PROBLEMY MECHATRONIKI
UZBROJENIE, LOTNICTWO, INŻYNIERIA BEZPIECZEŃSTWA

ISSN 2081-5891
E-ISSN 2720-5266

13, 3 (49), 2022, 43-66

PROBLEMS OF MECHATRONICS
ARMAMENT, AVIATION, SAFETY ENGINEERING

Stability of a Bomb with a Wind-Stabilised-Seeker

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Received: December 20, 2021. / Revised: January 6, 2022 / Accepted: July 22, 2022 / Published: September 30, 2022.

DOI 10.5604/01.3001.0016.0049

Abstract. The primary purpose of this article is to consider the main factors affecting the stability of a bomb with a Wind-Stabilised-Seeker (WSS). The article contains a full mathematical description of the bomb-WSS system motion. It allows you to calculate the spatial motion of the bomb and the motion of WSS in relation to the bomb. The method of calculating the aerodynamic forces is also described. The sample simulation results were presented and discussed. The equations of motion of the system were determined. They constituted the basis for the development of a computer program to simulate the motion of the system. Aerodynamic forces and moments were calculated using the Prodas software and based on the results of the bomb tests in the wind tunnel. The mass and geometrical data of the system relate to the tested prototype of an aerial bomb. The result of the research is a comprehensive assessment of the influence of the geometric parameters of the mobile tracking system on the dynamic stability of the bomb-WSS system. This knowledge is necessary for proper design of control laws based on the signals registered by the WSS. The main parameter influencing the WSS-bomb stability is the aerodynamic focus position of the tracking system. It should be behind the WSS attachment point.

The position of the WSS centre of mass in relation to the attachment point also has an adverse effect on its stability. The length of the stick connecting the WSS with the bomb is irrelevant.
Keywords: dynamic stability, laser guided bombs, WSS-bomb system motion equations

1. INTRODUCTION

The issue of active bomb targeting has been discussed for many years. The first attempts were made by the Germans. An example is the Fritz X bomb (Ruhrstahl SD 1400 X) – guided bomb designed to combat heavily armoured ships - Fig. 1 taken from *https://free3d.com/pl*. The bomb was dropped from a bomber and it was driven by a human operator with a remote radio. It was first used in combat in 1943. It sank an Italian battleship "Roma" and the British light cruiser HMS "Spartan".

Fig. 1. Fritz X - human guided bomb

A more perfect solution is to automate the control of the bomb. The position of the target is indicated and monitored by an external device, but the deflections of the rudders are calculated by the control system mounted inside the bomb. This solution requires the use of detection systems that detect the signal reflected from the target. Therefore, it is necessary to use precise guidance systems. One way is to use seekers, which are a combination of mechanical, electronic, and optical components. The optical subsystem is usually based on laser, radar, or infrared detectors. Mechanically, the seeker can be mounted on the vehicle body as strapdown or gimbaled systems.

Figures 2a and 2b show both solutions during tests conducted by the Air Force Institute of Technology (AFIT, Warsaw, Poland). Because the strapdown seeker is directly mounted and fixed on the bomb body (Fig. 2a) its mechanical configuration is simpler. But for this kind of seeker, a field-of-view of a target (FOV) is significantly limited due to various physical, optical, and electronic limitations [1]. Additionally, the position of a target is measured relative to the bomb body-fixed reference frame. Sometimes - to enlarge the FOV - a movable lens is utilized.

The gimbaled seeker is mounted inside the bomb's body on a platform consisting of two orthogonal gimbals [2], which are stabilised using rate gyro feedbacks with servo motors and rate sensors [3]. In this case, the FOV range is increased, and the position of the target can be measured independently of the bomb motion. This mechanical solution is more complicated. It should also provide mechanical isolation of the detectors from vehicle motion.

