Block-Structured Models Composed of Nonlinear Fuzzy Dynamic and Static Parts – a Case Study

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Abstract:
The paper addresses issues of the dynamic fuzzy Takagi-Sugeno models identification for multi-step ahead prediction. In the case of highly nonlinear models, standard Takagi-Sugeno models may be hard to identify if they should be designed for recurrent prediction generation. In such a case, alternative fuzzy block-structured models composed of fuzzy dynamic and fuzzy static parts may be useful. Two main benefits of the proposed models are: (1) possibility to speed-up model tuning procedure, (2) potential to fine-tune an already available, standard Takagi-Sugeno model. The benefits offered by the proposed models are illustrated using the example of identification of a nonlinear process – a system consisting of two tanks of different shapes (cylindrical and conical ones).

Keywords: *block-structured models, Takagi-Sugeno models, identification, modelling, fuzzy logic*

1. Introduction

Identification of dynamic systems is an important task, as it is crucial in many applications and fields of knowledge, including process control, robotics and economy [3]. Some processes can be efficiently modeled with linear models using Least Squares Method for identification [8]. However, linear dynamic models applied to the real systems tend to be often inaccurate, due to the nonlinear character of the processes. Tuning of such models is even more difficult in the case of multi-step ahead prediction. In order to improve modeling accuracy, nonlinear models can be used. Most popular nonlinear models are based on polynomials [4], neural networks [11] and fuzzy logic [12].

An example of the nonlinear dynamic model is a model based on fuzzy Takagi-Sugeno (TS) fuzzy system [14]. This model is based on fuzzy membership functions and local linear models. Identification of these models is not an easy task, especially in the case of multi-step ahead prediction [7]. There are several identification methods strictly designed for these models, like ANFIS (Adaptive Neuro-Fuzzy Inference System) [5]. This method determines the shape of fuzzy membership functions in the first place. Then, it solves quadratic programming problem in order to identify parameters of linear local models. The main disadvantage of this method is a high likelihood of failure, when multi-step ahead models are taken into consideration.

Methods of identification of TS models can be divided into global and local approaches [1]. The global approach generates highly accurate global models, but local models are not proper linearizations of the process in selected steady-state points. In the case of the local approach, the local models are in fact linearizations of the process in several steady-state points, on the other hand, global output of such a model may be not satisfactory. In order to overcome this issue, multi-objective identification methods have been introduced [6]. Despite many attempts towards creation of a universal and highly effective Takagi-Sugeno identification method, none of them gives a satisfactory result good enough in the case of multi-step ahead prediction for any plant. Thus, in the case of many processes, individually adapted identification procedures should be used.

Another example of a nonlinear model is the Wiener model, which consists of a linear dynamic block preceding a nonlinear static model [4]. Thus, the Wiener model is a composition of two different models. For example, linear dynamic part can be modeled as the ARX model and nonlinear static part can be provided as the TS fuzzy model. Such a structure can be efficiently used in Model Predictive Control (MPC) algorithms [10, 15]. In [2, 13] a Wiener model has been used to model a polymerization reactor and a distillation column respectively. In [9] an example of a fuzzy Wiener model has been given. Then, it has been shown that this model can be efficiently used in the MPC algorithms. These model belongs to the class of nonlinear block-oriented models. In general, several advantages of these models can be highlighted: low cost in identification, low computational complexity, possibility to approximate nearly all systems (with some exceptions) and block-oriented structure itself, which can be useful in for example control algorithms.

Both: Wiener and TS models have some drawbacks. Wiener models (especially tuned with traditional approach) are inefficient in the case of processes with highly nonlinear dynamics, because its dynamic part is linear, although other block-oriented nonlinear models can be successfully used inidentification of systems with nonlinear dynamics [18]. Moreover, new methods can be used to improve Wiener model tuning e.g. Maximum Likelihood methods [16, 17]. These methods can be used for the reduction of problems concerning bias. On the other hand, well-tuned TS model can be efficient for nearly all processes. However, these models may be very hard to identify in the case of models designed for a multi-step ahead prediction. Moreover, identification of fuzzy models often requires more heuristic approaches. It impedes procedure of model tuning and is strongly connected with the fact that often different heuristics have to be used for different problems. Despite these issues, TS models are still considered as useful in control engineering, because: (1) they are similar to linear models, so can be easily used in control algorithms, (2) they are considered as universal approximators and (3) TS model structure enables tuning of its individual elements (e.g. individual linear models).

In this paper, advantages of both block-structured and fuzzy models have been merged. The goal of this paper is to provide simple yet effective method for the improvement of Takagi-Sugeno model identification. Fuzzy block-structured models composed of a nonlinear dynamic part and of a nonlinear static part have been presented. Theyare combination of the two aforementioned models. In comparison to the Wiener model or the TS model, the block-structured model can be identified easier. It can be also used to improve quality of a roughly tuned TS fuzzy model. As there are many effective methods for identification of block-oriented models for strongly nonlinear systems [18, 19], their structure was used for improvement of TS models. Main goal of this paper is to show that advantages of the block-oriented models can be successfully used during facilitation of identification of other models.

In chapter 2, the block-structured model and its advantages are described. Chapter 3 contains description of an example process. In chapter 4, applications of ARX, Wiener and TS models to modeling of the example process are presented. All described models have been compared using Mean Square Error (MSE).Chapter 5 contains description of application of the proposed block-structured model. Additionally, advantages of block-structured models highlighted in chapter 2 are demonstrated. Finally, chapter 6 concludes the paper.

2. Block-Structured Models

The proposed block-structured models have been inspired by the Wiener models. They are composed of nonlinear models; see fig. 1. Both dynamic and static parts of the proposed model can be expressed as any nonlinear model (e.g. fuzzy TS models, multilayer perceptron neural models or polynomials). This work is focused on block-structured models composed of TS fuzzy dynamics and TS fuzzy statics.

