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Introduction to Fourier analysis using IT tools of Laboratory of Technical Aids of Teaching IF UJ

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Artykuł oryginalny

Abstrakt

The general aim of this article is to present didactic aspects of familiarizing students with the Fourier analysis. This didactic process issues through the use of IT tools of Laboratory of Technical Aids of Teaching is discussed. There are four separate computer stations, differing in software, where students learn step by step: composing signals from single harmonics, visualization through images of rotating vectors of harmonic signal composition, operation of a virtual generator of a given number of harmonics and elements of the CMA Coach system in the context of signal analysis. The applied didactic methods are aimed at students of biophysics for whom the Fourier analysis is of practical importance.

Keywords

- Fourier transform
- FFT
- didactics
- motoric aspects of approximation
- coach program

Authors contributions

A – Conceptualization
B – Methodology
C – Software
D – Investigation
E – Data duration
F – Visualization
G – Writing – original draft preperation
H – Writing, reviewing & editing
I – Funding acquisition

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None declared

Introduction

In science and technology and in everyday life, periodic quantities are common, i.e. those that reproduce in the same form in a specific time interval of length T – called a period. Examples include macroscopic state parameters of a thermodynamic machine factor (e.g. an ideal gas in the Carnot cycle) or the behavior of parameters describing alternating current (instantaneous voltage, instantaneous intensity), as well as mechanical vibrations of active elements of various musical instruments. This transformation is also an invaluable tool used in research related to environmental protection and ecology [1].

Issues related to the Fourier analysis are difficult mathematically and logically for beginners, which is why the PTŚN IF UJ has introduced didactic classes to introduce students to this issue. Currently, computer science allows for a relatively simple operation of harmonic functions and, through a kind of play, gaining intuition and experience in analysis.

The simplest model to describe periodic vibrations is a harmonic oscillator, reproduced by the dynamic Hook's law. Then harmonic vibrations of a single specific frequency arise. The situation described above is a kind of mathematical "comfort", consisting of the simplicity of describing the system's behavior (one harmonic). However, periodic functions representing the behavior of most real physical objects cannot be so easily reduced to single harmonic disturbances. It turns out, thanks to the works of Euler and Fourier [2, 3], that complex periodic signals can be represented by a Fourier series built in the base of harmonics with frequencies adjacent to each other, such as successive natural numbers:

$1, \cos(x), \sin(x), \cos(2x), \sin(2x), \ldots, \cos(nx), \sin(nx), \ldots$

The coefficients with which individual elements of the above base (weights) reproduce the real signal constitute a specific logotype of the periodic course and physically have a very important interpretation (e.g. timbre). It should be added here that there are several strictly mathematical requirements for the existence and convergence of the Fourier series of a given function. For a detailed analysis of the mathematical determinants of the problem, one should cite the developed criteria of the existence and convergence of the Fourier series (e.g. the Dirichlet and Dini criterion [4]). Generally, to ensure the compatibility of a function in the form of a Fourier series, it is sufficient that the mapping is absolutely integrable, has at most the first kind of discontinuities and a finite number of local maxima and minima.

Students are familiarized with the above-mentioned conditions of the existence of the Fourier series for a given periodic course during an introductory test. Among other things, the meaning of the first type of discontinuities so that it is understood that the analyzed function must be limited, an example of a square wave is given as not meeting this criterion and methods of modifying such a waveform so that this criterion is met (practically realized square signals are actually slightly trapezoidal signals).

Before starting work, we require students to know the basic rules of trigonometry, in particular, instead of using the sine function, you can use the cosine function, because they are functions of the same shape, only shifted in phase by $\pi/2$. Often, instead of operating the vibration phase, one consists of sinusoidal and cosine contributions, using the relation:

$$A sin(\omega \cdot t + \phi) = \mathrm{a} \cdot \cos(\omega \cdot t) + b \cdot \sin(\omega \cdot t) \qquad (1)$$

where:

$$A = \sqrt{a^2 + b^2}$$

$$\phi = \operatorname{atan}\left(\frac{a}{b}\right)$$

The mathematical consequence of the above relations is the following observation. The sum of harmonic functions with the same frequencies and different amplitudes remains a harmonic function. The nature of the sum can be amplifying, weakening (in particular, a constant function can be obtained), but the same component frequencies never lead to pulsations other than them. There is also an important conclusion that adding a signal with a frequency that is a natural multiple of the fundamental frequency retains the fundamental frequency of the resultant vibration as a result, the shape of which can be modified in this way.

In practice, we encounter the necessity to proceed as if the opposite to the one described above, because the analyzed signal (e.g. voice recording) is multimodal by nature and we need to answer the question of what basic components it can be composed of. The mathematical operation that answers this question is called the Fourier Transform.

