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Identification of complex technical system components reliability models

Keywords

system reliability, multistate system, statistical identification, transportation systems

Abstract

There is presented the contents of the training course addressed to industry. The curriculum of the course includes the methods, algorithms and procedures for identification of the reliability models of components of the complex technical systems and their applications in practice. It is based on the theoretical backgrounds concerned with the semi-markov modeling of the complex technical systems operation processes, on the complex technical systems and their components multistate reliability models and on the statistical methods of identification of the complex technical system components reliability models. The illustrations of the proposed methods and procedures practical application in port and shipyard transportation are included.

1. Introduction

The training course is concerned with methods, algorithms and procedures of identification of the reliability models the complex technical systems and their application in practice and it is based on the results given in [7], [2] and [1]. The participants of the course are provided training materials and a disk with the computer program included in [6]. Presented at the training course examples of practical applications are coming from [8] and [4]-[5].
The training course includes the following items:

- Theoretical backgrounds based on [3]: basic notions of the system multi-state reliability analysis, definition of the conditional multi-state reliability function of the system components, definition of the conditional multi-state exponential reliability function of the system components, definition of the system components conditional intensities of departure from the reliability state subsets;
- Methodology of fixing the subsystems and components of the complex technical
- systems in various operation states on [7] and [2]: defining the system operation states,
- fixing the subsystems of the system operating in various operation states, fixing and describing the components of the subsystems operating in various operation states;
- Methodology of defining the parameters of the system components multi-state reliability models based on [3]: fixing the number of different reliability states of the system components, defining the reliability states of the system components, fixing the possible transitions between the system components reliability states, fixing the set of unknown parameters of the system components reliability models;
- Procedure of the system components reliability data collection based on [3]: In the case of data coming from experts, fixing the approximate mean values of the system components lifetimes in the reliability states subsets; In the case of data coming from the system components reliability state changing processes, fixing the following experiment

kinds: *Case 1*. Observations of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Completed investigations, the same observation time on all experimental posts; *Case 2*. Observations of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, the same observation time on all experimental posts; *Case 3*. Observations of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, different observation times on particular experimental posts; *Case 4*. Observations of the realizations of the component simple renewal flow (stream) on one experimental post; *Case 5*. Observations of the realizations of the component simple renewal flows (streams) on several experimental posts – The same observation time on all experimental posts; *Case 6*. Observations of the realizations of the component simple renewal flows (streams) on several experimental posts – Different observation times on experimental posts; fixing the experiments duration times, fixing the realizations of the component lifetimes up to the first departure from the reliability states subsets, fixing the numbers of the observed realizations of the component lifetimes up to the first departure from the reliability states subsets in *Cases 1-6*;

- Procedure of evaluating the unknown system component conditional intensities of departures from the reliability states subset based on [1]: *Case 1*. The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Completed investigations, the same observation time on all experimental posts; *Case 2*. The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, the same observation time on all experimental posts; *Case 3*. The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, different observation times on particular experimental posts; *Case 4*. The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flow (stream) on one experimental post; *Case 5*. The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – The same observation time on all experimental posts; *Case 6*. The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – Different observation times on experimental posts; The pessimistic estimations of the components intensities of departures from the reliability states subsets in all *Cases 2-6*;
- Procedure of identifying the system components conditional multi-state exponential reliability functions based on [1]: constructing and plotting the realization of the histogram of the system component conditional lifetime in the reliability states subset, analyzing the realization of the histogram, comparing the histogram realization with the graph of the exponential density function and in the case of their good conformity formulating the hypothesis concerning the exponential form of the system component conditional multi-state reliability function;
- Procedure of applying the computer program for identification of system components reliability models based on [6];
- Application of the procedures and computer program for identification of the reliability models of the components of real complex technical systems operating in variable conditions: identification of the reliability of the components of the oil piping transportation system based on [8], identification of the reliability of the components of the ship-rope elevator based on [4], identification of the reliability of the components of the ground ship-rope transportation system based on [5].

2. Theoretical backgrounds

In the multi-state reliability analysis of non-repairable systems to define the system ageing (degrading) components we assume that:

- E is a component of a system,
- a components E has the reliability state set $\{0,1,...,z\}$, $z \geq 1$,
- the reliability states are ordered, the state 0 is the worst and the state z is the best,
- $T(u)$ is a random variable representing the lifetime of component E in the state subset $\{u,u+1,...,z\}$, while it was in the state z at the moment $t = 0$,
- the component reliability states degrade with time t without repair,
- $e(t)$ is a component E state at the moment t, $t \in <0,\infty)$, given that it was in the state z at the moment $t = 0$.

The above assumptions mean that the states of the system degrading components may be changed in time only from better to worse (see: *Figure 1*).

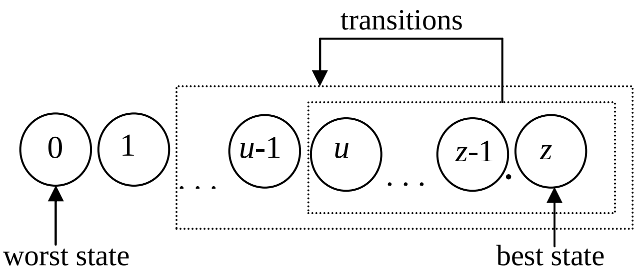

Figure 1. Illustration of reliability states changing of system ageing components

Under these assumption, a vector

$$R(t,\cdot) = [R(t,0),R(t,1),...,R(t,z)],\ t \in <0,\infty), \quad (1)$$

where

$$R(t,u) = P(e(t) \geq u \mid e(0) = z) = P(T(u) > t), \quad (2)$$

$$t \in <0,\infty),\ u = 0,1,...,z,$$

is the probability that the component E is in the state subset $\{u, u+1,..., z\}$ at the moment t, $t \in <0,\infty)$, while it was in the state z at the moment $t = 0$, is called the multi-state reliability function of a component E.

Particularly, for $u = 0$, in (1) and (2) we have

$$R(t,0) = P(e(t) \geq 0 \mid e(0) = z) = P(T(0) > t) = 1, \quad (3)$$

$$t \in <0,\infty).$$

We assume that the changes of operation states of the multistate system operation process $Z(t)$ have an influence on the reliability functions of the system components and we mark by $T^{(b)}(u)$ the conditional lifetime $T^{(b)}(u)$ of the system component in the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z.$ Consequently, we mark the conditional multistate reliability function of the system component when the system is in the operation state z_b, $b = 1,2,...,\nu$, by

$$[R(t,\cdot)]^{(b)} = [1,\ [R(t,1)]^{(b)},\ ...,\ [R(t,z)]^{(b)}\ ], \quad (4)$$

where

$$[R(t,u)]^{(b)} = P(T^{(b)}(u) > t \mid Z(t) = z_b) \quad (5)$$

for $t \in <0,\infty)$, $u = 1,2,..., z$, $b = 1,2,...,\nu$,

is the conditional reliability function standing the probability that the conditional lifetime $T^{(b)}(u)$ of the system component in the reliability states subset $\{u, u+1,..., z\}$ is greater than t, while the system operation process $Z(t)$ is in the operation state z_b, $b = 1,2,...,\nu$.

Further, we assume that the coordinates of the vector of the conditional multistate reliability function (4) are exponential reliability functions of the form

$$[R(t,u)]^{(b)} = \exp[-[\lambda(u)]^{(b)}t] \text{ for } t \in <0,\infty), \quad (6)$$

$$u = 1,2,..., z,\ b = 1,2,...,\nu.$$

Te above assumptions mean that the density function of the system component conditional life time $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z$, at the operation state z_b, $b = 1,2,...,\nu$, is exponential of the form

$$[f(t,u)]^{(b)} = [\lambda(u)]^{(b)} \exp[-[\lambda(u)]^{(b)}t] \quad (7)$$

for $t \in <0,\infty)$,

where $[\lambda(u)]^{(b)}$, $[\lambda(u)]^{(b)} \geq 0$, is an unknown intensity of departure from this subset of the reliability states.

3. Procedures of identification of complex technical system components reliability models

3.1. Methodology of fixing the subsystems and components of the complex technical systems in various operation states

To fix the subsystems and components of the system in various operation states, firstly, we should analyze the system operation process and to fix or to define its following general parameters:

- the number of the operation states of the system operation process ν,

- the operation states of the system operation process z_1, z_2, …, z_ν.

Next, we should do the following steps:

i) to fixing the subsystems of the system operating in particular operation states;

ii) to fix, to describe and to mark the components of the subsystems operating in particular operation states.

3.2. Methodology of defining the parameters of the system components multi-state reliability models

To make the estimation of the unknown parameters of the system components conditional multistate reliability functions the experiment delivering the necessary statistical data should be precisely planned.
Firstly, before the experiment, we should perform the following preliminary steps:

i) to analyze the processes of reliability states changing of all system components in different operation states;

ii) to fix or to define its following general parameters:

- the number of the reliability states of the system components z,

- the reliability states of the system components 0, 1, …, z;

iii) to fix the possible transitions between the system components reliability states;

iv) to fix the set of the unknown parameters of the system components reliability models.

3.3. Procedure of the system components reliability data collection

3.3.1. Data coming from experts

On the basis of the expert opinions the approximate values

$[\hat{\mu}(u)]^{(b)}$, $u = 1,2,...,z,\ \ b = 1,2,...,\nu,$

of the mean values

$[\mu(u)]^{(b)} = E[T(u)]^{(b)}$, $u = 1,2,...,z,\ \ b = 1,2,...,\nu,$

of the system components lifetimes $[T(u)]^{(b)}$, $u = 1,2,...,z,$ $b = 1,2,...,\nu,$ in the reliability states subsets $\{u, u+1,..., z\}$, $u = 1,2,...,z,$ while the system is operating in the operation state z_b, $b = 1,2,...,\nu,$ should be fixed.

3.3.2. Data coming from components reliability states changing processes

To estimate the unknown parameters of the system components multistate reliability models, during the experiment, we should collect necessary statistical data performing the following steps:

i) to fix the experiment kinds subjected to the defined below *Cases 1-6;*

ii) to fix and to collect, in *Cases 1-6*, the following statistical data necessary to evaluating the unknown intensity of departure from the reliability states subsets:

- the experiments duration times,

- the realizations of the component lifetimes up to the first departure from the reliability states subsets,

- the numbers of the observed realizations of the component lifetimes up to the first departure from the reliability states subsets.

The fixed kinds of the experiments and the collected statistical data are described below.

Case 1.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Completed

investigations, the same observation time on all experimental posts

We assume that during the time $\tau^{(b)}$, $\tau^{(b)} > 0$, we have been observing the realizations of the component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, at the operation state z_b, $b = 1,2,...,\nu$, on $n^{(b)}$ identical experimental posts. We assume that at the beginning of the experiment all components are new identical components staying at the best reliability state z and that during the fixed observation time $\tau^{(b)}$ all components have left the reliability states subset $\{1,2,...,z\}$, i.e. all observed components reached the worst reliability state 0 (*Figure 2*). It means that the number $m^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, is equal to $n^{(b)}$, i.e. $m^{(b)}(u) = n^{(b)}$, $u = 1,2,...,z$.

We mark by

$$A^{(b)}(u) = \{t_i^{(b)}(u) : i = 1,2,...,m^{(b)}(u)\},\ u = 1,2,...,z,$$

the set of the moments $t_i^{(b)}(u)$, $i = 1,2,...,m^{(b)}(u)$, $u = 1,2,...,z$, of departures from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, of the component on the $i-$th observational post, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,...,n^{(b)}$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (7).

