects of loading sequence, a new generic framework for fatigue life prediction under the multi-level cyclic (MLC) loading is developed, the method can calculate fatigue life of materials under any MLC loading [7].

The artificial intelligence (AI) is widely used to predict RUL [4, 27], whereas the machine learning methods need adequate training data. It is difficult to obtain adequate data for long-life equipment, but the machine learning model relies too much on training data. Therefore, the stochastic models (Wiener process, Gamma process and Inverse Gaussian process) are used to describe the degradation process and predict RUL, and the empirical knowledge and dynamic information are fused into the degradation model. In the earlier study, linear Wiener process was adopted to deal with the degradation process. Nevertheless, the degradation process of equipment is usually nonlinear. It is necessary to extend the linear Wiener process to nonlinear Wiener process. Therefore, a nonlinear model was established [21], and the variable threshold was used to characterize the nonlinear characteristics of the degradation process. The nonlinear Wiener process model is widely applied due to its better performance [33]. Considering the impact of degradation rate on RUL, a novel RUL prediction method was developed under time-varying temperature condition, and a stochastic degradation rate model based on Arrhenius temperature model was proposed [28]. A system reliability model with phase-type distribution considering the variation of the degradation rate was established [16]. Similarly, the degradation states of different parameters were estimated by information of operating modes of the system [18]. A degradation model was presented to address the stochastic processes with random initial degradation [22]. However, the historical degradation data was not considered into most models. In order to consider the historical data, a degradation model with an adaptive drift was proposed, and the nonlinear characteristics of temporal uncertainty, time-varying degradation and item-to-item variability were considered into the model [31]. To consider hidden states, a nonlinear-drifted Brownian motion model under multiple hidden states was established, and the model was applied to predict RUL of rechargeable batteries [24]. For reducing the prognostic uncertainty, a right-time prediction method was proposed, and hidden Markov model and proportional hazard model were used to map the degradation path [9]. An adaptive predictive maintenance model was developed to support the regular inspection, repair and replacement of the system [10]. Based on nonlinear-drift-driven Wiener process model, the multi-source uncertainties were built by an age-dependent state-space model for the RUL estimation of degrading systems [30].

It can be seen from the above analysis that the degradation model based on Wiener process is widely used in PHM of equipment. Nonetheless, there are few applications in cylinder liner wear. Furthermore, the service process of cylinder liner has the characteristics of multi-source uncertainty during the operational conditions. Most of the existing models focus on one or a few service conditions. In order to predict RUL of cylinder liner more accurately, an adaptive nonlinear degradation with multi-source uncertainty model based on Wiener

process is proposed. Different from the existing degradation models, the proposed model considers the adaptive process and stochastic effects of drift coefficient, and the Bayesian method is used to update the model parameters. On this basis, RUL of the cylinder liner wear is predicted, and the comparison of the proposed model with two classical degradation model is conducted.

## 2. Model description

### 2.1. Basic degradation model

Let  $X(t_k)$  denotes the degradation process of cylinder liner wear at time  $t_k$ . The general Wiener process model with constant drift coefficient can be expressed as [25]:

$$X(t_k) = X(0) + \mu t_k + \sigma B(t_k) \quad (1)$$

where  $X(0)$  is the degradation parameter at the initial time, it is usually assumed to 0.  $\mu$  is the drift coefficient,  $\sigma$  is the diffusion coefficient, and  $\sigma > 0$ .  $B(t)$  is the standard Brownian motion. In this model, the increments are assumed to be independent of each other and followed a normal distribution:  $(X(t_k) - X(t_{k-1})) \sim N(\mu(t_k - t_{k-1}), \sigma^2(t_k - t_{k-1}))$ . To express conveniently, the model is marked as Model 1.

If the degradation data  $X(t_k)$  exceeds the failure threshold  $W$ , the equipment is considered to be failure. The degradation process can be shown in Figure 1. Based on the concept of the first passage time (FPT), the lifetime of the equipment is defined as:

$$T = \inf \{t : X(t_k) \geq W\} \quad (2)$$

It is known that the lifetime  $T$  follows an inverse Gaussian distribution, PDF and cumulative distribution function (CDF) are formulated as:

$$f_T(t_k) = \frac{W}{\sqrt{2\pi\sigma^2 t_k^3}} \exp\left(-\frac{(W - \mu t_k)^2}{2\sigma^2 t_k}\right) \quad (3)$$

$$F_T(t_k) = \Phi\left(\frac{\mu t_k - W}{\sqrt{\sigma^2 t_k}}\right) + \exp\left(-\frac{2\mu W}{\sigma^2}\right) \cdot \Phi\left(-\frac{\mu t_k + W}{\sqrt{\sigma^2 t_k}}\right) \quad (4)$$

where  $\Phi(\cdot)$  is the standard normal CDF. Based on the above analysis, RUL can be predicted.