An alternative to the gimbaled seeker mounted inside the bomb body is a wind-stabilised-seeker (WSS) mounted outside of the body on the front of the vehicle [4, 5] - Fig. 2b. The use of this kind of seeker is cheaper than the use of an internal one. The seeker is connected to the body through a universal joint. The appropriate shape of the seeker ensures that the airflow adjusts the seeker to the relative direction of the wind, which is also the direction of motion.

a) b)

Fig. 2. Prototypes of guided bombs: a) inertial seeker; b) external wind stabilised seeker

In studies on guided bombs, the equations of motion of objects are often presented in a significantly simplified manner [6, 7], ignored [8], or discussed only in general terms [3, 9-11]. Often, they also do not comment on these simplifications. The attention is focused on the problem of the selection of appropriate control laws ensuring effective hitting the target [12, 13]. When analysing them, it is assumed that the parameters of the bomb's motion - angular velocities and linear accelerations - are known. During the flight, they can be obtained by measurements, and during computational works obtained from a separate module - a simulation computer [14].

Excessive simplification of the controlled object's model may mean that although the simulation results indicate the correctness of the control, it may be ineffective. This is especially important if the control system contains moving parts that have their dynamics. If the detection system is stationary, permanently attached to the bomb body (strapdown seeker), the problem is simpler. In this case, knowing the current parameters of the bomb's motion it is sufficient to calculate the controls necessary to hit the target. However, when the control system is movable, it becomes necessary to know its motion in relation to the bomb.

This motion depends on the method of mounting the control system to the bomb body, on its geometric and mass characteristics, and on the characteristics of the actuators stabilising the position of the control system. During the flight, mutual feedback appears between the bomb and the control system, which makes it difficult to control effectively.

The publications [4, 15-17] analyse the motion of the bomb in the vertical plane. The bomb is modelled as a material point, and the dynamics of WSS motion is taken into account by additional, stochastic components appearing in the equations of the bomb's motion. There, it is assumed that the WSS axis all the time coincides with the bomb velocity vector.

It was found in [18] that the impact of the WSS on the bomb is negligible due to the difference between the masses of the bomb and the WSS. At the same time, it was emphasised that the influence of the bomb on the SWW motion may be significant.

The main purpose of the work is to present the results of the analysis of the influence of WSS selected characteristics on its operation. The research concerned the guided bomb prototype developed at the Polish Air Force Institute of Technology (AFIT) - Fig. 2. All results were obtained using the developed mathematical model of the spatial motion of the mechanical system consisting of the bomb and WSS. The aerodynamic characteristics of the bomb and WSS were obtained as a result of numerical calculations [19] and measurements in the wind tunnel.

The location of the WSS mounting point, the location of its mass centre, and the position of the aerodynamic focus were changed. The performed calculations allow us for the assessment of the correctness of the WSS operation. This can be done by comparing the course of the bomb nutation angle and the SWW deflection angle in relation to the bomb. These angles coincide if the SWW remains parallel to the bomb velocity vector.

2. MATHEMATICAL MODEL OF THE WSS-BOMB SYSTEM

The mathematical model of the motion of the WSS-bomb system has been obtained using Newton's laws of motion. They have been applied to the bomb and the WSS, respectively, for both translational and rotational motions. The bomb and the WSS were assumed to be rigid bodies. Because the attachment point does not coincide with the centre of mass of the WSS, it is possible to study the effect of design errors or constraints on the dynamics of WSS motion and, as a result, the effectiveness of self-tracking.

The description of the WSS-bomb system motion must include the bomb motion in the inertial coordinate system and the WSS motion relative to the bomb. This model must allow simulations of spatial motion, and in particular the analysis of the impact of various mass and geometric characteristics on the dynamics of motion, especially on dynamic stability.

This is important due to the assumption that the WSS is a part of an automatic flight control system that should ensure effective bomb homing. The motion equations are determined by calculating respectively: the position, velocity, and acceleration vectors of the bomb and the WSS elementary masses.

2.1. Characteristic points and position vectors

According to Fig. 3, the following points and position vectors were defined: *B* - the mass centre of the bomb, *M* - the WSS attachment point to the bomb, *F* - the mass centre of the WSS, $\boldsymbol{l}_M$ - the position vector of the point *M* relative to *B*, $\boldsymbol{l}_F$ - the position vector of the point *F* relative to *M*, $\boldsymbol{r}$ - the position vector of an elementary mass of the WSS dm_f relative to *F*, $\boldsymbol{r}_f$ - the position vector of dm_f relative to *B*, $\mathbf{r}'_f$ - the position vector of dm_f relative to *M*.

