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MODELLING KINEMATICS OF A LOADING CRANE WITH A REDUNDANT KINEMATIC STRUCTURE

Key words: inverted kinematics, inverted Jacobian method, loading crane.

Abstract: The control of the loading crane in the classic method is based on the motion of individual kinematic pairs by changing the position of the valves of hydraulic cylinders. This article presents control methods for controlling the position of a loading crane end effector in Cartesian coordinates. The described mathematical models and simulation studies of the developed crane control algorithms have been implemented in the Matlab Simulink environment.

Modelowanie kinematyki żurawia przeładunkowego o redundantnej strukturze kinematycznej

Słowa kluczowe: kinematyka odwrotna, metoda Jakobianu odwrotnego, żuraw przeładunkowy.

Streszczenie: Sterowanie żurawiem przeładunkowym w klasyczny sposób polega na zadawaniu ruchu w poszczególnych parach kinematycznych konstrukcji nośnej za pomocą zmiany położenia zaworów kolejnych siłowników. W niniejszym artykule przedstawiono sposoby sterowania umożliwiające kontrolę ruchu końcówki roboczej żurawia przeładunkowego we współrzędnych kartezjańskich. Omawiane modele matematyczne oraz badania symulacyjne opracowanych algorytmów sterowania żurawiem zaimplementowano w środowisku Matlab Simulink.

Introduction

Loading cranes constitute a large branch of the crane industry. It is very common to integrate a transport vehicle with a loading crane. Therefore, the truck driver should have the appropriate authority and ability to operate the lifting devices. Efficient control of the automobile crane requires the operator to have a lot of practice and experience. The classic control system that is used for handling cranes is the control of individual arm joints using separate control levers (control in joint space) [1]. Most loading cranes are serial structures with a redundant number of degrees of freedom [2]. Figure 1 shows an example of a loading crane. Analysed HDS has nine degrees of freedom. Three of them are rotational and the rest of degrees are translational due to the application of the telescopic arm in the construction of the crane.

The control of the telescopic arm extensions is done using coupled actuators according to one of three strategies: synchronous, sequential, or arbitrary (random) movement. The work, according to the first strategy, consists in lifting all telescopic links simultaneously. According to the second strategy, only one link in sequential order extends at a given time. According to the third strategy, depending on the configuration and friction forces, a randomly selected link extends at a given time. In this study, a crane with a sequential output system was considered.

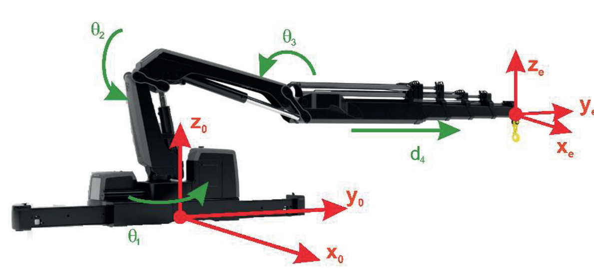

Fig. 1. Loading crane Hiab HS 111 in base position

The control of the loading crane must take into account the need to carry the load over an obstacle [2, 3], for example, the walls of buildings. One of the many problems encountered during controlling a load with HDS is the possibility of exceeding the safety zone for manipulating the load of a given weight. The design of the crane security system requires the development of a control system to calculate the position of the hook relative to the truck [3, 4, 5]. This article provides a mathematical description of simple kinematics and how to solve the problem of inverse kinematics of loading crane. For the considered construction of crane, the results of the simulation studies in the Matlab program are also presented.

1. Mathematic model of loading crane

1.1. Simple kinematics model

The mathematical model of simple kinematics has been developed based on the Denavit-Harteneberg notation [6]. A model with nine degrees of freedom was considered. Figure 2 shows the location of the coordinate system of the loading crane at base position.