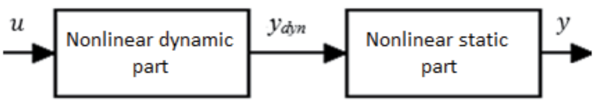

Fig. 1. Structure of a SISO block-structured model with nonlinear dynamic and nonlinear static parts; u – input, y – output, y_{dyn} – input of the static model

As Takagi-Sugeno model consists of many linear models, output of the dynamic part of the block-structured model can be expressed as the normalized weighted sum of outputs of linear dynamic models:

$$y_{dyn}(k) = \frac{\sum_{i=1}^{l} w_i \cdot y_{lin}^{i}(k)}{\sum_{i=1}^{l} w_i} \quad (1)$$

where l is the number of rules, w_i is firing strength (weighting factor) of the i-thrule and $y_{lin}^{i}(k)$ is the output of the i-thlocal dynamic model given by:

$$y_{lin}^{i}(k) = -a_1^i y_{dyn}(k-1) - \cdots - a_{n_a}^i y_{dyn}(k-n_a) + b_1^i u(k-1-\tau) + \cdots + b_{n_b}^i u(k-n_b-\tau) + c \quad (2)$$

where a_j^i and b_j^i are parameters of the dynamic model, n_a and n_b define model dynamics, τ denotes delay, c denotes constant value, is the output of the linear dynamic block in the k–m-th sampling instant, $u(k - m)$ is the input of the model in the k–m-th sampling instant. Output of the dynamic TS model is then used as an input to the static part described by:

$$(k) = \frac{\sum_{is=1}^{ls} \mu_{is}\left(y_{dyn}(k)\right) \cdot \left(a_1^{is} \cdot y_{dyn}(k) + a_0^{is}\right)}{\sum_{is=1}^{ls} \mu_{is}\left(y_{dyn}(k)\right)} \quad (3)$$

where ls is the number of fuzzy rules in the static part of the model, $\mu_{is}(\cdot)$ is a membership function in the is-th rule of the fuzzy TS static model, and a_0^{is} , a_1^{is} are parameters of linear models in the consequents of the fuzzy TS static model (i.e. parameters of the local models). It is worth to notice that consequents of the rules can be assumed constant. Then the model simplifies to:

$$y(k) = \frac{\sum_{is=1}^{js} \mu_{is}\left(y_{dyn}(k)\right) \cdot a_0^{is}}{\sum_{is=1}^{js} \mu_{is}\left(y_{dyn}(k)\right)} \quad (4)$$

After substituting (1) into (3), the proposed block structured model can be formulated as follows:

$$y(k) = \frac{\sum_{is=1}^{ls} \mu_{is}\left(\frac{\sum_{i=1}^{l} w_i \cdot y_{lin}^{i}(k)}{\sum_{i=1}^{l} w_i}\right) * \left(\frac{a_1^{is} * \sum_{i=1}^{l} w_i \cdot y_{lin}^{i}(k)}{\sum_{i=1}^{l} w_i} + a_0^{is}\right)}{\sum_{is=1}^{ls} \mu_{is}\left(\frac{\sum_{i=1}^{l} w_i \cdot y_{lin}^{i}(k)}{\sum_{i=1}^{l} w_i}\right)} \quad (5)$$

The main motivation for developing the block-structured models was improvement of roughly identified dynamic TS models by extending a model using low number of additional parameters. Wiener models have linear dynamic part, thus they may not be suitable for processes with highly nonlinear dynamics. Fuzzy TS models are universal approximators. However, in the case of fuzzy models, increase of parameters leads to the curse of dimensionality (though in the TS models it is, however, not as disruptive as in the case of Mamdani models). Moreover, TS models are often hard to identify in the case of models with recurrent prediction (multi-step ahead prediction).

2. Example control plant

Example identification of the block-structured model has been performed for the system consisting of two tanks of different shapes (cylindrical and conical ones, Fig. 2). Such a process can be described by the following set of equations:

$$\frac{dV_1}{dt} = F \quad + F_d - F_2(h_1)$$
$$\frac{dV_2}{dt} = F_2(h_1) - F_3(h_2)$$
$$F_2(h_1) = \alpha_1\sqrt{h_1}, F_3(h_2) = \alpha_2\sqrt{h_2},$$
$$V_1(h_1) = C_1 \cdot h_1^3, V_2(h_2) = A_2 \cdot h_2, F \quad (t) = F_{1\in(t-\tau)} \quad (6)$$

where h_1 and h_2 denote liquid levels in tanks *1* and *2* respectively, F_{1in} denotes input flow to the first tank, F_2 denotes input flow to the second tank, F_3 stands for output flow of the whole system and F_D stands for the disturbance flow. V_1 and V_2 denote volumes of liquid in tank *1* and *2*, respectively. Values of the process parameters are: $A_2 = 300$ cm², $C_1 = 0.75$, $\alpha_1 = 15.9$, $\alpha_2 = 20$, delay $\tau = 40$ s.

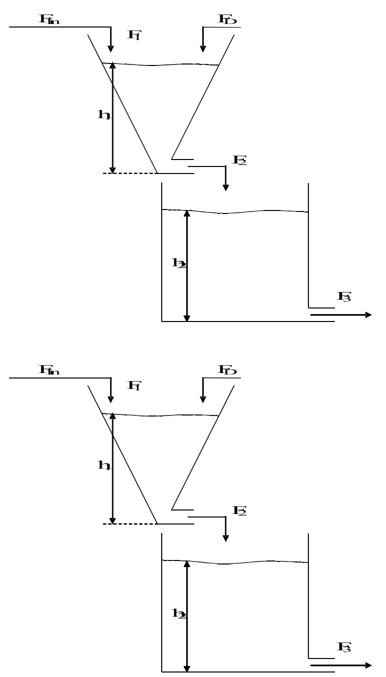

Fig. 2. System of two tanks

In Single Input Single Output (SISO) case, first flow F_1 is considered to be an input of the system. Output of the system is liquid level in the second tank, h_2. The process can be also treated as a MISO plant with disturbance input flow F_D taken into account. In order to calculate output value directly from the differential equations, equations (6) can be transformed into:

$$\frac{dh_1}{dt} = \frac{F_1 + F_d - \alpha_1\sqrt{h_1}}{3 \cdot h_1^2 \cdot C_1}$$
$$\frac{dh_2}{dt} = \frac{\alpha_1\sqrt{h_1} - \alpha_2\sqrt{h_2}}{A_2} \quad (7)$$

One should notice that $F_D$is not delayed. Thestatic characteristics of the control plant, and of the model linearized in the steady-state point $h_2 = 8.41$ cmand $F = 51$ cm³/sare presented in Fig. 3 (disturbance flow $F_D$was assumed constant and equal to 7 cm³/s). Step responses of the nonlinear model are compared with the ones of the linearized model in Fig. 4.

