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METHOD OF MEASURING FRICTIONAL RESISTANCE IN PENDULAR MOTION

METODA POMIARU OPORÓW TARCIA W RUCHU WAHADŁOWYM

Key words: friction, bearing, polymers, pendulum.

Abstract: This paper presents a methodology for conducting tribological sliding tests based on decaying vibrations in pendular motion. The proposed method of determining the (averaged) coefficient of friction in pendular motion is based on measuring the potential kinetic energy. The method is characterized by a short measuring time and enables a quick comparison of the friction coefficients of different materials.

Słowa kluczowe: tarcie, łożysko, polimery, wahadło.

Streszczenie: W artykule przedstawiono metodykę prowadzenia badań tribologicznych materiałów ślizgowych na podstawie drgań gasnących w ruchu wahadłowym. Zaproponowana metoda wyznaczania współczynnika tarcia (uśrednionego) w ruchu wahadłowym opiera się na pomiarze energii potencjalno-kinetycznej. Zaproponowana metoda cechuje się krótkim czasem prowadzenia pomiarów oraz umożliwia szybkie porównanie współczynników tarcia dla różnych materiałów.

INTRODUCTION

The ever-increasing requirements for modern design solutions and the more and more common use of polymeric materials in technical applications imply the need for a thorough knowledge of such materials. Some polymer materials have excellent sliding properties, whereby they can be used in tribological assemblies **[L. 1]**. Hence, they are a popular subject of tribological (but not exclusively) research. According to Lawrowski's estimates, about 30% of the energy produced in the world is lost due to frictional resistance. Therefore, there is an understandable interest in tribological testing methods. According to other forecasts, in the next 30 years, some areas of tribology will lose their significance, while others will gain it **[L. 3]**. The efforts to minimize energy losses associated with the friction process in sliding pairs will remain unchanged, mainly because of the technological progress and the care for the natural environment as well as for economic reasons.

Friction testing devices are divided primarily according to the geometry of the contact between the specimen and the counter-body. The basic types of contact are cylinder-plane (e.g., T-07), plane-surface (the T-01M pin-on-disc tester), ball-ball (the four-ball apparatus), and ball-plane (various scratch resistance testing devices). Tribometers are usually used to measure the friction force, the wear of friction joint elements, and the friction path length and velocity **[L. 4]**. Special tribometers are designed for specialized applications, such as hip joint endoprosthesis testing **[L. 5–7]**. Universal devices enabling testing in complex motion (e.g., rolling and sliding) conditions are popular **[L. 8]**. They are used to test the friction parameters of materials in various material pairs at different

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friction velocities, under different loads, and so on **[L. 9]**. They also enable one to control the motion parameters of a material pair in a wide range **[L. 10]**.

This paper presents a method of determining the frictional resistance of metal-polymer sliding pairs in pendular motion by means of a physical pendulum. Using this method, one can quickly compare the frictional resistances of different materials and determine the averaged friction coefficient value.

MATERIALS AND METHODS

Using the designed measuring stand, one can measure the energy dissipated in pendular motion by comparing pendulum swing angles in successive motion cycles. Assuming that all the dissipated energy was used to cover the friction losses and knowing the friction path length, one can determine the frictional resistance (the friction moment) and so the averaged friction coefficient value. As the idea was to maximally simplify the measuring stand design, the latter is not equipped with additional electronic sensors (e.g., force or velocity sensors); therefore, it is more reliable and resistant to damage. The only electronic component is an incremental encoder with a resolution of 1000 imp/rev.

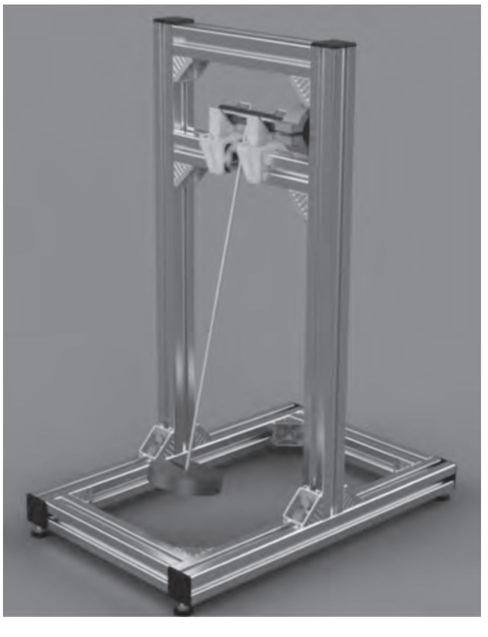

Fig. 1. Model of measuring stand used to develop test methodology
Rys. 1. Model stanowiska pomiarowego wykorzystanego do opracowania metodologii badań

