Scientific Journals
Maritime University of Szczecin

Zeszyty Naukowe
Akademia Morska w Szczecinie

2013, 33(105) pp. 69–74
ISSN 1733-8670

2013, 33(105) s. 69–74

The impact of the change of river channel geometry on the size and range of backwater from the receiving body

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Key words: backwater, backwater range, river water level rise, depth

Abstract

This study is an analysis of the possibility of harnessing backwater in open river channels to create waterways in the area of the river mouth. To accomplish this, simplified methods of analysing non-uniform flow have been assessed, with the use of Rühlmann's, Tolkmitt's and Bresse's equations. It was demonstrated that Bresse's method is the best of the three – the results obtained by using it to determine the range of backwater from the receiving body are much closer to true values than for the other two, and it is the only method that meets the physical criterion of backwater size with the boundary condition of Chezy depth tending to 0.

The carried out analyses demonstrated that it is possible to create high-class waterways in lower reaches of rivers by modifying the geometry of their channels, namely the depth and slope of their bottom.

Introduction

The effect of backwater from the receiving body occurs in lower reaches of rivers that flow into large receiving bodies. The size and range of backwater vary depending on the position of water in the receiving body.

This backwater effect occurs in the area of the estuary of the Oder and in the rivers that flow into Bay of Szczecin and Lake Dąbie. Backwater in lower reaches of rivers can be harnessed i.e. for the purposes of navigation: a high-class waterway can be created by dredging (instead of damming up). The determination of the size and range of backwater in the lower reaches of rivers by approximation is difficult in such conditions because of the selected method, as well as due to the definition of the values of key parameters such as the water level rise at the mouth cross-section, as well as filling "h" for uniform flow. Variable bottom slope is also often the case.

The subsequent section of the paper analyses three different methods of calculating backwater range (Rühlmann's, Tolkmitt's and Bresse's) and assesses their usefulness. It also includes a number of analyses on the example of a 0.00–12.00 km section of the selected river, taking into account certain parameters (hydrographic, hydrological and hydraulic) approximate to this section.

Assumptions simplifications and input data

Modifications of the geometry of a natural watercourse flowing into the receiving body, which causes backwater at the tributary, can be basically divided into three categories:

1) modification of the width of the watercourse (tributary);
2) modification of the depth (dredging – lowering the bottom);
3) modification of the slope.

Additionally, two or all three of these categories can be used simultaneously (mixed case).

The categories of modifications listed in points 1, 2 and 3 were considered in terms of the possibility of creating an international class waterway for a theorised case. This allowed to answer the question whether it is possible to obtain the desired geometric parameters of a high-class waterway in the lower ("mouth") reaches of a river by changing the geometrical parameters of the river channel.

The following assumptions and simplifications were used to analyse the calculations of backwater range:

a) The river flows into a large receiving body whose influence in the lower reaches of a river causes non-uniform flow (backwater).
b) The river has the channel of a constant width B (calculated for the assumed set of calculation parameters) and there is a constant flow Q and of constant slope I. Therefore, there is a uniform flow above the zone of influence of the receiving body.
   Given the conditions a) and b) it can be concluded and further assumed that for the mouth cross-section $Z + h = \text{const}$ (as long as the bottom is not lowered).
c) Modifying the width of the channel, slope of the bottom, position of the bottom and flow rates may influence the backwater range from the receiving body.
d) The influence of such modifications of channel geometry was analysed for the following data:
   – flows: $Q_1 = 10$ m^3/s and $Q_2 = 20$ m^3/s;
   – bottom slope: $I_1 = 0.0002$ and $I_2 = 0.0003$;
   – channel width: $B_1 = 5$ m, $B_2 = 10$ m, …, $B_{20} = 100$ m;
   – $Z + h = \text{const}$: const$_1$ = 2.0 m, const$_2$ = 3.0 m, const$_3$ = 4.0 m.
e) The assumed channel roughness coefficient was $n = 0.03$.
f) The analysis was based on the following simplified equations for non-uniform flow: Rühlmann's, Tolkmitt's and Bresse's. The reason was that these three equations are used if the detailed geometry of the watercourse is not available.