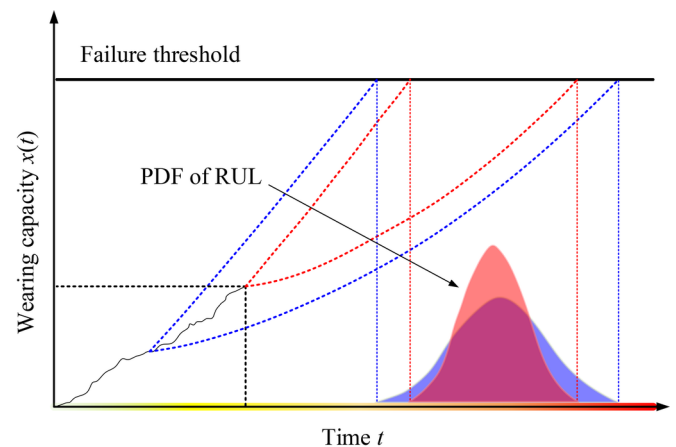


Fig. 1. Diagram of degradation process

### 2.2. Degradation model with individual influence

The influence of individual differences on RUL is not considered into the Model 1, and the drift coefficient is defined as a fixed con-

stant. However, the degradation rate changes with time going on. To describe the influence of changes on degradation process, the Model 2 is proposed, and it can be expressed as:

$$X(t_k) = X(0) + \lambda t_k + \sigma B(t_k) \quad (5)$$

where  $\lambda$  is the drift coefficient, it defined as the random function to describe the heterogeneity among the different individuals, and it is assumed to follow the normal distribution with parameters  $\lambda_\alpha$  and  $\lambda_\sigma$ .

The lifetime of equipment is defined as the formula (2). In order to derive PDF of RUL of the Model 2, the following Lemma is used.

**Lemma 1** if  $\alpha \sim N(\mu_\alpha, \sigma_\alpha^2)$ , and  $A, B \in R, C \in R^+$ , then [21]:

$$E_\alpha \left[ (A - \alpha) \exp\left(-\frac{(B - \alpha)^2}{2C}\right) \right] = \sqrt{\frac{C}{\sigma_\alpha^2 + C}} \times \left( A - \frac{\sigma_\alpha^2 B + \mu_\alpha C}{\sigma_\alpha^2 + C} \right) \times \exp\left(-\frac{(B - \mu_\alpha)^2}{2(\sigma_\alpha^2 + C)}\right) \quad (6)$$

According to the Lemma 1, PDF of RUL for Model 2 can be given as:

$$f_{L_k}(l_k) = \frac{W - x_k}{\sqrt{2\pi l_k^3 (\lambda_\sigma^2 l_k + \sigma^2)}} \exp\left\{-\frac{(W - x_k - \lambda_\alpha l_k)^2}{2l_k (\lambda_\sigma^2 l_k + \sigma^2)}\right\} \quad (7)$$

### 2.3. The proposed nonlinear model

The degradation rates of individuals are variable for the same batch of products, and the multi-source uncertainties of products in degradation process are usually caused by environmental factors and operating conditions. To describe the degradation process in this case, the nonlinear degradation model is proposed. It is given as:

$$X(t_k) = X(0) + \alpha \int_0^{t_k} \mu(\tau) d\tau + \sigma B(t_k) \quad (8)$$

where  $\alpha$  is the drift coefficient, and it is assumed to follow the normal distribution with mean  $\mu_\alpha$  and variance  $\sigma_\alpha^2$ , which can be written as  $\alpha \sim N(\mu_\alpha, \sigma_\alpha^2)$ .  $\mu(\tau)$  is the nonlinear function,  $\sigma$  is the diffusion coefficient. It is marked as Model 3.

Due to the characteristics of the wear, the exponential function is used to model the process of the wear. It can be expressed as follows:

$$\mu(\tau) = b \cdot \exp(b\tau) \quad (9)$$

In order to derive PDF of RUL, the following Lemma is given.

**Lemma 2:** For the degradation process  $\{X(t_k), t_k \geq 0\}$  given by Eq. (8), if  $\mu(\tau)$  is a continuous function at time  $t_k$  in  $[0, \infty]$ , PDF of the  $T$  of  $X(t_k)$  can be approximated as follows:

$$f_{T|\theta}(t_k|\theta) \cong \frac{1}{\sqrt{2\pi t_k}} \left( \frac{S_B(t_k)}{t_k} + \frac{\alpha}{\sigma} \mu(\tau) \right) \exp\left(-\frac{S_B(t_k)^2}{2t_k}\right) \quad (10)$$

where  $S_B(t_k) = (W - \alpha \int_0^{t_k} \mu(\tau) d\tau) / \sigma$ ,  $\theta$  is the unknown vector, and the proof of formula (10) can be referred to literature [21].

