

## TRIAxIAL BLOCK OF NAVIGATION ACCELEROMETER'S METROLOGICAL MODEL AND EXPERIMENTAL DETERMINATION OF ITS PARAMETERS ON THE UNIAXIAL ROTARY STAND

MYKOLA G. CHERNYAK, VICTOR A. PALYUSHOK, IRYNA M. BARANOVSKA

*Faculty of Aviation and Space Systems, National Technical University of Ukraine "Kyiv Polytechnic Institute",  
Peremogy av., 37, build. 28, Kyiv, Ukraine, [chernyak\\_dk@ukr.net](mailto:chernyak_dk@ukr.net), [viktor.palyushok@gmail.com](mailto:viktor.palyushok@gmail.com), [laifa@mail.ru](mailto:laifa@mail.ru)*

### Abstract

*Metrology model of triaxial navigation accelerometer unit as a vector transmitter, physical and mathematical foundations of a new method for its calibration on uniaxial rotary stand are considered.*

*The aim is to determine the coefficients of metrology model for triaxial accelerometer unit by the method mentioned above.*

*To ensure the identifiability of all coefficients of the metrological model, two installations of accelerometer unit on a platform of uniaxial rotary stand were introduced. Identification models for metrology model of triaxial accelerometer unit were produced.*

*Experimental validation of the obtained identification coefficients models was carried out. The ability to calibrate a triaxial accelerometer navigation unit with the given accuracy was confirmed.*

*Keywords: three-axis accelerometers navigation unit, calibration, metrology model.*

### INTRODUCTION

Triaxial block of navigation accelerometers (BA) within the inertial navigation system (INS) moving object converts an imaginary projection of the linear acceleration of the object to its measuring axis (MA) in the output electrical signals of three accelerometers (AC) established in BA. The calculation of the numerical values of these projections is in INS on metrological model (MM) BA, numerical values of individual factors pre-determined by the desired precision calibration of BA in the laboratory for appropriate accuracy Stand equipment (Lawrence, 2004) [1].

Traditionally, as described in articles written by Taranovsky, Yakovlev (2006) and Won, Golnaraghi (2009) [2, 3], during the initial calibration of BA as setpoint of test angular positions (TP) to the plane of the local horizon (PLH) precision (error less than 3...5 second of arc) biaxial (BRS) or triaxial (TRS), rotary stand that have a very high cost and need almost ideal laboratory conditions of use, which are only the manufacturers of BA are applied. This requires consuming



By this model in INS calculated imaginary projection acceleration on the BA MA by matrix expression

$$\begin{bmatrix} \hat{a}_x \\ \hat{a}_y \\ \hat{a}_z \end{bmatrix} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}^{-1} \begin{bmatrix} \hat{U}_x - U_{0x} \\ \hat{U}_y - U_{0y} \\ \hat{U}_z - U_{0z} \end{bmatrix} = \begin{bmatrix} N_{xx} & N_{xy} & N_{xz} \\ N_{yx} & N_{yy} & N_{yz} \\ N_{zx} & N_{zy} & N_{zz} \end{bmatrix} \begin{bmatrix} \hat{U}_x - U_{0x} \\ \hat{U}_y - U_{0y} \\ \hat{U}_z - U_{0z} \end{bmatrix}. \quad (2)$$

Model (1) contains 12 individual parameters of BA – three biases AC (ZB)  $U_{0i}(i = x,y,z)$ , three gain coefficients (GC)  $K_{ii}(i = x,y,z)$ , and six of its cross-axis coefficients (CAC)  $K_{ij}(i, j = x,y,z, i^1 j)$ , which are related in Fig. 1.b small angles  $\alpha_{ij}(i, j = x,y,z)$  BA MA and AC by formula  $K_{ij} = (-1)^n K_{ii} \alpha_{ji}$  where  $n = 2$ , if the index  $j$  follows index  $i$  in their cyclical permutation  $x \rightarrow y \rightarrow z$ , and  $n = 1$ , if this doesn't take place.

The numerical values of these coefficients are determined by BA calibration.

