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# Analysis of simple anti-windup compensation in pole-placement control of a second order oscillatory system

**Abstract**

The main aim of the paper is to present the analysis of anti-windup compensation impact on tracking performance for a second-order plant and continuous-time PID controller and five different simple anti-windup compensators. The performance of the system is compared on the basis of computing differences between the integrals of absolute and squared tracking errors for the system with and without compensation, as well as observing the excess of windup phenomena taking place. Parameters of the controller are computed according to pole-placement scheme. The control system quality is described on the basis of two quality indices for a stable oscillatory second-order plant and a square reference signal.

**Keywords**: anti-windup compensator, IAE, ISE, saturation, pole placement, PID controller.

## 1. Introduction

When we analyze a real-world tracking system, it is important to remember about constraints, e.g. the speed of opening a controller valve. The difference between constrained and computed control signals gives rise to *windup* phenomenon (Fig. 1), and has a negative influence on the system, by deteriorating tracking performance. For the calculated value of the computed control signal, violating its constraints, i.e. $|v(t)|>\alpha$, any further increase in the constrained control signal $u(t)=(\alpha\text{sign}(v(t))$ is not possible. The problem takes place in the case of controllers, particularly a PID controller, that have integrator blocks, where unnecessary integration operation starts having the undesirable effect when constraints become active.

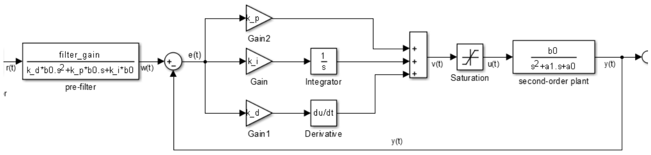


Fig. 1. Matlab-Simulink diagram of the tracking system with saturating controls

To minimize the *windup* effect on tracking performance, it is necessary to use *anti-windup compensator* (AWC) in the system.

In this paper there is presented the analysis of tracking performance for a continuous-time PID controller based on pole-placement [1], with five different *anti-windup compensators* compared for a square-wave reference input.

## 2. Quality indices

The analysis of tracking performance for the considered systems with PID pole-placement controllers is mainly based on two quality indices, namely: Integral of Absolute Error (IAE) defined as

$$J_1 = \int_0^\infty |e(t)|\, dt, \tag{1}$$

where $e(t) = r_M(t) - y(t)$ is the tracking error, $y(t)$ is the closed-loop output signal, and $r_M(t)$ is the reference signal from the desired closed-loop model and Integral of the Squared Error (ISE)

$$J_2 = \int_0^\infty [e(t)]^2\, dt, \tag{2}$$

The quality indices with subscripts $J_{1AWC}$ and $J_{2AWC}$ refer to systems with *anti-windup* compensators, contrary to $J_1$, $J_2$ which represent the systems without an *anti-windup* compensator. In the simulation, an infinite upper integral limit was replaced with the finite time $t_h$ assuring that transients were neglectable. The reference model is described by the transfer function:

$$\frac{am_0}{am_3 s^3 + am_2 s^2 + am_1 s + am_0}, \tag{3}$$

where $a_{M0}$, $a_{M1}$, $a_{M2}$, $a_{M3}$ are coefficients of its polynomials.

## 3. Shaping the desired closed loop response

### 3.1. Model of the plant

The plant is modeled by the second-order transfer function

$$G_{ob}(s) = \frac{b_0}{s^2 + a_1 s + a_0}, \tag{4}$$

where: $b_0 = K\varpi_N^2$, $a_1 = 2\xi\varpi_N$, $a_0 = \varpi_N^2$, $K$ is the gain, $\omega_N$ is the natural frequency, $\xi$ is the damping factor. All the coefficients are assumed to be known.

### 3.2. Pole-placement PID control

The PID controller takes the form:

$$G_{PID}(s) = \frac{s^2 k_d + s k_p + k_i}{s}, \tag{5}$$

and its parameters are tuned off-line according to a Diophantine equation.