Based on the Lemma 1 and 2, when the degradation status  $x_k$  is given at  $t_k$ , the PDF of RUL for Model 3 can be written as:

$$\begin{aligned} f_{L_k}(l_k) &= \int f_{L_k}(l_k|\alpha) p(\alpha) d\alpha \\ &= \frac{1}{\sigma \sqrt{2\pi l_k^3}} E_\alpha \left[ \exp\left(-\frac{(W - x_k - \alpha \varphi(l_k))^2}{2\sigma^2 l_k}\right) \right] \\ &= \frac{1}{\sqrt{2\pi l_k^2 (\sigma_\alpha^2 \cdot \xi(l_k)^2 + \sigma^2 l_k)}} \exp\left(-\frac{(W - x_k - \mu_\alpha \cdot \xi(l_k))^2}{2(\sigma_\alpha^2 \cdot \xi(l_k)^2 + \sigma^2 l_k)}\right) \\ &\quad \times \left[ W - x_k - \varphi(l_k) \frac{(W - x_k) \cdot \sigma_\alpha^2 \cdot \xi(l_k) + \mu_\alpha \cdot \sigma^2 l_k}{\sigma_\alpha^2 \cdot \xi(l_k)^2 + \sigma^2 l_k} \right] \end{aligned} \quad (11)$$

where  $\xi(l_k) = \exp(b(l_k + t_k)) - \exp(bt_k)$ ;  $\varphi(l_k) = (1 - bl_k) \exp(b(l_k + t_k)) - \exp(bt_k)$ .

By using Eq. (11), the PDF of RUL at time  $t_k$  can be obtained.

## 3. Parameter estimation

### 3.1. Off-line parameters estimation

In this section, the unknown parameters of PDF are evaluated. Let  $\theta = (\mu_\alpha, \sigma_\alpha^2, \sigma, b)$  expressed the unknown parameter vector. To obtain the maximum likelihood value of  $\theta$ , the  $i$ th system is monitored at the time series  $t_{i,1}, t_{i,2}, t_{i,3}, \dots, t_{i,j}$ . In this case, the Eq. (5) can be given by:

$$X_i(t_{i,j}) = \alpha \left[ \exp(bt_{i,j}) - 1 \right] + \sigma B(t_{i,j}) \quad (12)$$

where  $i=1, 2, 3, \dots, N$ ,  $N$  is the independent testing system.  $j=1, 2, 3, \dots, M_i$ ,  $M_i$  is the number of degradation data for each system.

Let's define the function  $\kappa_i(t_{i,j}) = \exp(bt_{i,j}) - 1$ ,  $\mathbf{T}_i = (T_{i,1}, T_{i,2}, T_{i,3}, \dots, T_{i,j})'$ , where  $T_{i,j} = \kappa_i(t_{i,j})$ ,  $\mathbf{X}_i = (x_i(t_{i,1}), x_i(t_{i,2}), x_i(t_{i,3}), \dots, x_i(t_{i,j}))'$ . According to the characteristics of mutual independence of the standard Brownian motion,  $\mathbf{X}_i$  obeys the multi-dimensional normal distribution. Its mean and variance can be expressed as:

$$\mu_i = \mu_\alpha \mathbf{T}_i, \quad \Delta = \sigma_\alpha^2 \mathbf{T}_i \mathbf{T}_i' \quad (13)$$

In the Eq. (13), the  $\Omega_i$  is written as:

$$\Omega_i = \sigma^2 \begin{bmatrix} t_{i,1} & t_{i,1} & \dots & t_{i,1} \\ t_{i,1} & t_{i,2} & \dots & t_{i,2} \\ \vdots & \vdots & \ddots & \vdots \\ t_{i,1} & t_{i,2} & \dots & t_{i,j} \end{bmatrix} \quad (14)$$

The degradation processes of the individual device are independent each other. Once the degradation data  $\mathbf{X}_i$  is given, the log-likelihood function of the  $\theta$  can be expressed as:

$$\begin{aligned} \ell(\theta|\mathbf{X}_i) &= -\frac{1}{2} \sum_{i=1}^N M_i \ln(2\pi) - \frac{1}{2} \sum_{i=1}^N \ln|\Delta| \\ &\quad - \frac{1}{2} \sum_{i=1}^N (\mathbf{X}_i - \mu_\alpha \mathbf{T}_i)' \Delta^{-1} (\mathbf{X}_i - \mu_\alpha \mathbf{T}_i) \end{aligned} \quad (15)$$

$$|\Delta| = |\Omega_i| (1 + \sigma_\alpha^2 \mathbf{T}_i' \Omega_i^{-1} \mathbf{T}_i) \quad (16)$$

$$\Delta^{-1} = \Omega_i^{-1} - \frac{\sigma_\alpha^2}{1 + \sigma_\alpha^2 \mathbf{T}_i' \Omega_i^{-1} \mathbf{T}_i} \Omega_i^{-1} \mathbf{T}_i \mathbf{T}_i' \Omega_i^{-1} \quad (17)$$

The first-order partial derivatives of  $\mu_\alpha$  and  $\sigma_\alpha$  of the Eq. (15) are written as:

$$\frac{\partial \ell(\Theta | X_i)}{\partial \mu_\alpha} = \sum_{i=1}^N T_i' \Delta^{-1} X_i - \mu_\alpha \sum_{i=1}^N T_i' \Delta^{-1} T_i \quad (18)$$

$$\frac{\partial \ell(\Theta | X_i)}{\partial \sigma_\alpha} = \sum_{i=1}^N \frac{\sigma_\alpha T_i' \Delta^{-1} T_i}{1 + \sigma_\alpha^2 T_i' \Delta^{-1} T_i} + \frac{\sigma_\alpha \sum_{i=1}^N (X_i - \mu_\alpha T_i) \Omega_i^{-1} T_i T_i' \Omega_i^{-1} (X_i - \mu_\alpha T_i)}{(1 + \sigma_\alpha^2 T_i' \Omega_i^{-1} T_i)^2} \quad (19)$$