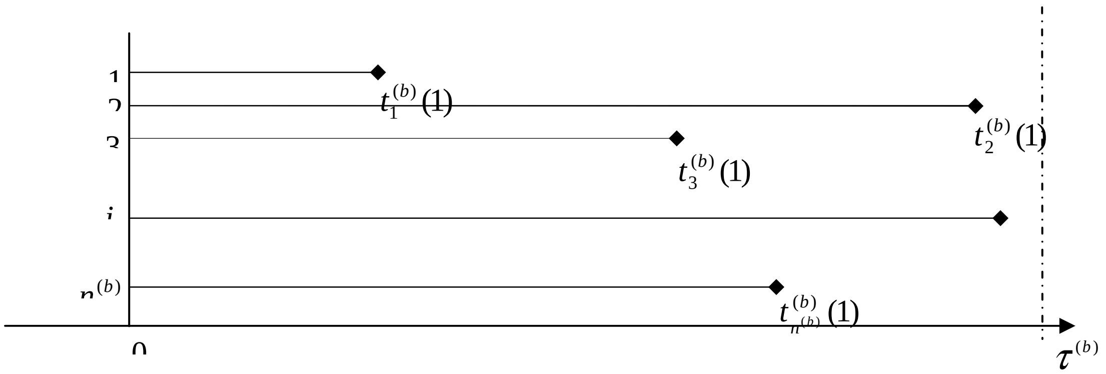

Figure 2. The scheme of the realizations of the component lifetimes up to the first departure from the reliability states subset on $n^{(b)}$ observational posts (completed investigations, the same observation time on all experimental posts)

Case 2.

The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, the same observation time on all experimental posts

We assume that during the time $\tau^{(b)}$, $\tau^{(b)} > 0$, we have been observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, at the operation state z_b, $b = 1,2,...,\nu$, on $n^{(b)}$ identical experimental posts. We assume that at the beginning of the experiment all components are new identical components staying at the best reliability state z and that during the fixed observation time $\tau^{(b)}$ not all components have left the reliability states subset $\{1,2,...,z\}$, i.e. $m^{(b)}$, $m^{(b)} < n^{(b)}$, observed components reached the worst reliability state 0 (*Figure 3*). It means that the number $m^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, is less or equal to $n^{(b)}$, i.e. $m^{(b)}(u) \le n^{(b)}$, $u = 1,2,...,z$.

We mark by

$$A^{(b)}(u) = \{t_i^{(b)}(u) : i = 1,2,...,m^{(b)}(u)\},\ u = 1,2,...,z,$$

the set of the moments $t_i^{(b)}(u)$, $i = 1,2,...,m^{(b)}(u)$, $u = 1,2,...,z$, of departures from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, of the component on the $i-$th observational post, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,...,n^{(b)}$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (7).

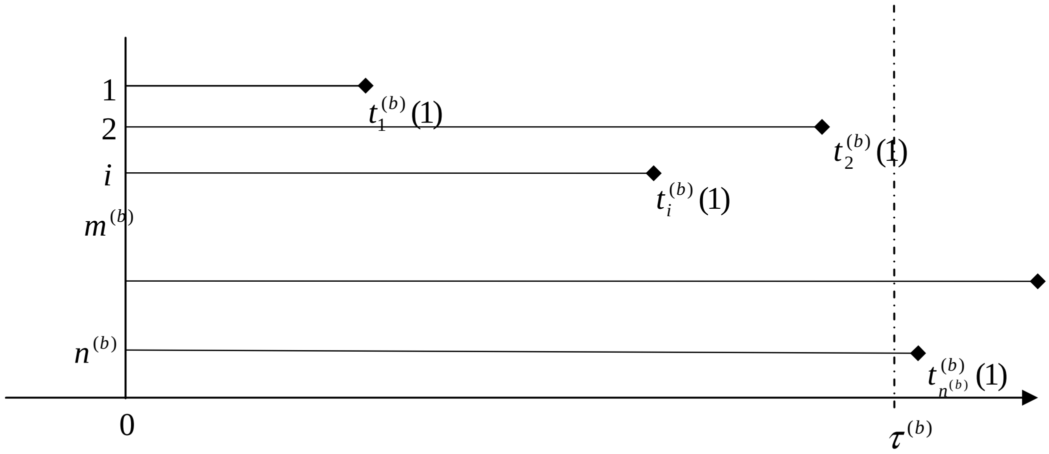

Figure 3. The scheme of the realizations of the component lifetimes up to the first departure from the reliability states subset on $n^{(b)}$ observational posts (non-completed investigations, the same observation time on all experimental posts)

Case 3.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, different observation times on particular experimental posts

We assume that we have been observing the realizations of the component lifetimes $T^{(b)}(u)$ in the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,..., z,$ at the operation state z_b, $b = 1,2,...,\nu$, on $n^{(b)}$ identical experimental posts. We assume that the observation times on particular experimental posts are different and we mark by $\tau_i^{(b)}$, $\tau_i^{(b)} > 0,$ $i = 1,2,..., n^{(b)}$, the observation time respectively on the *i*-th experimental post. We assume that at the beginning of the experiment all components are new identical components staying at the best reliability state z and that during the fixed observation times $\tau_i^{(b)}$ not all components have left the reliability states subset $\{1,2,..., z\}$, i.e. $m^{(b)}$, $m^{(b)} < n^{(b)}$, observed components reached the worst reliability state 0 (*Figure 4*). It means that the number $m^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,..., z,$ is less or equal to $n^{(b)}$, i.e. $m^{(b)}(u) \le n^{(b)}$, $u = 1,2,..., z$.

We mark by

$$A^{(b)}(u) = \{t_i^{(b)}(u) : i = 1,2,..., m^{(b)}(u)\},\ u = 1,2,..., z,$$

the set of the moments $t_i^{(b)}(u)$, $i = 1,2,..., m^{(b)}(u)$, $u = 1,2,..., z,$ of departures from the reliability states subset $\{u, u+1, ..., z\}$, $u = 1,2,..., z,$ of the component on the $i-$th observational post, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,..., n^{(b)}$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (7).

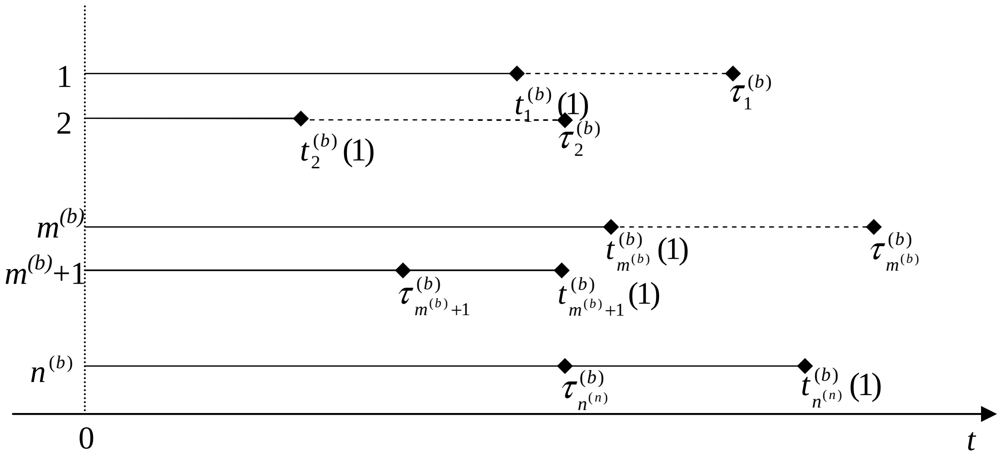

Figure 4. The scheme of the realizations of the component lifetimes up to the first departure from the reliability states subset on $n^{(b)}$ observational posts (non-completed investigations, different observation times on all experimental posts)

Case 4.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flow (stream) on one experimental post

We assume that during the time $\tau^{(b)}$, $\tau^{(b)} > 0$, we have been observing the realizations of the component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ at the operation state z_b, $b = 1,2,...,\nu$, on one experimental posts. We assume that at the moment when the component is leaving the reliability states subset $\{1,2,..., z\}$, i.e. the observed component reached the worst reliability state 0, it is replaced at once by the same new component staying at the reliability state z (*Figure 5*). It means that at the beginning all components are new identical components staying at the best reliability state z. We assume that during the fixed observation time $m^{(b)}$ components have left the reliability states subset $\{1,2,..., z\}$, i.e. $m^{(b)}$ observed components reached the worst reliability state 0. It means that the number $m^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ is equal either to $m^{(b)}$ or to $m^{(b)}+1$, i.e. $m^{(b)}(u) = m^{(b)}$ or $m^{(b)}(u) = m^{(b)}+1$, $u = 1,2,..., z$. We mark by

$$A^{(b)}(u) = \{t_i^{(b)}(u) : i = 1,2,..., m^{(b)}(u)\},\ u = 1,2,..., z,$$

the set of the moments $t_i^{(b)}(u)$, $i = 1,2,..., m^{(b)}(u)$, $u = 1,2,..., z,$ of departures from the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ of the component on the $i-$th observational post, i.e. the realizations of the identical component lifetimes $T_i^{(b)}(u)$, $i = 1,2,..., n^{(b)}$, to the first departure from the reliability states subsets, that are the independent random variables with the exponential distribution defined by the density function (7).

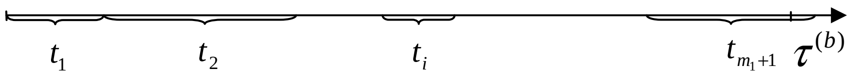

Figure 5. The scheme of the realizations of the component simple renewal flow (stream) on one experimental post

Case 5.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – The same observation time on all experimental posts

We assume that during the time $\tau^{(b)}$, $\tau^{(b)} > 0$, we have been observing the realizations of the component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,..., z,$ at the operation state z_b, $b = 1,2,...,\nu$, on $n^{(b)}$ experimental posts. We assume that, at each observation post, at the moment when the component is leaving the reliability states subset $\{1,2,...,z\}$, i.e. the observed component reached the worst reliability state 0, it is replaced at once by the same new component staying at the reliability state z (*Figure 6*). It means that, at each experiment post, at the beginning all components are new identical components staying at the best reliability state z. We assume that, at the j-th, $j = 1,2,...,n^{(b)}$, experimental post, during the fixed observation time $m_j^{(b)}$ components have left the reliability states subset $\{1,2,...,z\}$, i.e. $m_j^{(b)}$ observed components reached the worst reliability state 0. It means that the number $m_j^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ is equal either to $m_j^{(b)}$ or to $m_j^{(b)} + 1$, i.e. $m_j^{(b)}(u) = m_j^{(b)}$ or $m_j^{(b)}(u) = m_j^{(b)} + 1$, $u = 1,2,...,z$.

We mark by

$$A_j^{(b)}(u) = \{[t_i^{(b)}(u)]^{(j)} : i = 1,2,...,m_j^{(b)}(u)\},$$

$$u = 1,2,...,z, \quad j = 1,2,...,n^{(b)},$$

the sets of the times $[t_i^{(b)}(u)]^{(j)}$, $i = 1,2,...,m_j^{(b)}(u)$, to the components departures from the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ at the at the j-th, $j = 1,2,...,n^{(b)}$, experimental post, i.e. the realizations of the component lifetimes $T^{(b)}(u)$ to the first departure from the reliability states subsets, that is the random variable with the exponential distribution defined by the density function (7).