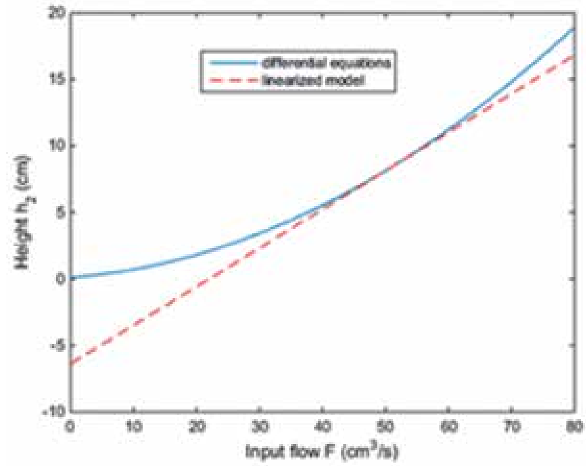

Fig. 3. Static characteristic of the process

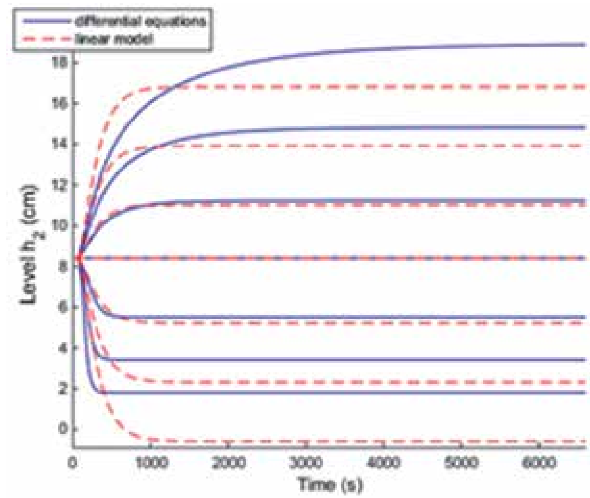

Fig. 4. Control plant dynamics

Observing Figs. 3 and 4 it is clear that the example process has both: nonlinear statics and nonlinear dynamics. During the experiments the input value F waschangedto the following values:20, 30, 40, 51, 60, 70, 80 cm³/s. It can be noticed that the bigger difference between actual input and the input at the steady-state point is, the bigger difference between responses of the linear and nonlinear models can be observed.

3. Application of Linear, Wiener and Takagi-Sugeno models

This chapter presents application of linear, Wiener and TS models to the example plant. Some assumptions concerning model identification have been done:

Control plant has been identified within input range $F \in$ *<0,80>*cm³/s.

During identification, control plant is considered to be a SISO process, with input flow F and liquid level h_2 being the output.

Only recurrent models with multi-step ahead predictions are considered.

As the considered control plant is slow, sampling period is equal to $T = 10$ s.

Disturbance flow F_D was constant and equal to $7\ cm^3/s$.

All of the models are compared with the ideal nonlinear model, received from the equations (6). Models have been evaluated and compared using the Mean Square Error (MSE)

$$\frac{E}{n} = \frac{1}{n}\sum_{k=1}^{n}\left(y_r(k) - y(k)\right)^2 \tag{8}$$

where $y_r(k)$ and $y(k)$ denote output values of the original model (6) in the discrete sample k and of an identified model, respectively.

3.1. Application of a Linear Model

The presented model has been identified by linearization of differential equations (6) in the steady-state point $F = 51\ cm^3/s$ and $h_2 = 8{,}41$ cm. It is given by:

$$\begin{aligned} y_{lin}(k) = & -a_1 y(k-1) - a_2 y(k-2) - a_3 y(k-3) + \\ & + b_1 u(k-5) + b_2 u(k-6) + b_3 u(k-7) \end{aligned} \tag{9}$$

Where
$a_1 = -2.83818251$, $a_2 = 2.68214541$,
$a_3 = -0.84396289$, $b_1 = 0.86185551 \cdot 10^{-3}$,
$b_2 = -0.04738442 \cdot 10^{-3}$, $b_3 = -0.81447108 \cdot 10^{-3}$.

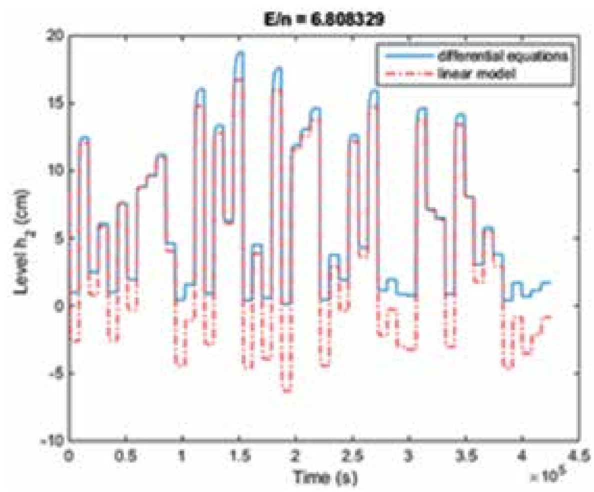

Fig. 5. Test of linear model

The linear model has been verified on the test data set obtained using the original equations (6); see Fig. 5. The linear model fails to imitate both statics and dynamics of the tanks at the satisfactory level. Imperfections in statics can be mostly seen at the lower range of input values. For the low values of the flow F, liquid level h_2 is negative. Bad representation of dynamics can be observed especially for the higher range of input values, where linear model achieves the steady state too quickly. The test shows that the linear model is not accurate enough and identification of the nonlinear process model should be done.

4.2. Application of a Wiener Model

The first simple and natural step towards improving the linear model (9) is to extend it with a nonlinear static part. Such an approach will lead to obtaining the Wiener model, with linear dynamics preceding nonlinear statics, which is given by:

$$\mu_1\left(y_{lin}(k)\right) = e^{\frac{-\left(y_{lin}(k)-c_1\right)^2}{2\cdot\sigma_1^2}}$$

$$\mu_2\left(y_{lin}(k)\right) = e^{\frac{-\left(y_{lin}(k)-c_2\right)^2}{2\cdot\sigma_2^2}}$$

$$y(k) = \frac{\sum_{i=1}^{2}\mu_i\left(y_{lin}(k)\right)\cdot\left(a_1^i \cdot y_{lin}(k) + a_0^i\right)}{\sum_{i=1}^{2}\mu_i\left(y_{lin}(k)\right)} \tag{10}$$

where $\mu_i(\cdot)$ denote the generalized Gaussian membership functions and $y_{lin}(k)$ denotes output of the linear dynamic model (9), the values of the parameters are as follows: $c_1 = -3.674$, $c_2 = 7.969$, $\sigma_1 = 18.61$, $\sigma_2 = 14.44$, $a_0^1 = 35.65$, $a_0^2 = -36.17$, $a_1^1 = 0.7923$, $a_1^2 = 2.053$. The fuzzy, static part of the model has been identified using Adaptive Neuro-Fuzzy Inference System (ANFIS) MATLAB tool. Nonlinear static part consists of two fuzzy rules, because bigger number of rules has not improved the model significantly. The test of the Wiener model using the test data set is presented in Fig. 6.