Let Eq. (18) equal zero, the maximum likelihood estimation of  $\mu_\alpha$  can be estimated by:

$$\hat{\mu}_\alpha = \frac{\sum_{i=1}^N T_i' \Delta^{-1} X_i}{\sum_{i=1}^N T_i' \Delta^{-1} T_i} \quad (20)$$

The profile log-likelihood function of  $\sigma_\alpha$ ,  $\sigma$  and  $b$  can be written as:

$$\ell(\sigma_\alpha, \sigma, b | X_i, \hat{\mu}_\alpha) = -\frac{1}{2} \sum_{i=1}^N M_i \ln(2\pi) - \frac{1}{2} \sum_{i=1}^N \ln |A| - \frac{1}{2} \left( \begin{array}{l} \left\{ \sum_{i=1}^N X_i' \Omega_i^{-1} X_i - 2 \frac{\sum_{i=1}^N T_i' \Delta^{-1} X_i}{\sum_{i=1}^N T_i' \Delta^{-1} T_i} \sum_{i=1}^N T_i' \Delta^{-1} X_i \right. \\ \left. + \left( \frac{\sum_{i=1}^N T_i' \Delta^{-1} X_i}{\sum_{i=1}^N T_i' \Delta^{-1} T_i} \right)^2 \sum_{i=1}^N T_i' \Delta^{-1} T_i \right\} \end{array} \right) \quad (21)$$

Based on the above analysis, the maximum likelihood estimation of  $\sigma_\alpha$ ,  $\sigma$  and  $b$  can be obtained using the three-dimensional search method by maximizing Eq. (21), and then substituting the  $\sigma_\alpha$ ,  $\sigma$  and  $b$  into Eq. (20), the  $\mu_\alpha$  can be obtained.

### 3.2. On-line parameter update

In order to predict RUL more accurately, the on-line parameter estimation method based on the off-line parameters is proposed. The historical degradation data and real-time data are considered to update the drift coefficient using the Bayesian method. The posterior distribution of  $\alpha$  at time  $t_k$  can be expressed as:

$$\begin{aligned} p(\alpha | X_{1:k}) &= \frac{p(X_{1:k} | \alpha) p(\alpha)}{p(X_{1:k})} \\ &= \frac{p(x_k | X_{1:k-1}, \alpha) p(\alpha | X_{1:k-1}) p(X_{1:k-1})}{p(X_{1:k})} \\ &= \frac{p(x_k | X_{1:k-1}, \alpha) p(\alpha | X_{1:k-1})}{p(x_k | X_{1:k-1})} \\ &\propto p(x_k | X_{1:k-1}, \alpha) p(\alpha | X_{1:k-1}) \end{aligned} \quad (22)$$

where  $p(\alpha | X_{1:k-1})$  denotes the posterior distribution of  $\alpha$  at time  $t_{k-1}$  with parameters  $(\mu_{\alpha, k-1}, \sigma_{\alpha, k-1})$ .

According to the characteristics of Wiener process,  $(x_k | X_{1:k-1}, \alpha)$  follows normal distribution, and PDF can be expressed as:

$$\begin{aligned} p(x_k | X_{1:k-1}, \alpha) &= \frac{1}{\sqrt{2\pi\sigma^2(t_k - t_{k-1})}} \\ &\times \exp\left(-\frac{(x_k - x_{k-1} - \alpha(t_k - t_{k-1}))^2}{2\sigma^2(t_k - t_{k-1})}\right) \end{aligned} \quad (23)$$

In this situation, the Eq. (22) can be expressed as:

$$\begin{aligned} p(\alpha | X_{1:k}) &\propto p(x_k | X_{1:k-1}, \alpha) p(\alpha | X_{1:k-1}) \\ &\propto \exp\left(-\frac{(x_k - x_{k-1} - \alpha(t_k - t_{k-1}))^2}{2\sigma^2(t_k - t_{k-1})}\right) \cdot \exp\left(-\frac{(\alpha - \mu_{\alpha, k-1})^2}{2\sigma_{\alpha, k-1}^2}\right) \\ &\propto \exp\left\{ \frac{1}{2\sigma^2(t_k - t_{k-1})\sigma_{\alpha, k-1}^2} \cdot \left[ \begin{array}{l} (x_k - x_{k-1} - \alpha(t_k - t_{k-1}))^2 \\ \times \sigma_{\alpha, k-1}^2 + (\alpha - \mu_{\alpha, k-1})^2 \\ \times \sigma^2(t_k - t_{k-1}) \end{array} \right] \right\} \\ &\propto \exp\left(-\frac{(\alpha - \mu_{\alpha, k})^2}{2\sigma_{\alpha, k}^2}\right) \end{aligned} \quad (24)$$