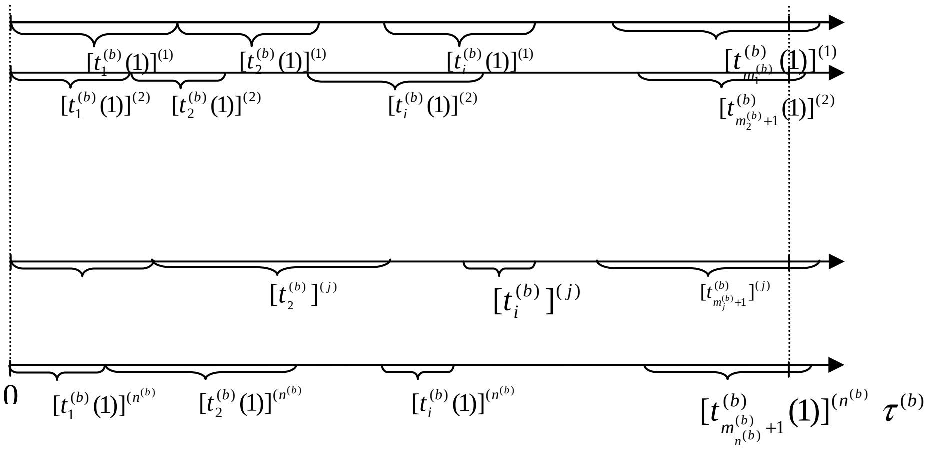

Figure 6. The scheme of the realizations of the component simple renewal flows (streams) on several experimental posts (the same observation time on all experimental posts)

Case 6.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – Different observation times on experimental posts

We assume that we have been observing the realizations of the component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ at the operation state z_b, $b = 1,2,...,\nu$, on $n^{(b)}$ experimental posts. We assume that the observation times on particular experimental posts are different and we mark by $\tau_j^{(b)}$, $\tau_j^{(b)} > 0, \quad i = 1,2,...,n^{(b)}$, the observation time respectively on the i-th experimental post. We assume that, at each observation post, at the moment when the component is leaving the reliability states subset $\{1,2,...,z\}$, i.e. the observed component reached the worst reliability state 0, it is replaced at once by the same new component staying at the reliability state z (*Figure 7*). It means that, at each experiment post, at the beginning all components are new identical components staying at the best reliability state z. We assume that, at the j-th, $j = 1,2,...,n^{(b)}$, experimental post, during the fixed observation time $m_j^{(b)}$ components have left the reliability states subset $\{1,2,...,z\}$, i.e. $m_j^{(b)}$ observed components reached the worst reliability state 0. It means that the number $m_j^{(b)}(u)$ of components that have left the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z,$ is equal either to $m_j^{(b)}$ or to $m_j^{(b)} + 1$, i.e. $m_j^{(b)}(u) = m_j^{(b)}$ or $m_j^{(b)}(u) = m_j^{(b)} + 1$, $u = 1,2,...,z$.

We mark by

$$A_j^{(b)}(u) = \{[t_i^{(b)}(u)]^{(j)} : i = 1,2,...,m_j^{(b)}(u)\},$$
$$u = 1,2,...,z, \quad j = 1,2,...,n^{(b)},$$

the sets of the times $[t_i^{(b)}(u)]^{(j)}$, $i = 1,2,...,m_j^{(b)}(u)$, to the components departures from the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,...,z,$ at the at the j-th, $j = 1,2,...,n^{(b)}$, experimental post, i.e. the realizations of the component lifetimes $T^{(b)}(u)$ to the first departure from the reliability states subsets, that is the random variable with the exponential distribution defined by the density function (7)

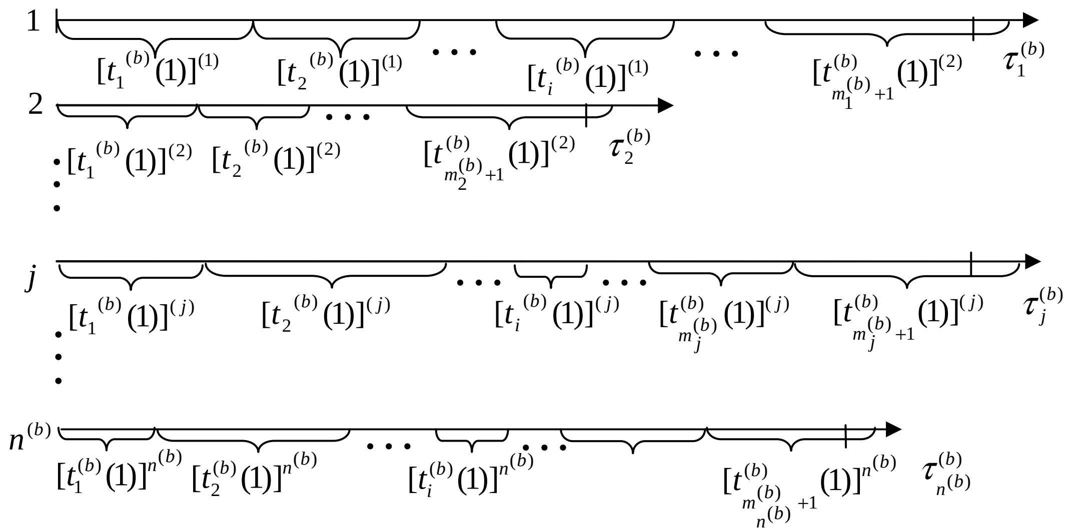

Figure 7. The scheme of the realizations of the component simple renewal flows (streams) on several experimental posts (different observation times on experimental posts)

3.4. Procedure of evaluating the system components unknown intensities of departure from the reliability state subsets

3.4.1. Data coming from experts

On the basis of the approximate values

$$[\hat{\mu}(u)]^{(b)},\ u = 1,2,...,z,\ \ b = 1,2,...,\nu,$$

of the mean values

$$[\mu(u)]^{(b)} = E[T(u)]^{(b)},\ u = 1,2,...,z,\ \ b = 1,2,...,\nu,$$

of the system components lifetimes $[T(u)]^{(b)}$, $u = 1,2,...,z,$ $b = 1,2,...,\nu,$ in the reliability states subsets $\{u, u+1,..., z\}$, $u = 1,2,...,z,$ while the system is operating in the operation state z_b, $b = 1,2,...,\nu,$ coming from experts and described in Section 3.3.1, we want to estimate the values $[\hat{\lambda}(u)]^{(b)}$ of the components unknown intensities $[\lambda(u)]^{(b)}$ of departure from the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,...,z,$ while the system is operating in the operation state z_b, $b = 1,2,...,\nu.$ The formula for all system components is given by the following approximate equation

$$[\lambda(u)]^{(b)} \cong [\hat{\lambda}(u)]^{(b)} = \frac{1}{[\hat{\mu}(u)]^{(b)}}, \tag{8}$$

$$u = 1,2,...,z,\ b = 1,2,...,\nu.$$

3.4.2. Data coming from components reliability states changing processes

On the basis of statistical data described in Section 3.3.2, we want to find the estimate $[\hat{\lambda}(u)]^{(b)}$ of the value $[\lambda(u)]^{(b)}$ of this unknown intensity of departure from the reliability states subset $\{u, u+1,..., z\}$, $u = 1,2,...,z.$ The formulae for all considered kinds of experiments are presented below.

Case 1.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Completed investigations, the same observation time on all experimental posts
In this case, the maximum likelihood evaluation of the unknown component intensity of departure

$[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z$, is

$$[\hat{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{n^{(b)}} t_i^{(b)}(u)}, \quad u=1,2,...,z. \tag{9}$$

Case 2.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, the same observation time on all experimental posts
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ is

$$[\hat{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \tau^{(b)}[n^{(b)} - m^{(b)}(u)]}, \tag{10}$$

$u=1,2,...,z.$

Assuming the observation time $\tau^{(b)}$ as the moment of departure from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the components that have not left this reliability states subset we get so called pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the form

$$[\hat{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \tau^{(b)}[n^{(b)} - m^{(b)}(u)]}, \tag{11}$$

$u=1,2,...,z.$

Case 3.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component lifetimes up to the first departure from the reliability states subset on several experimental posts – Non-completed investigations, different observation times on particular experimental posts
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ is

$$[\hat{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \sum_{i=m^{(b)}(u)+1}^{n^{(b)}} \tau_i^{(b)}}, \tag{12}$$

$u=1,2,...,z.$

Assuming the observation times $\tau_i^{(b)}$, $i=m^{(b)}(u), m^{(b)}(u)+1,...,n^{(b)}$, as the moment of departure from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the components that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $\lambda^{(b)}(u)$ from the reliability states subset of the form

$$[\hat{\lambda}(u)]^{(b)} = \frac{n^{(b)}}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + \sum_{i=m^{(b)}(u)+1}^{n^{(b)}} \tau_i^{(b)}}, \tag{13}$$

$u=1,2,...,z.$

Case 4.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flow (stream) on one experimental post
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ is

$$[\hat{\lambda}(u)]^{(b)} = \frac{m^{(b)}(u)}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + d^{(b)}(u)}, \quad u=1,2,...,z, \tag{14}$$

where

$$d^{(b)}(u) = \\ = \begin{cases} \tau^{(b)} - \sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(1) \quad if \quad m^{(b)}(u) = m^{(b)} \\ 0 \quad if \;\; m^{(b)}(u) = m^{(b)} + 1, \;\; u=1,2,...,z. \end{cases} \tag{15}$$

In the case if $m^{(b)}(u) = m^{(b)}$, $u=1,2,...,z,$ after assuming the observation time $\tau^{(b)}$ as the moment of departure from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the last component that has not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the form

$$[\hat{\lambda}(u)]^{(b)} = \frac{m^{(b)}+1}{\sum_{i=1}^{m^{(b)}(u)} t_i^{(b)}(u) + d^{(b)}(u)}, \ u=1,2,...,z. \quad (16)$$

Case 5.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – The same observation time on all experimental posts
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ is either

$$[\hat{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)}(u)}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} d_j^{(b)}(u)}, \quad (17)$$

$u=1,2,...,z,$

where for $j=1,2,...,n^{(b)}$

$$d_j^{(b)}(u) = \\ = \begin{cases} \tau^{(b)} - \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(1)]^{(j)} & if \ m_j^{(b)}(u) = m_j^{(b)} \\ 0 \quad if \ m_j^{(b)}(u) = m_j^{(b)}+1, \ u=1,2,...,z. \end{cases} \quad (18)$$

In the case if there exist j, $j \in \{1,2,...,n^{(b)}\}$, such that $m_j^{(b)}(u) = m_j^{(b)}$, $u=1,2,...,z,$ assuming the observation time $\tau^{(b)}$ as the moment of departures from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the last components on all experimental posts that have not left this reliability states subset we get so called pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the form

$$[\hat{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)} + n^{(b)}}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} d_j^{(b)}(u)}, \quad (19)$$

$u=1,2,...,z.$

Case 6.
The estimation of the component intensity of departure from the reliability states subset on the basis of the realizations of the component simple renewal flows (streams) on several experimental posts – Different observation times on experimental posts
In this case, the maximum likelihood evaluation of the unknown component intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ is either

$$[\hat{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)}(u)}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} \bar{d}_j^{(b)}(u)}, \quad (20)$$

$u=1,2,...,z,$

where for $j=1,2,...,n^{(b)}$

$$\bar{d}_j^{(b)}(u) = \\ = \begin{cases} \tau_j^{(b)} - \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(1)]^{(j)} & if \ m_j^{(b)}(u) = m_j^{(b)} \\ 0 \quad if \ m_j^{(b)}(u) = m_j^{(b)}+1, \ u=1,2,...,z. \end{cases} \quad (21)$$