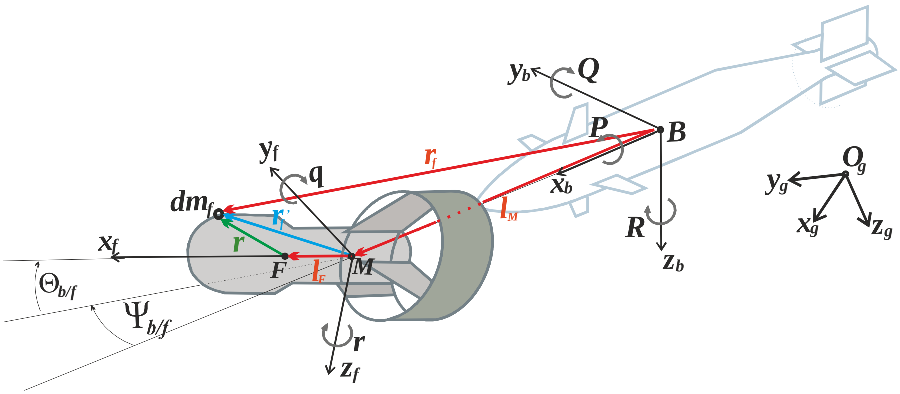

Fig. 3. Coordinate systems and position vectors

Figure 3 shows that there are the following relationships:

$$\mathbf{r}'_f = \mathbf{l}_F + \mathbf{r}, \ldots\ldots \mathbf{r}_f = \mathbf{l}_M + \mathbf{r}'_f = \mathbf{l}_M + \mathbf{l}_F + \mathbf{r} \tag{1}$$

2.2. Coordinate systems

To formulate the equations of motion, the following right-handed rectangular coordinate systems were introduced:

- $O_g x_g y_g z_g$ – the inertial system with origin on the Earth surface (topographic system); the $O_g z_g$ axis points to the Earth centre;
- $B x_g y_g z_g$ – the system with axes parallel to the $O_g x_g y_g z_g$ system. The origin of this system is at the bomb mass centre;
- $B x_b y_b z_b$ – the system fixed to the bomb. The $B x_b y_b$ and $B x_b z_b$ planes are planes of control surfaces;

- $Mx_f y_f z_f$ – the system fixed to the WSS. The axes of this system are parallel to the axis of the $Bx_b y_b z_b$ system if the Mx_f axis coincidences with the Bx_b axis.
- $Fx_f y_f z_f$ – the system fixed to the WSS originating at the WSS mass centre. Its axes remain parallel to the axis of the $Mx_f y_f z_f$ frame.

2.3. Transformations of the systems

Transformation matrices are necessary to convert forces and torques between different coordinate systems. They are analogous to the matrices used in flight mechanics [20]. So, they have the following forms:

- transformation matrix from the $Bx_g y_g z_g$ system to the $Bx_b y_b z_b$ system:

$$\mathbf{L}_{b/g} = \begin{bmatrix} c\Psi_b c\Theta_b & s\Psi_b c\Theta_b & -s\Theta_b \\ c\Psi_b s\Theta_b s\Phi_b - s\Psi_b c\Phi_b & s\Psi_b s\Theta_b s\Phi_b + c\Psi_b c\Phi_b & c\Theta_b s\Phi_b \\ c\Psi_b s\Theta_b c\Phi_b + s\Psi_b s\Phi_b & s\Psi_b s\Theta_b c\Phi_b - c\Psi_b s\Phi_b & c\Theta_b c\Phi_b \end{bmatrix} \tag{2}$$

where: Ψ_b - the azimuth angle of the bomb,
Θ_b - the pitch angle of the bomb,
Φ_b - the bank angle of the bomb,
$c = cos$, $s = sin$.

- transformation matrix from the $Mx_f y_f z_f$ system to the $Bx_b y_b z_b$ system:

$$\mathbf{L}_{f/b} = \begin{bmatrix} c\Psi_{b/f} c\Theta_{b/f} & s\Psi_{b/f} c\Theta_{b/f} & -s\Theta_{b/f} \\ -s\Psi_{b/f} & \cos\Psi_{b/f} & 0 \\ c\Psi_{b/f} s\Theta_{b/f} & s\Psi_{b/f} s\Theta_{b/f} & c\Theta_{b/f} \end{bmatrix} \tag{3}$$

where angles $\Psi_{b/f}$ and $\Theta_{b/f}$ determine the position of the WSS relative to the bomb. $\Phi_{b/f} = 0$ because there is no WSS rotation about the longitudinal axis Mx_f.