From the Eq. (24), the following equation can be obtained:

$$\mu_{\alpha, k} = \frac{\sigma^2 \mu_{\alpha, k-1}}{\sigma_{\alpha, k-1}^2 (t_k - t_{k-1}) + \sigma^2} + \frac{\sigma_{\alpha, k-1}^2 (x_k - x_{k-1})}{\sigma_{\alpha, k-1}^2 (t_k - t_{k-1}) + \sigma^2} \quad (25)$$

$$\sigma_{\alpha, k} = \sqrt{\frac{\sigma^2 \sigma_{\alpha, k-1}^2}{\sigma_{\alpha, k-1}^2 (t_k - t_{k-1}) + \sigma^2}} \quad (26)$$

It can be seen that the  $\mu_{\alpha, k}$  and  $\sigma_{\alpha, k}$  are dependent on the parameters  $\mu_{\alpha, k-1}$ ,  $\sigma_{\alpha, k-1}$  and the current degradation data  $x_k$ . In this case, the historical data is introduced into the model to estimate the drift parameters adaptively. The off-line estimated results of the  $\mu_\alpha$ ,  $\sigma_\alpha$  are regarded as the prior distribution parameters at the initial time.

PDF of RUL of adaptive model at time  $t_k$  is:

$$\begin{aligned} f_{L_k | \Theta}(l_k | \Theta) &= \frac{1}{\sqrt{2\pi l_k^2 (\sigma_{\alpha, k}^2 \cdot \xi(l_k)^2 + \sigma^2 l_k)}} \\ &\times \exp\left(-\frac{(W - x_k - \mu_{\alpha, k} \cdot \xi(l_k))^2}{2(\sigma_{\alpha, k}^2 \cdot \xi(l_k)^2 + \sigma^2 l_k)}\right) \\ &\times [W - x_k - \varphi(l_k)] \cdot \frac{(W - x_k) \cdot \sigma_{\alpha, k}^2 \cdot \xi(l_k) + \mu_{\alpha, k} \cdot \sigma^2 l_k}{\sigma_{\alpha, k}^2 \cdot \xi(l_k)^2 + \sigma^2 l_k} \end{aligned} \quad (27)$$

## 4. RUL prediction

### 4.1. Data description

As shown in Figure 2, the TDC of cylinder liner is sharply worn due to the soot particles, wear particles and thermal load. In order to monitor the health status, the wear of cylinder liner at TDC is measured to evaluate the operational status. In this section, the wear data were collected from cylinder liners of two 8-cylinder SULZER RTA 58 single acting two-stroke diesel engines from January 1999 to August 2006 [8]. The diesel engines were equipped on three identical ships of the Grimaldi Group, and they worked on the same routes during the whole year. The load, environment and operational conditions were almost the same. Because the caliper sensitivity is 0.05 mm, the measured values are rounded to the nearest multiple of 0.05. The wear paths are shown in Figure 3. It can be seen that the wear rates change randomly, and the paths of the wear are different during monitoring time. In order to facilitate the comparison of the established model, the average wear path is used to verify the model. The interpolation method is used to obtain the average wear of cylinder liner at the same time interval, and the detailed expression can be found in reference [8].

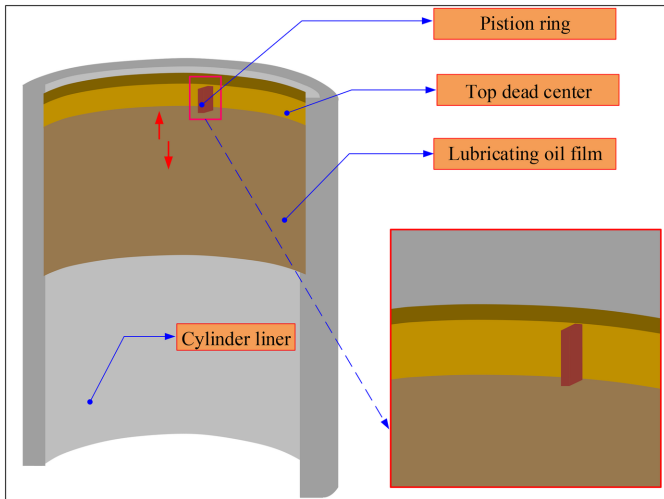


Fig. 2. The wear diagram of the cylinder liner

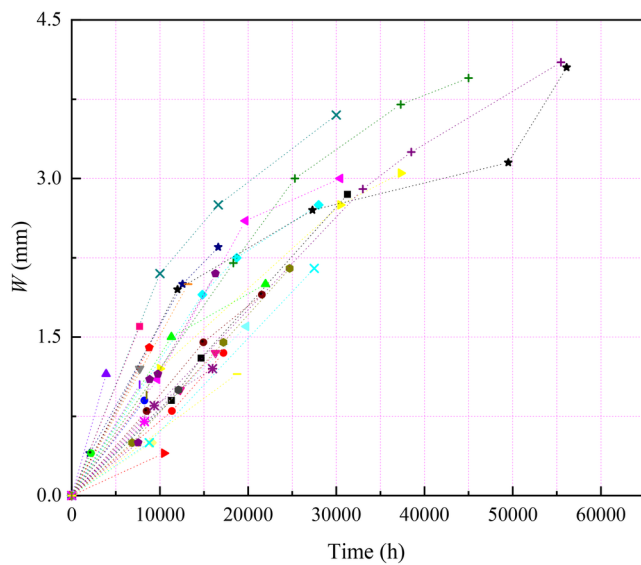


Fig. 3. Wear path of the cylinder liners [13]