Based on location of coordinate systems presented in Figure 2, a relation (1) describing the position of the crane tip with respect to its base was developed:

$$ {}^{0}_{9}F = \prod_{i=1}^{9} {}^{i-1}_{i}F \quad (1)$$

where

${}^{i-1}_{i}F$ – matrix of homogeneous transformations between individual links.

1.2. Model of extension

The arm extension model was described according to the sequential motion convention. Extending successive links is described in an iterative way with the following equation (2):

$$ D_i = \begin{cases} 0 \; for \; d_4 \le L_{i-1} \\ d_4 - L_{i-1} \; for \, L_{i-1} < d_4 < L_i \quad i = 1 \ldots 6 \\ L_i - L_{i-1} \quad for \, d_4 \ge L_i \end{cases} \quad (2)$$

where

D_i – length of the i-th link extension,

L_i – maximum distance of link end from the point B (see Fig. 2).

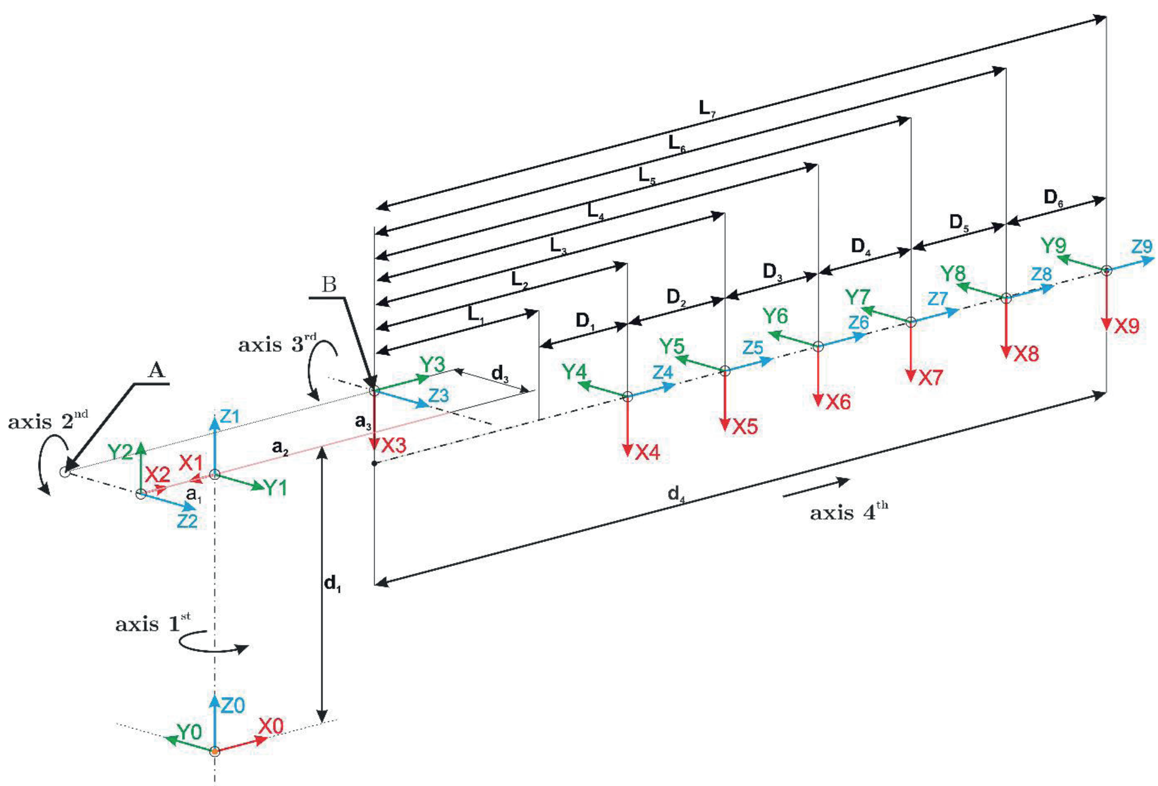

Fig. 2. Location of coordinate systems according to notation Denavita-Hartenberg for the crane with 9 degrees of freedom in base position

Parameters used in the model are presented in Table 1. The telescopic arm diagram is shown in Figure 2.