2.4. Angular velocities

The following angular velocities were taken into account (see Fig.3):

$\boldsymbol{\Omega}$ - the angular velocity of the bomb relative to the inertial system. It has the following components in the $Bx_b y_b z_b$ system: $\boldsymbol{\Omega} = [P, Q, R]^T$;

$\boldsymbol{\omega}$ - the angular velocity of the $Mx_f y_f z_f$ system relative to the $Bx_b y_b z_b$ system. It has two non-zero components in the $Mx_f y_f z_f$ system: $\boldsymbol{\omega} = [0, q, r]^T$.

2.5. Translational velocities

The absolute linear velocity of any WSS particle is the sum of the bomb mass centre velocity $\boldsymbol{V}$ and velocities which are a result of angular motion of the bomb at $\boldsymbol{\Omega}$ and angular motion of the WSS relative to the bomb at $\boldsymbol{\omega}$. We have:

$$\mathbf{V}_f = \mathbf{V} + \mathbf{\Omega} \times \mathbf{r}_f + \boldsymbol{\omega} \times \mathbf{r}'_f = \mathbf{V} + \mathbf{\Omega} \times \mathbf{l}_M + (\mathbf{\Omega} + \boldsymbol{\omega}) \times \mathbf{r}'_f \tag{4}$$

On this basis, the absolute velocity of the WSS mass centre can be calculated $(\mathbf{r}'_f = 0)$:

$$\mathbf{V}_F = \mathbf{V} + \mathbf{\Omega} \times \mathbf{l}_M + (\mathbf{\Omega} + \boldsymbol{\omega}) \times \mathbf{l}_F \tag{5}$$

In the $Bx_b y_b z_b$ frame, the velocity vector $\boldsymbol{V}$ of the point B has the following components: $\boldsymbol{V} = [U,V,W]^T$.

2.6. Absolute accelerations

Applying the laws of classical mechanics, the following expressions describing the absolute accelerations are obtained:
- for the bomb mass centre:

$$\mathbf{a}_B = \dot{\mathbf{V}} + \mathbf{\Omega} \times \mathbf{V} \tag{6}$$

- for the WSS mass element dm_f:

$$\mathbf{a}_f = \dot{\mathbf{V}} + \mathbf{\Omega} \times \mathbf{V} + \boldsymbol{\varepsilon} \times \mathbf{r}_f + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_f) + \boldsymbol{\varepsilon}_f \times \mathbf{r}'_f + 2\mathbf{\Omega} \times (\boldsymbol{\omega} \times \mathbf{r}'_f) \tag{7}$$

- for the WSS mass centre (point F):

$$\mathbf{a}_F = \dot{\mathbf{V}} + \mathbf{\Omega} \times \mathbf{V} + \boldsymbol{\varepsilon} \times (\mathbf{l}_M + \mathbf{l}_F) + \mathbf{\Omega} \times (\mathbf{\Omega} \times (\mathbf{l}_M + \mathbf{l}_F)) + \boldsymbol{\varepsilon}_f \times \mathbf{l}_F + 2\mathbf{\Omega} \times (\boldsymbol{\omega} \times \mathbf{l}_F) \tag{8}$$

where: $\dot{\mathbf{V}}$ is the linear acceleration of the point B in a non-inertial coordinate system, $\boldsymbol{\varepsilon} = \dot{\mathbf{\Omega}}$ and $\boldsymbol{\varepsilon}_f = \dot{\boldsymbol{\omega}}$ are the angular accelerations of the bomb and the WSS, respectively.

2.7. Equations of the bomb translational motion

According to the d'Alambert principle, the sum of forces acting on the bomb, and its inertia is equal to zero:

$$\mathbf{F}_{b\_b} + \mathbf{F}_{b\_a} + \mathbf{F}_{b\_g} + \mathbf{F}_{b\_R} = 0 \tag{9}$$

where: $\mathbf{F}_{b\_b} = -m_b \mathbf{a}_B$ - the inertia force,

$\mathbf{F}_{b\_a}$ – the bomb aerodynamic force,

$\mathbf{F}_{b\_g} = m_b \mathbf{g}$ – the bomb weight,

$\mathbf{F}_{b\_R}$ – the reaction force from the WSS.