Mathematical model of the effect

Two sections were considered: I and II located in a very small distance of ds from each other [1].

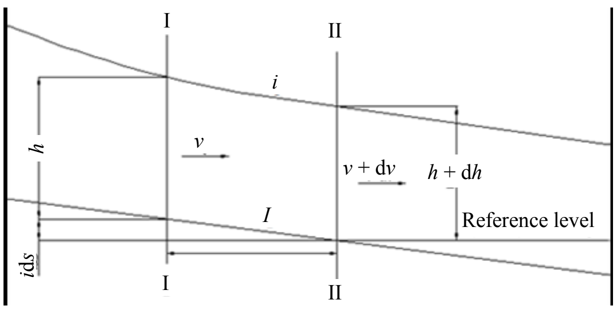

Fig. 1. Graphical calculation scheme for non-uniform flow

Bernoulli's equation for the two analysed cross-sections can be expressed as follows:

$$h + i\,\mathrm{d}s + \frac{\alpha v^2}{2g} = h + \mathrm{d}h + \frac{\alpha(v+\mathrm{d}v)^2}{2g} + \Sigma h_{str} \quad (1)$$

If it is assumed that: pressure $p = \text{const}$ and $\alpha = \text{const}$, and $v = Q/A$ and $Q = \text{const}$, $\mathrm{d}v = -(Q/A^2)\mathrm{d}A$, $da = BDH$ (B – the width of the table of liquid for the analysed cross-section).

This can be transformed into:

$$\frac{\mathrm{d}h}{\mathrm{d}s} = \frac{1 - \dfrac{Q^2}{c^2 R_h A^2}}{1 - \dfrac{\alpha Q^2 B}{g A^3}} \quad (2)$$

or

$$\frac{\mathrm{d}h}{\mathrm{d}s} = i \cdot \frac{1 - \dfrac{Q^2}{i \cdot c^2 R_h A^2}}{1 - \dfrac{\alpha Q^2 B}{g A^3}} \quad (3)$$

Equation (2) and (3) are the general equations of steady non-uniform flow in open channels.

For the special case of uniform flow where $I = i$, and $\mathrm{d}h/\mathrm{d}s = 0$ we obtain:

$$1 - \frac{Q^2}{I c^2 R_h A^2} = 0 \quad (4)$$

It can be transformed to obtain Chezy equation for uniform flow:

$$Q = Ac\sqrt{IR_h} \quad (5)$$

The following assumptions and simplification are used to analyse the course of the table of liquid for non-uniform flow in a simple channel with constant cross-section, where the table of the liquid is parallel to the bottom and the slope of the bottom is in the direction of the flow.

The case where $i > 0$ was considered.

The following designations were added:

$$K_0 = A_0 c_0 \sqrt{R_{h0}} \quad (6)$$

that allowed to obtain the following relation:

$$Q^2 = K_0^2 i \quad \text{or} \quad K_0^2 = \frac{Q^2}{i} \quad (7)$$

The value of K_0 does not depend on the slope but only on the shape and the filling of the cross-section. Thus, the following designation was used by analogy:

$$K = Ac\sqrt{R_h} \qquad \frac{K}{K_0} = \kappa \quad \text{and} \quad \frac{h}{H} = \eta$$

where:

H – normal depth,

K_0 – flow rate corresponding to the depth.

Therefore:

$$\frac{\mathrm{d}h}{\mathrm{d}s} = i \cdot \frac{1 - \dfrac{Q^2}{i c^2 A^2 R_h}}{1 - \dfrac{\alpha Q^2 B}{g A^3}} = i \cdot \frac{1 - \dfrac{K_0^2}{K^2}}{1 - j \cdot \dfrac{K_0^2}{K^2}} \quad (8)$$

When the following formula is introduced:

$$\left(\frac{K_1}{K_2}\right)^2 = \left(\frac{h_1}{h_2}\right)^x \tag{9}$$

where:

h_1 and h_2 – filling of the cross-section corresponding to coefficients K_1 and K_2,

x – a fixed power coefficient.