Table 1. Parameters of the telescopic arm

i	D_i (mm)	L_i (mm)
0	0	2706
1	1650	4356
2	1900	6256
3	2000	8256
4	1200	9456
5	2100	11556
6	2100	13656

1.3. Velocity Jacobian of Loading Crane

Based on the matrix of homogeneous transformations for the simplified model, velocity Jacobian was determined according to the following formula:

$$J_4 = \begin{bmatrix} J_{v1} & J_{v2} & J_{v3} & J_{v4} \\ J_{\omega 1} & J_{\omega 2} & J_{\omega 3} & J_{\omega 4} \end{bmatrix} \tag{3}$$

Linear and angular velocities dependent on the speed of the rotation of kinematic rotational pairs are given by the following equations (4–5):

$$J_{vi} = z_i \times \left({}_4^0P - {}_i^0P \right) \quad \text{for } i = 1 \ldots 3 \tag{4}$$

$$j_{\omega i} = z_i \quad \text{for } i = 1 \ldots 3 \tag{5}$$

where

$$z_i = {}_i^0F_{1:3,3} \tag{6}$$

$${}_i^0P = {}_i^0F_{1:3,4} \tag{7}$$

For a prismatic joint, velocities are determined according to equations (6) and (7).

$$J_{v4} = z_4 \tag{8}$$

$$j_{\omega 4} = 0 \tag{9}$$

1.4. Inverse kinematics model

The reloading crane is a nine-degree redundancy system that can be simplified to a four-degree freedom device by replacing six collinear prismatic joints with one based on dependence (2). In spite of the reduction of degrees of freedom for the manipulator with a given kinematic structure (Fig. 2), we did not get a clear solution. In order to achieve an unambiguous solution, one of several methods known in the literature [2] and [7] should be used. For the control of the loading crane, the most effective control methods are as follows:

a. Control with blocking certain kinematic pairs in order to achieve the manipulator characterized by fewer degrees of freedom, and
b. Control using the inverse Jacobian control method.

1.4.1. Inverse kinematics model

Controlling the 4-DOF manipulator by setting the XYZ coordinates is not straightforward therefore, to obtain a clear solution, we block one of the three degrees of freedom (θ_2, θ_3, d_4). For the loading crane, three models of inverse kinematics were obtained:

a. Model 1 – blocked extension of the 4th axis (d_4 = const),
b. Model 2 – blocked rotation of the 3rd axis (θ_3 = const), and
c. Model 3 – blocked rotation of the 2nd axis (θ_2 = const).

For each of the three models, the angle of the column (first link) rotation is calculated according to the following formula (10):

$$\theta_1 = \text{arctg}\left(\frac{X}{Y}\right) - \text{arctg}\left(\frac{d_3}{\sqrt{X^2 + Y^2 - d_3^2}}\right) \tag{10}$$

where
x, y – the crane tip coordinates,
θ_1 – configuration angle of the 1st link.

The position of the crane tip relative to point A (see Fig. 2) is described by equations (11–13):

$$X_A = X\cos(\theta_1) + Y\sin(\theta_1) + a_2 \tag{11}$$

$$Y_A = -X\sin(\theta_1) + Y\cos(\theta_1) - d_3 \tag{12}$$

$$Z_A = Z - a_1 \tag{13}$$

where
Z – coordinate of the working tip of the crane.