**Triaxial accelerometers navigation unit calibration method**

As a basis we take uniaxial navigation AC calibration method on precision URS by turning in the gravitational field of the Earth presented in the work by Chernyak, Hazynedarlu (2009) [6]. Based on the recommendations in this paper, similar to the AC, we choose eight point calibration test with three passing eight test point of BA relative PLH, located on the URS platform as shown in Fig. 2 [own elaboration]. Relevant BA TP defined by angle URS shaft  $\varphi_{l+1} = \varphi_l + \Delta\varphi$  (where  $j_1 = j_9 = 0, \Delta\varphi = 45^\circ, l = 1,2...9$  – test point number that is given) which is measured from the URS horizontal position.

Using two installations (U1 and U2) shown in Fig. 2, BA on a URS platform needed to provide observability of all 12 BA MM coefficients, because if the BA calibration using only in the U1 installation, it is impossible to determine the MM coefficients  $K_{xy}, K_{yy}$  and  $K_{zy}$  by using measurements results of its output signals in the aforementioned TP, also if BA is set only in the installation U2,  $K_{xz}, K_{yz}$  and  $K_{zz}$  coefficients are not determined.

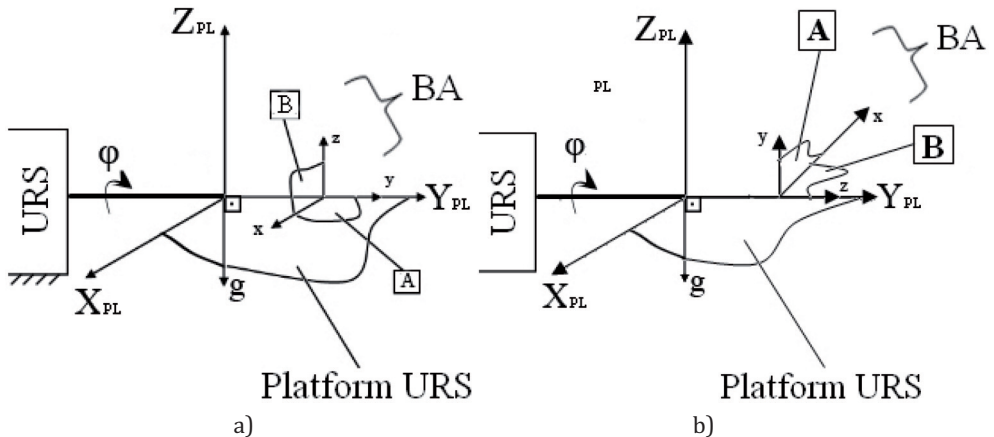


Fig. 2. BA installation U1 (a) and installation U2 (b) on URS platform:  $Ox_{PL}y_{PL}z_{PL}$  – coordinate system which is linked to URS platform on which BA installed;  $OXYZ$  – coordinate system bound according to Fig. 1, with BA MA;  $\vec{g}$  – Vector of gravity [Chernyak, Palyushok, 2014]

According to the BA model (1), BA output signals in specified TP, according to Fig. 2, depends on the projection vector of gravity  $\vec{g}$  on the BA MA. To find a mathematical expression for calculating these projections, consider BA MA orientation relative PLH, shown in Fig. 3 [own elaboration], rotated to angle  $\varphi$  by URS, and relatively vector  $\vec{g}$ , with the following components of small angular error installing BA in appropriate TP – regular (which are the same in each TP) errors  $\alpha_1$  and  $\alpha_2$  of initial leveling URS platform, error  $\alpha_3$  of BA installing on URS platform, and random error  $\alpha_4$  of set the angle rotation for URS shaft. Taking into account these errors is necessary to obtain further mathematical model of BA instrumental calibration errors due to this method.

According to Fig. 3 desired projection vector of gravity on BA MA at BA location in installation U1 is defined by matrix equation (3), while BA in installation U2 – expression (4)

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos \varphi - \alpha_4 \sin \varphi & \alpha_3 - \alpha_1 \sin \varphi & \sin \varphi + \alpha_4 \cos \varphi \\ -\alpha_3 \cos \varphi & 1 & \alpha_1 - \alpha_3 \sin \varphi \\ -\sin \varphi - \alpha_4 \cos \varphi & -\alpha_1 \cos \varphi & \cos \varphi - \alpha_4 \sin \varphi \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}, \quad (3)$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} -\cos \varphi + \alpha_4 \sin \varphi & \alpha_1 \sin \varphi - \alpha_3 & -\sin \varphi - \alpha_4 \cos \varphi \\ -\sin \varphi - \alpha_4 \cos \varphi & -\alpha_1 \cos \varphi & \cos \varphi - \alpha_4 \sin \varphi \\ -\alpha_3 \cos \varphi & 1 & \alpha_1 - \alpha_3 \sin \varphi \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}. \quad (4)$$