Taking into account the relation:

$$\kappa^2 = \eta^2,$$

the following can be derived:

$$\frac{\mathrm{d}h}{\mathrm{d}s} = i\,\frac{\eta^x - 1}{\eta^x - j} \tag{10}$$

and

$$j = \frac{i\,\alpha\,c^2}{g} \tag{11}$$

$$\mathrm{d}h = H\,\mathrm{d}\eta .$$

The final equation is obtained as [2]:

$$\begin{aligned}\frac{i\,\mathrm{d}s}{H} &= \frac{\eta^x - j}{\eta^x - j}\mathrm{d}\eta = \frac{\eta^x - 1 + 1 - j}{\eta^x - 1}\mathrm{d}\eta = \\ &= \mathrm{d}\eta + (1-j)\frac{\mathrm{d}\eta}{\eta^x - 1}\end{aligned} \tag{12}$$

If const is:

$$\frac{i}{H}(s_2 - s_1) = \eta_2 - \eta_1 + (1-j)\int_{\eta_1}^{\eta_2}\frac{\mathrm{d}\eta}{\eta^x - 1} \tag{13}$$

and where:

$$l = s_2 - s_1 \quad \text{and} \quad -\int\frac{\mathrm{d}\eta}{\eta^x - 1} = \varphi(\eta),$$

the following is derived:

$$\frac{il}{H} = \eta_2 - \eta_1 - (1-j)\left[\varphi(\eta_2) - \varphi(\eta_1)\right] \tag{14}$$

or

$$\begin{aligned}\frac{il}{H} &= \eta_2 - \eta_1 - \left[\varphi(\eta_2) - \varphi(\eta_1)\right] = \\ &= \varphi_1(\eta_2) - \varphi_1(\eta_1)\end{aligned} \tag{15}$$

where:

$$\varphi_1(\eta) = \eta - \varphi(\eta) \tag{16}$$

If j is omitted, that is, if it is assumed that $j = 0$, the formula (16) can be used. The value of $j = 0$ only slightly differs from $j \neq 0$ for the equations, therefore it can be omitted in the calculations.

To analyse the impact of modifications of the geometry of the river channel on the size and range of backwater, the following calculation scheme was adopted (Fig. 2) [3]:

- the river flows into the receiving body causing backwater at the tributary;
- the level of the receiving body water for the calculation scheme is constant;
- the cross-section of the channel is rectangular;
- the slope of the bottom of the channel is constant.

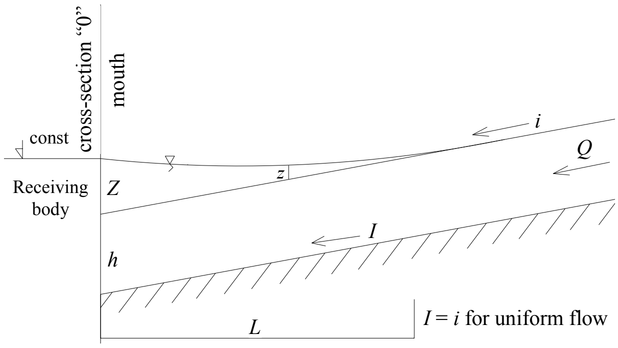

Fig. 2. Calculation scheme

The adopted constant level of the water table in the receiving body means that the value of $Z + h$ = const, for various values of channel width B, flow rate Q and bottom slope "I" gives different values of Z but their sum is always constant. Changing const value equals to lowering the bottom (dredging). Approximated equations were used to calculate the range of backwater:

a) Rühlmann's [4]:

$$\frac{i \cdot L}{h} = f\left(\frac{Z}{h}\right) - f\left(\frac{z}{h}\right) \tag{17}$$

if: $z = 0$ and $f\left(\frac{z}{h}\right) = 0 \Rightarrow$

$$L = \frac{h}{I}\cdot f\left(\frac{Z}{h}\right) \rightarrow \text{backwater range} \tag{18}$$

b) Tolkmitt's [4]:

$$\frac{i \cdot L}{h} = f\left(\frac{Z+h}{h}\right) - f\left(\frac{z+h}{h}\right) \tag{19}$$

if: $z = 0$ and $f\left(\frac{z+h}{h}\right) = 0 \Rightarrow$

$$L = \frac{h}{I}\cdot f\left(\frac{Z+h}{h}\right) \tag{20}$$

c) Bresse's [4, 5]:

$$L = \frac{1}{I}\left[Z + \left(h - \frac{v^2}{2g}\right)\phi\left(\frac{h+Z}{h}\right)\right] \tag{21}$$

where:

v – velocity acc. to Chezy [m/s] [6],

$$v = \frac{Q}{A} = \frac{Q}{Bh} \tag{22}$$

Therefore:

With $h \to 0$, $\phi\left(\frac{h+Z}{h}\right) \to 0$,

$$v \to 0, \left(h - \frac{v^2}{2g}\right) \to 0, \text{ thus } L = \frac{Z}{I} \quad (23)$$

The following calculation scheme was adopted for cases a), b) and c):

h – acc. to Chezy [m],

$$v = \frac{Q}{A} = \frac{1}{n} h^{\frac{1}{6}} \sqrt{Ih} = \frac{1}{n} h^{\frac{2}{3}} I^{\frac{1}{2}} = \frac{Q}{Bh} \quad (24)$$

therefore:

$$h = \left(\frac{Qn}{I^{\frac{1}{2}} B}\right)^{\frac{3}{5}} \quad (25)$$

Although Rühlmann's and Tolkmitt's equations apply only to rectangular and parabolic channels, they can be successfully used for natural channels with cross-sections similar to a rectangle or a parabola.

According to various authors, if the cross-section of the channel is more or less regular for the analysed area, the obtained results are sufficiently accurate for practical calculations. Thus, Rühlmann's equation applies to channels with steep banks, whereas Tolkmitt's equation to channels with flat banks.

Example calculations

The following parameters were assumed for further calculations:

$Q_1 = 10$ m^3/s and $Q_2 = 20$ m^3/s;
$I_1 = 0.0002$ and $I_2 = 0.0003$;
$Z + h = \text{const}$, thus: $Z = \text{const} - h$.

The calculations were performed for different values of $Z + h = \text{const}$;

$\text{const}_1 = Z + h = 2.0$ m,
$\text{const}_2 = Z + h = 3.0$ m,
$\text{const}_3 = Z + h = 4.0$ m.

First, the changes in the channel depth versus the changes in the width (i.e. widening the channel) were calculated. Figure 3 presents the calculated results showing that for an increase of B there is a corresponding decrease of h and increase of Z (with $Z + h = \text{const}$).

Figures 3a and 3b present the relation between the change of the depth of the river channel and its width for various slopes of the bottom and the various flow rates Q in a rectangular channel.

It can be observed that, naturally, with the increase of the channel depth, its width decreases. Nonetheless, it should be emphasized that above certain width values (for the assumptions made in this paper – above 60–80 m) depth changes are minimal.

The subsequent part is a calculation of backwater with the use of the different specified above methods.

