

Grzegorz SŁOŃ*
Alexander YASTREBOV*

APPLICATION OF FUZZY RELATIONAL COGNITIVE MAPS IN INTELLIGENT MODELLING THE TECHNICAL SYSTEMS

Certain approach to the modelling dynamic states in technical systems is presented in this paper. This approach lies in a replacing classic differential model with a model based on fuzzy relational cognitive map. Described method is illustrated by practical example of simple electrical RLC circuit. The method of the normalized reference data preparation was described. The process of building a cognitive map with concepts crucial from the modelling purposes point of view was presented also results of such a map teaching process were shown. In the end a partial comparison of simulation results of work of models: classic – based on the set of differential equations and proposed – based on fuzzy relational cognitive map was performed.

1. INTRODUCTION

Cognitive maps in variants: crisp and fuzzy can be applied in intelligent modelling of imprecise objects. Different aspects of such an approach were presented e.g. in [1]-[9], where possibilities of a synthesis [2, 3, 8, 9] as well as analysis [1, 4, 6] of such type models (building, teaching, fuzzyfication, etc.) were taken into consideration.

The paper is devoted to designing and applying the fuzzy relational cognitive maps at the modelling of precise technical systems on the example of a simple electro technical circuit with transient states.

Application of traditional modelling methods (based on e. g. sets of differential equations) sometimes involves, at improper parameters selection, a risk of arising instability of described process and, in consequence, difficulties in the analysis. Dynamic relational cognitive maps, especially in fuzzy grasp, can represent a finiteness of real processes, which fulfils (thank to description using fuzzy operators) real object stability conditions.

It needs to be stressed that some aspects of applying the fuzzy cognitive maps in modelling electro technical systems were presented in [7].

* Kielce University of Technology.

Comparative results of the application of a cognitive map and classic method of the modelling, on the example of a specific RLC circuit, and conducted simulation analysis short summary are presented.

2. COGNITIVE MODEL

Modelling the technical systems generally doesn't differ from the modelling other types systems. First of, all it should be defined whether the modelling purpose is gaining the information on the model state at certain point of time or mapping the time courses of values of selected quantities. Then, concepts crucial from the modelling purpose point of view should be selected. In extreme cases, the number of these concepts can correspond with the number of quantities existing in real system, but usually the assumed purpose can be reached for significantly lower number of considered concepts.

Characteristic feature of a model based on fuzzy relational cognitive map is possibility of the creation relations (connections) between concepts, which are purely abstract, not reflecting physical flow of signals between elements of a real system. It results from passing over concepts unimportant for the given modelling purpose. At this point, it should be noticed that, dependently on assumed purposes, cognitive maps containing different groups of concepts can be built for the same technical system. Moreover, these concepts can be physical quantities occurring in real system as well as quantities with qualitative nature "added" to the model for the needs of achieving the specific purpose. Independently of the number and the nature of concepts, proper selection of relations between them plays a crucial role. These relations can be assigned to initial values basing on expert knowledge, but mostly it is necessary to additionally teach the model (e.g. through the comparison of the model work results with reference results obtained from a real system).

Below, on a selected example of simple RLC circuit (shown in Fig. 1), certain approach to the building the technical object model based on fuzzy relational cognitive map, is presented.

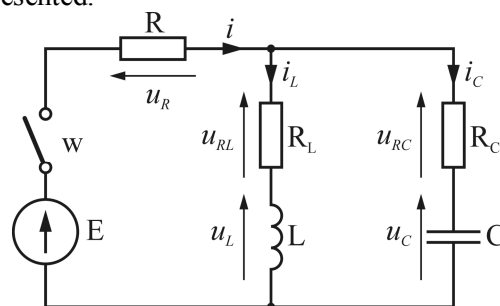


Fig. 1. Tested RLC circuit. $R = 10 \Omega$; $R_L = 0.5 \Omega$; $R_C = 0.5 \Omega$; $L = 0.08 \text{ H}$; $C = 0.03 \text{ F}$; $E = 10 \text{ V}$

2.1. Reference courses

For purposes of a cognitive map supervised teaching the access to reference data is necessary. It can be obtained through measurement or gathering statistical data. For the purpose of this article the method consisting in a building precise mathematical model of tested system and then simulating its work was chosen. Such an approach is especially advantageous for the created method tests because it makes representation of the system work's different states easier.