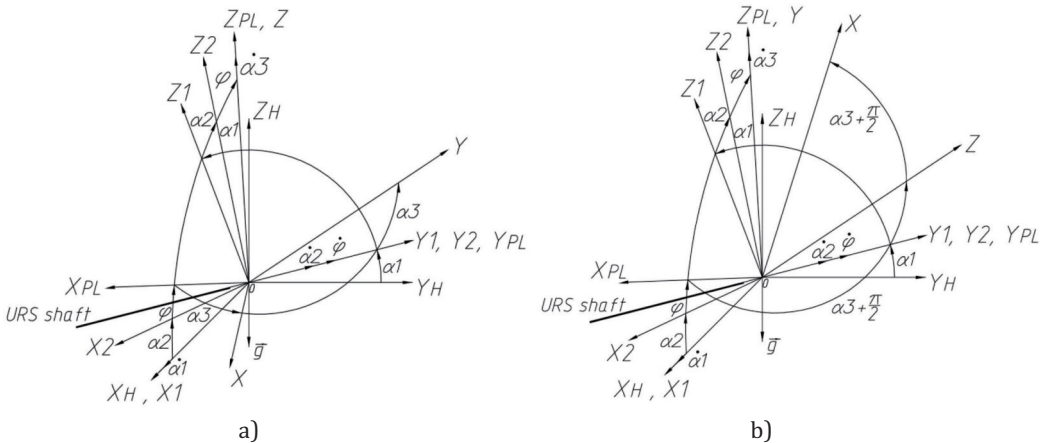


Fig. 3. Communications between the coordinate system  $OX_H Y_H Z_H$ , which is associated with PLH ( $\vec{g} \wedge OX_H Y_H, \vec{g} \parallel OZ_H$ ), and linked to BA MA coordinate system  $OXYZ$  while BA to installation U1 (a) and installation U2 (b) [Chernyak, Palyushok, 2014]

Substitution of projections vector  $\vec{g}$  on the BA MA established to appropriate TP, calculated by formulas (3) and (4), to metrological model (1) can get expressions for the BA output signals

$U_{mii}$  (where  $m = 1,2$  - BA installation number according to Fig. 2,  $i = x,y,z$  - signs AC installed in BA,  $l = \overline{1..9}$  - number TP) in this TP. These expressions are quite numerous (54 terms) and large, therefore, as an example, we give only the formula for the BA output signals in test positions TP3 and TP7 installation U1.

$$\begin{cases} U_{1x3} = K_{xx}g + K_{xy}g(\alpha_1 - \alpha_3) - K_{xz}\alpha_{\dot{a}}g + U_{0x} \pm \overset{\circ}{\Delta}U_M \\ U_{1y3} = K_{yx}g + K_{yy}g(\alpha_1 - \alpha_3) - K_{yz}\alpha_{\dot{a}}g + U_{0y} \pm \overset{\circ}{\Delta}U_M, \\ U_{1z3} = K_{zx}g + K_{zy}g(\alpha_1 - \alpha_3) - K_{zz}\alpha_{\dot{a}}g + U_{0z} \pm \overset{\circ}{\Delta}U_M \end{cases} \quad (5)$$

$$\begin{cases} U_{1x7} = -K_{xx}g + K_{xy}g(\alpha_1 + \alpha_3) + K_{xz}\alpha_{\dot{a}}g + U_{0x} \pm \overset{\circ}{\Delta}U_M \\ U_{1y7} = -K_{yx}g + K_{yy}g(\alpha_1 + \alpha_3) + K_{yz}\alpha_{\dot{a}}g + U_{0y} \pm \overset{\circ}{\Delta}U_M, \\ U_{1z7} = -K_{zx}g + K_{zy}g(\alpha_1 + \alpha_3) + K_{zz}\alpha_{\dot{a}}g + U_{0z} \pm \overset{\circ}{\Delta}U_M \end{cases} \quad (6)$$

where  $\alpha_{\dot{a}} = \alpha_2 \pm \alpha_4$ ;  $\overset{\circ}{\Delta}U_M$  - BA output signals random measurement error.