The closed loop characteristic polynomial was chosen to be of second-order, i.e., the same as the order of the plant (Fig. 1), and it was compared to the desired characteristic equation described as:

$$\begin{aligned} &am_3 s^3 + am_2 s^2 + am_1 s + am_0 = \\ &= s^3 + s^2(a_1 + b_0 k_d) + s(a_0 + b_0 k_p) + b_0 k_i . \end{aligned} \tag{6}$$

After computations, the solution of the Diophantine equation for this pole-placement problem [2] is

$$\begin{aligned} k_i &= \frac{am_0}{b_0}, \\ k_p &= \frac{am_1 - a_0}{b_0}, \\ k_d &= \frac{am_2 - a_1}{b_0}. \end{aligned} \tag{7}$$

The cut-off level of the computed control signal $v(t)$ (see Fig. 1) set at $\pm\alpha_{\min}$ allows asymptotic tracking in the closed-loop system and is used to give a relative measure of the hardness of the constraints.

## 4. Pre-filter

It should be noted that the pre-filter is added in series with the closed-loop system to perfectly cancel its zero-dynamics. Its transfer function is

$$\frac{k_{filter}}{s^2 k_d b_0 + s k_p b_0 + k_i b_0}, \tag{8}$$

where $k_{filter}$ is a pre-filter gain. Its aim is to decrease the speed of transients, as well as to reduce the probability of consecutive resaturations of the control signal.

An important aspect is to make sure that the pre-filter remains stable. The following necessary and sufficient conditions of stability on the basis of Routh criterion

$$\begin{aligned} k_d b_0 &> 0, \\ k_p b_0 &> 0, \\ k_i b_0 &> 0. \end{aligned} \tag{9}$$

can be met by adjusting closed-loop reference model parameters.

## 5. Anti-windup compensators

### 5.1. Tracking system with limitation of Integrator (AWC1)

The block diagram of the system with an AWC1 compensator is presented in Figure 2 [3], as a Matlab-Simulink diagram.

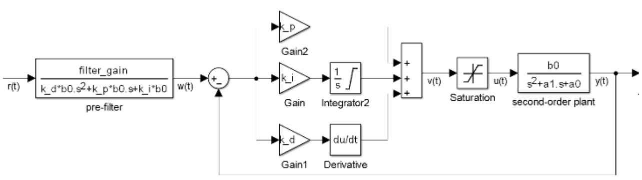


Fig. 2. Matlab-Simulink diagram with AWC1 compensator

The main disadvantage of this compensation is that the integral part of the controller can be stopped even if the signal $v(t)$ is not saturated [3].

### 5.2. Tracking system with limitation of Integrator with error signal e(t) (AWC2)

The block diagram of the system with an AWC2 compensator is shown in Figure 3 [4]. In this method, the signal fed to the integrator is the difference between the reference signal $r(t)$ and the output signal $y(t)$. When the signal $v(t)$ or $u(t)$ resaturates, the integration should be stopped.

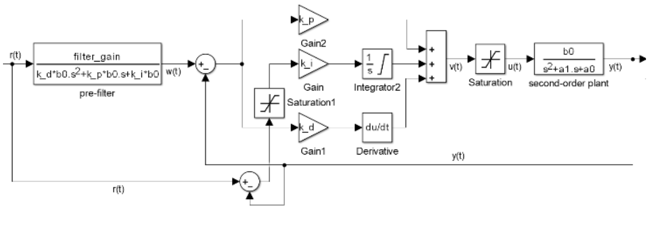


Fig. 3. Matlab-Simulink diagram with AWC2 compensator

### 5.3. Tracking system with control signal conditioning (AWC3)

The block diagram of the system with an AWC3 compensator is presented in Figure 4 [4]. In this case to improve damping in the output signal $y(t)$ [4], the integrator block is fed with the sum of $w(t)$ and the difference between the computed $v(t)$ and constrained $u(t)$ control signals multiplied by $K_f T_N$. The input signal of the integrator is described by

$$e_i(t) = e(t)k_i + \Delta u(t) K_f T_N, \tag{10}$$

where $K_f$, $T_N$ are constant during the simulation process.

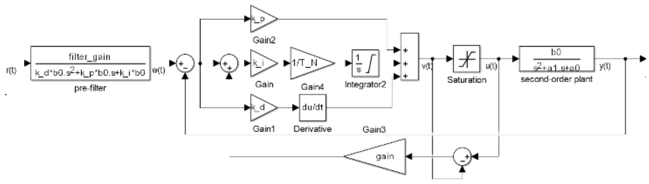


Fig. 4. Matlab-Simulink diagram with AWC3 compensator

### 5.4. Tracking system with dead-zone of the control signal (AWC4)

The block diagram of the system with an AWC4 compensator is shown in Figure 5 [3].