The circuit from Fig. 1 can be modeled using a set of equations (1):

$$\left\{ \begin{array}{l} \frac{di_L(t)}{dt} = \frac{1}{L} [E(t) - Ri(t) - R_L i_L(t)] \\ \frac{du_C(t)}{dt} = \frac{1}{CR_C} [E(t) - Ri(t) - u_C(t)] \\ i_C = \frac{E(t) - Ri_L(t) - u_C(t)}{R + R_C} \\ i(t) = i_L(t) + i_C(t) \\ u_R(t) = Ri(t) \\ u_L(t) = E(t) - u_R(t) - R_L i_L(t) \end{array} \right. \quad (1)$$

The modeling subject are time courses of selected currents and voltages in a situation when, in earlier stabilized circuit, disturbances, consisting in step lowering the voltage E value from 10 V to 5 V, appear.

Exemplary course of $i_L(t)$ current in time of 1 second is presented in Fig. 2.

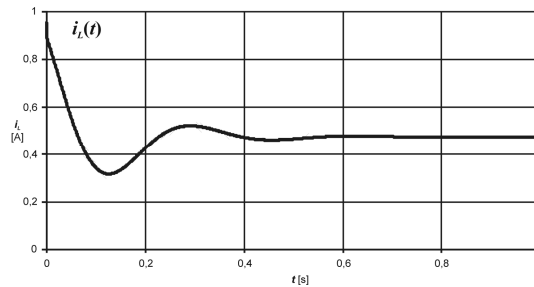


Fig. 2. Exemplary course of $i_L(t)$ current value obtained from classic differential model

The course from Fig. 2 can't be yet a reference for the cognitive map teaching. Firstly, it was made in a simulation process with time step equal to 1 ms, which means that to obtain it 1000 steps were made, which is too many from cognitive modeling point of view. Secondly, similar courses were obtained also for other quantities and there were calibrated in different units, which makes them poorly

useful to build dimensionless relations. To solve these problems discrete time was introduced, where one step corresponds to 50 ms of real time. Moreover, dimensionless normalization was applied in which, for each considered quantity, value 0 corresponds to real value of stationary state (from before the appearance of the disturbance) and values 1 and -1 correspond to limitary real values anticipated for given quantities. In such a depiction exemplary course from Fig. 2 get the form like in Fig. 3.

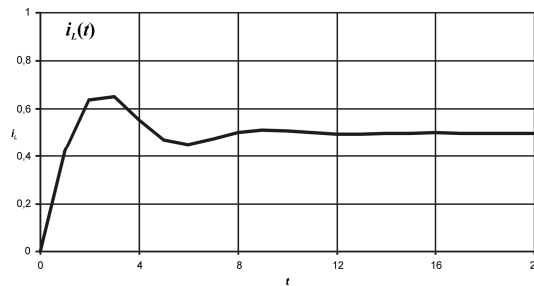


Fig. 3. Current $i_L(t)$ value course transformed do the normalized form for the needs of the model teaching, where: t – discrete time

2.2. The model based on fuzzy relational cognitive map

Relational cognitive map is a kind of mathematical model, where physical quantities of the modeled system are substituted by concepts crucial for the modeling purposes and the "flow" of the information between quantities runs through so called relations that connect individual concepts. Its specific attribute is using normalized values of individual concepts as well as of relations. General form of relational cognitive map can be written like in (2).

$$\langle \mathbf{X}, \mathbf{R} \rangle \quad (2)$$

where: \mathbf{X} – set of values of concepts ($\mathbf{X} = [X_1, \dots, X_N]^T$); \mathbf{R} – set of relations between concepts ($\mathbf{R} = \{R_{ij}\}_{i \neq j}$); $i, j = 1, \dots, N$; N – number of a cognitive map concepts.