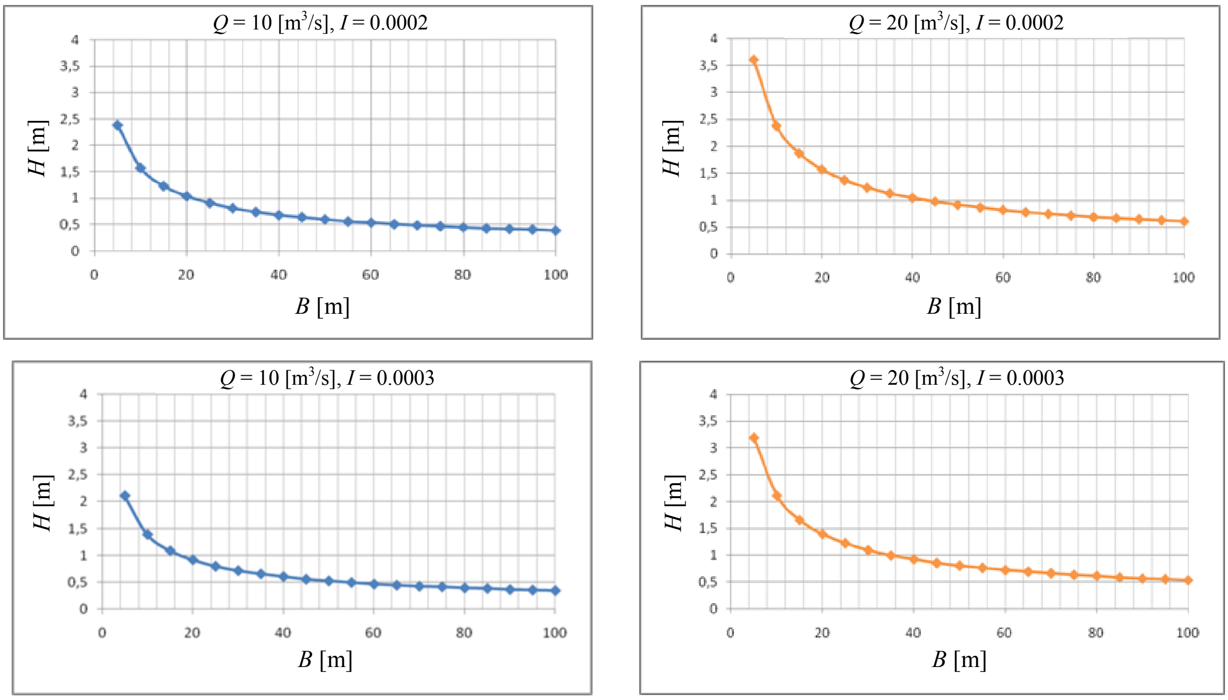

Fig. 3. Changes of filling H for different widths B acc. to Chezy-Manning

The results are shown as graphs in figures 4–9 [3].

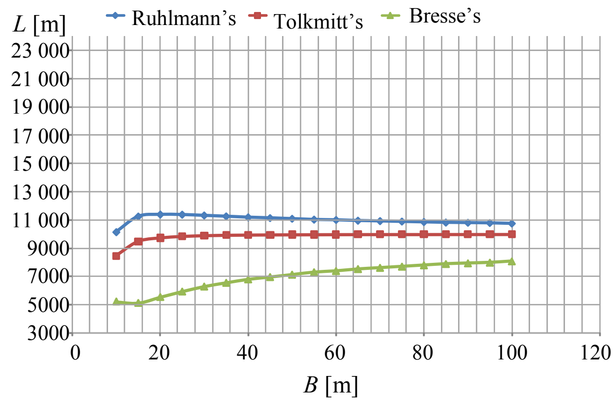

Fig. 4. Backwater range acc. to Rühlmann, Tolkmitt and Bresse – $Z + h = 2$ m, $Q = 10$ m^3/s, $I = 0.0002$

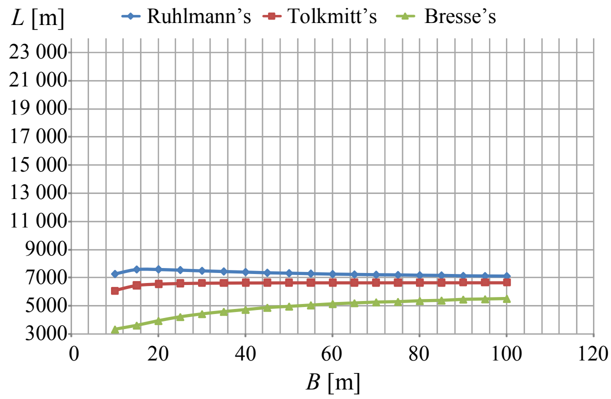

Fig. 5. Backwater range acc. to Ruhlmann, Tolkmitt and Bresse – $Z + h = 2$ m, $Q = 10$ m^3/s, $I = 0.0003$