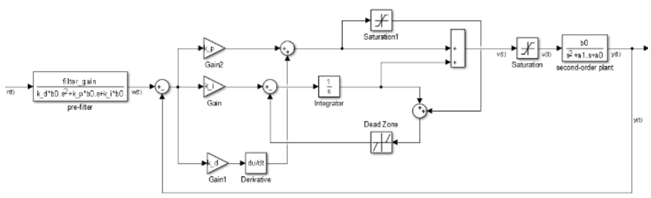


Fig. 5. Matlab-Simulink diagram with AWC4 compensator

In this system with AWC4, large initial values of $v(t)$ signal are caused by P and D actions of the controller due to step changes in the reference signal. In due course of the control process, the influence of P and D actions is decreasing so that the I element is not able to compensate for this change fast enough. The AWC4 compensator eliminates this problem by adding another saturation to P and D elements.

### 5.5. Tracking system with external reset of Integrator (AWC5)

The block diagram of the system with an AWC5 compensator is presented in Figure 6 [4].

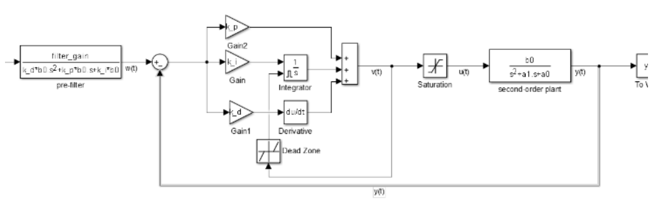


Fig. 6. Block diagram with AWC5 compensator

The fifth method is based on resetting the integral action by feeding the signal $v(t)$ to dead-zone function and the reset port of the integrator. The range of dead-zone is equal to the doubled cut-off limit $2\alpha$ in the constrained control signal $u(t)$. When the value of $v(t)$ exceeds a given limit, the signal on the additional input of the integrator resets the integration. If the $v(t)$ gets desaturated, the normal integration action is restored.

## 6. Performance analysis

The analysis of AWC performance of the systems with the proposed PID controller and the second-order plant (4) is presented below.

The output signal $y(t)$ of the analyzed systems and the square reference signal are presented in Figure 7 for the following closed-loop parameters: $a_1$=2, $a_0$=6, $b_0$=6, $\omega_n=\sqrt{2}$ , $\xi=1/\sqrt{2}$ , $K$=1, $K_f$=5, $T_n$=0.1, saturation of the integrator for AWC1 equal to ±5/2, ±0.3 for AWC2, for AWC3, AWC4 and AWC5 equal to $\pm\alpha_{\min}$=5 and the filter_gain=1800. The simulation time and the period of the reference signal are set to 30 seconds.

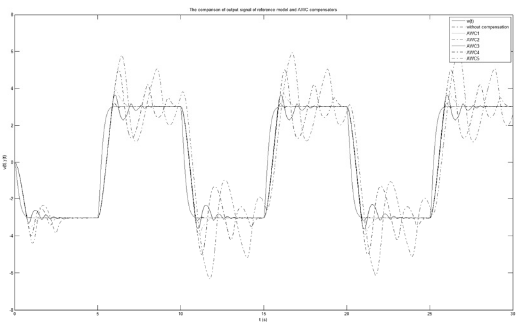

Fig. 7. Tracking performance of non-compensated and compensated systems

An important question is how much of the control signal is cut-off during simulation. The results are shown in Figure 8.

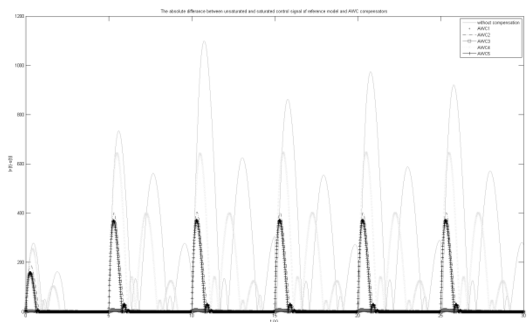

Fig. 8. Absolute difference between the computed and constrained control signals for the system with and without compensation

The results of simulations are shown in Figures 9-13 as surface plots of the functions:

$$\Delta J_1 = J_1 - J_{1AWC}, \quad \Delta J_2 = J_2 - J_{2AWC}, \tag{11}$$

referring to the difference between control performance for the uncompensated and AWC-compensated system. Positive values in such plots depict the cases when the AWC improves the control quality and negative values if the contrary.