In the case if there exist j, $j \in \{1,2,...,n^{(b)}\}$, such that $m_j^{(b)}(u) = m_j^{(b)}$, $u=1,2,...,z,$ assuming the observation times $\tau_j^{(b)}$, $j=1,2,...,n^{(b)}$, as the moments of departures from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the last components on experimental posts that have not left this reliability states subset we get so called a pessimistic evaluation of the intensity of departure $[\lambda(u)]^{(b)}$ from the reliability states subset $\{u,u+1,...,z\}$, $u=1,2,...,z,$ of the form

$$[\hat{\lambda}(u)]^{(b)} = \frac{\sum_{j=1}^{n^{(b)}} m_j^{(b)} + n^{(b)}}{\sum_{j=1}^{n^{(b)}} \sum_{i=1}^{m_j^{(b)}(u)} [t_i^{(b)}(u)]^j + \sum_{j=1}^{n^{(b)}} \bar{d}_j^{(b)}(u)}, \quad (22)$$

$u=1,2,...,z.$

3.5. Procedure of identifying the system components conditional multistate exponential reliability functions

To formulate and next to verify the non-parametric hypothesis concerning the exponential form of the coordinate

$$[R(t,u)]^{(b)} = \exp[-[\lambda(u)]^{(b)} t] \text{ for } t \in <0,\infty), \quad (23)$$

$$u = 1,2,...,z,\ b = 1,2,...,\nu,$$

of the vector

$$[R(t,\cdot)]^{(b)} = [1,\ [R(t,1)]^{(b)}, ...,\ [R(t,z)]^{(b)}\ ], \quad (24)$$

of the conditional multistate reliability function of the system component when the system is at the operation state z_b, $b = 1,2,...,\nu$, it is necessary to act according to the scheme below:

- to fix the numbers $n^{(b)}$ of realizations of the system component conditional lifetimes $T^{(b)}(u)$, $b = 1,2,...,\nu$, in the reliability states subsets $\{u, u+1,...,z\}$, $u = 1,2,...,z$,

- to fix the realizations $t_1^{(b)}(u),\ t_2^{(b)}(u),\ \ldots,\ t_{n^{(b)}}^{(b)}(u)$, $u = 1,2,...,z$, of the system component conditional lifetimes $T^{(b)}(u)$, $b = 1,2,...,\nu$, in the reliability states subsets $\{u, u+1,...,z\}$, $u = 1,2,...,z$,

- to determine the number $\bar{r}^{(b)}$ of the disjoint intervals $I_j^{(b)} =< x_j^{(b)}, y_j^{(b)})$, $j = 1,2,...,\bar{r}^{(b)}$, that include the realizations $t_1^{(b)}(u),\ t_2^{(b)}(u),\ \ldots,\ t_n^{(b)}(u)$ of the system component conditional lifetimes $T^{(b)}(u)$ in the reliability states subset, according to the formula

$$\bar{r}^{(b)} \cong \sqrt{n^{(b)}},$$

- to determine the length $d^{(b)}$ of the intervals $I_j^{(b)} =< x_j^{(b)}, y_j^{(b)})$, $j = 1,2,...,\bar{r}^{(b)}$, according to the formula

$$d^{(b)} = \frac{\bar{R}^{(b)}}{\bar{r}^{(b)} - 1},$$

where

$$\bar{R}^{(b)} = \max_{1\le i\le n} t_i^{(b)}(u) - \min_{1\le i\le n} t_i^{(b)}(u),$$

- to determine the ends $x_j^{(b)}$, $y_j^{(b)}$, of the intervals $I_j^{(b)} =< x_j^{(b)}, y_j^{(b)})$, $j = 2,3,...,\bar{r}^{(b)}$, , according to the formulae

$$x_1^{(b)} = \max\{\min_{1\le i\le n} t_i^{(b)}(u) - \frac{d^{(b)}}{2},\ 0\},$$

$$y_j^{(b)} = x_1^{(b)} + jd^{(b)},\ j = 2,3,...,\bar{r}^{(b)}, ,$$

$$x_j^{(b)} = y_{j-1}^{b)},\quad j = 2,3,...,\bar{r}^{(b)},$$

in the way such that

$$I_1^{(b)} \cup I_2^{(b)} \cup ... \cup I_{\bar{r}^{(b)}}^{(b)} =< x_1^{(b)}, y_{\bar{r}^{(b)}}^{(b)}),$$

and

$$I_i^{(b)} \cap I_j^{(b)} = \emptyset \text{ for all } i \ne j,\ i,j \in \{1,2,...,\bar{r}^{(b)}\},$$

- to determine the numbers of realizations $n_j^{(b)}$ in particular intervals $I_j^{(b)}$, $j = 1,2,...,\bar{r}^{(b)}$, according to the formula

$$n_j^{(b)} = \#\ \{i : t_i^{(b)}(u) \in I_j^{(b)},\ i \in \{1,2,...,n\}\},$$
$$j = 1,2,...,\bar{r}^{(b)},$$

where

$$\sum_{j=1}^{\bar{r}} n_j^{(b)} = n^{(b)},$$

whereas the symbol # means the number of elements of a set,

- to evaluate the value of the unknown intensity of the component departure $[\lambda(u)]^{(b)}$, from the reliability states subset, applying suitable formula from Section 3.4.2,

- to construct and to plot the realization of the histogram of the conditional system component lifetime $T^{(b)}(u)$, $b = 1,2,...,\nu$, in the reliability states subset $\{u, u+1,...,z\}$, $u = 1,2,...,z$, at the system operation state z_b, $b = 1,2,...,\nu$,

$$\bar{f}_n^{(b)}(t,u) = \frac{n_j^{(b)}}{n^{(b)}} \text{ for } t \in I_j,$$

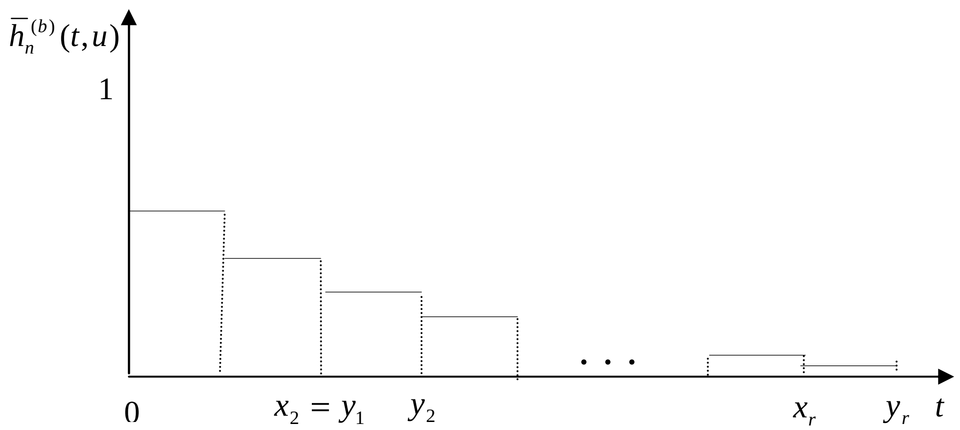

Figure 8. The realization of the histogram of the conditional system component lifetime in the reliability states subset

- to analyze the realization of the histogram, comparing it with the graph of the exponential density function

$$[f(t,u)]^{(b)} = [\lambda(u)]^{(b)} \exp[-[\lambda(u)]^{(b)} t]$$

for $t \in <0, \infty)$,

of the system component lifetime $T^{(b)}(u)$ in the reliability states subset $\{u, u+1, ..., z\}$ at the operation state z_b, corresponding the reliability function coordinate (20) of the vector of the conditional multistate reliability function of the system component (21) and to formulate the null hypothesis H_0 and the alternative hypothesis H_A, concerned with the form of the component multistate reliability $[R(t,\cdot)]^{(b)}$ in the following form:

H_0 : The conditional multistate reliability function of the system component

$$[R(t,\cdot)]^{(b)} = [1, [R(t,1)]^{(b)}, ..., [R(t,z)]^{(b)}],$$

has the exponential reliability functions coordinates of the form

$$[R(t,u)]^{(b)} = \exp[-[\lambda(u)]^{(b)} t] \text{ for } t \in <0, \infty),$$

H_A : The conditional multistate reliability function of the system component has different from the exponential reliability functions coordinates,

- to join each of the intervals $I_j^{(b)}$, that has the number $n_j^{(b)}$ of realizations less than 4 either with the neighbor interval $I_{j+1}^{(b)}$ or with the neighbor interval $I_{j-1}^{(b)}$, this way that the numbers of realizations in all intervals are not less than 4,

- to fix a new number of intervals $\overline{\overline{r}}^{(b)}$,

- to determine new intervals

$$\overline{I}_j^{(b)} = <\overline{x}_j^{(b)}, \overline{y}_j^{(b)}), \quad j = 1,2,..,\overline{\overline{r}}^{(b)},$$

- to fix the numbers $\overline{n}_j^{(b)}$ of realizations in new intervals $\overline{I}_j^{(b)}$, $j = 1,2,..,\overline{\overline{r}}^{(b)}$,

- to calculate the hypothetical probabilities that the variable $T^{(b)}(u)$ takes values from the interval $\overline{I}_j^{(b)}$, under the assumption that the hypothesis H_0 is true, i.e. the probabilities

$$p_j^{(b)} = P(T^{(b)}(u) \in \overline{I}_j^{(b)}) = P(\overline{x}_j^{(b)} \le T^{(b)}(u) < \overline{y}_j^{(b)})$$

$$= [R(\overline{x}_j^{(b)}, u)]^{(b)} - [R(\overline{y}_j^{(b)}, u)]^{(b)}, \; j = 1,2,...,\overline{\overline{r}}^{(b)},$$

where $R^{(b)}(\overline{x}_j^{(b)}, u)$ and $R^{(b)}(\overline{y}_j^{(b)}, u)$ are the values of the coordinate reliability function $R^{(b)}(t,u)$ of the multistate reliability function defined in the null hypothesis H_0,

- to calculate the realization of the χ^2 (*chi-square*)-Pearson's statistics U_n, according to the formula

$$u_n = \sum_{j=1}^{\overline{\overline{r}}^{(b)}} \frac{(\overline{n}_j^{(b)} - n^{(b)} p_j^{(b)})^2}{n^{(b)} p_j^{(b)}},$$

- to assume the significance level α ($\alpha = 0.01$, $\alpha = 0.02$, $\alpha = 0.05$ or $\alpha = 0.10$) of the test,

- to fix the number $\overline{\overline{r}}^{(b)} - l - 1$ of degrees of freedom, substituting $l = 1$,

- to read from the Tables of the χ^2 – Pearson's distribution the value u_α for the fixed values of the significance level α and the number of degrees of freedom $\bar{\bar{r}} - l - 1$ such that the following equality holds

$$P(U_n > u_\alpha) = 1 - \alpha,$$

and next to determine the critical domain in the form of the interval $(u_\alpha, +\infty)$ and the acceptance domain in the form of the interval $< 0, u_\alpha >$,

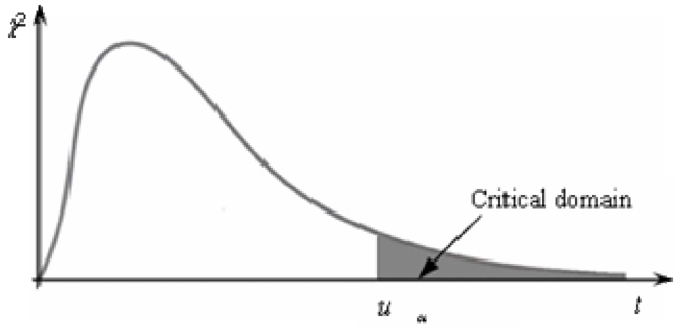

Figure 9. The graphical interpretation of the critical interval and the acceptance interval for the chi-square goodness-of-fit test

- to compare the obtained value u_n of the realization of the statistics U_n with the read from the Tables critical value u_α of the chi-square random variable and to verify previously formulated the null hypothesis H_0 in the following way: if the value u_n does not belong to the critical domain, i.e. when $u_n \le u_\alpha$, then we do not reject the hypothesis H_0, otherwise if the value u_n belongs to the critical domain, i.e. when $u_n > u_\alpha$, then we reject the hypothesis H_0 in favor of the hypothesis H_A.