Fourier Transform

The Fourier transform is an integral transform that for mapping $f(x)$ takes the form $F(z)$.

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{izu} du \qquad (2)$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(z) e^{-ixz} dz \qquad (3)$$

It is clear that the function $F(z)$ is generally complex even when the function $f(x)$ is real, so the mappings $f(x)$ and $F(z)$ are in different domains. Inverse mapping (3) allows us to return to the original $f(x)$ in the case when we know its transformed image. Physically, this means the ability to transform signals from the time domain to the frequency domain and possibly recreate the actual signal when we know its spectral shape. The Fourier transform is a kind of generalization of the complex form of the Fourier series in the sense that it gives the entire spectrum of the distribution, i.e. all Euler-Fourier coefficients. Since it is mathematically difficult to expect such a complex integral to have an analytical form, numerical methods of its quick estimation (FFT) have been developed, based on the intuitively obvious possibility of replacing a convergent series with its terms giving a significant input.

The specificity of teaching how to proceed with periodic runs

Although the idea of presenting a vector distribution on an orthogonal basis is essentially simple, in the case of periodic waveforms distributed in the basis (1), it faces the problem of computational complexity. Formally, the decomposition problem:

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(nx) + b_n \sin(nx)) \tag{4}$$

solves the knowledge of character factors:

$$a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(mx)\,dx \ \ (m = 1, 2, 3, \cdots) \tag{5}$$

$$b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(mx)\,dx \ \ (m = 1, 2, 3, \cdots) \tag{6}$$

In practice, however, even for analytically simple formulas defining the mapping $f(x)$, the calculus of integrals appearing in formulas (4), (5), and (6) may pose serious difficulties. Formulas (5) and (6) can be unified when considering the Fourier series in the complex domain [4].

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(u)e^{-nui}du \ \ (n = 0, \pm 1, \pm 2, \cdots) \tag{7}$$

In order to go through the tedious calculations and appreciate the power of the apparatus of such analysis, it is necessary to offer the student such IT tools that will take over the painstaking work, emphasizing the final effect of decomposition.

It is with this in mind that computer didactic sets have been set up, which offer students the opportunity to visually familiarize themselves with the formation of the $f(x)$ signal mapping from successively applied harmonics.

Thanks to the real-time tracking of successive approximations with the increasing number of harmonics of the expected function, students get an insight into the improving quality of this function [5].

In addition, the software offers students the opportunity to observe animations in which the influence of individual harmonics is presented as multiple complex (7) motions of circles with radii in relation to the frequencies of successive harmonics (smaller circles circulate larger - similarly to the pre-Keppler model, epicycles and difference complemented the orbital motion of planets).

Practical implementation of computer-aided learning harmonic analysis

As part of the Laboratory of Technical Teaching Means, four independent computer and didactic workstations were set up. The students then fill these positions in teams of two or three, moving through them in such order that each team has the opportunity to participate in all four experiments. These experiments are: 1) composing a signal from single harmonics, 2) animations of rectangular and triangular signal formation, 3) creating a signal using a virtual generator, 4) introduction to the detection, recording, and analysis of signals using the Coach system.

1. Composing a signal from single harmonics

In the menu of this program, the offer includes a base signal in the form of a sine function and a harmonic signal selectable for it with the possibility of adjusting the amplitude, frequency, and initial phase. The result of superposing both signals against the background of themselves (three different lines) can be observed by the student over a wide period of time. Thanks to this, the person performing the exercise gains experience and physical intuition regarding the submission of vibrations. In particular, you can observe selective gains and fades as well as rumble by manipulating the parameters of the regulated harmonics.

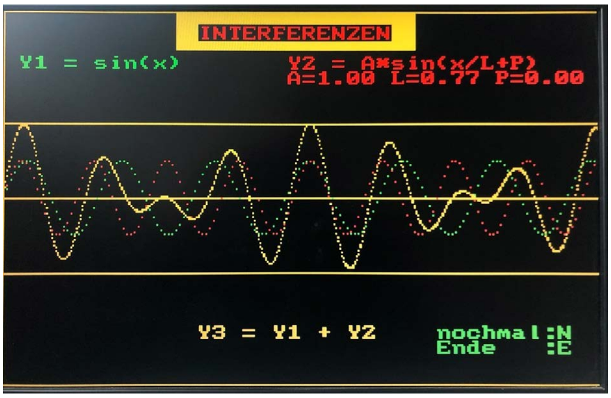

Figure 1. Visualization of beating formation by adding two sine waves with slightly different frequencies

Another option is offered by a variant of the program, in which the student observes the formation of rectangular, triangular, sawtooth, and semi-sine signals from adding harmonics. A step before the observer is familiarized with the decomposition of the signal into harmonics according to the Fourier series. Here you can pay attention to even and odd components in the Fourier series, which is reflected in the nature of the signal itself.