The top plots represent $\Delta J_1$ and $\Delta J_2$ while the bottom plots represent the amount of time (in seconds) when the computed control signal exceeds the cut-off limits.

The results in Figure 9 show that improvement of the control quality is obtained for the AWC1 compensator for the object analyzed in this paper in the range of 2< $\omega_N$ <3.2 and 0< $\xi$ <0.3. In other situations the tracking performance is similar to the model without compensation.

It was proven that for high oscillations in the closed loop system without cut-off limit of the integrator and control signal, the system was stable.

a)

b)

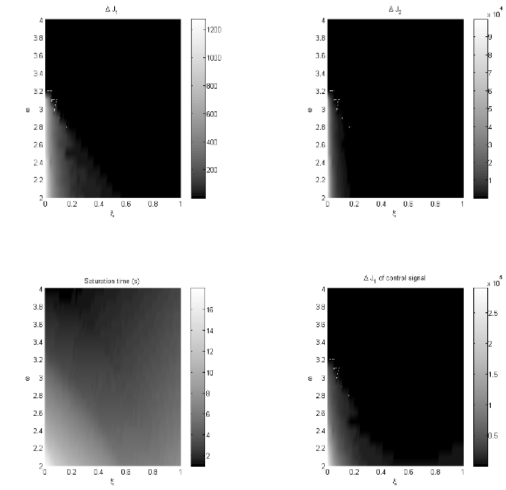

Fig. 9. Tracking performance for a) AWC1 $\Delta J_1$ and $\Delta J_2$ and b) time (s) when the control signal is saturated

a)

b)

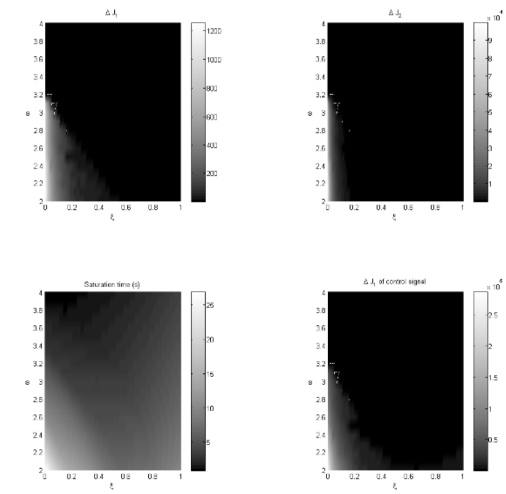

Fig. 10. Tracking performance for AWC2 a) $\Delta J_1$ and $\Delta J_2$ and b) time (s) when the control signal is saturated

a)

b)

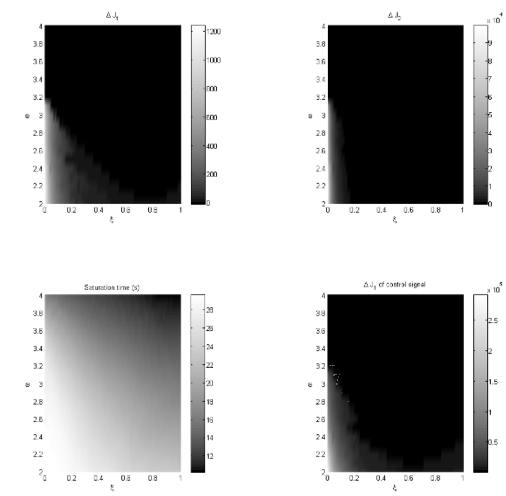

Fig. 11. Tracking performance for AWC3 a) $\Delta J_1$ and $\Delta J_2$ and b) time (s) when the control signal is saturated

For the AWC2 compensator, the improvement can be noted but time where the control signal is saturated is up to 25 seconds (Fig. 10b). The consequence is that the Integral part of the PID controller is saturated for a longer period of time. The reason why the control signal is saturated for such a long period of time is that there are no instruments in the system with AWC2 which cause to overstep the saturation limit of the control signal.