4. Procedure of applying the computer program for identification of the system components reliability models

Training material is given in [6]

5. Identification of the components reliability models of real complex technical systems – using procedures

5.1. Statistical identification of the port oil piping transportation system components reliability models

5.1.1. The subsystems and components of the port oil piping transportation system in various operation states

The considered terminal is composed of three parts A, B and C, linked by the piping transportation systems with the pier. The scheme of this terminal is presented in *Figure 10.*

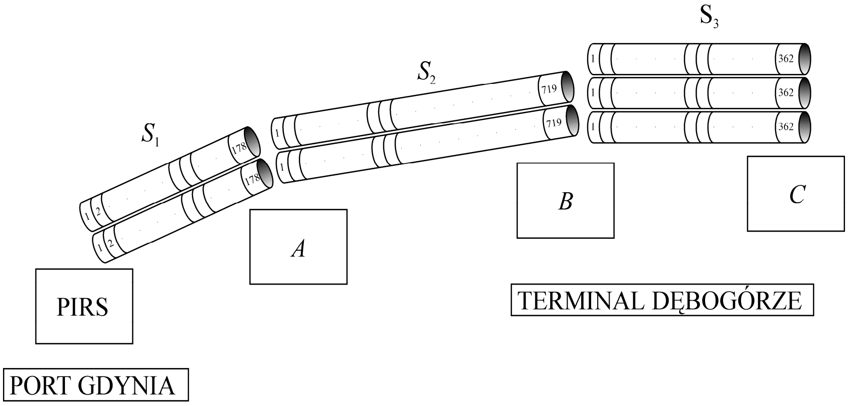

Figure 10. The scheme of the port oil piping transportation system.

The oil pipeline transportation system consists of three subsystems S_1, S_2, S_3:

- the subsystem S_1 is composed of two identical pipelines, each composed of 176 pipe segments of length 12m and 2 valves, denoted respectively by $E_{ij}^{(1)}$, $i = 1,2$, $j = 1,2,...,178$,

- the subsystem S_2 is composed of two identical pipelines, each composed of 717 pipe segments of length 12m and 2 valves, denoted respectively by $E_{ij}^{(2)}$, $i = 1,2$, $j = 1,2,...,719$,

- the subsystem S_3 is composed of two identical and one different pipelines, each composed of 360 pipe segments of either 10 m or 7,5 m length and 2 valves, denoted respectively by $E_{ij}^{(3)}$, $i = 1,2,3$, $j = 1,2,...,362$.

The subsystems S_1, S_2, S_3, indicated in *Figure 10* are forming a general port oil pipeline system structure presented in *Figure 11*.

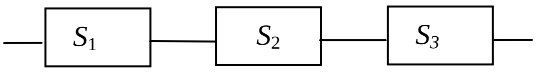

Figure 11. General scheme of port oil pipeline transportation system

However, the pipeline system structure and the subsystem components reliability depend on its changing in time operation states.

Taking into account the expert opinion on the operation process of the considered port oil pipeline transportation system we fix the number of the pipeline system operation process states $\nu = 7$ and we distinguish the following as its seven operation states:

- an operation state z_1 – transport of one kind of medium from the terminal part B to part C using two out of three pipelines in subsystem S_3,
- an operation state z_2 – transport of one kind of medium from the terminal part C (from carriages) to part B using one out of three pipelines in subsystem S_3,
- an operation state z_3 – transport of one kind of medium from the terminal part B through part A to pier using one out of two pipelines in subsystem S_2 and one out of two pipelines in subsystem S_1,
- an operation state z_4 – transport of two kinds of medium from the pier through parts A and B to part C using one out of two pipelines in subsystem S_1, one out of two pipelines in subsystem S_2 and two out of three pipelines in subsystem S_3,
- an operation state z_5 – transport of one kind of medium from the pier through part A to B using one out of two pipelines in subsystem S_1 and one out of two pipelines in subsystem S_2,
- an operation state z_6 – transport of one kind of medium from the terminal part B to C using two out of three pipelines in subsystem S_3, and simultaneously transport one kind of medium from the pier through part A to B using one out of two pipelines in parts S_1 and one out of two pipelines in subsystem S_2,
- an operation state z_7 – transport of one kind of medium from the terminal part B to C using one out of three pipelines in part S_3, and simultaneously transport second kind of medium from the terminal part C to B using one out of three pipelines in part S_3.

At the system operational state z_1, the system is composed of the subsystem S_3, with the scheme showed in *Figure 12*.

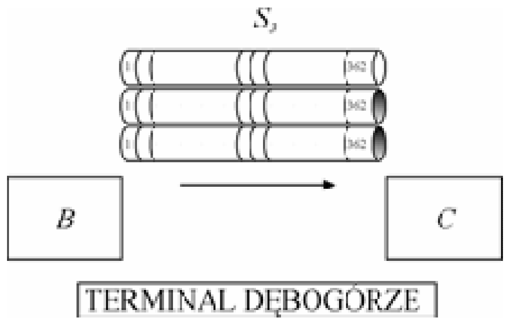

Figure 12. The scheme of port oil transportation system at operation state z_1

At the system operational state z_2, the system is composed of the subsystem S_3, which contains three pipelines with the scheme showed in *Figure 13*.

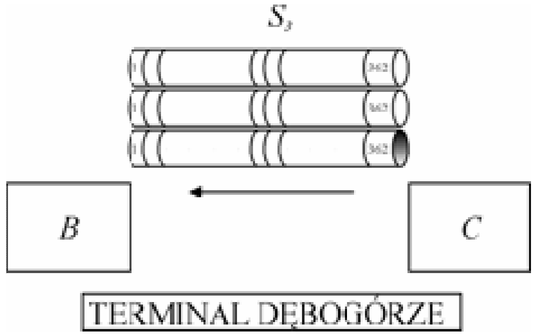

Figure 13. The scheme of port oil transportation system at operation state z_2

At the system operational state z_3, the system is series and composed of two subsystems S_1, S_2, each containing two pipelines with the structure showed in *Figure 14*.

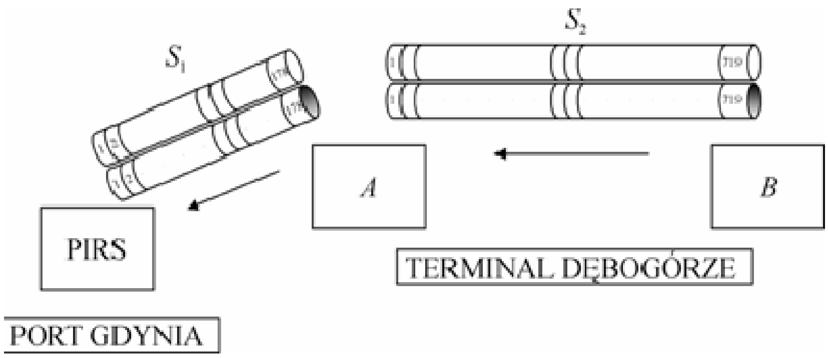

Figure 14. The scheme of port oil transportation system at operation state z_3

At the system operational state z_4, the system is composed of two subsystems S_1, S_2, each containing two pipelines and subsystem S_3 with the scheme showed in *Figure 15*.

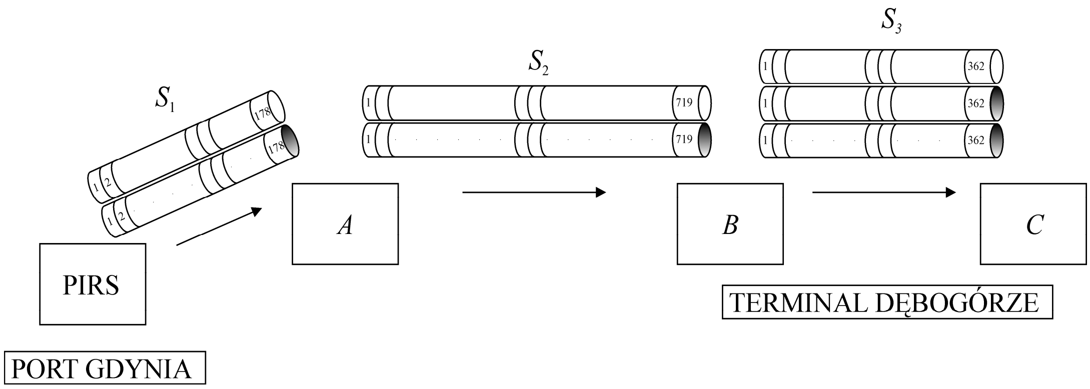

Figure 15. The scheme of port oil transportation system at operation state z_4

At the system operational state z_5, the system is composed of two subsystems S_1, S_2, each containing two pipelines with the scheme showed in *Figure 16*.

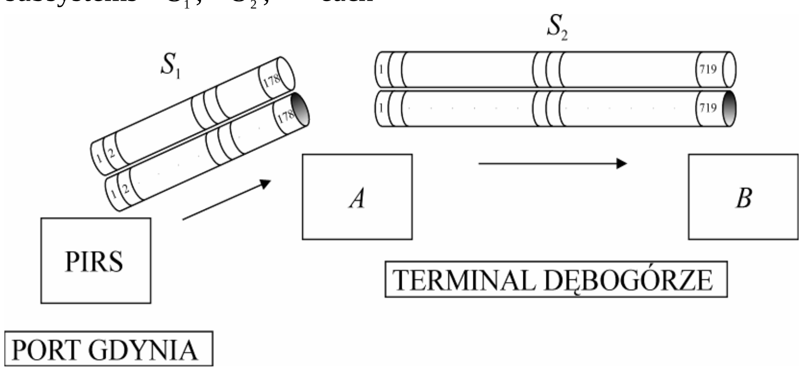

Figure 16. The scheme of port oil transportation system at operation state z_5

At the system operational state z_6, the system is composed of two subsystems S_1, S_2, each containing two pipelines and one subsystem S_3 composed of three pipelines with the scheme showed in *Figure 17*.

