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THE NOTATION OF MECHANISM STRUCTURE IN STRUCTURAL RESEARCH

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Abstract. Different methods of notation are in use when describing the construction of mechanisms in structural research. This work deals with the issue of notation and presents:
– constructional drawing in which the construction of the mechanism is detailed by the use of the Monge's projection (views, projections), axonometric or perspective projections;
– kinematic diagram in which kinematic pairs and links are described by symbols. Given dimensions of individual links allow to specify the positioning of specific parts of the mechanism depending on the movement of input links;
– structural diagram in which the class of each kinematic pair is specified;
– structural graph in a form of a polygon in which its sides and vertices correspond with kinematic pairs and links respectively;
– adjacency matrix in a form of a square matrix in which the number of rows and columns corresponds with the number of links in the mechanism. Its elements are: 0 when links are not joined, or 1 when links are joined;
– loop notation in which links and kinematic pairs that form individual closed contours of the mechanism are given;
– author's notation of the construction of spatial mechanisms, in which the classes of kinematic pairs are presented in a form of labels next to their graphic representation.

Keywords: structure, structural diagram, graph, adjacency matrix, loop notation

1 Introduction

The structure of mechanisms is a part of Machine and Mechanism Theory (MMT), in which all basic terms of the discipline are being defined then systematized and classified. Terms such as a mechanism, a machine, Assur group, a link, a kinematic pair are being introduced. Links are defined as parts which while forming a given mechanism are moving in relation to each other. They are frequently in a form of rigid links in which a load-related strain has no significant impact on a movement performed. Kinematic pair is always created by two links, which allows some motion. A free link motion in a 3D space can be described by six independent parameters i.e. three rotations and three translations along the axis of coordinate system. These quantities determine the number of degrees of freedom (DOF) for a given link. Any connection of two links by a kinematic joint restricts the number of DOF by 1 at least and 5 at most. Hence the kinematic pairs can be classified accordingly: class I – the DOF number decreased by 1, class II – by 2, class III – by 3, class IV – by 4, class V – by 5. The class of a kinematic pair that forms a given mechanism depends on a specific families[1] the mechanism belongs to.

[1] This classification into families results from a structural equation: $W' = (6-H_W)n_r - \sum_{i=H_W+1}^{i=5}(i-H_W)p_i$, where n_r – the number of motor links, p_i – the number of kinematic joints of i – th class. For the zero family, at the

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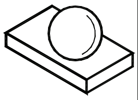
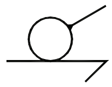
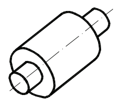
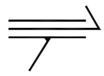
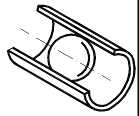
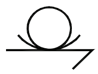
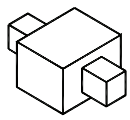
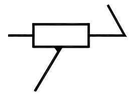
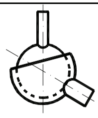
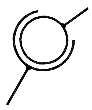
Most TMM studies give the classification of kinematic joints together with their construction solutions. Each construction-related drawing comes with its symbol notation. Table 1 gives examples of such a notation.

Table 1: Kinematic pairs – classification and symbols

Class	Name	Graphical form	Symbol notation	Joint's class	Joint's name	Example	Symbol notation
I	Ball–semi–plane			IV	Cylindrical		
II	Ball–cylinder			V	Prismatic		
III	Spherical						

The constructional notation of a kinematic joint is used while making a construction drawing of a mechanism whereas symbols are used in kinematic diagrams.

2 The notation of the mechanism's construction

The most detailed notation of the mechanism's construction is a constructional drawing. In a specific scale, the mechanism is presented with the use of the descriptive geometry well–known notation for three–dimensional objects on the drawing's plane such as Monge's projections, axonometric or perspective projections. Those notations can be found in studies (from MMT) dating back to the 19th century (Figure 1a) as well as more recent ones from 21st century (Figure 1c).

a) b) c)

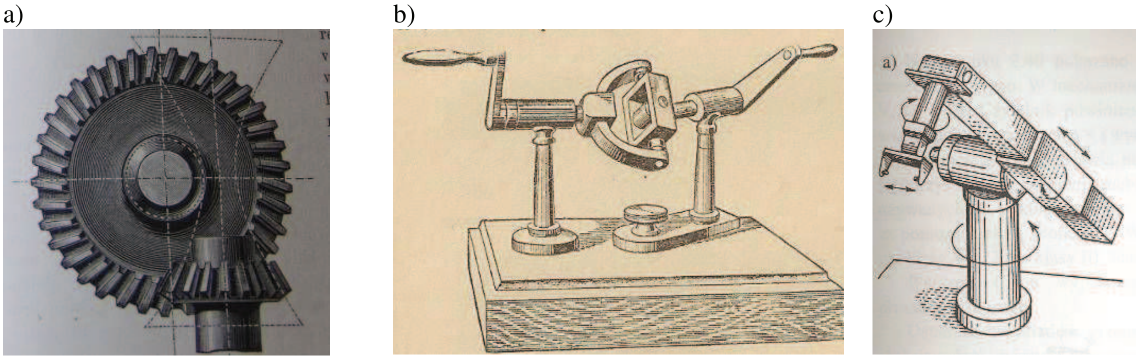


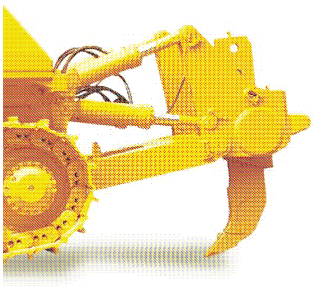
Figure 1: Constructional drawing: a) gear mechanism – front view, 1875[2]; b) spherical mechanism – perspective projection, 1951[3]; c) Stanford manipulator – axonometric projection, 2002[4]

assumption $H_W = 0$: $W' = 6n_r - 5p_5 - 4p_4 - 3p_3 - 2p_2 - p_1$, for the first family, at the assumption $H_W = 1$: $W' = 5n_r - 4p_5 - 3p_4 - 2p_3 - p_2$, for the second family, at the assumption $H_W = 2$: $W' = 4n_r - 3p_5 - 2p_4 - p_3$, for the third family, at the assumption $H_W = 3$: $W' = 3n_r - 2p_5 - p_4$, for the fourth family, at the assumption $H_W = 4$: $W' = 2n_r - p_5$.

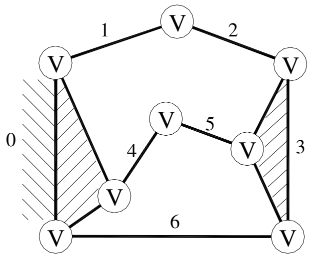
[2] Reuleaux F.: *Theoretische Kinematik. Grundzüge einer Theorie des Maschinenwesens*. Braunschweing, 1875, p.85.

For the purpose of structural research regarding both the analysis and synthesis, a simplified notation of a mechanism in a form of structural and kinematic diagrams has been introduced. In the structural notation the links and their connecting kinematic joints are presented by symbols. Binary links are pictured as segments whereas k–conjunctive links as polygon with k vertices. The kinematic joint takes a form of a circle, with its class number noted inside (Figure 2b).

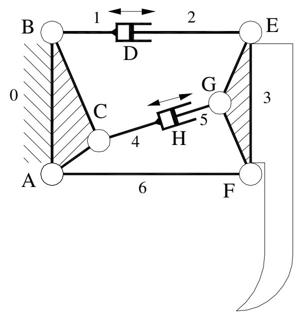
a)



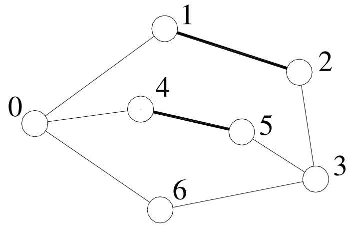
b)



c)



d)



e)

$$M_p = \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

f)

Outer loop	Inner loops	
0	0	0
B	B	C
1	1	4
D	D	H
2	2	5
E	E	G
3	3	3
F	G	F
6	5	6
A	H	A
0	4	0
	C	
	0	

Figure 2: Ripper KAMATSU D475A-5: a) photo; b) structural diagram; c) kinematic diagram; d) structural graph; e) adjacency matrix; f) loop notation

As in the structural diagram, the links in the kinematic diagram are noted as line segments and polygons (Figure 2b). Kinematic joints are represented by symbols shown in Table 1. This type diagram, specifies geometric quantities which allow the determination of the position of the propelled links depending on the movement of the input links. For the planar mechanism in Figure 2c the following were given:

– coordinates of the A, B, C points,

– lengths: l_1, l_2, l_4, l_5, l_6, l_{EF}, l_{EG},

– angle κ_{3E} in 3-conjunctive link 3.

Another method for noting the mechanisms' construction is a structural graph, which is frequently used in structural research. A structural graph is a polygon with its sides and vertices corresponding to kinematic joints and links of the mechanism respectively (Figure 2d). This notation is suitable only for planar mechanisms in which there are revolute kinematic joints (fine line) and prismatic kinematic joints (boldface line).

[3] Artobolevskij I., I.: *Teoria mechanizmov i maszin*. Gosudarstviennoje Izdatielstvo Techniko-Teoreticzeskoj Literatury, Moskva, 1951, p.53.
[4] Morecki A., Knapczyk J., Kędzior K.: *Teoria mechanizmów i manipulatorów*. WNT, Warszawa 2002, p.71.

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An adjacency matrix, shown in Figure 2e, presents yet another method of the mechanism structure notation, a non-graphic one this time. It is a square matrix of order n (n – the number of links in the mechanism), in which the numbers of rows and columns correspond to the links' numbers. If a mechanism is composed of kinematic joints connected two links only (first order pin joints), the matrix contains just two elements i.e. 0 and 1[5]. If links i and j are joined, the adjacency matrix element $a_{ij}=1$, whereas if the links are not joined that element takes the 0 value. When mechanisms contain multiple joints (e.g. second order pin join – three links joined), the adjacency matrix gives the number k indicating the order of the kinematic pair[6].

The loop notation of the mechanism construction, features the links and kinematic joints that form individual closed contours (loops). Loops need to be determined in such a way that they include all links and kinematic joints and all loops are independent. In the mechanism, shown in Figure 2, it is possible to determine one outer loop and two inner loops. In order to describe the mechanisms unequivocally two of these suffice, i.e. two inner loops or one outer and one inner loop. This notation is mainly used in the loop method of determining the number of mobility degree of the mechanism [5] and in determining rigid and over-rigid sub-chains existing in the mechanism [3,6,7].

The identification of isomorphic mechanisms constitutes a very important structure-related issue. This procedure aims at eliminating recurring structures and continue the research for different non-recurring mechanisms. One of the solutions that solves this problem uses a variety of graphs, loop notation and adjacency matrixes [4]. For instance to verify isomorphism of two 8–link kinematic chains shown in Figure 3a, there have been determined structural graphs (Figure 3b), which are the source of information about the loops that build chains. Next, the longest loop is taken as an outer one and constitutes the basis for creating perimeter graphs (Figure 3c). Renumerating vertices so that the vertices of the external loop take consecutive natural numbers, transforms a perimeter graph into a canonical perimeter graph (Figure 3d). Comparing perimeter graphs and appropriate adjacency matrixes is essential in gathering information on the isomorphism of mechanisms. In this particular case the kinematic chains are structurally identical.

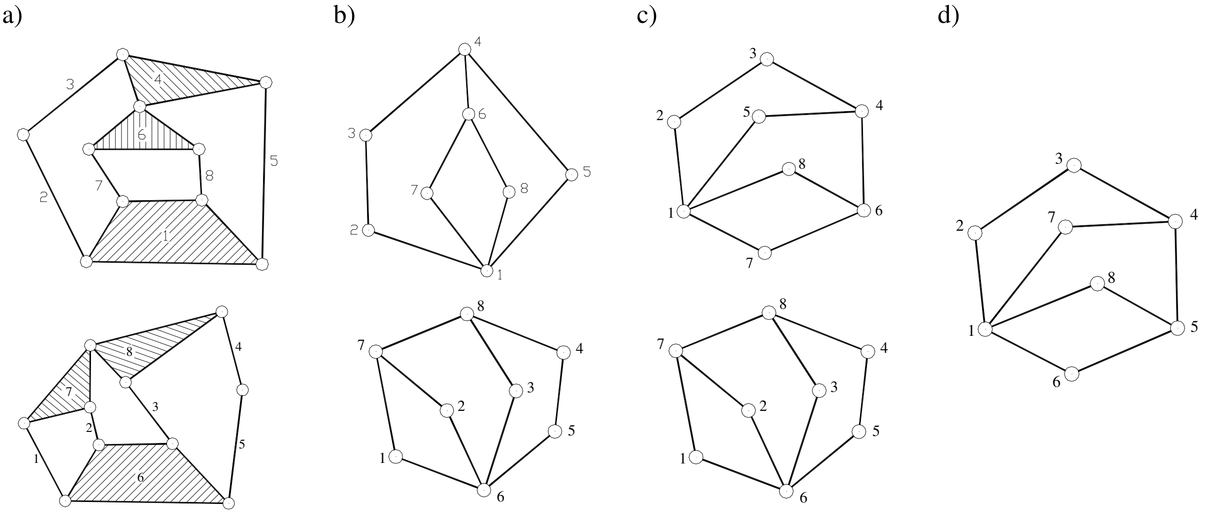


Figure 3: a) Eight-link kinematic chains; b) topological graphs; c) perimeter graphs; d) canonical perimeter graph

[5] Norton R. L.: *Design of Machinery. An introduction to the Synthesis and Analysis of Mechanisms and Machines*. Mc Craw–Hill, Worcester Polytechnic Institute, 2011, p. 34.
[6] Ibidem, pp. 34, 40-41.

In the work [2] the authors used the mechanism’s notation in a form of contracted graph (Figure 4) which is made by eliminating the two–degree vertices from the topological graph. The structural synthesis of contracted graphs was carried out with the computer program created by the authors.

a) b) c) d)

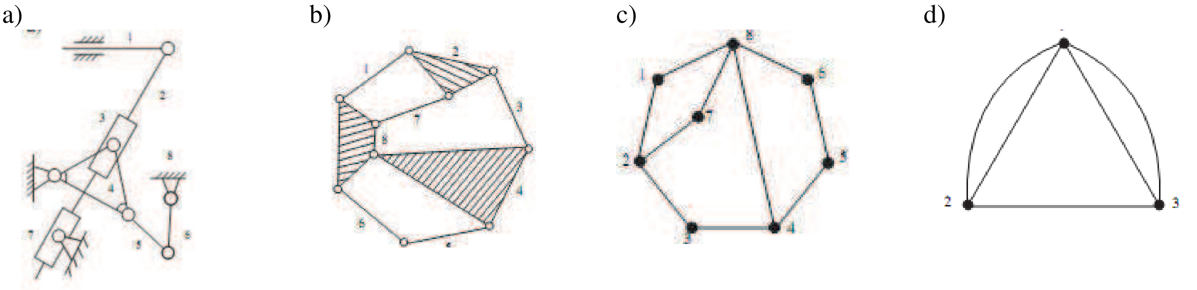


Figure 4: a) Nine-link mechanism; b) structural diagram; c) topological graph; d) contracted graph

Examples of solutions obtained by the authors, have been presented in Figure 5.

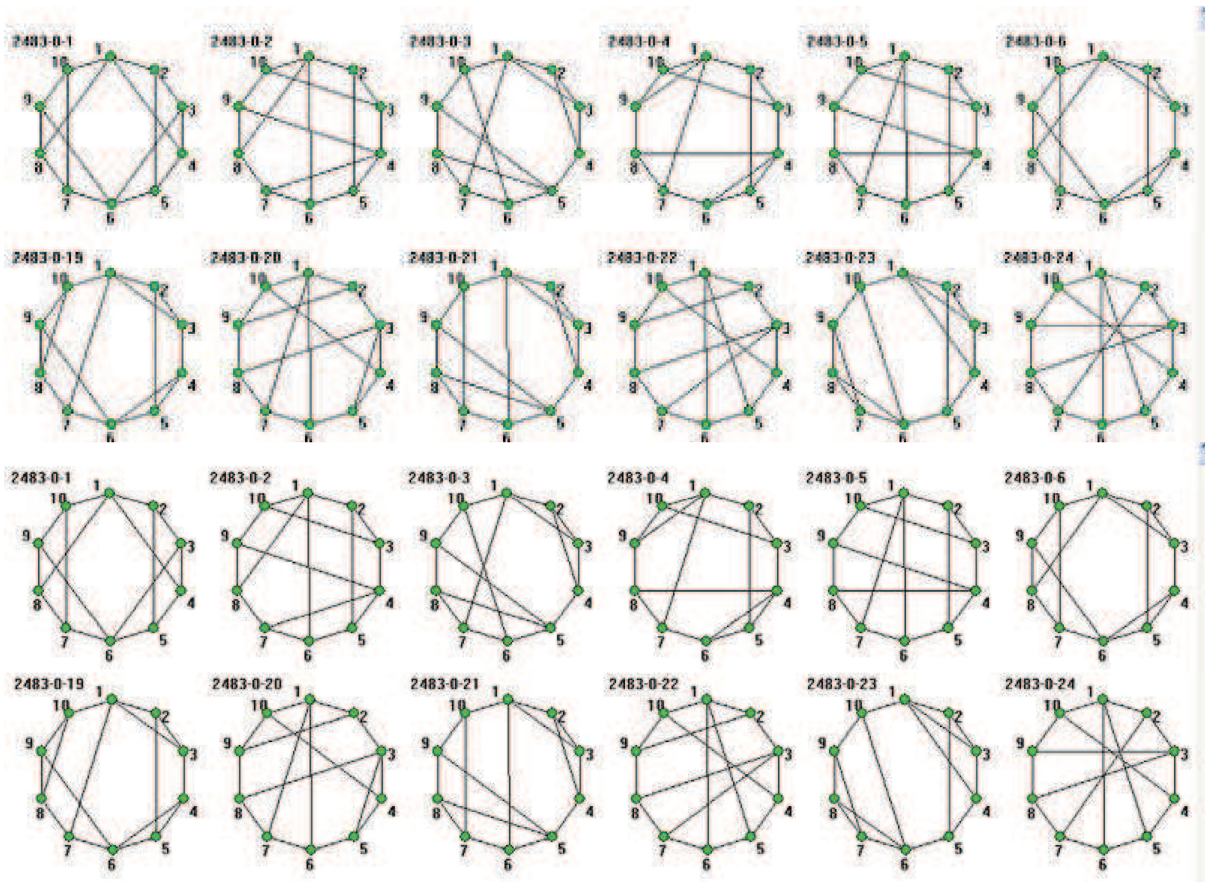


Figure 5: Examples of contracted graphs synthesis, obtained for 18–link mechanisms

In structural analysis of a spatial mechanism and when its diagram needs to be drawn, kinematic or structural diagrams can be used to suit that purpose (Figure 6b). In structural synthesis when all possible solutions fulfilling specific structural assumptions have to be generated, a symbolic notation leads to many diagrams identical graphically but different in classes of kinematic pairs. A new method of notation has been proposed, in which classes of kinematic joints are presented in a form of labels next to their graphic representation (Figure 6c) [11].

a) b) c)

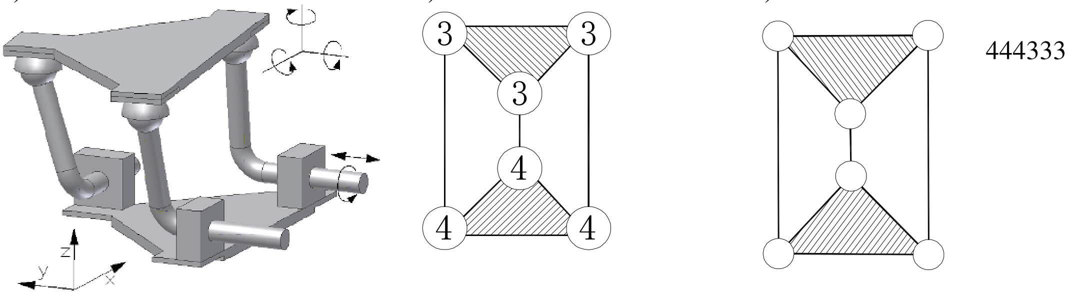
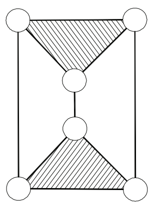


Figure 6: a) Parallel mechanism with ternary platform; b) structural diagram; c) diagram with an ascribed label

To determine a total set of all solutions of a parallel mechanism with a ternary platform, with arms made of binary links, containing kinematic joints of third, fourth and fifth class, the sets of labels in each individual family need to be determined (Table 2).

Table 2

Graphic diagram	Labels			
	Family 0	Family I	Family II	Family III
	444333 543333	554333 544433 444443	555433 554443 544444	555444

The notation of mechanisms presented in Table 2 has an advantage of enabling notation many of mechanisms, built out of the same number of links, of the same connectivity, and the same number of kinematic pairs differing in classes. It is a synthetic notation which provides, by inserting classes of kinematic pairs into structural scheme, set of all possible mechanism solutions. For example, for the mechanism of group 0 and label 543333 (Figure 7a) there have been obtained 3 structurally different solutions (Figure 7b).

a) b)

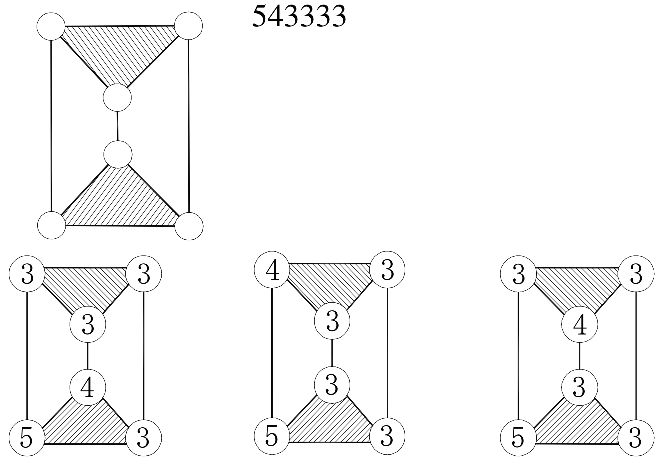


Figure 7: Parallel mechanism with ternary platform: a) diagram with an ascribed label; b) solutions in structural notation

The hereby presented, simplified structural notation of three–dimensional mechanisms facilitates research on isomorphism of three–dimensional mechanisms. For the diagrams any method created for planar mechanisms can be used.

3 Conclusions

While researching the structure of mechanisms a variety of notation forms are used. They range from very precise construction drawings, through a notation based on diagrams and graphs and adjacency matrixes and loop notation to complete the list. Literature review on structural research indicates that in recent years most frequently used structural notation of mechanisms takes form of graphs and adjacency matrixes. These forms of notation proved the easiest for implementation in a computer environment. Research in the area of structure isomorphism prompted the introduction and the use of different types of structural graphs and adjacency matrixes which restricts the range of mechanisms in question to flat mechanisms only. This new structural notation of the mechanisms enables the research in the area of their isomorphism to be carried out also for spatial mechanisms.

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ZAPIS BUDOWY MECHANIZMÓW W ASPEKCIE BADAŃ STRUKTURALNYCH

Praca poświęcona jest różnym sposobom zapisu mechanizmów, używanych w badaniach strukturalnych. Przedstawiono:
– zapis konstrukcyjny, w którym używając rzuty prostokątne (widoki, przekroje) oraz aksonometrię przedstawia się szczegółowo budowę mechanizmu;

– schemat kinematyczny – pary kinematyczne i ogniwa przedstawione są w nim w sposób symboliczny, podane długości poszczególnych ogniw, umożliwiają określenie położenia poszczególnych części mechanizmu zależnie od przemieszczenia ogniw napędzających;
– schemat strukturalny, w którym określona jest klasa każdej pary kinematycznej;
– graf strukturalny – wielokąt, którego boki odpowiadają parom kinematycznym, a wierzchołki ogniwom;
– macierz połączeń – macierz kwadratowa, której liczba wierszy i kolumn odpowiada liczbie wierszy w mechanizmie, jej elementami są 0 (ogniwa nie są ze sobą połączone) lub 1 (gdy ogniwa są ze sobą połączone);
– notację konturową, w której zapisane są ogniwa i pary kinematyczne tworzące poszczególne kontury zamknięte mechanizmu;
– autorski zapis budowy mechanizmów przestrzennych, w którym klasy par kinematycznych przedstawione są w postaci etykiet, obok schematu graficznego.