Concepts values as well as relations from (2) can take crisp or fuzzy nature depending on the degree of imprecision of the owned accessed data. More precise description of cognitive map design methods, concepts values and relations fuzzyfication methods and arithmetic operations on fuzzy quantities are contained, among others, in [1, 9].

For the needs of this article, dynamic model of fuzzy relational cognitive map considering 4 concepts: E , i_L , u_C and i , was chosen. These concepts values were marked: X_1 , X_2 , X_3 and X_4 , and the cognitive map based on them is schematically presented in Fig. 4.

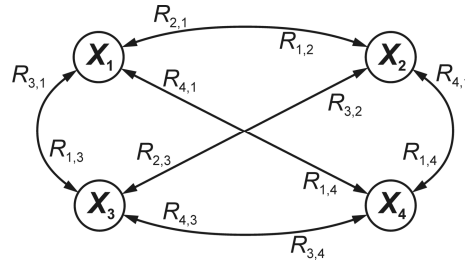


Fig. 4. General diagram of the cognitive map chosen for the method testing, where: R_{ij} – fuzzy relation between concepts i and j

The model constructed in this way had fuzzy nature, and concepts were fuzzyfied basing on function type (3), whereas relations – on function type (4) (for defuzzyfication weighted average method was used) [6]

$$\mu_{\bar{X}_i}(x) = e^{-\left(\frac{x-\bar{X}_i}{\sigma_i}\right)^2} \tag{3}$$

$$\mu_{R_{i,j}}(x_1, x_2) = e^{-\left(\frac{x_2-r_{i,j}(x_1)}{\sigma_{i,j}}\right)^2} \tag{4}$$

where: \bar{X}_i – the i -th linguistic variable’s member function’s center; $i = 1, \dots, K$; K – number of linguistic variables; σ_i – fuzziness coefficient of the i -th linguistic variable; x – the point of universum, where the function $\mu_{\bar{X}_i}$ is calculated; $\mu_{R_{i,j}}$ – member function of fuzzy relation between concepts i and j ; x_1, x_2 – axes of universum for fuzzy relation; $\sigma_{i,j}$ – fuzziness coefficient of fuzzy relation between concepts i and j ; r_{ij} – power of fuzzy relation between concepts i and j ($r_{ij} \in [-1, 1]$).

The work of fuzzy relational cognitive map dynamic model was simulated on the basis of equation (5) [9]:

$$X_i(t+1) = X_i(t) \oplus \bigoplus_{j=1}^4 [(X_j(t) \ominus X_j(t-1)) \circ R_{j,i}] \tag{5}$$

where: i – the number of considered output concept ($i = 1, \dots, 4$); t – discrete time; \oplus – fuzzy addition operation; \ominus – fuzzy subtraction operation; $R_{j,i}$ – single fuzzy relation between fuzzy concepts j and i ; \circ – maxmin fuzzy composition operation.

The constructed fuzzy cognitive map teaching procedure (i.e. adaptation of power coefficients and fuzziness coefficients in fuzzy relations) was performed with ”trial and error” method with variable length of changes step. Initial values of the teaching process were common for all relations and they were amounted to: $r = 0$ and $\sigma = 0.4$. Fuzziness coefficient for concepts had identical value for each and was amounted to 0.6. There were also assumed universum with the range $[-2, 2]$ and the number of linguistic variables $K = 17$.