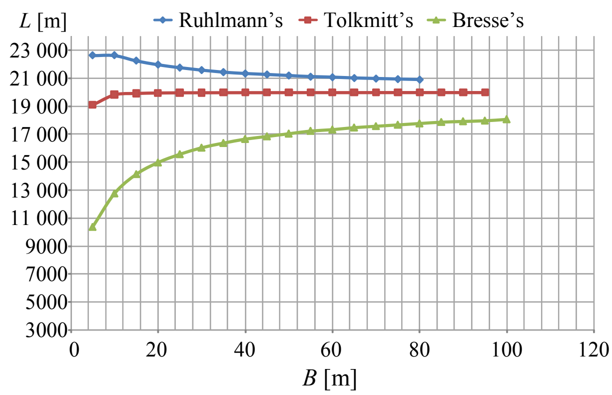

Fig. 6. Backwater range acc. to Rühlmann, Tolkmitt and Bresse – $Z + h = 4$ m, $Q = 10$ m^3/s, $I = 0.0002$

The results clearly show that changing the geometry of the watercourse in its the lower reaches ("mouth" section) has a significant impact on the size and range of the backwater, and prove that deepening and widening the channel may allow to achieve the desired characteristics of the waterway for a certain section of the watercourse.

An analysis of the results allows to draw the conclusion that different calculation methods yield

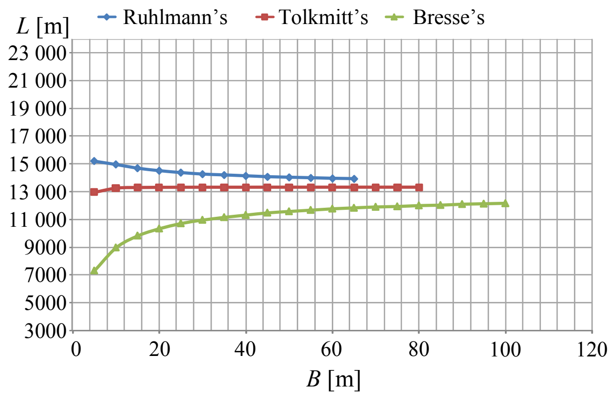

Fig. 7. Backwater range acc. to Ruhlmann, Tolkmitt and Bresse – $Z + h = 4$ m, $Q = 10$ m^3/s, $I = 0.0003$

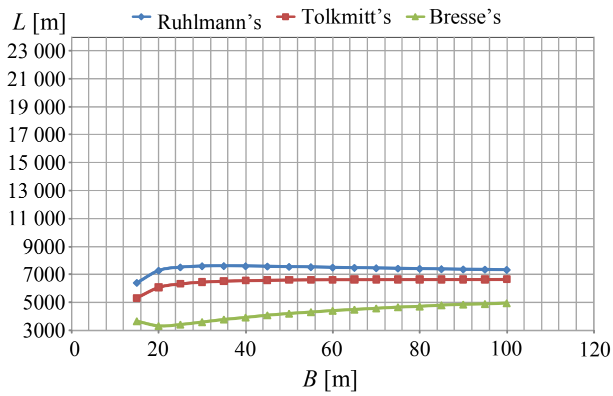

Fig. 8. Backwater range acc. to Ruhlmann, Tolkmitt and Bresse – $Z + h = 2$ m, $Q = 20$ m^3/s, $I = 0.0003$

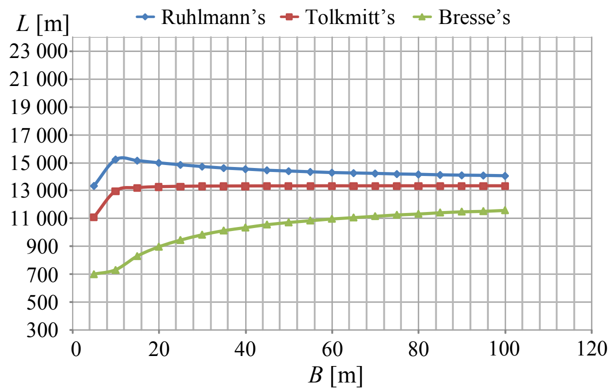

Fig. 9. Backwater range acc. to Ruhlmann, Tolkmitt and Bresse – $Z + h = 4$ m, $Q = 20$ m^3/s, $I = 0.0003$

highly different values of backwater range – the greatest ranges were obtained by using Rühlmann's method, whereas the results for Bresse's method were the lowest. This begs the question – what method to use in the absence of detailed geometric data?

To answer this question it was analysed whether the following criteria were met for all cases:

– supercritical flow (Froude number $F_r < 1$) [6],
– turbulent flow (Reynolds number $R_e > 6000$) [7].

The values of Froude number (F_r) were determined for different widths and a flow rate $Q = 10$

m^3/s and $I = 0.0002$. F_r value varied from 0.25 to 0.01.

This allows to draw the conclusion that the criteria for supercritical flow are met. Additionally, Reynolds numbers were calculated, assuming that the temperature of water $t = 10°C$, and thus by assuming a kinematic viscosity coefficient $\vartheta = 1.31 \cdot 10^{-6}$.

In all cases, even at a depth of 0.02 m, the obtained values of $R_e > 6000$. Thus, the criteria for turbulent flow are also met.

Interesting results were obtained when calculating the backwater range for a flow rate $Q = 10$ m^3/s and $I = 0.0002$ and if extreme conditions were assumed – width of 5000 m and depth h tending to "0". The calculated backwater range is then:

– acc. to Rühlmann L = 95,414 km
– acc. to Tolkmitt L = 76,598 km
– acc. to Bresse L = 9,800 m = 9.8 km

This makes it clear that the results obtained using Rühlmann's and Tolkmitt's methods are completely unrealistic.

There exist a very simple method of determining the maximum range of backwater for extreme conditions. Assuming that the depth of the channel tends to zero ($H \to 0$), the backwater range will be equal to the length of the section where the water table of the receiving body (horizontal) "penetrates" the bottom.

That is:

$$\text{with} \quad H \to 0, \; L_{\max} = \frac{Z}{I} \tag{26}$$

which is shown as a graph in figure 11.

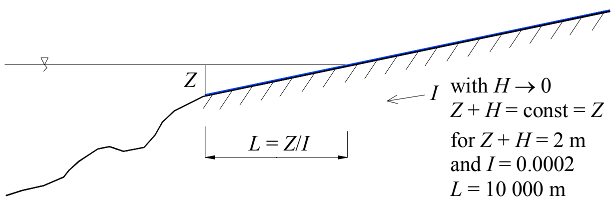

Fig. 11. Boundary conditions diagram for $H \to 0$

With the conditions thus defined, it is possible to verify whether the equations (methods) satisfy this condition. It is easy to demonstrate that only Bresse's equation (method) satisfies this boundary condition.

Therefore, it was decided that Rühlmann's and Tolkmitt's methods should not be used due to the following:

– irregular course of the water level rise curve, for which there is no physical explanation;
– too great value for backwater ranges, for which there is no practical explanation;
– obtaining completely unrealistic results for extreme conditions (which affects the quality of the equation);
– failure to meet the boundary condition for $H \to 0$.

Conclusions

The theoretical analysis presented in this paper demonstrates that by changing the geometry of a watercourse at its mouth it is possible to achieve the desired waterway parameters by harassing the backwater effect caused by the receiving body.

The results of the calculations clearly show that the size and range of backwater increase with the increase of channel width and lowering of the bottom – the increase of $Z + H$. On the other hand, greater slope and increased flow rate reduce the range of backwater.

It was found that Rühlmann's and Tolkmitt's methods should not be used in practice, as they completely fail to model the physics of the effect. It is preferred to use Bresse's method as a specific case of Bakhmeteff method.

Modifying roughness coefficient "n" changes the filling of the channel h, and in consequence affects the size and range of backwater.

A practical case for the lower reaches of the Ina analysed in another paper [8] demonstrated that by changing the parameters of the river channel (width, slope and position of the bottom) it is possible to obtain, at the analysed section, a IV international class waterway, by changing the size and range of backwater.

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