The last element of the program is the ability to independently select the nature of the signal, the range in which the signal is to be approximated and the cut-off amplitude, i.e. the minimum value with which we also take into account the series elements.

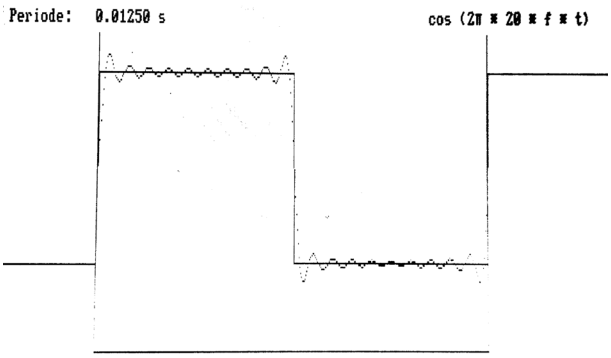

Figure 2. Square signal approximation by a series of sinusoidal signal

In this part of the experiment, the possibility of observing the influence of the given range of decomposition on the nature of the distribution seems to be particularly valuable didactically (the same signal spread over the symmetric and asymmetric ranges reveals significantly different distribution coefficients). Moreover, it is clearly visible how important for the quality of the distribution is the task of the cut-off amplitude; if its value is high, even a single harmonic approximation of a given waveform can be obtained; in turn, its too low value results in a breakdown of up to twenty harmonics (Figures 2, 3). It is also difficult to overestimate the experience that students acquire here in terms of the quality of the signal distribution, in particular, the distribution over a narrow range does not give an approximation as efficiently as the distribution in at least one-period domain.

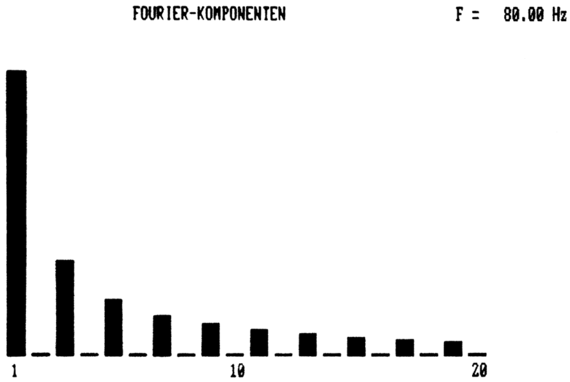

Figure 3. The share of sinusoidal signals - components of the series approximating the square signal

At this stage, when composing the signals, the student selects the amplitude, wavelength, and phase of the signal. Then, when moving to Fourier analysis, the student enters the cut-off amplitude below which the contribution to the contribution of a given harmonics is considered insignificant.

2. Animations of rectangular and triangular signal formation

Another computer station is dedicated to the dynamics of visualization in real time. The thing is that it is worth observing the effect of adding successive elements of the Fourier series on the quality of the approximation of the given signal. When we are able to see better and better progress towards it against the background of the original course, it is a huge cognitive value that speaks in favor of the usefulness of the Fourier series [6].

Another type of animation is the image of a point orbiting on the circumference of a circle, additionally disturbed by the rotation of additional circles with smaller radii (Figure 4). This reflects the successive decreasing Euler Fourier coefficients (we have an analogy here with orbital motion modified by differentials and epicycles in the model of the pre Keppler dynamics of the planets of the Solar System). On the smallest radius of the circle, there is a virtual pen that draws the shape of the composite of these movements. The effect is very spectacular.

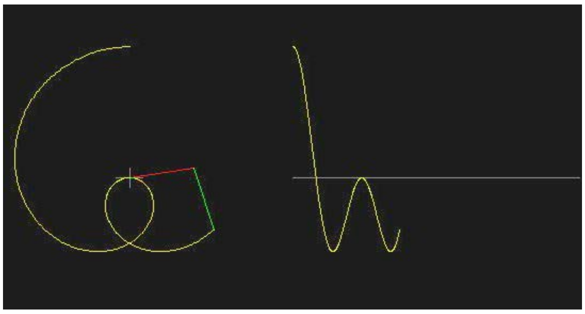

Figure 4. Square wave signal generation simulation

The above visualization (Figure 4) is complemented by an applet that allows you to see the composition of selected (from the set of the first five most important) harmonics against the background of the original signal. You can clearly see how the significance of successive higher harmonics is diminished (e.g. by adding the 1st and 2nd harmonics together we already have the original signal in outline, while adding the 2nd harmonics, 3rd, 4th, and 5th harmonics together, there is no clear agreement with the original yet). At this stage, the role of the student is limited to observing the animation of spinning vectors.