Having reviewed the relevant linear combinations of all the above derived expressions for the BA output signals (for example, difference  $U_{1x3} - U_{1x7}$  to determine GC  $K_{1xx}$  in BA installation U1) and received zero all components of the previously entered assignment BA error in the appropriate TP ( $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ ) and the measurement error of its output signals  $\overset{\circ}{\Delta}U_M$ , one can get formula for determining the coefficients of model (1) in BA installations U1 and U2. (in these formulas, the first index in the corresponding coefficients of the model (1) determine the number of BA installation on a URS platform according to Fig. 2)

$$\begin{aligned} U_{10x} &= \frac{1}{4}(U_{1x1} + U_{1x9} + 2U_{1x5}), U_{10z} = \frac{1}{2}(U_{1z3} + U_{1z7}), K_{1xz} = \frac{1}{4g}(U_{1x1} + U_{1x9} - 2U_{1x5}), \\ K_{2xx} &= \frac{-1}{2(1+\sqrt{2})g}[U_{2x3} - U_{2x7} + U_{2x2} + U_{2x4} - U_{2x6} - U_{2x8}], U_{20y} = \frac{1}{2}(U_{2y3} + U_{2y7}), \\ K_{1xx} &= \frac{1}{2(1+\sqrt{2})g}[U_{1x3} - U_{1x7} + U_{1x2} + U_{1x4} - U_{1x6} - U_{1x8}], K_{1yx} = \frac{1}{2g}(U_{1y3} - U_{1y7}), \\ K_{2xy} &= \frac{1}{4g}(U_{2x1} + U_{2x9} - 2U_{2x5}), K_{2zy} = \frac{1}{4g}(U_{2z1} + U_{2z9} - 2U_{2z5}), U_{20z} = \frac{1}{8} \overset{\circ}{\Delta} U_{2zi}, \end{aligned}$$

$$\begin{aligned}
K_{1zx} &= \frac{1}{2g}(U_{1z3} - U_{1z7}), K_{1yz} = \frac{1}{4g}(U_{1y1} + U_{1y9} - 2U_{1y5}), K_{2zx} = \frac{1}{2g}(U_{2z7} - U_{2z3}), \\
K_{1zz} &= \frac{1}{8g}[U_{1z1} + U_{1z9} - 2U_{1z5} + \sqrt{2}(U_{1z2} + U_{1z8} - U_{1z4} - U_{1z6})], U_{10y} = \frac{1}{8} \sum_{i=1}^8 U_{1yi}, \\
K_{2yy} &= \frac{1}{8g}[U_{2y1} + U_{2y9} - 2U_{2y5} + \sqrt{2}(U_{2y2} + U_{2y8} - U_{2y4} - U_{2y6})], \\
U_{20x} &= \frac{1}{4}(U_{2x1} + U_{2x9} + 2U_{2x5}), K_{2yx} = \frac{1}{2g}(U_{2y7} - U_{2y3}). \tag{7}
\end{aligned}$$

From (7) we can write the following final expressions for the BA coefficients of model (1) from measurements of its output signals in all TP, who subsequently we assume as mathematical model of determining this coefficient by developed BA calibration method.

$$\begin{aligned}
K_{xx} &= 0,5(K_{1xx} + K_{2xx}), K_{xy} = K_{2xy}, U_{0x} = 0,5(U_{10x} + U_{20x}), K_{zy} = K_{2zy}; \\
K_{yx} &= 0,5(K_{1yx} + K_{2yx}), K_{zx} = 0,5(K_{1zx} + K_{2zx}), U_{0z} = 0,5(U_{10z} + U_{20z}), \\
U_{0y} &= 0,5(U_{10y} + U_{20y}), K_{zz} = K_{1zz}, K_{yy} = K_{2yy}, K_{yz} = K_{1yz}, K_{xz} = K_{1xz}. \tag{8}
\end{aligned}$$

Formulas (8) are obtained analytically without any approximations, so their use is no methodological error identification coefficients of model (1). The identification instrumental error sources of model (1) coefficients in these formulas, of the above error  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ , and  $\Delta U_M$  of bench equipment used for BA calibration. A mathematical model of the instrumental error can be obtained by expressions (5...8).

### Experimental quality verification of developed BA calibration method

Experimental quality verification of the BA developed calibration method on URS is made by comparing calculated by BA model (2) (which is pre-calibrated by the developed method and installed on a fixed basis in an arbitrary position relative to PLH) module of the vector of gravity

$$g_p = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{9}$$

and the true value of the module in place BA calibration. The difference between these values  $\Delta g$ , as determined by identification instrumental errors of the BA coefficients model (1), by its calibration, is considered to be integrated as a BA calibration criterion described by authors Taranovsky, Yakovlev (2006) and Chernyak, Gryshchenko (2011) [2, 5].

If this difference satisfies the requirement

$$\Delta g = g_p - g_i \leq \Delta g_A, \tag{10}$$

where  $\Delta g_A$  – permissible under condition of maintenance the specified accuracy BA calibration calculation error module  $\bar{g}$  for model (2) by the BA output signal, we will assume that criterion as a BA calibration performed and developed BA calibration method so that it provides the required accuracy of calibration. Regarding the numerical values of the error  $\Delta g_A$  of BA, for example, BA of GINS in launcher “Cyclone-4” this value (approximately) is  $(3\sigma) \Delta g_A \approx \pm 3 \cdot 10^{-4}g$ .

Stand equipment necessary to BA calibration by developed method is shown in Fig. 4 [own elaboration].

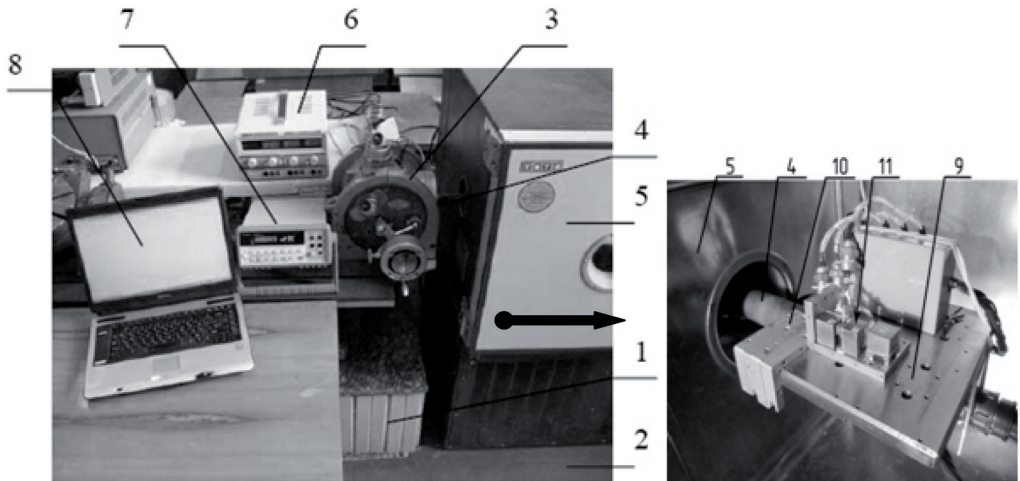


Fig. 4. Stand for BA calibration: 1 – foundation; 2 – building; 3 – dividing head; 4 – dividing head shaft; 5 – heat chamber; 6 – power supply; 7 – voltmeter; 8 – PC; 9 – dividing head platform; 10 – angle to install BA on platform of dividing head; 11 – BA that will be calibrated, mounted on platform 9 by Fig. 2.a. [Chernyak, Palyushok, 2014]

The equipment includes: foundation 1 with the basics daily fluctuations no more than 1..2 arcsec, unleashed from the building 2 in which conducts research; URS 3 – optic dividing heads ODH-10 (ODH) which provides task accuracy rotation angle of the shaft 4  $\alpha_4 = \pm 10$  arcsec; bubble level (not shown in the figure), which provides an initial ODH platform 9 levelling with errors  $\alpha_1 = \alpha_2 = \pm 5$  arcsec; L-steel 10 through which the BA is set at ODH platform according to Fig. 2 with an error about the  $\alpha_3 = \pm 90$  arcsec; heat chamber 5 type TWT-2 provides tasks temperature error throughout the range of operating BA temperatures (necessary to determine the temperature dependence of the BA coefficient model (1)); precision multichannel voltmeter 7 such as Agilent 34401A, voltmeter accuracy class 0.003% of the sub-band measurements  $\pm 10V$ .

Result of BA calibration, shown in Fig. 1, by the developed method, is following coefficient model (1):

$$\begin{aligned}
 &K_{xx} = -2,177087 \text{ V/g}; K_{yy} = -2,139892 \text{ V/g}; K_{zz} = -2,144367 \text{ V/g}; \\
 &K_{xy} = -0,018243 \text{ V/g}; K_{xz} = 0,025027 \text{ V/g}; K_{yx} = 0,026542 \text{ V/g}; \\
 &K_{yz} = 0,032755 \text{ V/g}; K_{zx} = 0,006754 \text{ V/g}; K_{zy} = 0,006026 \text{ V/g}; \\
 &U_{0x} = 0,071469 \text{ V}; U_{0y} = -0,196485 \text{ V}; U_{0z} = -0,411789 \text{ V}.
 \end{aligned}
 \tag{11}$$

Comprehensive experimental verification of BA quality calibration by developed method was performed by determining error  $\Delta g$  according to the formula (10), the value  $g_p$  calculated by the formula (9), which components have been determined by the model (2), by using BA coefficients given in (11) of model (1). For that BA was established on ODH platform as shown in Fig. 5 [own elaboration], then using ODH there were given the four verification points (VP) relative PLH by rotating the platform about an axis  $Y_{PL}$  at an angle  $\theta_{j+1} = \theta_j + \Delta\theta$  (here  $\theta_1 = 45^\circ$ ,  $\Delta\theta = 90^\circ$ ,  $j = 1, 4$  - number BA verification points).

Comprehensive experimental verification quality calibration method developed by BA was performed by determining the formula (10) error, provided that the value calculated by the formula (9), which included a pre-determined by the model (2) using the expressions given in (11) the coefficients of the model (1) asthma.

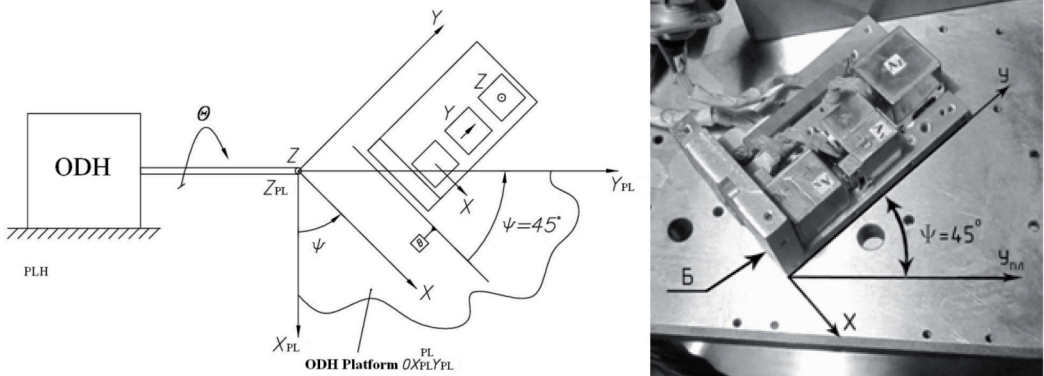


Fig. 5. Installing BA on ODH platform to verify the quality of its calibration:  $\psi$  - deviation angle of the BA axis  $OY$  from axis  $OY_{PL}$  of ODH platform [Chernyak, Palyushok, 2014]

In these positions voltmeter measured BA outputs to be used in the model (2) for the calculation  $g_{\delta}$  in BA verification points.

The results of the error calculation  $\Delta g$  in BA VP are shown in Fig. 6 [own elaboration].