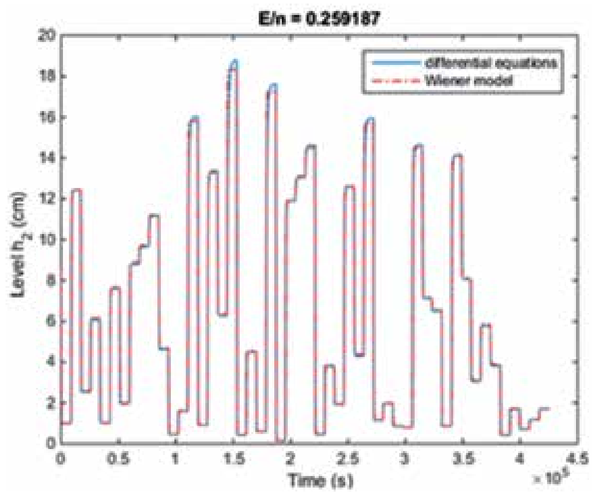

Fig. 6. Test of the Wiener model

Although the Wiener model performs much better than linear model (6), it is still not perfect enough. It can be noticed that at the higher range of input flow the Wiener model not only achieves steady-state values too fast, but there is also a considerable deviation between steady-state values.

4.3 Application of a Takagi-Sugeno Model

In the case of the example plant, identification of a TS fuzzy model working in a satisfactory way with multi-step ahead prediction is not easy. The aim was to identify a model, which would be able to properly predict output h_2 in the whole range of inputs, with as low number of parameters (and of fuzzy rules) as possible. First of all, standard identification tools like ANFIS are not able to identify a stable recurrent model for the considered system of tanks. In order to identify a properly working model, a lot of optimization procedure calls have been performed. After many experiments, the following model consisting of three local linear models and three membership functions has been identified

$$y_{lin_1}(k) = -a_1^1 y_{dyn}(k-1) - a_2^1 y_{dyn}(k-2) - a_3^1 y_{dyn}(k-3) + b_1^1 u(k-5) + b_2^1 u(k-6) + b_3^1 u(k-7) + a_0^1$$

$$y_{lin_2}(k) = -a_1^2 y_{dyn}(k-1) - a_2^2 y_{dyn}(k-2) - a_3^2 y_{dyn}(k-3) + b_1^2 u(k-5) + b_2^2 u(k-6) + b_3^2 u(k-7) + a_0^2$$

$$y_{lin_3}(k) = -a_1^3 y_{dyn}(k-1) - a_2^3 y_{dyn}(k-2) - a_3^3 y_{dyn}(k-3) + b_1^3 u(k-5) + b_2^3 u(k-6) + b_3^3 u(k-7) + a_0^3$$

$$\mu_1\left(y_{dyn}(k-1)\right) = \frac{1}{1+\left|\frac{y_{dyn}(k-1)-c_{d1}}{a_{d1}}\right|^{2b_{d1}}}$$

$$\mu_2\left(y_{dyn}(k-1)\right) = \frac{1}{1+\left|\frac{y_{dyn}(k-1)-c_{d2}}{a_{d2}}\right|^{2b_{d2}}}$$

$$\mu_3\left(y_{dyn}(k-1)\right) = \frac{1}{1+\left|\frac{y_{dyn}(k-1)-c_{d3}}{a_{d3}}\right|^{2b_{d3}}}$$

$$y_{dyn}(k) = \frac{\sum_{i=1}^{3}\mu_i\left(y_{dyn}(k-1)\right)\cdot y_{lin_i}(k)}{\sum_{i=1}^{3}\mu_i\left(y_{dyn}(k-1)\right)} \qquad (11)$$

Parameters of Takagi-Sugeno model are presented in Table 1. In this particular case, $\mu_i(\cdot)$ are the generalized bell membership functions. Results of the test of the fuzzy model are presented in Fig. 7.

Table 1. Parameters of Takagi-Sugeno model

$a_1^1 = -2.99269755$	$a_2^1 = 3.09986829$	$a_3^1 = -1.10786562$
$b_1^1 = -0.00265196$	$b_2^1 = 0.00097673$	$b_3^1 = 0.00144840$
$a_0^1 = 0.02325236$	$a_1^2 = -2.07214459$	$a_2^2 = 1.40937893$
$a_3^2 = -0.31914971$	$b_1^2 = 0.00743793$	$b_2^2 = -0.00209157$
$b_3^2 = -0.00149540$	$a_0^2 = -0.05903962$	$a_1^3 = -0.49863498$
$a_2^3 = -1.35260971$	$a_3^3 = 0.85935076$	$b_1^3 = 0.02694692$
$b_2^3 = -0.02428196$	$b_3^3 = -0.00135914$	$a_0^3 = 0.03383882$
$a_{d_1} = 2.38310951$	$b_{d_1} = 1.20259040$	$c_{d_1} = 9.65702467$

The TS fuzzy model is very well tuned. Some imperfections can be noticed only in the magnified plot. However, the differences are very small and typical for such identification tasks. In comparison to the Wiener model, the MSE coefficient is of order of magnitude better. One should note that procedure of TS model identification was very time-consuming, especially in comparison to the rapid identification of Wiener model. Moreover, TS model required heuristic approach, which was strictly defined for the presented system of tanks. As the heuristic approach was used, it is hard to strictly define convergence rate. However, TS model identification time was significantly longer than in case of Wiener model, where strict mathematical rules can be used for system identification. The goal of the proposed fuzzy block-structured models presented in the next chapter is to connect their structure with fuzzy approach, in order to significantly improve procedure of fuzzy models identification.

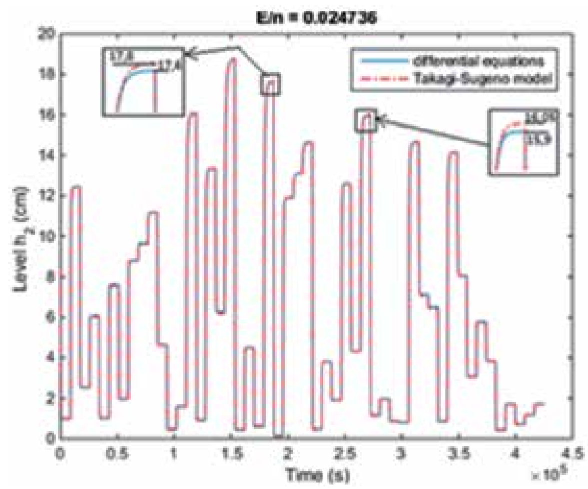

Fig. 7. Test of the Takagi-Sugeno model

5. Application of the Proposed Block-Structured Model

In the proposed block-structured model, a nonlinear fuzzy static model follows a nonlinear dynamic model. The ANFIS tool has been used to identify nonlinear static part, input of which is the output of the dynamic model (11). The model consists of two generalized Gaussian membership functions and of two linear models

$$\mu_4\left(y_{dyn}(k)\right) = e^{\frac{-\left(y_{dyn}(k)-c_4\right)^2}{2\cdot\sigma_4^2}}$$

$$\mu_5\left(y_{dyn}(k)\right) = e^{\frac{-\left(y_{dyn}(k)-c_5\right)^2}{2\cdot\sigma_5^2}}$$

$$y(k) = \frac{\sum_{i=4}^{5}\mu_i\left(y_{dyn}(k)\right)\cdot\left(a_1^i\cdot y_{dyn}(k)+a_0^i\right)}{\sum_{i=4}^{5}\mu_i\left(y_{dyn}(k)\right)} \qquad (12)$$

where $c_5 = -2.103$, $\sigma_5 = 1.818$, $c_6 = 19.59$, $\sigma_6 = 3.257$, $a_1^5 = 0.9964$, $a_0^5 = -0.002777$, $a_1^6 = 0.9908$, $a_0^6 = 0.1114$. Results of the test of the model are presented in Fig. 8. The obtained model is very well tuned and almost perfectly mimics the original equations (6).