## 4.2. Results and discussion

In this section, the three different RUL models are used to evaluate the health status of cylinder liner. Based on the collected data, the parameters of the models are evaluated by proposed method, and the estimated parameters are shown in Table 1. The evaluating performance of three RUL models can be compared and analyzed once the unknown parameters are obtained. To verify the accuracy of the models, four monitoring data at different operational time are chosen to evaluate RUL, and the maximum value of wear is 4 mm, the failure time is set as 55000 h. The selected monitoring data are  $t=12000$  h,  $x=1.95$  mm;  $t=27300$  h,  $x=2.70$  mm;  $t=49500$  h,  $x=3.15$  mm;  $t=52000$  h,  $x=3.50$  mm, respectively. The cylinder liner wear degradation paths of three models are described at 12000 h, as shown in Figure 4. Comparing the estimated wear path with the average wear path, it can be seen that the degradation path of Model 3 is closer to the average wear path, which can indirectly illustrate the superiority of Model 3.

To illustrate the evaluation performance in more detail, the PDFs of RUL for three models are analyzed at four different operation time. As shown in Figure 5, it can be seen from Figure 5(a) to (c) that the PDFs become narrow with time increases. The narrowing rates of PDF from Model 1 to Model 3 increase. It indicates that the uncertainty of the model decreases with the increase of collecting data. Comparing with Figures 5(a) and (b), the PDF curve of Figure 5(c) is more compact around the RUL. It means that the uncertainty is considered in the Model 3 to estimate the RUL. Based on the above analysis, it can be

Table 1. Estimated results of unknown parameters by different models

Model	parameters	values
1	$\mu$	$1.0163 \times 10^{-4}$
	$\sigma$	$5.200 \times 10^{-3}$
2	$\lambda_{\alpha}$	$1.0133 \times 10^{-4}$
	$\lambda_{\sigma}$	0.0005
3	$\mu_a$	0.0160
	$\sigma_a$	0.0011
	$\sigma$	0.0034
	$b$	0.0001

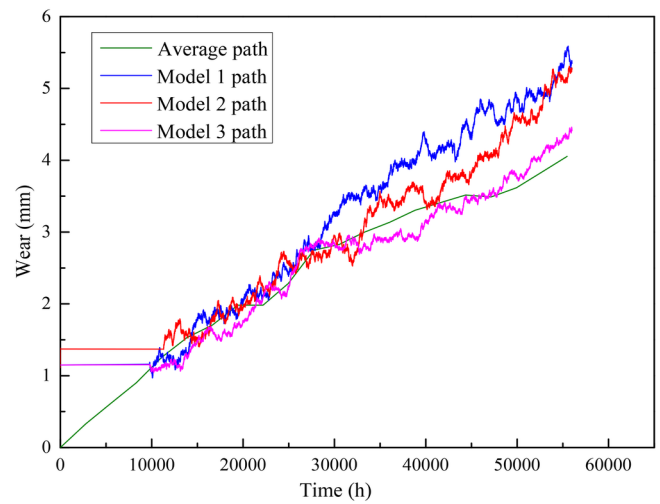


Fig. 4. Evaluated wear path

clearly derived that the random characteristics of models are gradually reduced, and the evaluation results are more accurate. The Model 3 has a better performance in the evaluation of RUL.

To further illustrate the evaluation performance of three models, the estimated useful lifetimes at different moments are compared, as shown in Figure 6. It can be seen that when the monitoring time away from the target value, the prediction accuracy of Model 1 and Model 2 is very low. When the monitoring time close to the target value, the prediction result exceeds the actual life. In this case, the prediction can lead to monitoring delays and accidents. For Model 3, the prediction accuracy is more accurate when the monitoring time is closer to the target value. It can effectively predict the life of equipment and prevent accidents. This is mainly because that more influencing factors are added sequentially from Model 1 to Model 3, which makes the evaluation model more perfect and effectively reduces the impact of uncertainty on the evaluation performance. According to the calculation formula of percentage error, the percentage errors of evaluated results are listed in Table 2. The calculation formula can be expressed as:

$$\text{percentage errors} = \frac{|ES - TA|}{TA} \times 100\% \quad (28)$$

where  $ES$  is the evaluated value,  $TA$  is the target value.