The calculation of the consecutive values of the configuration coordinates for each model is done by a separate algorithm. The configuration coordinates for the 1st inverse kinematics model (d_4 = const) were described by formulas (14–17):

$$R = \sqrt{X_A^2 + Y_A^2} \tag{14}$$

$$r = \sqrt{d_4^2 + a_4^2} \tag{15}$$

$$\theta_3 = \text{arctg}\left(\frac{a_4}{d_4}\right) - arccos\left(\frac{Z_1^2 + R^2 - a_3^2 - r^2}{2a_3 r}\right) \quad (16)$$

$$\theta_2 = \text{arctg}\left(\frac{r\sin(\theta_3)}{a_3 + r\cos(\theta_3)}\right) + arctg\left(\frac{Z_A}{R}\right) \quad (17)$$

The configuration coordinates for the 2nd inverse kinematics model (θ_3 = const) were described by formulas (18–25):

$$R = \sqrt{X_A^2 + Y_A^2 + Z_A^2} \quad (18)$$

$$r = \sqrt{d_4^2 + a_4^2} \quad (19)$$

$$b_1 = 2a_3 cos(\theta_3) \quad (20)$$

$$b_2 = a_4^2 + a_3^2 + 2a_4 a_3 sin(\theta_3) - R \quad (21)$$

$$\Delta = b_1^2 - 4b_1 \quad (22)$$

$$d_4 = \frac{-b_1 + \sqrt{\Delta}}{2} \quad (23)$$

$$\theta_{31} = \theta_3 + arctg\left(-\frac{a_4}{d_4}\right) \quad (24)$$

$$\theta_2 = \text{arctg}\left(\frac{r\sin(\theta_{31})}{a_3 + r\cos(\theta_{31})}\right) + \text{arctg}\left(\frac{Z_A}{R}\right) \quad (25)$$

Next, for the 3rd inverse kinematics model (θ_2 = const), the position of Point B (see Fig. 2) was obtained by equations (26–28):

$$X_B = X_A - a_3\cos(\theta_2) \quad (26)$$

$$Y_B = Y_A \quad (27)$$

$$Z_B = Z_A - a_3\sin(\theta_2) \quad (28)$$

Finally, the configuration coordinates (29–30) were determined:

$$d_4 = \sqrt{X_B^2 + Y_B^2 + Z_B^2 - a_4^2} \quad (29)$$

$$\theta_3 = \text{arctg}\left(\frac{a_4}{d_4}\right) - \theta_2 - \text{arctg}\left(\frac{Z_B}{X_B}\right) \quad (30)$$

1.4.2. Inverted Jacobian method

Calculating the position of successive joints while moving on a given trajectory can also be accomplished by applying the following equation (31). The pseudo-inverse Moore-Penrose matrix is used in the presented dependence (33).

$$\dot{\theta}(t) = J(\theta)^{\dagger} \nu(t) \quad (31)$$

where

$$\nu = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} \quad (32)$$

$$J(\theta)^{\dagger} = J(\theta) * \left(J(\theta) J(\theta)^T\right)^{-1} \quad (33)$$

By integrating the equation (33), we obtain a relation (34) describing the change of configuration variables at the assumed starting point.

$$\theta(t) = \int_{t_0}^{t} J(\theta)^{\dagger} \nu(t) dt + \theta(t_0) \quad (34)$$

2. Results of the analysis

Figure 3 show the values of the configuration coordinates while the end effector move after a straight line from Point P_1 (X = 4000 mm, Y = 2000 mm, Z = 500 mm) to Point P_2 (X = 7000, Y = –2000 Z = 4000). The duration of the motion is 100 s.

Controlling a loading crane with Cartesian coordinates is an ambiguous issue due to redundant degrees of freedom, although the use of a temporary blocking of one of the configuration coordinates allows for efficient control of the crane. The proposed approach involves the alternate use of three inverse kinematics models of the crane. The choice of the inverse kinematics model of the device is based on the limitations of the working space and the motion ranges of the individual joints. Switching between operating modes can be performed automatically or forced manually. During the controlling of the position of the XYZ work tip, it is possible to change the position of the locked joint. Thus, we get the ability to control with four coordinates, i.e. XYZ and the position of the limited joint. This approach is required to achieve a configuration that allows you to bypass an obstacle. As a result of the work, we have obtained the algorithm of switching the inverse kinematics modes and the limits of movement of individual crane joints (movement ranges).