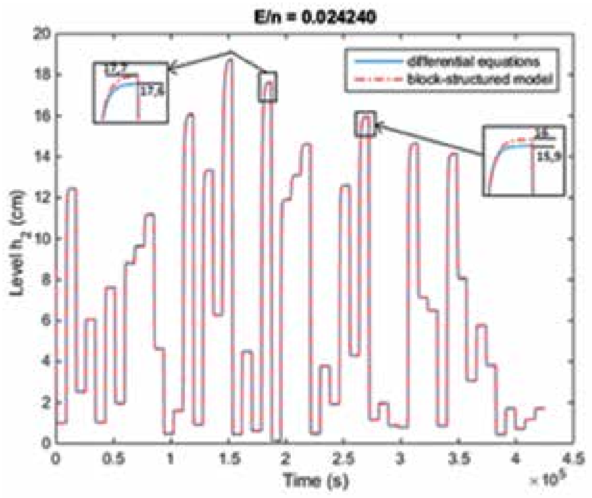

Fig. 8. Test of the block-structured model

5.1. Comparison with the Takagi-Sugeno and Wiener Models

The proposed block-structured model has slightly improved already identified Takagi-Sugeno model. It can be observed after comparing the magnified fragments of Figs. 7 and 8 that modeling at the higher values of level h_2 has been improved. Difference in values of the MSE coefficient is equal to $\Delta E/n = 0.0005$. Such a small difference could have been foreseen, as TS fuzzy model has been already very accurate and well-tuned.

In comparison to the Wiener model, the block-structured model is about 11 times better (comparing the MSE). Still, the block-structured model has Wiener-specific structure, what can be beneficial in the case of some special applications (like predictive control cooperating with the set-point optimization). The tests confirm that the block-structured models can offer better performance comparing to the Wiener models).

5.2. Improvement of roughly tuned Takagi-Sugeno models

One of the benefits gained from using block-structured models is possibility to improve already existing TS fuzzy models. Comparing results from subsection 4.3 and section 5, it is hard to clearly confirm such an advantage. However, two factors should be taken into consideration:

Firstly, TS fuzzy model has already been very well tuned.

Secondly, it was mentioned that TS fuzzy model for the proposed plant has been hard to identify. It required hundreds of optimization procedure calls (Sequential Quadratic Programming, Active-set and Genetic Algorithm optimization methods) and considerable computational effort. Thus, the identification process is very time-consuming.

TS fuzzy models can be especially hard to identify in the case of models with multi-step ahead prediction. When one-step ahead prediction is considered, standard tools and methods, like ANFIS, are usually sufficient. It is because they are adjusted to the non-recurrent problems. This is due to the objective function, which consists of already defined membership functions (treated as constants in the latter optimization steps) and linear consequents of TS fuzzy model. Such an approach leads to the quadratic programming problem (a convex function). In multi-step ahead prediction, membership functions cannot be treated as constants, what leads to the nonlinear optimization problem.

In the case of the process under consideration, ANFIS was unable to determine a stable model for the system of two tanks. An approach tailored to the given problem was required.

In such cases, block-structured models can be found useful. When fine-tuning of a TS fuzzy model is too hard, it can be stopped and current best model can be improved by adding a nonlinear static part. Such an approach can considerably improve performance of the fuzzy model, without significant increase of computation complexity.

Now, a TS fuzzy model obtained during the identification procedure of the fine-tuned model presented in subsection 4.3 will be improved using the proposed approach.

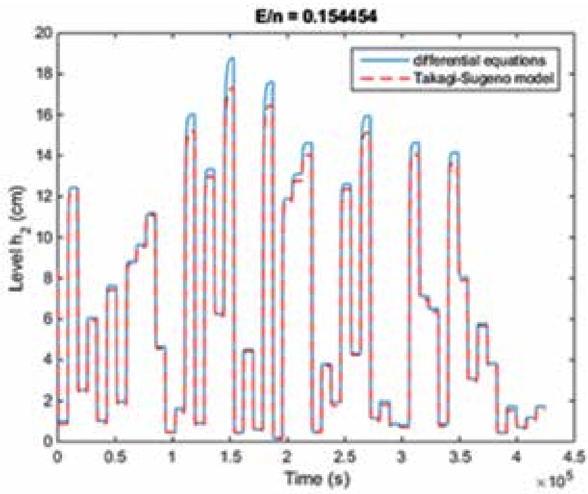

Fig. 9 The first roughly tuned TS fuzzy model

The model was obtained pretty fast, but the value of the MSE coefficient (E/n=0.1544) is better than in the case of the Wiener model, though visually Wiener model test looks better (compare Figs. 6 and 9). It is caused by the better adjustment of control plant dynamics in the TS fuzzy model, which greatly impacts the MSE coefficient. The Wiener model is in fact better only in prediction of process statics. Improvement has been done by identifying the nonlinear static part using ANFIS tool. Several variants of the static part have been tested: with linear or constant consequents and 2, 3 or 4 Gaussian membership functions. The obtained results are presented in Table 2.

The initially not too well tuned TS fuzzy model has been improved almost 4 times. In the case of linear consequents of the fuzzy model, two membership functions are sufficient to significantly improve TS fuzzy model. In the case of constant consequents three membership functions are sufficient to provide similar quality of the model, as in the case of the model with linear consequents. Selection of constant consequents may be useful in the case, when computation time matters. Test of the block-structured model with 2 membership functions and linear consequents in the static part is presented in Fig. 10.

Table 2. Improvement of first TS fuzzy model

num. of mem. functions \ conseq. type	linear	constant
2	$\frac{E}{n} = 0.044$	$\frac{E}{n} = 0.054$
3	$\frac{E}{n} = 0.043$	$\frac{E}{n} = 0.045$
4	$\frac{E}{n} = 0.043$	$\frac{E}{n} = 0.044$

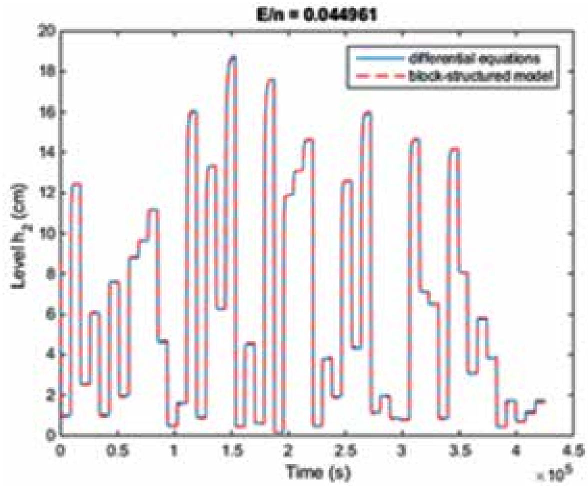

Fig. 10 The first roughly tuned TS fuzzy model followed by the nonlinear fuzzy static part

The second roughly tuned TS fuzzy model is even worse than the first one. Test of this model is presented in Fig. 11. The MSE coefficient of this model is equal E/n=0.888.