3. SELECTED SIMULATION RESULTS

Teaching a map procedure for the t -th cycle of signals circulation inside the dynamic model (the t -th step of discrete time) has a form of an algorithm consisting in recurrent execution of the following consecutive steps (similar algorithm was presented in [6]):

1. Select the initial values of coefficients of changes of parameters: Δr and $\Delta \sigma$.
2. Execute t cycles of circulation of signals in fuzzy model.
3. Calculate aberrations of defuzzified values of concepts from reference values.
4. For each tested concept (e.g. X_p) and each relation, which introduces to it signals from other concepts (e.g. $R_{i,p}$) check out whether it's possible to reach the concept value closer to the reference value after increasing or decreasing the relation power $r_{i,p}$ with Δr . If yes, take new $r_{i,p}$ value.
5. For each tested concept (e.g. X_p) and each relation, which introduces to it signals from other concepts (e.g. $R_{i,p}$) check out whether it's possible to reach the concept value closer to the reference value after increasing or decreasing the fuzziness coefficient $\sigma_{i,p}$ with $\Delta \sigma$. If yes, take new $\sigma_{i,p}$ value.
6. After modification of all fuzzy relations check out whether assumed model accuracy was reached. If not, go back to step 2. Also values of parameters Δr and $\Delta \sigma$ can be changed (decreased) if it's judged necessary.
7. Repeat above steps until assumed criterion of the algorithm end is reached.

The above algorithm, applied to created cognitive map, resulted in stabilization of power and fuzziness coefficients of relations on final levels like in Table 1.

Table 1. Power (a)) and fuzziness coefficients (b)) of relations in cognitive map, achieved as a result of the teaching process

r	X_1	X_2	X_3	X_4
X_1	0	0.85	0.95	0.85
X_2	0	0	-0.25	0.15
X_3	0	0.25	0	0.05
X_4	0	0.10	-0.20	0

σ	X_1	X_2	X_3	X_4
X_1	0.40	0.25	0.15	0.24
X_2	0.39	0.40	0.30	0.36
X_3	0.39	0.29	0.40	0.38
X_4	0.39	0.29	0.36	0.40

Remark 1. For the improvement of complete time courses representation, teaching process for each consecutive step of discrete time was starting from optimal values obtained for previous step.

Then computer simulations with use of a cognitive map model from Fig. 4, where relations were built on the basis of parameters from Table 1, were performed. The time courses obtained in this way, put together with reference courses obtained from classic model, are presented in Fig. 5.

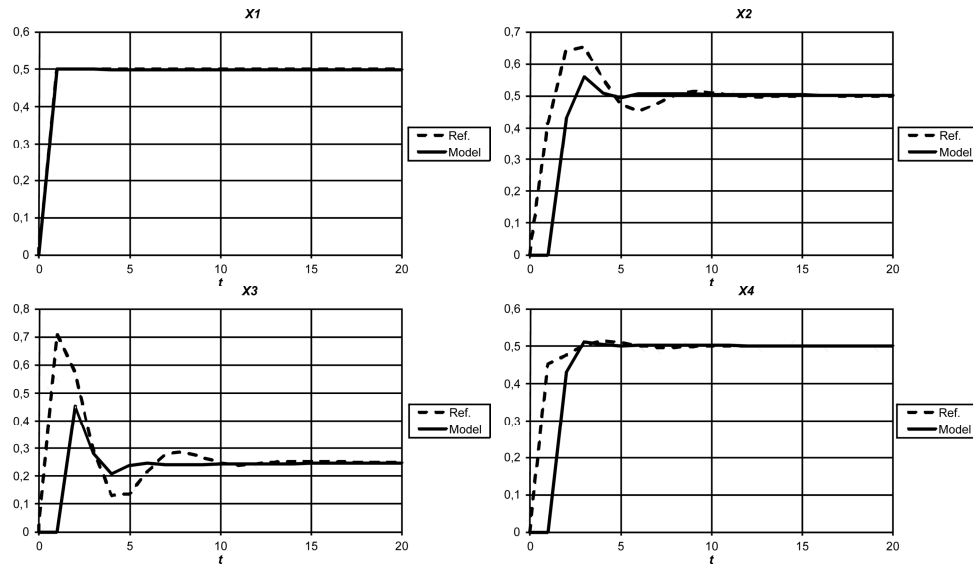


Fig. 5. Comparison of time courses (for normalized values) of selected quantities for classic (Ref.) and based on fuzzy relational cognitive map (Model) modeling

Remark 2. It should be noticed that courses from Fig. 5 are presented in normalized (dimensionless) form. Transition to dimension (physical) form is a simple arithmetic procedure, which can be performed with use of the same coefficients as normalization process.