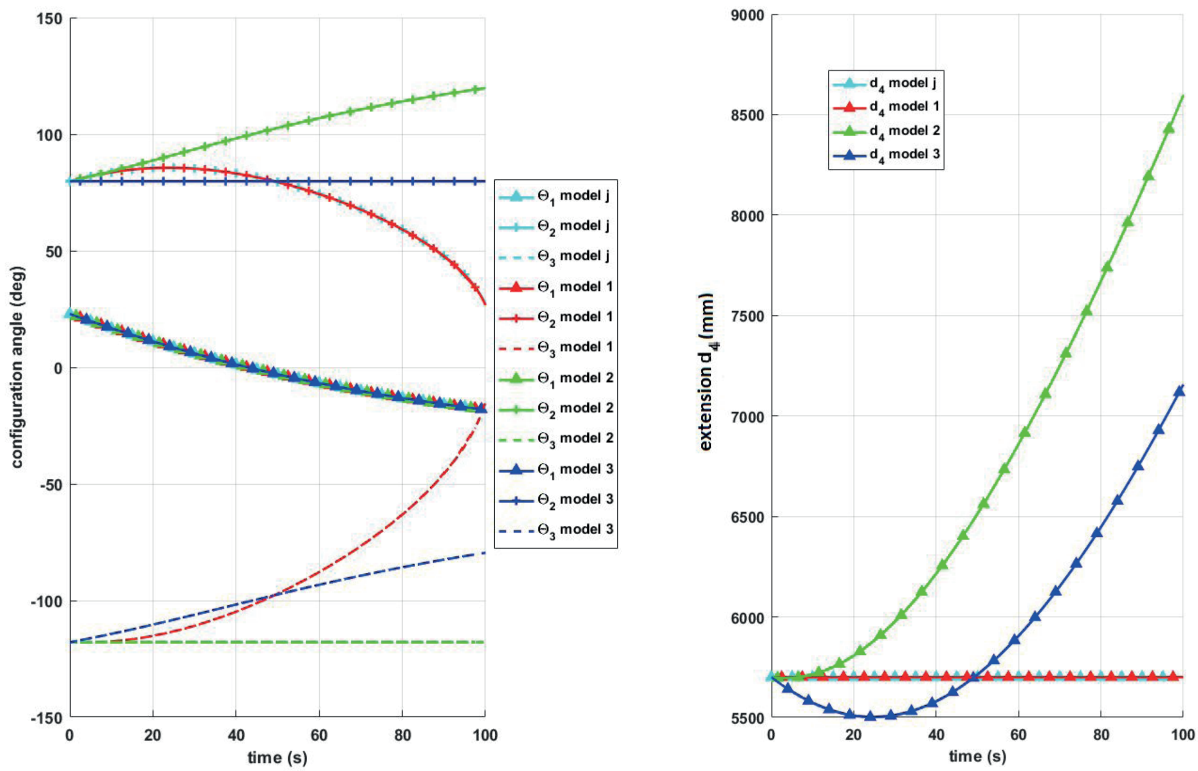

Fig. 3. Value of the configuration coordinates determined using inverse kinematics models with locking joints and the inverse Jacobian method

Conclusions

The paper describes a methodology of modelling a simple and inverse kinematic of a loading crane with a redundant structure. The obtained equations enable controlling the crane workpiece in a Cartesian system, regardless of its configuration. The use of inverse kinematics to control the loading crane restricts the crane movement in a specific direction with respect to the truck. This becomes extremely important, as the loading cranes must meet the relevant safety standards [1]. The paper shows how to model simple and inverse kinematics. Thanks to the presented approach, it is possible to easily modify the system and to expand the model with further modules such as dynamic models and hydraulic models of crane cylinders.

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