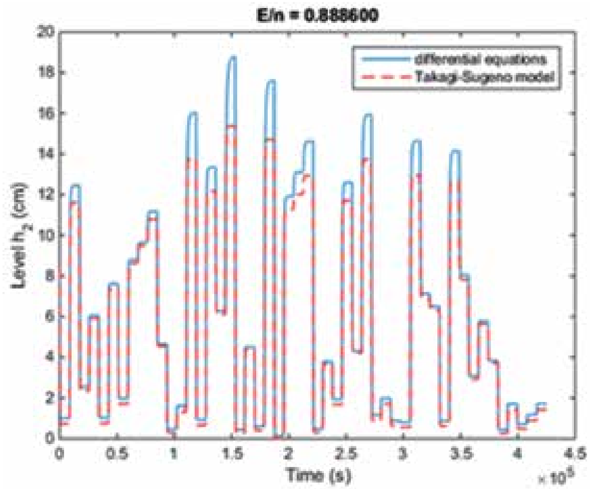

Fig. 11. The second roughly tuned TS fuzzy model

Similar tests have been performed as in the case of the first roughly tuned TS fuzzy model. The static part has been also identified by using the ANFIS tool. Gaussian membership functions have been selected. Table 3 presents results of tests of different block-structured models based on the above mentioned TS fuzzy model.

Table 3. Improvement of second TS fuzzy model

num. of mem. functions \ conseq. type	linear	constant
2	$\frac{E}{n} = 0.206$	$\frac{E}{n} = 0.238$
3	$\frac{E}{n} = 0.203$	$\frac{E}{n} = 0.207$
4	$\frac{E}{n} = 0.203$	$\frac{E}{n} = 0.205$

Like in the previous case, the static model with 2 Gaussian membership functions and linear consequents was enough to significantly improve TS fuzzy model. The model with constant consequents needed 3 Gaussian membership functions in order to achieve quality similar to the one offered by the model with linear consequents. Test of the model with 2 linear consequents in the static part of the model is presented in Fig. 12.

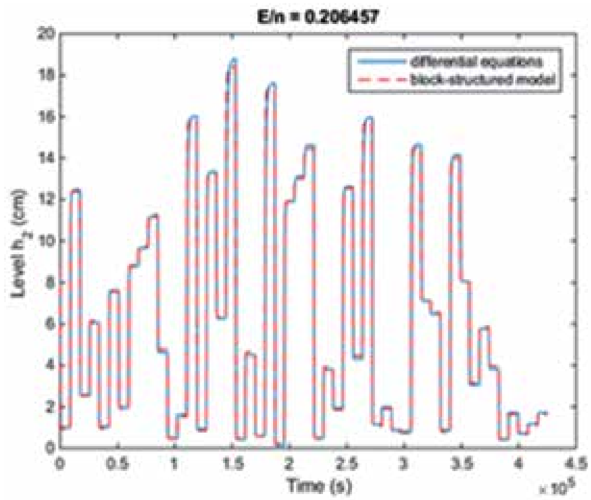

Fig. 12. The second roughly tuned TS fuzzy model followed by the nonlinear fuzzy static part

In the considered example, the block-structured model is about 4.5 times better than the initial TS fuzzy model. Using ANFIS tool, identification of the nonlinear static model is almost instant, therefore the identification procedure of the model is much faster and effortless than in the case of the TS fuzzy model described in Sect. 4.3. One should notice that usage of different identification techniques for static part may provide even better model.

To sum up, this subsection proves that a roughly tuned TS fuzzy model can be significantly improved by adding and identifying a nonlinear static block, input of which is an output of the nonlinear dynamic model. In the case of a highly nonlinear processes, and multi-step ahead prediction, such an approach may turn out to be very useful.

5.3. Disturbance Modeling in Block-Structured Models

So far, only SISO models have been considered. However, liquid level h_2 depends not only on the input flow F, but also on the disturbance flow F_D. Taking the flow F_D intoconsideration as one of the system inputs, leads to the MISO model. In the current work, disturbance F_D has been changed in the range from *0* to *15* cm^3/s. The static characteristic of the plant considered as a MISO system is presented in Fig. 13.

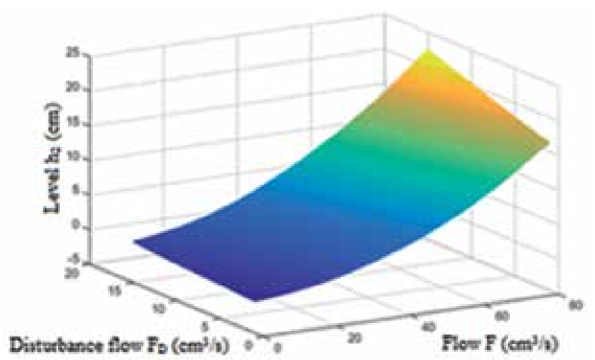

Fig. 13. Static characteristic of the process with two input flows: F and F_D

In block-structured models the additional disturbance input can be included in different ways. Two approaches to MISO modeling have been thus investigated. In the first of them, a simplified one, it is assumed that the disturbance is taken into consideration only in the nonlinear static part of the model. The structure of the model in this approach is presented in Fig. 14.