Based on the Model 3, an adaptive nonlinear degradation model of cylinder liner wear is established. The evaluated parameters of Model 3 are used as initial values to update the evaluation parameters by Bayesian theory. Figure 7 is the wear paths of adaptive model at different time. It can be seen from Figure 7 that the predicted wear

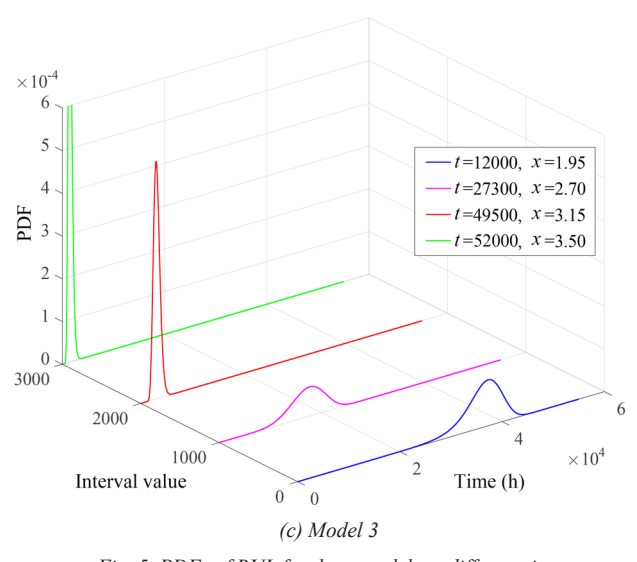
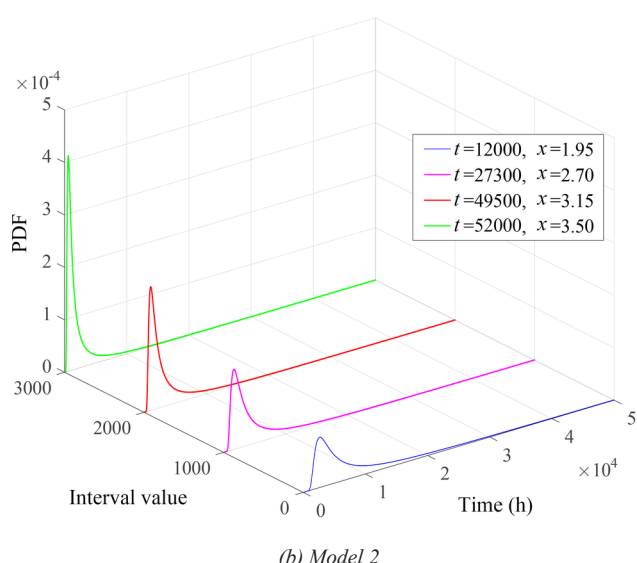
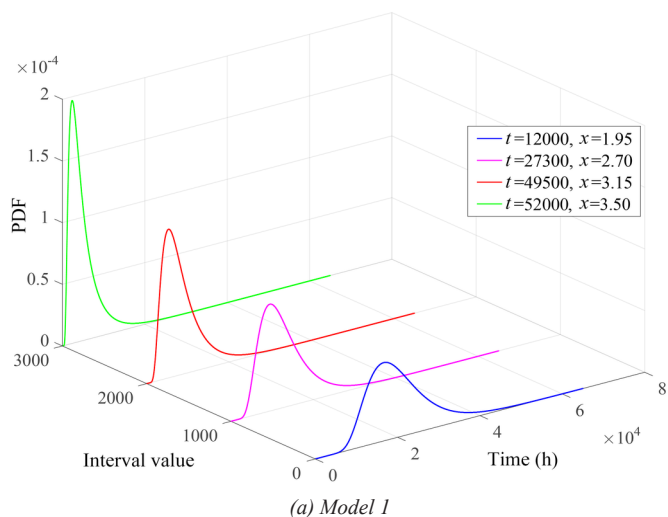


Fig. 5. PDFs of RUL for three models at different time

paths are roughly consistent with the average wear path. However, the initial value has an influence on prediction of wear path, and the adaptive model can effectively reduce the influence of the initial value on the system.

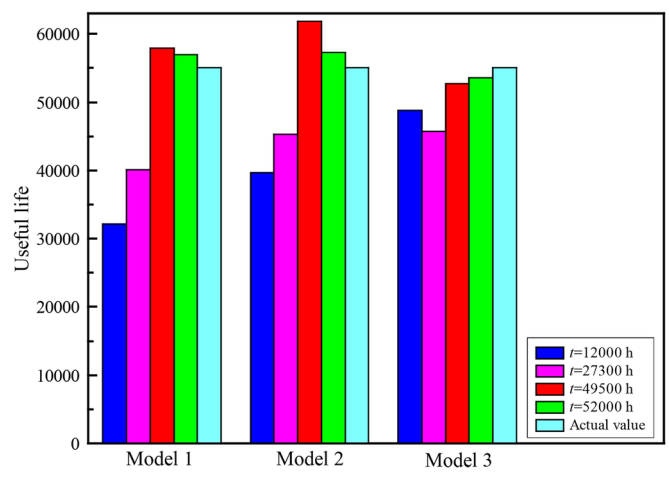


Fig. 6. Estimated useful life at different time

Table 2. Percentage error of estimating lifetime

t	Model 1	Model 2	Model 3
27300 h	26.7%	17.7%	16.9%
49500 h	5.2%	11.9%	4.2%
52000 h	3.5%	7.8%	2.5%