The model of fuzzy relational cognitive map was built on the basis of only four concepts. From this reason, even though all concepts have their reflections in physical quantities, relations nature is purely abstract. Moreover, the number of applied linguistic variables (17 on universum $[-2, 2]$, which means 5 linguistic variables at standard universum range $[0, 1]$) is relatively low. Despite these limitations, proper adaptation of fuzzy relations parameters allowed to achieve good representation of modeled courses.

4. CONCLUSIONS

The application of fuzzy relational cognitive maps for modeling technical systems on the example of simple RLC circuit was described. Selected results of simulation comparison with classic model described with differential equations were quoted. The analysis of the results shows that simple (with small number of concepts) fuzzy relational cognitive maps can well approximate the work of technical systems, at this they are comparable with results achieved in classic

models. Currently performed works on the refinement of methods of a cognitive map teaching should lead to continued improvement of such models precision.

REFERENCES

- [1] Borisov, V.V., Kruglov, V.V., Fedulov, A.C., Fuzzy Models and Networks. Publishing house "Telekom", Moscow, Russia, 2004 (in Russian).
- [2] Kosko, B., Fuzzy cognitive maps. In: *Int. Journal of Man-Machine Studies*, Vol. 24. pp. 65-75, 1986.
- [3] Papageorgiou, E.I., Stylios, C.D., Fuzzy Cognitive Maps. In: Pedrycz W., Skowron A., Kreinovich V.: In: Pedrycz W., Skowron A., Kreinovich V. (eds.), *Handbook of Granular Computing*, pp. 755-774, Publication Atrium, JohnWiley & Son Ltd, Chichester, England, 2008.
- [4] Pedrycz, W., Fuzzy Sets as a User-Centric Processing of Granular Computing. In: Pedrycz W., Skowron A., Kreinovich V. (eds.), *Handbook of Granular Computing*, pp. 97-140, Publication Atrium, John Wiley & Son Ltd, Chichester, England, 2008.
- [5] Silov, V.B.: Assuming strategical solutions in fuzzy environment. INPRO-RES, Moscow, Russia, 1995 (in Russian).
- [6] Słon, G., Yastrebov A.: Optimization and Adaptation of Dynamic Models of Fuzzy Relational Cognitive Maps. In: Kuznetsov S.O. et al. (eds.): *RSFDGrC 2011, Lecture Notes in Artificial Intelligence 6743*, pp. 95-102, Springer-Verlag, Berlin, Heidelberg, 2011.
- [7] Styblinski, M.A., Meyer, B.D.: Fuzzy cognitive maps signal flow graphs and qualitative circuit analysis. In: *Proc. of the 2nd IEEE International Conference on Neural Network (ICNN) 1987*, pp. 549-556, San Diego, California, USA, 1988.
- [8] Yastrebov, A., Gad, S., Słóń, G., Fuzzy cognitive maps in decisional monitoring of technical objects. In: *Biuletyn Wojskowej Akademii Technicznej*, Vol. LIX, No. 4/2010, pp. 209-219, Warsaw, Poland, 2010 (in Polish).
- [9] Yastrebov, A., Słóń, G.: Optimization of models of fuzzy relational cognitive maps. In: Jastriebow A., Raczynska M. (eds.) *Computers in scientific and educational activity*. Institute for Sustainable Technologies - National Research Institute, pp. 60-71, Radom, Poland, 2011.