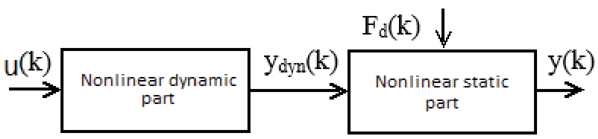

Fig. 14. MISO model with disturbance taken into consideration only in the static part of the model

Therefore, in the first model, the nonlinear dynamic part is the same as in eq. (11) and the nonlinear static part is formulated as follows:

$$\mu_5\left(y_{dyn}(k)\right) = e^{\frac{-\left(y_{dyn}(k)-c_5\right)^2}{2\cdot\sigma_5^2}}$$

$$\mu_6\left(y_{dyn}(k)\right) = e^{\frac{-\left(y_{dyn}(k)-c_6\right)^2}{2\cdot\sigma_6^2}}$$

$$\mu_7\left(y_{dyn}(k)\right) = e^{\frac{-\left(y_{dyn}(k)-c_7\right)^2}{2\cdot\sigma_7^2}}$$

$$y_{lin_5}(k) = \left(a_2^5\cdot y_{dyn}(k) + a_1^5\cdot F_d(k) + a_0^5\right)$$

$$y_{lin_6}(k) = \left(a_2^6\cdot y_{dyn}(k) + a_1^6\cdot F_d(k) + a_0^6\right)$$

$$y_{lin_7}(k) = \left(a_2^7\cdot y_{dyn}(k) + a_1^7\cdot F_d(k) + a_0^7\right)$$

$$y\quad(k) = \frac{\sum_{i=5}^{7}\mu_i\left(y_{dyn}(k)\right)\cdot y_{lin_i}(k)}{\sum_{i=5}^{7}\mu_i\left(y_{dyn}(k)\right)}(12) \qquad (13)$$

Parameters of the model are presented in Table 4. It is worth to notice that the disturbance F_D is not fuzzified and it is present only in the consequents of the static TS fuzzy model.

Table 4. Parameters of block-structured model with disturbance included in the static part

c_5 = 0.09089	σ_5 = 2.009	c_6 = 8.124
σ_6 = 4.001	c_7 = 10.26	σ_7 = 1.925
a_2^5 = 1.067	a_1^5 = 0.1034	a_0^5 = –0.7514
a_2^6 = 1.044	a_1^6 = 0.3213	a_0^6 = –2.606
a_2^7 = 1.061	a_1^7 = 0.4334	a_0^7 = –4.206

The second approach to obtain a MISO block-structured model is more complex. The model contains a couple of nonlinear dynamic models. The structure of this MISO model is presented in Fig. 15.

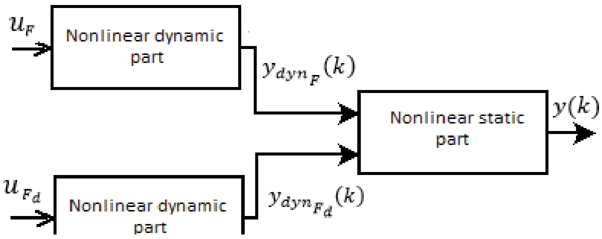

Fig. 15. MISO model with two nonlinear dynamic models

As in the previous approach, the nonlinear dynamic part for the flow F is the same as in eq. (11). However, another nonlinear dynamic model has to be identified, describing influence of the disturbance F_D on the plant:

$$\mu_5\left(y_{dynF_d}(k-1)\right) = e^{\frac{-\left(y_{dynF_d}(k-1)-c_5\right)^2}{2\cdot\sigma_5^2}}$$

$$\mu_6\left(y_{dynF_d}(k-1)\right) = e^{\frac{-\left(y_{dynF_d}(k-1)-c_6\right)^2}{2\cdot\sigma_6^2}}$$

$$\mu_7\left(y_{dynF_d}(k-1)\right) = e^{\frac{-\left(y_{dynF_d}(k-1)-c_7\right)^2}{2\cdot\sigma_7^2}}$$

$$y_{lin_5}(k) = -a_1^5 y_{dynF_d}(k-1) - a_2^5 y_{dynF_d}(k-2) - a_3^5 y_{dynF_d}(k-3) + b_1^5 F_d(k-1) + b_2^5 F_d(k-2) + b_3^5 F_d(k-3) + a_0^5$$

$$y_{lin_6}(k) = -a_1^6 y_{dynF_d}(k-1) - a_2^6 y_{dynF_d}(k-2) - a_3^6 y_{dynF_d}(k-3) + b_1^6 F_d(k-1) + b_2^6 F_d(k-2) + b_3^6 F_d(k-3) + a_0^6$$

$$y_{lin_7}(k) = -a_1^7 y_{dynF_d}(k-1) - a_2^7 y_{dynF_d}(k-2) - a_3^7 y_{dynF_d}(k-3) + +b_1^7 F_d(k-1) + b_2^7 F_d(k-2) + b_3^7 F_d(k-3) + a_0^7$$

$$y_{dynF_d}(k) = \frac{\sum_{i=5}^{7} \mu_i \left(y_{dynF_d}(k-1)\right) \cdot y_{lin_i}(k)}{\sum_{i=5}^{7} \mu_i \left(y_{dynF_d}(k-1)\right)} \quad (14)$$

Parameters of the disturbance model are presented in Table 5.

Table 5. Parameters of the disturbance model

$c_5 = 6.461$	$\sigma_5 = 2.009$	$c_6 = 8.124$
$\sigma_6 = 1.458$	$c_7 = 10.26$	$\sigma_7 = 1.925$
$a_1^5 = -1.09$	$a_2^5 = -0.3266$	$a_3^5 = 0.4494$
$b_1^5 = 0.002509$	$b_2^5 = 0.003863$	$b_3^5 = 0.00137$
$a_0^5 = 0.21$	$a_1^6 = -1.226$	$a_2^6 = -0.3538$
$a_3^6 = 0.5818$	$b_1^6 = 0.0001568$	$b_2^6 = 0.0003302$
$b_3^6 = 0.0002166$	$a_0^6 = 0.01735$	$a_1^7 = -1.229$
$a_2^7 = 0.01735$	$a_3^7 = 0.01735$	$b_1^7 = 0.01735$
$b_2^7 = 0.0005275$	$b_3^7 = 0.0001986$	$a_0^7 = 0.02841$

In the model, the nonlinear static part is composed of four fuzzy rules:

$$\mu_8\left(y_{dyn_F}(k)\right) = e^{\frac{-\left(y_{dyn_F}(k)-c_8\right)^2}{2\cdot\sigma_8^2}}$$

$$\mu_9\left(y_{dyn_F}(k)\right) = e^{\frac{-\left(y_{dyn_F}(k)-c_9\right)^2}{2\cdot\sigma_9^2}}$$

$$\mu_{10}\left(y_{dyn_{F_d}}(k)\right) = e^{\frac{-\left(y_{dyn_{F_d}}(k)-c_{10}\right)^2}{2\cdot\sigma_{10}^2}}$$

$$\mu_{11}\left(y_{dyn_{F_d}}(k)\right) = e^{\frac{-\left(y_{dyn_{F_d}}(k)-c_{11}\right)^2}{2\cdot\sigma_{11}^2}}$$

$$w_8 = \mu_8\left(y_{dyn_F}(k)\right) \cdot \mu_{10}\left(y_{dyn_{F_d}}(k)\right)$$

$$w_9 = \mu_8\left(y_{dyn_F}(k)\right) \cdot \mu_{11}\left(y_{dyn_{F_d}}(k)\right)$$

$$w_{10} = \mu_9\left(y_{dyn_F}(k)\right) \cdot \mu_{10}\left(y_{dyn_{F_d}}(k)\right)$$