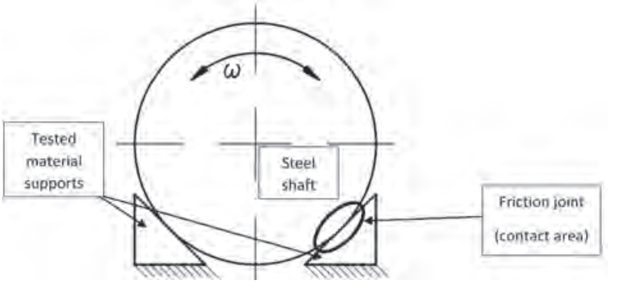

Fig. 2. Schematic of friction joint in considered stand
Rys. 2. Schemat węzła tarcia w rozpatrywanym stanowisku

As the pendulum moves, both the velocity in the contact zone and the pressure distribution change. One should note that motion energy dissipation results not only from friction in the area where the axles are in contact with the supports, but also from the air resistance of the moving weight suspended from the string.

Two basic types of pendulums are distinguished in mechanics:

- Mathematical – a material particle moving along a circle in the vertical plane in a homogeneous gravitational field, and
- Physical – a rigid body suspended from a fixed horizontal axis in a homogeneous gravitational field.

The equation of motion of a mathematical pendulum is as follows:

$$\theta(t) = \theta_0 \sin(\omega_0 t + \varphi), \tag{1}$$

where:
θ_0 – the vibration amplitude,
$\omega_0 = \sqrt{\frac{g}{l}}$ – the circular frequency of vibrations $\left[\frac{1}{s}\right]$,
φ – the initial phase of vibration.

However, the above equation is valid for only small pendulum deflections (where sinα ≈ α). The general formula for the vibration period of a mathematical pendulum for any vibration amplitude has this form:

$$T(\theta_0) = 2\pi\sqrt{\frac{l}{g}}\sum_{n=0}^{\infty}\left[\left(\frac{(2n)!}{(2^n \times n!)^2}\right)^2 \times \sin^{2n}\left(\frac{\theta_0}{2}\right)\right] \tag{2}$$

The equation of the motion dynamics of a physical pendulum is as follows:

$$I_z\ddot{\varphi} = -mgh\sin\varphi \tag{3}$$

The period of vibrations for small deflections is expressed as follows:

$$T = 2\pi\sqrt{\frac{l_{red}}{g}} = 2\pi\sqrt{\frac{I_z}{mgh}} \tag{4}$$

It can be assumed that to each physical pendulum vibrating around given point O with period T corresponds to a mathematical pendulum with length L0, which is also vibrating with the same period T.

Harmonic damping consists in the vibration amplitude diminishing over time, and it is connected with energy losses in vibrating systems. The general equation for a damped harmonic oscillator has the following form:

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0, \tag{5}$$

where: β – the damping factor.

The damping factor is directly proportional to the damping decrement. A parameter referred to as the damping power factor is also known:

$$\zeta = \frac{\beta}{\omega_0} \tag{6}$$

Depending on the values assumed by the damping power factor, three types of damping can be distinguished:

- Strong damping ($\zeta > 1$) – the system does not vibrate, striving for an equilibrium position;
- Critical damping ($\zeta = 1$) – the system does not vibrate, striving to reach the equilibrium in the shortest time possible; and,
- Poor damping ($0 < \zeta < 1$) – the system vibrates and the vibration amplitude decreases exponentially.

Taking into consideration the above equations (1–5) and the assumed simplicity of testing (yielding a small amount of data), it was decided to develop two methods of analysing the results of tests carried out on the measuring stand. When an encoder is used for measuring, only the value of the pendulum deflection angle over time is measured. However, it is possible to take measurements using an angular ruler and observing the maximum pendulum deflection angles in successive periods.