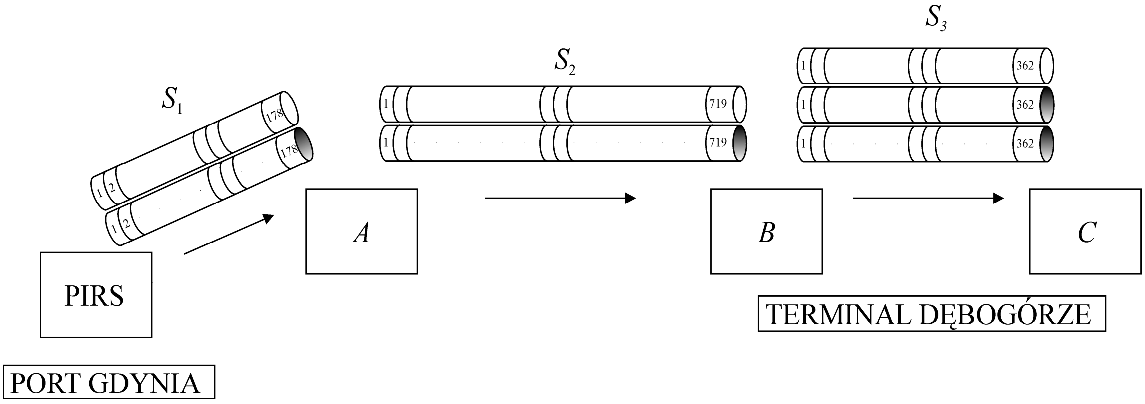

Figure 17. The scheme of port oil transportation system at operation state z_6

At the system operational state z_7, the system is composed of the subsystem S_3, which contains three pipelines with the scheme showed in *Figure 18*.

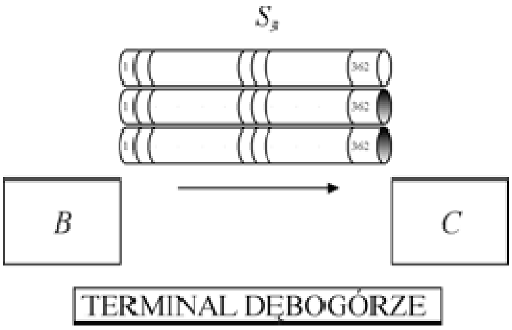

Figure 18. The scheme of port oil transportation system at operation state z_7

5.1.2. The parameters of the port oil piping transportation system components multi-state reliability models

After discussion with experts, taking into account the operation conditions influence on the reliability of the oil pipeline transportation system, in all operation states z_b, $b = 1,2,...,7$, we distinguish the following three reliability states $(z = 2)$ of the system and its components:

- a reliability state 2 – piping operation is fully safe,
- a reliability state 1 – piping operation is less safe and more dangerous because of the possibility of environment pollution,
- a reliability state 0 – piping is destroyed.

Moreover, we fix that there are possible the transitions between the components reliability states only from better to worse ones.

From the above, the oil pipeline transportation subsystems S_k, $k = 1,2,3$, are composed of three-state, i.e. $z = 2$, components $E_{ij}^{(k)}$, $k = 1,2,3$, with the conditional multi-state reliability functions

$$[R_{ij}^{(k)}(t,\cdot)]^{(b)} = [1, [R_{ij}^{(k)}(t,1)]^{(b)}, [R_{ij}^{(k)}(t,2)]^{(b)}],$$

$$b = 1,2,...,7,$$

with exponential co-ordinates $[R_{ij}^{(k)}(t,1)]^{(b)}$ and $[R_{ij}^{(k)}(t,2)]^{(b)}$ different in various operation states z_b, $b = 1,2,...,7.$

More precisely, from the performed in Section 3.4.2 analysis, the unknown reliability parameters of the system components reliability models in various system operation states are:

i) at the system operation states z_1:

- the reliability functions of the subsystem S_3 components

$$[R_{ij}^{(3)}(t,\cdot)]^{(1)} = [1, [R_{ij}^{(3)}(t,1)]^{(1)}, [R_{ij}^{(3)}(t,2)]^{(1)}],$$

$$i = 1,2,3, \quad j = 1,2,...,362,$$

coordinates

$$[R_{ij}^{(3)}(t,1)]^{(1)} = \exp[-[\lambda_{ij}^{(3)}(1)]^{(1)} t],$$

$$[R_{ij}^{(3)}(t,2)]^{(1)} = \exp[-[\lambda_{ij}^{(3)}(2)]^{(1)} t],$$

$$i = 1,2,3, \quad j = 1,2,...,362,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(1)}, \ [\lambda_{ij}^{(3)}(2)]^{(1)}, \ i = 1,2,3, \quad j = 1,2,...,362;$$

ii) at the system operation states z_2:

- the reliability functions of the subsystem S_3 components

$$[R_{ij}^{(3)}(t,\cdot)]^{(2)} = [1, [R_{ij}^{(3)}(t,1)]^{(2)}, [R_{ij}^{(3)}(t,2)]^{(2)}],$$

$$i = 1,2,3, \quad j = 1,2,\ldots,362,$$

coordinates

$$[R_{ij}^{(3)}(t,1)]^{(2)} = \exp[-[\lambda_{ij}^{(3)}(1)]^{(2)} t],$$

$$[R_{ij}^{(3)}(t,2)]^{(2)} = \exp[-[\lambda_{ij}^{(3)}(2)]^{(2)} t],$$

$$i = 1,2,3, \quad j = 1,2,\ldots,362,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(2)}, \ [\lambda_{ij}^{(3)}(2)]^{(2)}, \ i = 1,2,3, \quad j = 1,2,\ldots,362;$$

iii) at the system operation states z_3:

- the reliability functions of the subsystem S_1 components

$$[R_{ij}^{(1)}(t,\cdot)]^{(3)} = [1, [R_{ij}^{(1)}(t,1)]^{(3)}, [R_{ij}^{(1)}(t,2)]^{(3)}],$$

$$i = 1,2, \quad j = 1,2,\ldots,178,$$

coordinates

$$[R_{ij}^{(1)}(t,1)]^{(3)} = \exp[-[\lambda_{ij}^{(1)}(1)]^{(3)} t],$$

$$[R_{ij}^{(1)}(t,2)]^{(3)} = \exp[-[\lambda_{ij}^{(1)}(2)]^{(3)} t],$$

$$i = 1,2, \quad j = 1,2,\ldots,178,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(1)}(1)]^{(3)}, \ [\lambda_{ij}^{(1)}(2)]^{(3)}, \ i = 1,2, \quad j = 1,2,\ldots,178,$$

- the reliability functions of the subsystem S_2 components

$$[R_{ij}^{(2)}(t,\cdot)]^{(3)} = [1, [R_{ij}^{(2)}(t,1)]^{(3)}, [R_{ij}^{(2)}(t,2)]^{(3)}],$$

$$i = 1,2, \quad j = 1,2,\ldots,719,$$

coordinates

$$[R_{ij}^{(2)}(t,1)]^{(3)} = \exp[-[\lambda_{ij}^{(2)}(1)]^{(3)} t],$$

$$[R_{ij}^{(2)}(t,2)]^{(3)} = \exp[-[\lambda_{ij}^{(2)}(2)]^{(3)} t],$$

$$i = 1,2, \quad j = 1,2,\ldots,719,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(2)}(1)]^{(3)}, \ [\lambda_{ij}^{(2)}(2)]^{(3)}, \ i = 1,2, \quad j = 1,2,\ldots,719;$$

iv) at the system operation states z_4:

- the reliability functions of the subsystem S_1 components

$$[R_{ij}^{(1)}(t,\cdot)]^{(4)} = [1, [R_{ij}^{(1)}(t,1)]^{(4)}, [R_{ij}^{(1)}(t,2)]^{(4)}],$$

$$i = 1,2, \quad j = 1,2,\ldots,178,$$

coordinates

$$[R_{ij}^{(1)}(t,1)]^{(4)} = \exp[-[\lambda_{ij}^{(1)}(1)]^{(4)} t],$$

$$[R_{ij}^{(1)}(t,2)]^{(4)} = \exp[-[\lambda_{ij}^{(1)}(2)]^{(4)} t],$$

$$i = 1,2, \quad j = 1,2,\ldots,178,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(1)}(1)]^{(4)}, \ [\lambda_{ij}^{(1)}(2)]^{(4)}, \ i = 1,2, \quad j = 1,2,\ldots,178,$$

- the reliability functions of the subsystem S_2 components

$$[R_{ij}^{(2)}(t,\cdot)]^{(4)} = [1, [R_{ij}^{(2)}(t,1)]^{(4)}, [R_{ij}^{(2)}(t,2)]^{(4)}],$$

$$i = 1,2, \quad j = 1,2,\ldots,719,$$

coordinates

$$[R_{ij}^{(2)}(t,1)]^{(4)} = \exp[-[\lambda_{ij}^{(2)}(1)]^{(4)} t],$$

$$[R_{ij}^{(2)}(t,2)]^{(4)} = \exp[-[\lambda_{ij}^{(2)}(2)]^{(4)} t],$$

$$i = 1,2, \quad j = 1,2,\ldots,719,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(2)}(1)]^{(4)},\ [\lambda_{ij}^{(2)}(2)]^{(4)},\ i=1,2,\quad j=1,2,\ldots,719,$$

- the reliability functions of the subsystem S_3 components

$$[R_{ij}^{(3)}(t,\cdot)]^{(4)} = [1, [R_{ij}^{(3)}(t,1)]^{(4)}, [R_{ij}^{(3)}(t,2)]^{(4)}],$$

$$i=1,2,3,\quad j=1,2,\ldots,362,$$

coordinates

$$[R_{ij}^{(3)}(t,1)]^{(4)} = \exp[-[\lambda_{ij}^{(3)}(1)]^{(4)} t],$$

$$[R_{ij}^{(3)}(t,2)]^{(4)} = \exp[-[\lambda_{ij}^{(3)}(2)]^{(4)} t],$$

$$i=1,2,3,\quad j=1,2,\ldots,362,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(4)},\ [\lambda_{ij}^{(3)}(2)]^{(4)},\ i=1,2,3,\quad j=1,2,\ldots,362;$$

v) at the system operation states z_5:

- the reliability functions of the subsystem S_1 components

$$[R_{ij}^{(1)}(t,\cdot)]^{(5)} = [1, [R_{ij}^{(1)}(t,1)]^{(5)}, [R_{ij}^{(1)}(t,2)]^{(5)}],$$

$$i=1,2,\quad j=1,2,\ldots,178,$$

coordinates

$$[R_{ij}^{(1)}(t,1)]^{(5)} = \exp[-[\lambda_{ij}^{(1)}(1)]^{(5)} t],$$

$$[R_{ij}^{(1)}(t,2)]^{(5)} = \exp[-[\lambda_{ij}^{(1)}(2)]^{(5)} t],$$

$$i=1,2,\quad j=1,2,\ldots,178,$$

ith the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(1)}(1)]^{(5)},\ [\lambda_{ij}^{(1)}(2)]^{(5)},\ i=1,2,\quad j=1,2,\ldots,178,$$

- the reliability functions of the subsystem S_2 components

$$[R_{ij}^{(2)}(t,\cdot)]^{(5)} = [1, [R_{ij}^{(2)}(t,1)]^{(5)}, [R_{ij}^{(2)}(t,2)]^{(5)}],$$

$$i=1,2,\quad j=1,2,\ldots,719,$$

coordinates

$$[R_{ij}^{(2)}(t,1)]^{(5)} = \exp[-[\lambda_{ij}^{(2)}(1)]^{(5)} t],$$

$$[R_{ij}^{(2)}(t,2)]^{(5)} = \exp[-[\lambda_{ij}^{(2)}(2)]^{(5)} t],$$

$$i=1,2,\quad j=1,2,\ldots,719,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(2)}(1)]^{(5)},\ [\lambda_{ij}^{(2)}(2)]^{(5)},\ i=1,2,\quad j=1,2,\ldots,719;$$

vi) at the system operation states z_6:

- the reliability functions of the subsystem S_1 components

$$[R_{ij}^{(1)}(t,\cdot)]^{(6)} = [1, [R_{ij}^{(1)}(t,1)]^{(6)}, [R_{ij}^{(1)}(t,2)]^{(6)}],$$

$$i=1,2,\quad j=1,2,\ldots,178,$$

coordinates

$$[R_{ij}^{(1)}(t,1)]^{(6)} = \exp[-[\lambda_{ij}^{(1)}(1)]^{(6)} t],$$

$$[R_{ij}^{(1)}(t,2)]^{(6)} = \exp[-[\lambda_{ij}^{(1)}(2)]^{(6)} t],$$

$$i=1,2,\quad j=1,2,\ldots,178,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(1)}(1)]^{(6)},\ [\lambda_{ij}^{(1)}(2)]^{(6)},\ i=1,2,\quad j=1,2,\ldots,178,$$

- the reliability functions of the subsystem S_2 components

$$[R_{ij}^{(2)}(t,\cdot)]^{(6)} = [1, [R_{ij}^{(2)}(t,1)]^{(6)}, [R_{ij}^{(2)}(t,2)]^{(6)}],$$

$$i=1,2,\quad j=1,2,\ldots,719,$$

coordinates

$$[R_{ij}^{(2)}(t,1)]^{(6)} = \exp[-[\lambda_{ij}^{(2)}(1)]^{(6)} t],$$

$$[R_{ij}^{(2)}(t,2)]^{(6)} = \exp[-[\lambda_{ij}^{(2)}(2)]^{(6)} t],$$

$$i = 1,2, \quad j = 1,2,\ldots,719,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(2)}(1)]^{(6)}, \ [\lambda_{ij}^{(2)}(2)]^{(6)}, \ i = 1,2, \quad j = 1,2,\ldots,719,$$

- the reliability functions of the subsystem S_3 components

$$[R_{ij}^{(3)}(t,\cdot)]^{(6)} = [1, [R_{ij}^{(3)}(t,1)]^{(6)}, [R_{ij}^{(3)}(t,2)]^{(6)}],$$

$$i = 1,2,3, \quad j = 1,2,\ldots,362,$$

coordinates

$$[R_{ij}^{(3)}(t,1)]^{(6)} = \exp[-[\lambda_{ij}^{(3)}(1)]^{(6)} t],$$

$$[R_{ij}^{(3)}(t,2)]^{(6)} = \exp[-[\lambda_{ij}^{(3)}(2)]^{(6)} t],$$

$$i = 1,2,3, \quad j = 1,2,\ldots,362,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(6)}, \ [\lambda_{ij}^{(3)}(2)]^{(6)}, \ i = 1,2,3, \quad j = 1,2,\ldots,362;$$

vii) at the system operation states z_7:

- the reliability functions of the subsystem S_3 components

$$[R_{ij}^{(3)}(t,\cdot)]^{(7)} = [1, [R_{ij}^{(3)}(t,1)]^{(7)}, [R_{ij}^{(3)}(t,2)]^{(7)}],$$

$$i = 1,2,3, \quad j = 1,2,\ldots,362,$$

coordinates

$$[R_{ij}^{(3)}(t,1)]^{(7)} = \exp[-[\lambda_{ij}^{(3)}(1)]^{(7)} t],$$

$$[R_{ij}^{(3)}(t,2)]^{(7)} = \exp[-[\lambda_{ij}^{(3)}(2)]^{(7)} t],$$

$$i = 1,2,3, \quad j = 1,2,\ldots,362,$$

with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(7)}, \ [\lambda_{ij}^{(3)}(2)]^{(7)}, \ i = 1,2,3, \quad j = 1,2,\ldots,362.$$

5.1.3. The port oil piping transportation system components reliability data collection

5.1.3.1. Data coming from experts

In the *Tables 1-4* there are given the approximate realizations

$$[\hat{\mu}_{ij}^{(k)}(u)]^{(b)}, \ k = 1,2,3 \ \ u = 1,2, \ b = 1,2,\ldots,7,$$

of the mean values

$$[\mu_{ij}^{(k)}(u)]^{(b)} = E[[T_{ij}^{(k)}(u)]^{(b)}], \qquad k = 1,2,3$$
$$u = 1,2, \ b = 1,2,\ldots,7,$$

of the conditional lifetimes $[T_{ij}^{(k)}(u)]^{(b)}$, $k = 1,2,3$, $u = 1,2$, $b = 1,2,\ldots,7$, in reliability states of the component $E_{ij}^{(k)}$ of the oil pipeline subsystems S_k, $k = 1,2,3$, in particular operation states z_b, $b = 1,2,\ldots,7$, estimated on the basis of the expert opinions.

Table 1. The approximate mean values $[\hat{\mu}_{ij}^{(1)}(u)]^{(b)}$ of the subsystem S_1 components conditional lifetimes $[T_{ij}^{(1)}(u)]^{(b)}$ in particular operation states z_b

Subsystem S_1 components	$E_{ij}^{(1)}$ $i = 1,2,$ $j = 1,2,\ldots,176,$ $u = 1$	$E_{ij}^{(1)}$ $i = 1,2,$ $j = 1,2,\ldots,176,$ $u = 2$	$E_{ij}^{(1)}$ $i = 1,2,$ $j = 177,178,$ $u = 1$	$E_{ij}^{(1)}$ $i = 1,2,$ $j = 361,362,$ $u = 2$
Operation state z_b	The approximate mean values $[\hat{\mu}_{ij}^{(1)}(u)]^{(b)}$ of the conditional lifetimes $[T_{ij}^{(1)}(u)]^{(b)}$ of the component $E_{ij}^{(1)}$ (*in years*)			
z_1				
z_2				
z_3	161	114	60	55

z_4	161	114	60	55
z_5	161	114	60	55
z_6	161	114	60	55
z_7				

Table 2. The mean values $[\hat{\mu}_{ij}^{(2)}(u)]^{(b)}$ of the subsystem S_2 components conditional lifetimes $[T_{ij}^{(2)}(u)]^{(b)}$ in particular operation states z_b

Subsystem S_2 components	$E_{ij}^{(2)}$ $i=1,2,$ $j=1,2,...,717,$ $u=1$	$E_{ij}^{(2)}$ $i=1,2,$ $j=1,2,...,717,$ $u=2$	$E_{ij}^{(2)}$ $i=1,2,$ $j=718,719,$ $u=1$	$E_{ij}^{(2)}$ $i=1,2,$ $j=718,719,$ $u=2$
Operation state z_b	The mean values $[\hat{\mu}_{ij}^{(2)}(u)]^{(b)}$ of the conditional lifetimes $[T_{ij}^{(2)}(u)]^{(b)}$, of the component $E_{ij}^{(2)}$ (*in years*)			
z_1				
z_2				
z_3	161	114	60	55
z_4	161	114	60	55
z_5	161	114	60	55
z_6	161	114	60	55
z_7				

Table 3. The mean values $[\hat{\mu}_{ij}^{(3)}(u)]^{(b)}$ of the subsystem S_3 components conditional lifetimes $[T_{ij}^{(3)}(u)]^{(b)}$ in particular operation states z_b

Subsystem S_3 components	$E_{ij}^{(3)}$ $i=1,2,$ $j=1,2,...,360,$ $u=1$	$E_{ij}^{(3)}$ $i=1,2,$ $j=1,2,...,360,$ $u=2$	$E_{ij}^{(3)}$ $i=1,2,$ $j=361,362,$ $u=1$	$E_{ij}^{(3)}$ $i=1,2,$ $j=361,362,$ $u=2$
Operation state z_b	The mean values $[\hat{\mu}_{ij}^{(3)}(u)]^{(b)}$ of the conditional lifetimes $[T_{ij}^{(3)}(u)]^{(b)}$ of the component $E_{ij}^{(3)}$ (*in years*)			
z_1	170	135	60	55
z_2	170	135	60	55
z_3				
z_4	170	135	60	55
z_5				
z_6	170	135	60	55
z_7	170	135	60	55

Table 4. The mean values $[\hat{\mu}_{ij}^{(3)}(u)]^{(b)}$ of the subsystem S_3 components conditional lifetimes $[T_{ij}^{(3)}(u)]^{(b)}$ in particular operation states z_b

Subsystem S_3 components	$E_{ij}^{(3)}$ $i=3,$ $j=1,2,...,360,$ $u=1$	$E_{ij}^{(3)}$ $i=3,$ $j=1,2,...,360$ $u=2$	$E_{ij}^{(3)}$ $i=3,$ $j=361,362,$ $u=1$	$E_{ij}^{(3)}$ $i=3,$ $j=361,362,$ $u=2$
Operation state z_b	The mean values $[\hat{\mu}_{ij}^{(3)}(u)]^{(b)}$ of the conditional lifetimes $[T_{ij}^{(3)}(u)]^{(b)}$ of the component $E_{ij}^{(3)}$ (*in years*)			
z_1	140	127	60	55
z_2	140	127	60	55
z_3				
z_4	140	127	60	55
z_5				
z_6	140	127	60	55
z_7	140	127	60	55

5.1.3.2. Data coming from components reliability states changing processes

There are no data collected from the port oil piping transportation system components reliability states changing processes.

5.1.4. Statistical identification of the port oil piping transportation system components reliability

5.1.4.1. Statistical identification of the port oil piping transportation system components reliability on the basis of data coming from experts

To identify the parameters of multistate reliability functions of the port oil pipeline system components the statistical data coming from their failure processes are needed. The statistical data that has been collected is given in *Tables 1-4*.

From data given in *Tables 1–4*, of the basis of the resulting from (8) formula

$$[\hat{\lambda}_{ij}^{(k)}(u)]^{(b)} = \frac{1}{[\hat{\mu}_{ij}^{(k)}(u)]^{(b)}},\ k=1,2,3,\ u=1,2,$$

$$b=1,2,...,7,$$

we get the approximate values $[\hat{\lambda}_{ij}^{(k)}(u)]^{(b)}$ of the subsystems S_k, $b=1,2,3,$ components unknown intensities $[\lambda_{ij}^{(k)}(u)]^{(b)}$ of departure from the reliability states subset $\{1,2\}$, $\{2\}$, while the system is operating in the operation state z_b, $b=1,2,...,7.$ The results are presented below.

At the system operation state z_1, the system is composed of the subsystem S_3 containing three pipelines with the structure showed in *Figure 12*.