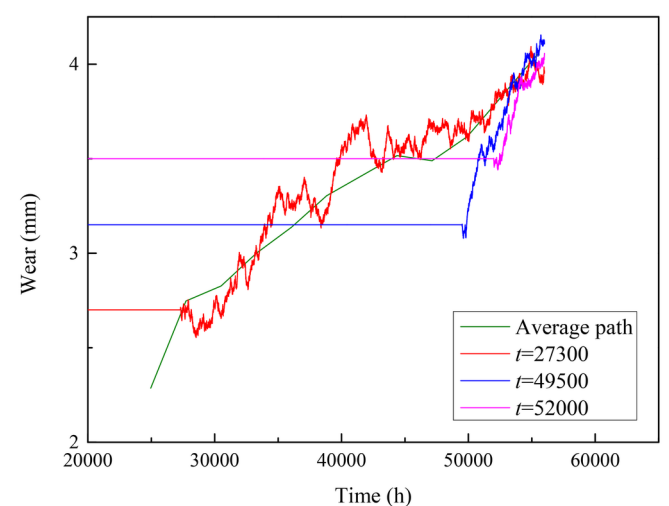


Fig. 7. Wear path under adaptive model

Figure 8 shows PDFs of RUL of adaptive model at different time. Comparing with the Figure 5, the change of the PDF is relatively small in Figure 8. It means that the adaptive method can effectively reduce the uncertainty of the system. The main reason is that real-time test data is used to update the evaluation parameters adaptively, and it can reduce the errors caused by uncertainty. By comparing the PDF at  $t=49500$  h and  $t=52000$  h, it is shown that the adaptive model is more effective when the time interval is small. To quantitative compare the evaluated performance, the useful lives of adaptive model and Model 3 are analyzed at different time, as shown in Figure 9. It can be seen that the accuracy of adaptive model is more accurate than Model 3, it increases 12.32%, 2.04%, and 1.27% at different evaluation points respectively. The smaller time interval is, the higher prediction accuracy of the adaptive model is acquired. When the parameters are updated adaptively, the prediction results are more accurate. The adaptive model can monitor the cylinder liner wear and health status.

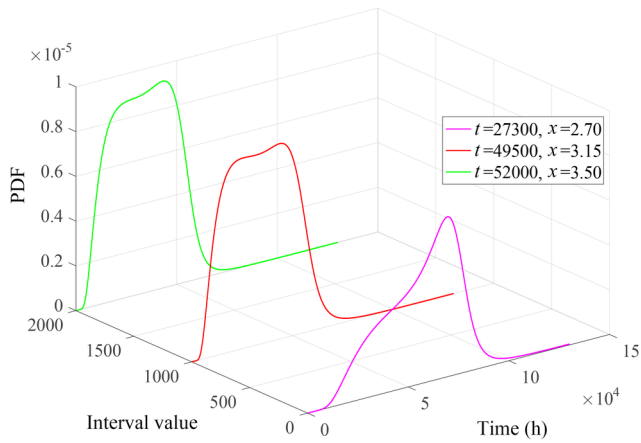


Fig. 8. PDF of adaptive model at different time

## 5. Conclusion

The reliability of ICEs is an important indicator of safe operation, and the cylinder liner wear directly affects the reliability of ICEs. In order to improve operation reliability, a nonlinear degradation model of the cylinder liner wear is established in this paper. The different influence factors are considered into the model by a random function, and the variation of wear is characterized as an exponential function. The PDF of RUL of cylinder liner wear is derived, and the unknown parameters are estimated by the maximum likelihood estimation method. An adaptive updating model of RUL is proposed based on the degradation model, and it can implement the prediction of wear life effectively. The main conclusions are as follows:

- 1) Comparing with the classical stochastic degradation model, the proposed model has a better evaluation performance.

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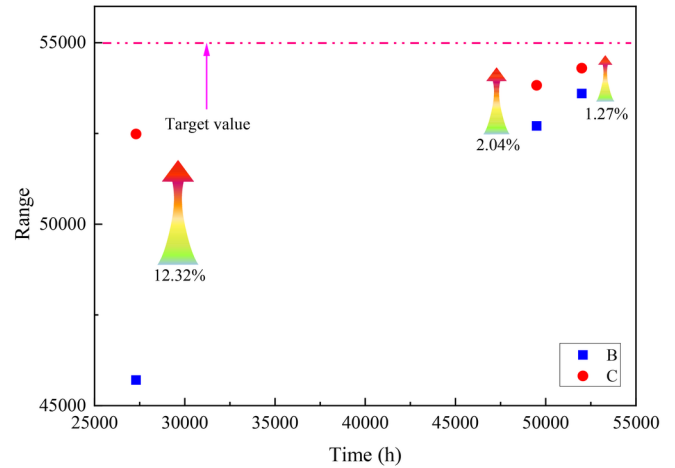


Fig. 9. Estimated useful life at different time for adaptive model

- 2) The adaptive nonlinear degradation model estimates the operation life more accurately. In addition, the more historical data have, the higher accuracy becomes.
- 3) An adaptive model can appropriately reduce the influence of random factors on life prediction of the system.
- 4) The proposed model can provide a reference for monitoring the cylinder liner wear, and reduce the monitoring time as well as save costs.

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