Measurement based on dissipated energy

Measurement based on dissipated energy consists in comparing the potential energy before the measurement and after a cycle of several pendulum motions.

$$E = m \times g \times \Delta h, \tag{7}$$

where:
m – the mass of the pendulum [kg],
g – the acceleration of gravity $\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$,
Δh – the change in the height of the pendulum’s centre of gravity [m].

In the equilibrium position, the height of the pendulum’s centre of gravity is equal to the difference between the position of the pendulum’s axis of rotation (a) and its length (l). For a pendulum deflected from the equilibrium position by a known angle α, the height is equal to the following:

$$h = a - (l \times \cos\alpha) \tag{8}$$

A simplification, consisting in omitting the weight of the tendon from which the load is suspended, is adopted here. The tendon’s weight is negligible in comparison with the weight suspended from it, and so its influence on the position of the centre of gravity can be neglected. The change in the energy of the pendulum is expressed by the following formula:

$$\Delta E = m \times g \times l(\cos\alpha_1 - \cos\alpha_2) \tag{9}$$

In order to determine the friction path, the following transformed formula for the length of the arc of a circle was used:

$$s = \frac{3}{2} \times k \times (\alpha_0 + \alpha_k) \times r, \tag{10}$$

where:
k – the number of periods of oscillation of the pendulum,
r – the radius of the cylinder constituting the pendulum’s axis [m],
$\alpha_0 + \alpha_k$ – the pendulum’s initial and final swing angle [rad].

Finally, the formula for the value of the friction coefficient assumes the following form:

$$\mu = \frac{\Delta E}{m \times g \times \cos\gamma \times s \times n} = \frac{l(\cos\alpha_0 - \cos\alpha_k)}{\cos\gamma \times s \times n} \quad (11)$$

where:
m – the mass of the pendulum [kg],
g – the acceleration of gravity $\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$,
γ – the angle of inclination of the support prism [rad],
s – the length of the friction path [m],
n – the number of contact nodes.

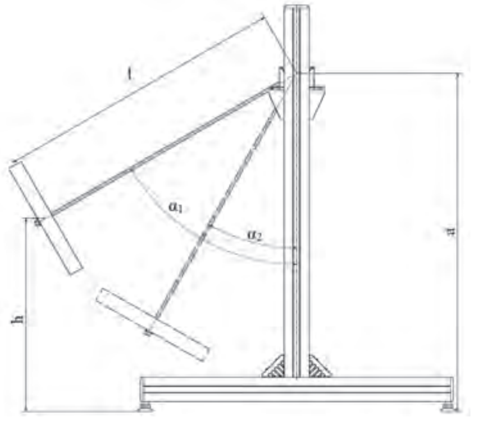

Fig. 3. Schematic of stand with key dimensions marked
Rys. 3. Schemat stanowiska z zaznaczonymi kluczowymi wymiarami

Measurement based on damping decrement

A damping decrement is a ratio of two successive amplitudes in the damped motion. Also a parameter called the logarithmic decrement of damping, which is the natural logarithm of a damping decrement, is often used.

$$\delta = \frac{A_n}{A_{n+1}} \quad (12)$$

$$\Lambda = ln\left(\frac{A_n}{A_{n+1}}\right) \quad (13)$$

In the case of harmonic vibrations, the value of the damping decrement does not change over time. Thus, in order to determine the value of the damping decrement, it is enough to know the amplitudes of the mth and nth vibration of the system. In this case, the formula assumes the following form:

$$\Lambda = \frac{1}{m-n} ln\left(\frac{A_n}{A_m}\right) = ln\frac{Ae^{(-\beta t)}}{Ae^{[-\beta(t+T)]}} = ln\left(e^{\beta t}\right) = \beta t \quad (14)$$