Introduction of the third compensator proposed in the paper results in improvement for similar ranges of the natural frequency and damping factor as of its two predecessors. In Figure 11b, it can be seen that for the whole range of variables $\omega_N$ and $\xi$ the control signal is saturated at a half time of the simulation time.

a) b)

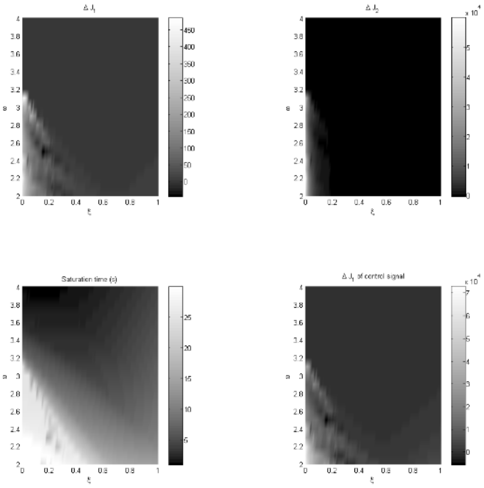

Fig. 12. Tracking performance for AWC4 a) $\Delta J_1$ and $\Delta J_2$, and b) time (s) when the control signal is saturated

For the AWC4 compensator it can be noted that improvement of the control signal is as good as for the other compensators. In Figure 7, it can be observed that the output signal of the system with AWC4 is most oscillatory from all the five compensators analyzed it this paper.

a) b)

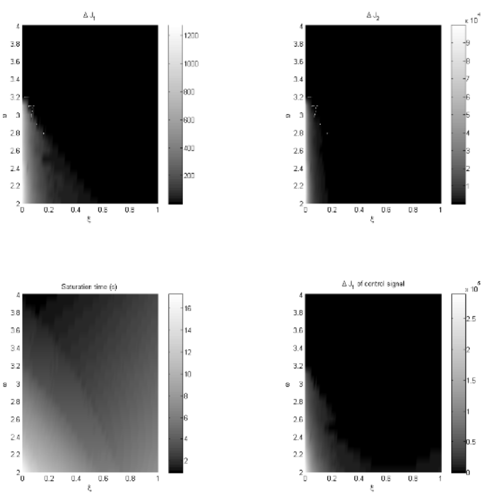

Fig. 13. Tracking performance for AWC5 a) $\Delta J_1$ and $\Delta J_2$, and b) time (s) when the control signal is saturated

It can be observed that the AWC5 compensator gives least oscillations in the output signal $y(t)$ and the time when the control signal is saturated is rarely bigger than 16 seconds (Fig. 13b). It is important because the system does not need large values of the control signal to have good tracking performance.

## 7. Conclusions

The results presented in Figures 9-13 are only selected examples of the performance of the proposed pole-placement AWC-compensated control system. It was observed that during the simulations increase in the cut off level of the control signal caused decrease in the control quality improvement when using the AWC compensators. The saturation time of the control signal for five AWC compensators in all the ranges of simulations described in this paper are presented below:

Tab. 1. Saturation time and approximate improvement of AWC compensators

| | AWC1 | AWC2 | AWC3 | AWC4 | AWC5 |
|---|---|---|---|---|---|
| Saturation time of control signal (s) | 0-18 | 0-26 | 10-30 | 0-26 | 0-18 |
| Approximate improvement of control signal based on $\Delta J_1$ and $\Delta J_2$ surfaces (%) | 15-20 | 15-20 | 25 | 30-35 | 15-20 |

It can be noted that for an underdamped system the AWC compensators give improvement for $\xi$<0.2 and $\omega_N$<3. To sum up, if a system needs to be less oscillatory, it is better to use an AWC1 or AWC5 compensator where the saturation time of the control signal is the shortest. On the other side, if the quality indices are considered, the best solution will be an AWC4 compensator. It should be noted that these results are for the underdamped system and low dynamics with dominant pole of the reference model at -6.6.

## 8. References


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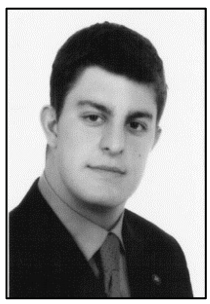

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