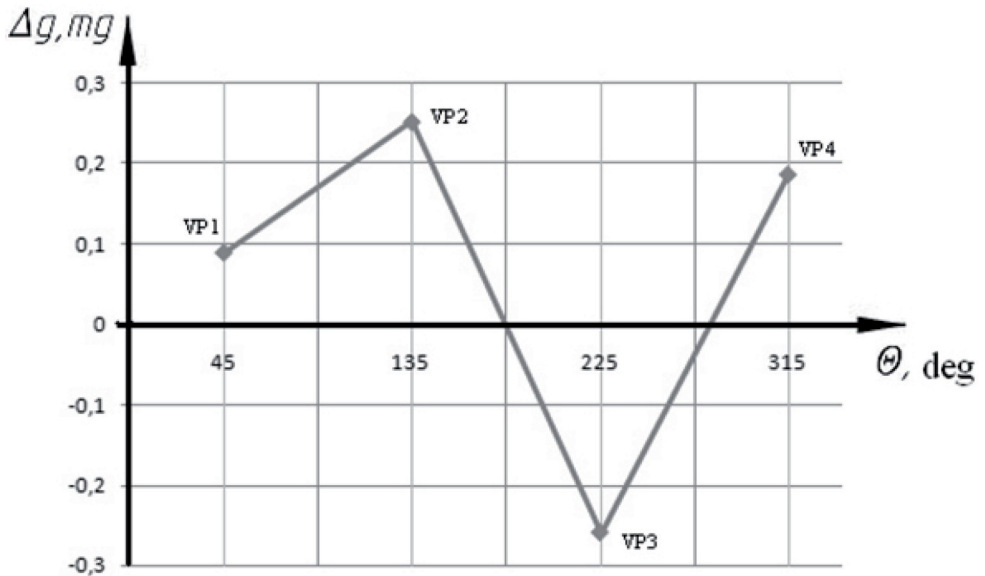


Fig. 6. The value of error calculation in BA verification points [Chernyak, Palyushok, 2014]

The experimental results show that the error calculation module  $\bar{g}$  for model (2) by the BA output signal that is pre-calibrated by the developed method, does not exceed the value  $\pm 3 \cdot 10^{-4}g$ , that shows in the high quality of the BA calibration and confirms the possibility of triaxial block of navigation accelerometers calibration on URS by the developed method with sufficiently high accuracy.

According to the authors, in this case the current dependence of the modulus error from VP number, i.e. from BA orientation relative to PLH, is occurred due to existing instrumental error of the coefficients BA model (1) sources which, as noted above, is the corresponding error  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$  and  $\mathcal{A}_M^0$  bench equipment that used for BA calibration.

CONCLUSIONS

It is confirmed that the proposed BA calibration method on URS provides the identification by expressions (8) all the coefficients of the linear MM (1) BA by the use of precision bench equipment with a guaranteed supply of precision.

Quality check BA calibration by a combined criterion (10) suggests only guaranteed BA suitability to measure the acceleration  $a \leq 1g$  with error which satisfies the condition (10). When measuring the another acceleration, the resulting error of this measure, could be significantly higher, since the numerical values of instrumental errors identify of each BA coefficients of the model (1) are unknown, which, as noted above, are determined by  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$  and  $\mathcal{A}_M^0$  bench equipment error.

Later, for the research development it is necessary to develop a mathematical model of identification errors for all coefficients of BA model (1) with its calibration by developed method, which will calculate the resulting measurement error of any acceleration in the range of BA measurement and make demands to bench equipment (to values errors  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$  and  $\overset{\circ}{X}_{M}$ ) based on the condition of the given value the resulting measurement error of any acceleration in the range of BA measurement.

#### LITERATURE

- [1] Lawrence, A. (2004). *Modern Inertial Technology: Navigation, Guidance and Control* – Springer-Verlag. New York, 280.
- [2] Taranovsky, G. O., Yakovlev, E. A. (2006). Calibration of the three-way accelerometer comprising an inertial navigation system / Taranovsky, G. O., Yakovlev, E. A. // Proceedings of the VIII Conference. Young Scientists „Navigation and Motion Control” – St. Petersburg: NRI „electrical appliance”, 29-37.
- [3] Won, S. P., Golnaraghi, F. (2009). *A triaxial accelerometer calibration using a mathematical model* // *IEEE transactions on instrumentation and measurement*, 8.
- [4] Vavilova, N. B., Parusnikov, N. A., Sazonov, I. Y. (2009). *Calibration of strapdown inertial navigation systems using a single-stage coarse stands* / Vavilov, N. B., Sailboats, N. A., Sazonov, I. Y. // *Modern Problems of Mathematics and Mechanics*, Volume I. - Moscow: Moscow State University, 212-222.
- [5] Chernyak, M. G., Gryshchenko, O. M. (2011). *Calibration of navigation pendulous accelerometer by testingrotating method in terrestrial gravitational field* // *Inform. system mechanics and control: scientific-technical. Collected.* - Vol. 7, 105-116.
- [6] Chernyak, M. G., Hazynedarlu, E. (2009). *Navyhatsyonnoho pendulum accelerometer calibration method testovih povorotov in hravytatsyonnom field land* / Chernyak, M. G., Hazynedarlu, E. // *Mechanics of gyroscopic systems.* - Nauchn. - Sc. collection. - Kyiv. - Vol. 20, 81-91.