The subsystem S_3 consists of 2 pipelines of the first type and 1 pipeline of the second type, each composed of 362 components. In each pipeline of the first type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(1)} = 0.0059,\ [\lambda_{ij}^{(3)}(2)]^{(1)} = 0.0074,$$

$$i=1,2,\ \ j=1,2,...360,$$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(1)} = 0.0166,\ [\lambda_{ij}^{(3)}(2)]^{(1)} = 0.0181$$

$$i=1,2,\ \ j=361,362.$$

In the pipeline of the second type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}, \{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(1)} = 0.0071, \; [\lambda_{ij}^{(3)}(2)]^{(1)} = 0.0079,$

$i = 3, \quad j = 1,2,...360,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}, \; \{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(1)} = 0.0166, \; [\lambda_{ij}^{(3)}(2)]^{(1)} = 0.0181,$

$i = 3, \; j = 361,362.$

At the system operation state z_2, the system is composed of the subsystem S_3, which contains 3 pipelines with the structure showed in *Figure 13*.
The subsystem S_3 consists of 2 pipelines of the first type and 1 pipeline of the second type, each composed of 362 components. In each pipeline of the first type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}, \; \{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(2)} = 0.0059, \; [\lambda_{ij}^{(3)}(2)]^{(2)} = 0.0074,$

$i = 1,2, \; j = 1,2,...,360,$

- 2 valves with the intensities of departure fom the reliability states subsets $\{1,2\}, \; \{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(2)} = 0.0166, \; [\lambda_{ij}^{(3)}(2)]^{(2)} = 0.0181,$

$i = 1,2, \; j = 361,362.$

In the pipeline of the second type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}, \; \{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(2)} = 0.0071, \; [\lambda_{ij}^{(3)}(2)]^{(2)} = 0.0079,$

$i = 3, \; j = 1,2,...,360,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}, \; \{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(2)} = 0.0166t, \; [\lambda_{ij}^{(3)}(2)]^{(2)} = 0.0181,$

$i = 3, \; j = 361,362.$

At the system operation state z_3, the system is composed of two subsystems S_1, S_2, each containing 2 pipelines with the structure showed in *Figure 14*.
The subsystem S_1 consists of 2 identical pipelines, each composed of 178 components. In each pipeline there are:

- 176 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}, \; \{2\}$, respectively

$[\lambda_{ij}^{(1)}(1)]^{(3)} = 0.0062, \; [\lambda_{ij}^{(1)}(2)]^{(3)} = 0.0088,$

$i = 1,2, \; j = 1,2,...,176,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}, \; \{2\}$, respectively

$[\lambda_{ij}^{(1)}(1)]^{(3)} = 0.0166, \; [\lambda_{ij}^{(1)}(2)]^{(3)} = 0.0181,$

$i = 1,2, \; j = 177,178.$

The subsystem S_2 consists of 2 identical pipelines, each composed of 719 components. In each pipeline there are:

- 717 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}, \; \{2\}$, respectively

$[\lambda_{ij}^{(2)}(1)]^{(3)} = 0.0062, \; [\lambda_{ij}^{(2)}(2)]^{(3)} = 0.0088,$

$i = 1,2, \; j = 1,2,...,717,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}, \; \{2\}$, respectively

$[\lambda_{ij}^{(2)}(1)]^{(3)} = 0.0166, \; [\lambda_{ij}^{(2)}(2)]^{(3)} = 0.0181,$

$i = 1,2, \; j = 718,719.$

At the system operational state z_4, the system is composed of two subsystems S_1, S_2, each containing 2 pipelines and one subsystem S_3 containing 3 pipelines with the structure showed in *Figure 15*.

The subsystem S_1 consists of 2 identical pipelines, each composed of 178 components. In each pipeline there are:

- 176 pipe segments the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(1)}(1)]^{(4)} = 0.0062,\ [\lambda_{ij}^{(1)}(1)]^{(4)} = 0.0088,$$

$$i = 1,2,\ j = 1,2,...,176,$$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(1)}(1)]^{(4)} = 0.0166,\ [\lambda_{ij}^{(1)}(1)]^{(4)} = 0.0181,$$

$$i = 1,2,\ j = 177,178.$$

The subsystem S_2 consists of 2 identical pipelines, each composed of 719 components. In each pipeline there are:

- 717 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(2)}(1)]^{(4)} = 0.0062,\ [\lambda_{ij}^{(2)}(2)]^{(4)} = 0.0088,$$

$$i = 1,2,\ j = 1,2,...,717,$$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(2)}(1)]^{(4)} = 0.0166,\ [\lambda_{ij}^{(2)}(2)]^{(4)} = 0.0181,$$

$$i = 1,2,\ j = 718,719.$$

The subsystem S_3 consists of 2 pipelines of the first type and 1 pipeline of the second type, each composed of 362 components. In each pipeline of the first type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(4)} = 0.0059,\ [\lambda_{ij}^{(3)}(2)]^{(4)} = 0.0074,$$

$$i = 1,2,\ j = 1,2,...,360,$$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(4)} = 0.0166,\ [\lambda_{ij}^{(3)}(2)]^{(4)} = 0.0181,$$

$$i = 1,2,\ j = 361,362.$$

In the pipeline of the second type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(4)} = 0.0071,\ [\lambda_{ij}^{(3)}(2)]^{(4)} = 0.0079,$$

$$i = 3\ \ j = 1,2,...,360,$$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(3)}(1)]^{(4)} = 0.0166,\ [\lambda_{ij}^{(3)}(2)]^{(4)} = 0.0181,$$

$$i = 3,\ j = 361,362.$$

At the system operational state z_5, the system is series and composed of two subsystems S_1, S_2, each containing 2 pipelines with the structure showed in *Figure 16*.

The subsystem S_1 consists of 2 identical pipelines, each composed of 178 components. In each pipeline there are:

- 176 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(1)}(1)]^{(5)} = 0.0062,\ [\lambda_{ij}^{(1)}(2)]^{(5)} = 0.0088,$$

$$i = 1,2,\ j = 1,2,...,176,$$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$$[\lambda_{ij}^{(1)}(1)]^{(5)} = 0.0166,\ [\lambda_{ij}^{(1)}(2)]^{(5)} = 0.0181,$$

$$i = 1,2,\ j = 177,178.$$

The subsystem S_2 consists of 2 identical pipelines, each composed of 719 components. In each pipeline there are:

- 717 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(2)}(1)]^{(5)} = 0.0062,\ [\lambda_{ij}^{(2)}(2)]^{(5)} = 0.0088,$

$i = 1,2,\ j = 1,2,...,717,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(2)}(1)]^{(5)} = 0.0166,\ [\lambda_{ij}^{(2)}(2)]^{(5)} = 0.0181,$

$i = 1,2,\ j = 718,719.$

At the system operational state z_6, the system is composed of two subsystems S_1, S_2, each containing 2 pipelines and one subsystem S_3 containing 3 pipelines with the structure showed in *Figure 17*.
The subsystem S_1 consists of 2 identical pipelines, each composed of 178 components. In each pipeline there are:

- 176 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(1)}(1)]^{(6)} = 0.0062,\ [\lambda_{ij}^{(1)}(2)]^{(6)} = 0.0088,$

$i = 1,2,\ j = 1,2,...,176,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(1)}(1)]^{(6)} = 0.0166,\ [\lambda_{ij}^{(1)}(2)]^{(6)} = 0.0181,$

$i = 1,2,\ j = 177,178.$

The subsystem S_2 consists of 2 identical pipelines, each composed of 717 components. In each pipeline there are:

- 717 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(2)}(1)]^{(6)} = 0.0062,\ [\lambda_{ij}^{(2)}(2)]^{(6)} = 0.0088,$

$i = 1,2,\ j = 1,2,...,717,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(2)}(1)]^{(6)} = 0.0166,\ [\lambda_{ij}^{(2)}(2)]^{(6)} = 0.0181,$

$i = 1,2,\ j = 718.719.$

The subsystem S_3 consists of 2 pipelines of the first type and 1 pipeline of the second type, each composed of 362 components. In each pipeline of the first type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(6)} = 0.0059,\ [\lambda_{ij}^{(3)}(2)]^{(6)} = 0.0074,$

$i = 1,2,\ j = 1,2,...,360,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(6)} = 0.0166,\ [\lambda_{ij}^{(3)}(2)]^{(6)} = 0.0181,$

$i = 1,2,\ j = 361,362.$

In the pipeline of the second type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(6)} = 0.0071,\ [\lambda_{ij}^{(3)}(2)]^{(6)} = 0.0079,$

$i = 3,\ j = 1,2,...,360,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(6)} = 0.0166,\ [\lambda_{ij}^{(3)}(2)]^{(6)} = 0.0181,$

$i = 3,\ j = 361.362.$

At the system operational state z_7, the system is composed of the subsystem S_3, which contains 3 pipelines with the structure showed in *Figure 18*.
The subsystem S_3 consists of 2 pipelines of the first type and 1 pipeline of the second type , each composed of 362 elements. In each pipeline of the first type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}$, $\{2\}$, respectively

$[\lambda_{ij}^{(3)}(1)]^{(7)} = 0.0059,\ [\lambda_{ij}^{(3)}(2)]^{(7)} = 0.0074,$

$i = 1,2, \ j = 1,2,...,360,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}, \ \{2\},$ respectively

$[\lambda_{ij}^{(3)}(1)]^{(7)} = 0.0166, \ [\lambda_{ij}^{(3)}(2)]^{(7)} = 0.0181,$

$i = 1,2, \ j = 361,362.$

In the pipeline of the second type there are:

- 360 pipe segments with the intensities of departure from the reliability states subsets $\{1,2\}, \ \{2\},$ respectively

$[\lambda_{ij}^{(3)}(1)]^{(7)} = 0.0071, \ [\lambda_{ij}^{(3)}(2)]^{(7)} = 0.0079,$

$i = 3, \ j = 1,2,...,360,$

- 2 valves with the intensities of departure from the reliability states subsets $\{1,2\}, \ \{2\},$ respectively

$[\lambda_{ij}^{(3)}(1)]^{(7)} = 0.0166, \ [\lambda_{ij}^{(3)}(2)]^{(7)} = 0.0181,$

$i = 3, \ j = 361,362.$

5.1.4.2. Statistical identification of the port oil piping transportation system components reliability on the basis of data coming from their reliability states changing processes

As there are no data collected from the system components failure processes their reliability models identification using the methods of Section 3.4.2 and Section 3.5 is not possible.

5.1.5. Identifying the port oil piping transportation system components conditional multistate exponential reliability functions

As there are no data collected from the port oil piping transportation system components reliability states changing processes, then it is not possible to verify the hypotheses on the exponential forms of the port oil transportation system components conditional reliability functions. We arbitrarily assume that these reliability functions are exponential and using the results of the previous section and the relationships given in Section 5.1.2 we fix heir forms.

At the system operation state z_1, the system is composed of the subsystem S_3 containing three pipelines with the structure showed in *Figure 12*.
The subsystem S_3 consists of 2 pipelines of the first type and 1 pipeline of the second type, each composed of 362 components. In each pipeline of the first type there are:

- 360 pipe segments with the conditional reliability functions co-ordinates

$[R_{ij}^{(3)}(t,1)]^{(1)} = \exp[-0.0059t],$

$[R_{ij}^{(3)}(t,2)]^{(1)} = \exp[-0.0074t], i = 1,2, \ j = 1,2,...360,$

- 2 valves with the conditional multi-state reliability functions co-ordinates

$[R_{ij}^{(3)}(t,1)]^{(1)} = \exp[-0.0166t],$

$[R_{ij}^{(3)}(t,2)]^{(1)} = \exp[-0.0181t],$

$i = 1,2, \ j = 361,362.$

In the pipeline of the second type there are:

- 360 pipe segments with the conditional reliability functions co-ordinates

$[R_{ij}^{(3)}(t,1)]^{(1)} = \exp[-0.0071t],$

$[R_{ij}^{(3)}(t,2)]^{(1)} = \exp[-0.0079t],$

$i = 3, \ \ j = 1,2,...360,$

- 2 valves with the conditional reliability functions co-ordinates

$[R_{ij}^{(3)}(t,1)]^{(1)} = \exp[-0.0166t],$

$[R_{ij}^{(3)}(t,2)]^{(1)} = \exp[-0.0181t],$

$i = 3, \ j = 361,362.$

At the system operation state z_2, the system is composed of the subsystem S_3, which contains 3 pipelines with the structure showed in *Figure 13*.
The subsystem S_3 consists of 2 pipelines of the first type and 1 pipeline of the second type, each composed of 362 components. In each pipeline of the first type there are: