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MAXIMUM AMPLITUDES IN TRANSIENT RESONANCE OF DISTRIBUTED-PARAMETER SYSTEMS

AMPLITUDY MAKSYMALNE W REZONANSIE PRZEJŚCIOWYM UKŁADÓW O PARAMETRACH ROZŁOŻONYCH

The application of the kinetic energy balance for the estimation maximum amplitudes of continuous systems in the transient resonance excited by the free coasting of unbalanced rotor or piston machines placed on the continuous system – was proposed in the study.

The exact as well as the approximate methods were shown. For the typical one- and two-dimensional systems the calculation formulae, useful for the engineering practice, were given.

Keywords: transient resonance, distributed-parameter systems

W pracy wskazano na możliwość zastosowania metody bilansu energii kinetycznej dla oszacowania amplitud maksymalnych w rezonansie przejściowym podczas wybiegu ciężkich maszyn wirnikowych lub tłokowych posadowionych na układach o ciągłym rozkładzie masy.

W szczególności rozważono drgania okołorezonansowe przesiewaczy i przenośników wibracyjnych o znacznej długości oraz płytowych i belkowych układów podporowych maszyn o dużym zredukowanym momencie bezwładności, jak wirówki odwadniające czy kruszarki młotkowe, stosowanych w zakładach przeróbki kopalin.

Słowa kluczowe: rezonans przejściowy, układy o parametrach rozłożonych

1. Introduction

The transient resonance constitutes one of the most dangerous dynamic states in case of unbalanced rotor and piston machines placed on flexible floors, supporting structures or vibroinsulating systems. Exceeding by an unbalanced rotor the natural frequencies range of the system, during start-up and coasting, generates loading of the supporting system several times higher than in the

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steady state. Especially dangerous is the transient resonance during coasting due to its long-lasting action and for machines of a high rotor inertia such as e.g. hydroextractors, hammer crushers and vibratory machines in mineral processing plants, blast fans in metallurgical plants, etc.

The transient resonance analysis performed in papers (Lewis, 1932; Kac, 1947; Markert & Seidler, 2001; Cieplok, 2009) is limited to systems of one degree of freedom and except for the last cited paper, based on assumption that the rotor angular velocity is *a priori* known. In reality we are dealing with systems of several degrees of freedom (Goliński, 1979) or of a continuous mass distribution. In addition, a decisive influence on the rotor angular velocity has its feedback with machine vibrations (Kononienko, 1964). Such coupling causes, that the rotor in the transient resonance returns the majority of its kinetic energy for the increase of the machine vibrations amplitude (Agranowskaja & Blechman, 1960; Michalczyk,1995). This allows to determine the maximum amplitudes on the basis of the energy balance in the rotor-machine system. This method was used for the lumped-parameter systems in papers (Agranowskaja & Blechman, 1969; Michalczyk, 1993). Formulae – suitable for the engineering practice – for maximum amplitudes during the transient resonance, of elastically supported bodies of 6 degrees of freedom, were derived in paper (Michalczyk, 2012).

The aim of this study is an indication of the possibility of applying this method for the distributed-parameter systems.

2. Accurate analysis

The starting point of the analysis, based on the kinetic energy balance of the rotor and machine body, is the matrix equation (Michalczyk, 2010):

$$\kappa \cdot \frac{1}{2} J_r \omega_{0n}^2 = \frac{1}{2} \dot{\mathbf{q}}_{\max n}^{\mathbf{T}} \cdot \mathbf{M} \cdot \dot{\mathbf{q}}_{\max n} \tag{1}$$

where:

κ — number of identical synchronously running driving systems,

J_r — moment of inertia of the driving system reduced on the rotor shaft of unbalanced vibrator,

ω_{0n} — angular velocity at which the energy exchange occurs (generally different (Lewis, 1932) in a certain range from the n^{th} frequency of natural vibrations ω_n of the machine body supported on an elastic suspension system),

$\mathbf{q} = \text{col}\{x_s, y_s, z_s, \varphi_x, \varphi_y, \varphi_z\}$ — coordinates vector, describing the system vibrations, assumed $\mathbf{q} \cong \mathbf{q}_\text{n}$ in the vicinity of the n^{th} resonance,

$\dot{\mathbf{q}}_{\max n}$ — velocity vector, determined for the moment of the maximum amplitude $\mathbf{q}_{\max n}$ of the n^{th} form in the n^{th} resonance, $\dot{\mathbf{q}}_{\max n} = \omega_{0n} \cdot \mathbf{q}_{\max n}$.

In case of distributed-parameters systems a further modus operandi is formally slightly different. In order to demonstrate it, let us consider a machine e.g. vibratory screen or conveyer of a diagram illustrated in Fig. 1.

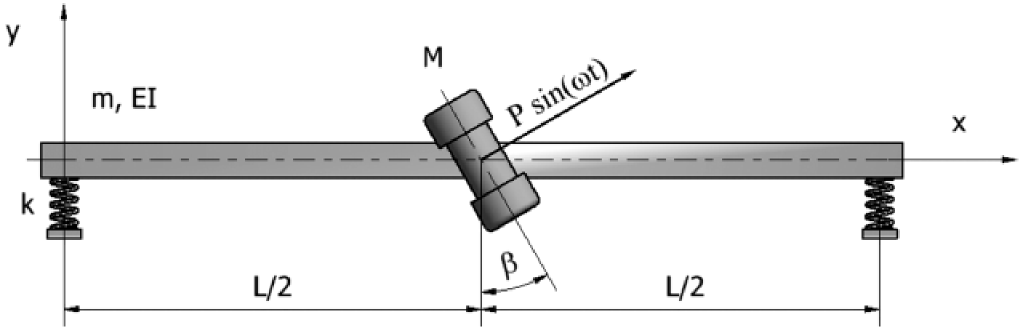

Fig. 1. Computational diagram of the system

Disregarding, small in this case, an influence of the horizontal forces component on transverse vibrations allows to separate transverse vibrations problems from the body motion along the coordinate x. Vibrations in the horizontal direction x were not analysed since usually $k_x \ll k_y$, which causes that the transient resonance of vertical vibrations does not occur together with the resonance in the horizontal direction and is much more dangerous. In addition, antisymmetric vibrations were not analysed, since the central way of applying force $\overline{P}(t)$ causes that there are no direct excitations of antisymmetric forms, (this problem is much more complicated in case of machines utilising the self-synchronisation effect (Michalczyk & Czubak, 2010) and for machines, in which the centre of elasticity is not coinciding with the mass centre (Michalczyk, 2012)).

Equation of machine body free vibrations is of a form:

$$EI\frac{\partial^4 y}{\partial x^4} + \overline{m}\frac{\partial^2 y}{\partial t^2} = 0 \tag{2}$$

where:

EI — flexural rigidity,
$\overline{m} = m/L$ — mass for the body length unit.

Boundary conditions, on account of the system symmetry, were assumed for the ‘half’ model:

$0 \le x \le \frac{L}{2}$:

$$-ky(0,t) = EI\frac{\partial^3 y}{\partial x^3}(0,t) \tag{3a}$$

$$\frac{\partial^2 y}{\partial x^2}(0,t) = 0 \tag{3b}$$

$$\frac{\partial y}{\partial x}\left(\frac{L}{2},t\right) = 0 \tag{3c}$$

$$\frac{1}{2} M \frac{\partial^2 y}{\partial t^2}\left(\frac{L}{2}, t\right) = EI \frac{\partial^3 y}{\partial x^3}\left(\frac{L}{2}, t\right) \tag{3d}$$

where $k = k_y$ — coefficient of vertical rigidity of support springs.

After separation of variables:

$$y(x, t) = f_x(x) \cdot f_t(t) \tag{4}$$

and substituting (4) to (2) the solution can be obtained in a form:

$$y(x,t) = \sum_{n=1}^{\infty} \begin{pmatrix} C_{1n} \sin\left(\frac{2}{L}\alpha_n x\right) + C_{2n} \cos\left(\frac{2}{L}\alpha_n x\right) + \\ + C_{3n} \sinh\left(\frac{2}{L}\alpha_n x\right) + C_{4n} \cosh\left(\frac{2}{L}\alpha_n x\right) \end{pmatrix} \cdot \left(\sin\left(\frac{4}{L^2}\sqrt{\frac{EJ}{\overline{m}}}\alpha_n^2 t + \delta\right)\right) \tag{5}$$

Substituting the above into boundary conditions (3) it is possible to obtain the equations system for C_{1n}, C_{2n}, C_{3n}, C_{4n}. From the condition of non-zero solution of these equations (zeroing of the main determinant) the equation for eigenvalues α_n can be obtained:

$$\begin{aligned} &\frac{8EIM}{\overline{m}L^4}\alpha^4(1 + \cosh\alpha \cdot \cos\alpha) + \frac{8EI}{L^3}\alpha^3(\sin\alpha \cdot \cosh\alpha + \sinh\alpha \cdot \cos\alpha) + \\ &+ \frac{kM}{\overline{m}L}\alpha\,(\cosh\alpha \cdot \sin\alpha - \sinh\alpha \cdot \cos\alpha) - 2k \cdot \cosh\alpha \cdot \cos\alpha = 0 \end{aligned} \tag{6}$$

The form of the n^{th} vibration, it means the set of ratios C_{1n}: C_{2n}: C_{3n}: C_{4n}, can be determined as the ratio of algebraic complements of the selected matrix row coefficients in the above equations, for the given $\alpha = \alpha_n$:

$$\begin{aligned} C_{1n} : C_{2n} : C_{3n} : C_{4n} = &\left[\frac{k}{4EI}\left(\frac{L}{\alpha}\right)^3 \cosh\alpha + \sin\alpha - \sinh\alpha\right] : [\cosh\alpha + \cos\alpha] : \\ &: \left[\sin\alpha - \sinh\alpha - \frac{k}{4EI}\left(\frac{L}{\alpha}\right)^3 \cos\alpha\right] : [\cosh\alpha + \cos\alpha] \end{aligned} \tag{7}$$

This allows to write the main vibrations of the symmetric type in a form containing only one unknown parameter C_{1n}:

$$f_{xn}(x) = C_{1n} \begin{bmatrix} \sin(\frac{2}{L}\alpha_n x) + \frac{C_{2n}}{C_{1n}} \cos(\frac{2}{L}\alpha_n x) + \\ + \frac{C_{3n}}{C_{1n}} \sinh(\frac{2}{L}\alpha_n x) + \frac{C_{4n}}{C_{1n}} \cosh(\frac{2}{L}\alpha_n x) \end{bmatrix}, \quad 0 \le x \le L/2 \tag{8}$$

Value of coefficients C_{1n} and δ,(5), can be estimated on the basis of initial conditions. An example of the symmetric main vibrations is shown in Fig. 2.

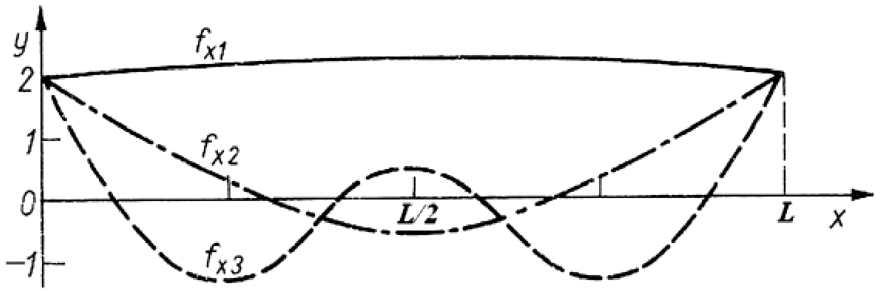

Fig. 2. Symmetric forms of the main vibrations

Angular frequencies corresponding to main vibrations can be determined from the dependence:

$$\omega_n = \frac{4}{L^2}\sqrt{\frac{EI}{\overline{m}}}\alpha_n^2 \tag{9}$$

Since the considered systems are tuned to have: $\omega_1 < \omega < \omega_2, \omega_3, ...$, the transient resonance, in their case, means passing via ω_1.

The equation of the kinetic energy balance (1) obtains a general form:

$$\kappa\frac{1}{2}J_r\omega_{0n}^2 = 2\int_0^{L/2}\frac{1}{2}\overline{m}[\omega_{0n}f_{xn}(x)]^2dx + \frac{1}{2}M[\omega_{0n}f_{xn}(L/2)]^2 \tag{10}$$

where:

κ — number of unbalanced rotors running synchronously,
J_r — moment of inertia of the driving system reduced on the rotor shaft,
ω_{0n} — frequency, at which the energy exchange occurs (slightly different than the natural frequency) ω_n, $n = 1$.

After reduction and taking into account (8) value of C_{11} can be determined for $n = 1$:

$$C_{11}^2 = \frac{\kappa J_r}{2\overline{m}\int_0^{L/2}[\sin(\frac{2}{L}\alpha_1 x) + \frac{C_{21}}{C_{11}}\cos(\frac{2}{l}\alpha_1 x) + \frac{C_{31}}{C_{11}}\sinh(\frac{2}{L}\alpha_1 x) + \frac{C_{41}}{C_{11}}\cosh(\frac{2}{L}\alpha_1 x)]^2dx + \Delta} \tag{11}$$

where:

$$\Delta = M[\sin(\alpha_1) + \frac{C_{21}}{C_{11}}\cos(\alpha_1) + \frac{C_{31}}{C_{11}}\sinh(\alpha_1) + \frac{C_{41}}{C_{11}}\cosh(\alpha_1)]^2 \tag{11a}$$

After calculating C_{11} a form of $f_{x1}(x)$, (8), determining the maximum of the first symmetric form of vibrations in the transient resonance, is completely known. The maximum amplitude of the machine body, occurring for $x = L/2$ equals:

$$A_{\max} = C_{11}[\sin(\alpha_1) + \frac{C_{21}}{C_{11}}\cos(\alpha_1) + \frac{C_{31}}{C_{11}}\sinh(\alpha_1) + \frac{C_{41}}{C_{11}}\cosh(\alpha_1)] \quad (12)$$

Let us also consider the case of a simple-supported beam of a constant cross-section, in the centre of which a concentrated mass M – being the source of forces of variable frequencies – was placed. The previously analysed system can be used for the solution of the above problem, assuming $k \to \infty$.

Then the frequency equation (6) becomes:

$$\frac{M}{\overline{m}L}\alpha(\cosh\alpha\cdot\sin\alpha - \sinh\alpha\cdot\cos\alpha) - 2\cdot\cosh\alpha\cdot\cos\alpha = 0 \quad (13)$$

This simplifies the n^{th} vibration form:

$$C_{1n} : C_{2n} : C_{3n} : C_{4n} = [\cosh\alpha] : [0] : [\cos\alpha] : [0] \quad (14)$$

$$f_{xn}(x) = C_{1n}[\sin(\frac{2}{L}\alpha_n x) + \frac{\cos\alpha}{\cosh\alpha}\sinh(\frac{2}{L}\alpha_n x)],\ 0 \le x \le L/2 \quad (15)$$

Equations (11) and (12) are still correct, but they do not contain terms: C_{21}, C_{41}.

3. Approximate estimations of maximum amplitudes of continuous systems one- and two-dimensional

Performed above considerations indicate that, even in relatively simple cases, exact analytical determinations of maximum amplitudes is difficult due to the necessity of integration of complicated expressions.

In order to obtain simpler, useful in engineering practice, dependencies the exact solution can be substituted by its approximate form.

a) Beam of a constant cross-section, length L and mass m, simple-supported at the end points, loaded by inertial excitation generated by the unbalanced shaft of the reduced moment of inertia J_r, applied to the concentrated mass M in the beam centre.

The real form of vibrations (15), given above, can be substituted by the approximate one:

$$f_{x1}(x) = A_{\max}\sin(n\pi\frac{x}{L}),\quad n = 1 \quad (16a)$$

exact for $M/m \to 0$, or

$$f_{x1}(x) = A_{\max}[3\left(\frac{x}{L}\right) - 4\left(\frac{x}{L}\right)^3],\quad (0 < x < L/2) \quad (16b)$$

exact for $M/m \to \infty$.

The equation of the maximum kinetic energy balance for $n = 1$:

$$\kappa \frac{1}{2} J_r \omega_{01}^2 = \int_0^L \frac{1}{2} [f_{1x}(x) \cdot \omega_{01}]^2 \bar{m} dx + \frac{1}{2} M [f_{1x}(L/2) \cdot \omega_{01}]^2, \quad \text{where } \bar{m} = m/L \tag{17}$$

leads for the shape function (16a) to the dependence:

$$A_{\max} = \sqrt{\frac{\kappa J_r}{M + \frac{1}{2} m}} \tag{18a}$$

while for the shape function (16b) to:

$$A_{\max} = \sqrt{\frac{\kappa J_r}{M + \frac{17}{35} m}} \tag{18b}$$

The transient resonance occurs in such cases for:

$$\omega_{01} \approx \omega_1 \approx \sqrt{\frac{48 EI}{L^3 (M + \frac{1}{2} m)}} \tag{18c}$$

b) The same system as in point (a), but the beam on both sides fixed to supports. In such case, for the approximated vibration form, acc. to Rayleigh, equation (17) leads to:

$$A_{\max} = \sqrt{\frac{\kappa J_r}{M + 0.4m}} \tag{19}$$

$$\omega_{01} \approx \omega_1 \approx \sqrt{\frac{192 EI}{L^3 (M + 0.4m)}} \tag{19a}$$

c) The same system as in point (a), but the cantilever beam, mass M on the free end- point of the beam. In such case:

$$A_{\max} = \sqrt{\frac{\kappa J_r}{M + \frac{33}{140} m}} \tag{20}$$

$$\omega_{01} \approx \omega_1 \approx \sqrt{\frac{3 EI}{L^3 (M + \frac{33}{140} m)}} \tag{20a}$$

d) Rectangular plates, with lumped mass $M(x_M, y_M)$ attached.
In the case of flat elements of a constant mass for the area unit, $\overline{m} = \dfrac{m}{a \cdot b}$

thickness h and a cylindrical flexural rigidity $D = \dfrac{Eh^3}{12(1-\upsilon^2)}$, υ – Poisson's ratio,
The equation of the kinetic energy balance is of a form:

$$\kappa \frac{1}{2} J_r \omega_{01}^2 = \iint_S \frac{1}{2} \left[f_{1xy}(x,y) \cdot \omega_{01} \right]^2 \overline{m} dx dy + \frac{1}{2} M \left[f_{1xy}(x_M, y_m) \cdot \omega_{01} \right]^2 \tag{21}$$

In the case of the rectangular plate of dimensions $a \times b$ and $x_M = a/2$, $y_M = b/2$:
d1) Simply – supported in the perimeter:

$$f_{1xy}(x,y) = A_{\max} \sin(\pi \frac{x}{a}) \cdot \sin(\pi \frac{y}{b}) \tag{22}$$

equation (21) leads to:

$$A_{\max} = \sqrt{\frac{\kappa J_r}{M + \frac{1}{4} m}} \tag{23}$$

$$\omega_{01} \approx \omega_1 \approx \frac{\pi^2}{2} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) \sqrt{\frac{Dab}{M + \frac{1}{4} m}} \tag{23a}$$

d2) Fixed in the perimeter:

$$f_{1xy}(x, y) = \frac{A_{\max}}{4} \left[1 - \cos\left(2\pi \frac{x}{a} \right) \right] \cdot \left[1 - \cos\left(2\pi \frac{y}{b} \right) \right] \tag{24}$$

equation (21) leads to:

$$A_{\max} = \sqrt{\frac{\kappa J_r}{M + \frac{9}{64} m}} \tag{25}$$

$$\omega_{01} \approx \omega_1 \approx \frac{2}{a} \sqrt{\frac{3D}{\beta \left(M + \frac{9}{64} m \right)}} \tag{25a}$$

where shape coefficient $\beta = 0.0624$ for $b/a = 1$, $\beta = 0.0816$ for $b/a = 2$ and $\beta = 0.072$ for $b/a = 4$ (Niezgodziński, 1975).

4. Conclusions

A possibility of the estimation maximum amplitudes in the transient resonance of systems of a continuous mass distribution – on the basis of the kinetic energy balance of the unbalanced rotor of the driving system and the kinetic energy of the vibrating system – was indicated in the study.

The exact as well as the approximate methods were shown, together with the calculation formulae for the basic cases of one- and two-dimensional systems.

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