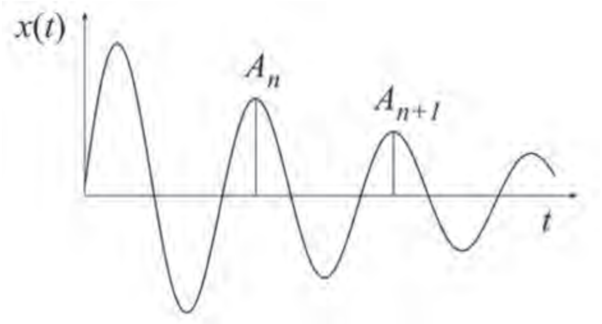

Fig. 4. Deflection (x) versus time (t) for decaying vibrations, with marked amplitudes
Rys. 4. Schemat węzła tarcia w rozpatrywanym stanowisku

Knowing the damping decrement, one can determine the difference in potential energy in the motion of the pendulum. By transforming the formulas, one obtains the following formula for the value of the friction coefficient:

$$\mu = \frac{l\left(\cos\alpha_1 - \cos\left(e^{-\Lambda + \ln\alpha_1}\right)\right)}{\cos\gamma \times s \times n} \quad (15)$$

CONCLUSIONS

Combined with the stand for testing friction in pendular motion, the proposed measuring method enables a very quick comparison of the values of frictional resistance of various materials. In terms of tribology, the stand does not enable the precise measurement of the value of the static or kinetic friction coefficient, nor does it enable assigning a specific pressure value in the contact zone. The determined value of the friction coefficient is averaged for the following:

- A variable slip speed – the slip speed is maximum when the pendulum passes through the equilibrium point and equal to zero for extreme deflections; and,
- A variable pressure value in the contact zone – when the pendulum deflects to one side, the pressure force acting on the support from the opposite side increases, while the pressure acting on the support from the deflection side decreases.

Despite certain limitations of the stand and the testing method, their greatest advantage is the speed and simplicity of testing sliding materials. When the resistances of different sliding materials are to be compared within an averaged range in a short time, the pendulum test is irreplaceable.

REFERENCES

1. Wieleba W.: Bezobsługowe łożyska ślizgowe z polimerów termoplastycznych, 2013.
2. Lawrowski Z.: Tribologia: tarcie, zużywanie i smarowanie, Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław 2009.
3. Spikes H.: Tribology research in the twenty-first century, “Tribol.” Int. 2001, 34, pp. 789–799, https://doi.org/10.1016/S0301-679X(01)00079-2.
4. Capanidis D., Kowalewski P., Tadeusz L., Paszkowski M., Wieleba W.: Rola badań tribologicznych w aspekcie zwiększania trwałości i niezawodności eksploatacyjnej maszyn i urządzeń użytkowanych w Zagłębiu Miedziowym, „Zesz. Nauk. DWSPiT. Stud. z Nauk Tech.” 2015, 4, pp. 47–64.
5. Wiśniewski T., Mróz A., Magda J., Wielowiejska-Giertuga A.: Stand for tribological testing of hip endoprotheses, “Tribologia” 2016, pp. 167–177.
6. Wiśniewski T., Wielowiejska-Giertuga A., Rubach R., Łapaj Ł., Magda J.: Research on Friction Wear of Hip Joint Endoprostheses of Polyethylene-Metal Material Combination on Hip Joint Simulator, “Tribologia” 2018, 281, pp. 153–158, https://doi.org/10.5604/01.3001.0012.7668.
7. Gierzyńska-Dolna M., Rybak T., Wendland J., Wiśniewska-Weinert H.: Metodyka badań materiałów i elementów endoprotez stawu biodrowego w instytucie obróbki plastycznej (Methodology of Materials and Hip Endoprosthesis Components Investigations in Inop), “Tribologia” 2010, 4, pp. 107–116.
8. Kowalewski P., Wieleba W., Leśniewski T.: Stanowisko do badań tribologicznych w złożonym ruchu cyklicznym toczno-ślizgowym, „Tribologia” 2007, 2, pp. 303–311.
9. Capanidis D.: Selected aspects of the methodology of tribological investigations of polymer materials, “Arch. Civ. Mech. Eng.” 2007, 7, pp. 39–55, https://doi.org/10.1016/S1644-9665(12)60224-6.
10. Kowalewski P.: Tarcie polimerów termoplastycznych w warunkach złożonego ruchu, Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław 2019.