Creating a signal using a virtual generator

In this part of the Laboratory, the student has the opportunity to independently select the harmonics in a given number (based on the book's Fourier series) and observe the effect of their composition. The results of these selections, saved in the form of separate files, can then be compared and compared with each other, which allows drawing conclusions about the increasing quality of the approximation with the number of harmonics. It is also possible to observe the result of placing any periodic signals, e.g. beating (Figure 1).

The program installed on this stand already offers students the possibility to perform Fourier analysis of the generated signal and thus recreating / filtering the parameters of previously combined harmonics (Figure 5, 6). Such a procedure is an opportunity for students to learn the process of transition from the time domain to the frequency domain (formula with the Fourier integral), which is typical for Fourier's analysis. At the same time, the spectrum of the signal in the time domain, the frequency domain and the share of harmonics making up the signal are displayed on the screen.

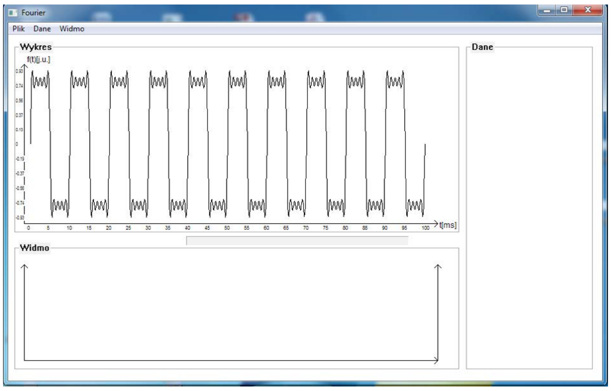

Figure 5. Fourier program's screen with square wave visualization. The spectrum area (in Polish „Widmo") is empty yet

This fragment of the laboratory's work carries the greatest didactic value, as the student selects the number of harmonics taken for analysis and controls the quality of the approximation. The virtual generator allows you to enter information about the amplitude, frequency, initial phase, measurement duration, and sampling frequency. The resulting signal can be visually presented and then subjected to a Fourier transformation. The student can also choose the duration range of the signal to be transformed.

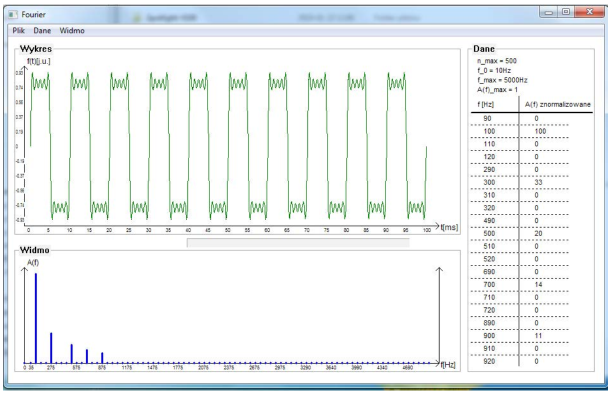

Figure 6. Fourier program's screen with square wave visualization and its Fourier transform. The table shows the share of individual component harmonics

3. Introduction to the detection, recording, and analysis of signals using the Coach system

In the final position, students have the opportunity to become familiar with the operation of the Coach integrated measurement system. This system consists of a universal console, to which it is possible to connect various types of measurement sensors compatible with this system, and dedicated software for data collection and analysis [7]. The essence of the activities performed at this stand is the detection, recording, and analysis of the image of acoustic vibrations originating from standardized tuning forks. This program offers a wide range of possibilities for manipulating both the settings of the signal detection and the recorded image, its, geometric and algebraic analysis. The student is also slowly introduced to the issue of the Fourier transform of real signals.

Conclusion

The above-mentioned cascade of measuring stations located in the Laboratory of Teaching Techniques is a unique didactic offer for a wide range of recipients, including physicists, naturalists, biophysicists, and technicians. Starting from the initial understanding of the essence of harmonic vibration assembly (part I) through an insight into the geometric aspect of vibration assembly (animations - part II), and ending with the Fourier analysis, step by step the student can become familiar with the modern technique of signal analysis.

The student learns how a periodic function can be presented as a sum of harmonic functions with frequencies: the analyzed function and being its integer multiple. Classes allow students to acquire intuition, which makes it easier for students to familiarize themselves with the mathematical formalism of the above-mentioned Fourier Transform.

The didactic overtone of the above is difficult to overestimate: the measurement cascade is helpful in educating medical physicists (image and sound analysis), engineers of broadly understood spectroscopy, IT technicians (data compression methods), cryptographers, and employees of related fields.

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