$$w_{11} = \mu_9\left(y_{dyn_F}(k)\right) \cdot \mu_{11}\left(y_{dyn_{F_d}}(k)\right)$$

$$y_{st_8}(k) = w_8 \cdot \left(a_2^8 \cdot y_{dyn_F}(k) + a_1^8 \cdot y_{dyn_{F_d}}(k) + a_0^8\right)$$

$$y_{st_9}(k) = w_9 \cdot \left(a_2^9 \cdot y_{dyn_F}(k) + a_1^9 \cdot y_{dyn_{F_d}}(k) + a_0^9\right)$$

$$y_{st_{10}}(k) = w_{10} \cdot \left(a_2^{10} \cdot y_{dyn_F}(k) + a_1^{10} \cdot y_{dyn_{F_d}}(k) + a_0^{10}\right)$$

$$y_{st_{11}}(k) = w_{11} \cdot \left(a_2^{11} \cdot y_{dyn_F}(k) + a_1^{11} \cdot y_{dyn_{F_d}}(k) + a_0^{11}\right)$$

$$y(k) = \frac{\sum_{i=8}^{11} y_{st_i}(k)}{\sum_{i=8}^{11} w_i} \quad (15)$$

Parameters of the nonlinear static part are presented in Table 6.

Table 6. Parameters of the nonlinear static part in block-structured model with disturbance included in the dynamic model

$c_8 = 0.0751$	$\sigma_8 = 8.018$	$c_9 = 18.84$	$\sigma_9 = 8.073$
$c_{10} = 6.724$	$\sigma_{10} = 2.076$	$c_{11} = 11.13$	$\sigma_{11} = 2.195$
$a_2^8 = 0.8044$	$a_1^8 = 0.4272$	$a_0^8 = -3.234$	$a_2^9 = 1.299$
$a_1^9 = 0.4489$	$a_0^9 = -4.359$	$a_2^{10} = 0.8786$	$a_1^{10} = 1.427$
$a_0^{10} = -9.973$	$a_2^{11} = 1.164$	$a_1^{11} = 1.13$	$a_0^{11} = -12.57$

Both MISO models have been verified using a test data set. This data set is very similar to the one used earlier, but now it includes also changes of the disturbance F_D. In the case of a simplified approach with disturbance considered only in the static part of the model, the MSE is equal to $E/n = 0.0178$. In the case of the more complex model with two dynamic parts, $E/n = 0.0064$, thusthe MSE coefficient of the MISO model with two dynamic parts is about 2.5 times better than in the case of the simplified MISO model.

Fig. 16 presents comparison between two fragments of test data set for both MISO models. In this piece of test, changes of disturbance F_D occur. It can be noticed that the model with disturbances included only in the static part has significant problems with modeling influence of F_D on system dynamics. On the contrary, the model with two separate nonlinear dynamic models reconstructs dynamic behavior of the process appropriately. To sum up, the MISO model with two dynamic parts is very accurate and it can be used as a multi-step ahead predictor for the considered control plant.

6. Conclusions

In this paper, block-structured models based on TS fuzzy systems have been proposed. The models have been tested during modeling of the example process, consisting of two tanks (cylindrical and conical ones). All models have been designed for multi-step ahead prediction. The designed model has been compared with other widely used dynamical models, such as linear ARX, the Wiener model and the standard TS fuzzy model. The goal of the method proposed in this paper was to merge advantages of block-structured models and TS models. Block-structured models can be used for rapid identification of many systems, while Takagi-Sugeno models can be used to relatively easy synthesize control algorithms.

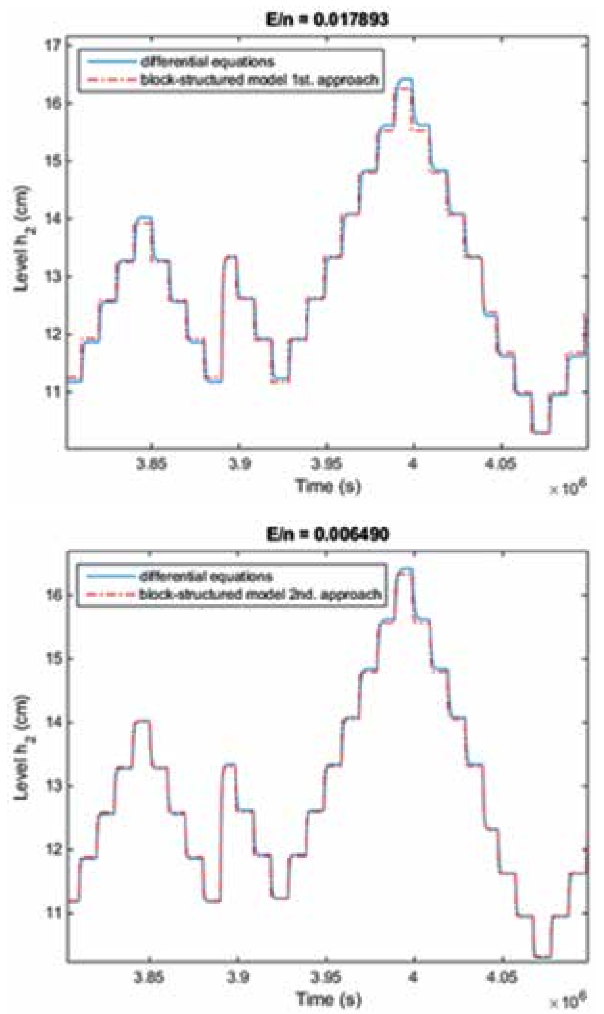

Fig. 16. Comparison of MISO models with one (up) and two (bottom) dynamic parts

The tests have shown that drawbacks of the other models can be relatively easy reduced by means of the proposed block-structured models with the nonlinear static model following the TS fuzzy dynamic model. Thus, the following benefits of the block-structured models (on contrary to Wiener or dynamic TS fuzzy models) can be listed:

- Better performance than the one offered by the Wiener models, in the case of systems with highly nonlinear dynamics;
- Possibility to improve already identified TS fuzzy model. This benefit is especially noticeable when TS fuzzy model is hard to identify;
- Possibility to significantly shorten identification time;
- Possibility to include another input in the model without much effort;
- Possibility to identify separate dynamic models for different inputs and connect them with a single (or multiple) nonlinear static model(s). (Versatility of the proposed approach.)

The structure of the proposed models can be easily used in the Model Predictive Control algorithms cooperating with the set-point optimization. Future work will concern application of the block-structured models in such control algorithms. To sum up, fuzzy block-structured models can be a good alternative to TS fuzzy models, especially when it is hard to find proper heuristic approach for the system identification. They can significantly simplify model identification procedure without sacrificing quality of modeling. Moreover, their structure can be easily extended to the MISO models, if needed.

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