Considering that for the bomb mass centre the position vector is equal to zero ($\boldsymbol{r}_b$ = **0**) and using (6), the equation of the linear motion for the bomb is obtained:

$$m_b \dot{\mathbf{V}} = \mathbf{F}_{b\_a} + m_b \mathbf{g} + \mathbf{F}_{b\_R} - m_b \mathbf{\Omega} \times \mathbf{V} \tag{10}$$

If the equation is solved in the $Bx_b y_b z_b$ system, then all the vectors occurring here should be determined in this system.

2.8. Equations of the WSS translational motion

For the WSS of the mass m_f, the equation of the translational motion has been obtained by balancing all forces. This balance has the form:

$$\mathbf{F}_{f\_b} + \mathbf{F}_{f\_a} + \mathbf{F}_{f\_g} + \mathbf{F}_{f\_R} = \mathbf{0} \tag{11}$$

where: $\mathbf{F}_{f\_b} = -m_f \mathbf{a}_F$ - the inertia force,

$\mathbf{F}_{f\_a}$ – the WSS aerodynamic force,

$\mathbf{F}_{f\_g} = m_f \mathbf{g}$ – the WSS weight,

$\mathbf{F}_{f\_R}$ – the reaction force from the bomb.

To calculate the inertia force, relation (8) referring to the WSS mass centre is used. Equation (11), written in the $Bx_b y_b z_b$ system, has the following form:

$$m_f\left(\dot{\mathbf{V}} + \boldsymbol{\varepsilon}\times(\mathbf{l}_M + \mathbf{l}_F) + \boldsymbol{\varepsilon}_f \times \mathbf{l}_F\right) = \mathbf{F}_{f\_a} + m_f \mathbf{g} + \mathbf{F}_{f\_R} - m_f\left(\boldsymbol{\Omega}\times\mathbf{V} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times(\mathbf{l}_M + \mathbf{l}_F)) + 2\boldsymbol{\Omega}\times(\boldsymbol{\omega}\times\mathbf{l}_F)\right) \tag{12}$$

2.9. Equations of the WSS-bomb system linear motion

To obtain the final form of the linear motion equations of the WSS-bomb system, Equations (10) and (12) should be added to each other and it should be taken into account that the reaction forces $\mathbf{F}_{b\_R}$ and $\mathbf{F}_{f\_R}$ are connected by the relation $\mathbf{F}_{f\_R} = -\mathbf{F}_{b\_R}$. The final equation written in the $Bx_b y_b z_b$ system is obtained:

$$\begin{aligned}(m_b + m_f)\dot{\mathbf{V}} + \mathbf{m}_\Omega \boldsymbol{\varepsilon} + \mathbf{m}_\omega \boldsymbol{\varepsilon}_f &= \mathbf{F}_{b\_a} + \mathbf{L}_{b/f}\mathbf{F}_{f\_a} + (m_b + m_f)\mathbf{L}_{b/g}\mathbf{g} - (m_b + m_f)\boldsymbol{\Omega}\times\mathbf{V} + \\ &- m_f\left(\boldsymbol{\Omega}\times\left(\boldsymbol{\Omega}\times(\mathbf{l}_M + \mathbf{L}_{b/f}\mathbf{l}_F)\right) + 2\boldsymbol{\Omega}\times\left(\mathbf{L}_{b/f}(\boldsymbol{\omega}\times\mathbf{l}_F)\right)\right)\end{aligned} \tag{13}$$

where matrices are marked as follows[1]:

$$\mathbf{m}_\Omega = m_f \begin{bmatrix} 0 & L_{13}l_F & -L_{12}l_F \\ -L_{13}l_F & 0 & l_M + L_{11}l_F \\ L_{12}l_F & -(l_M + L_{11}l_F) & 0 \end{bmatrix}, \qquad \mathbf{m}_\omega = m_f l_F \begin{bmatrix} 0 & -L_{31} & L_{21} \\ 0 & -L_{32} & L_{22} \\ 0 & -L_{33} & L_{23} \end{bmatrix}$$

In equation (13), it was considered that the vector $\boldsymbol{\omega}$ has only two non-zero components in the $Fx_f y_f z_f$ system. Therefore, we have $\boldsymbol{\varepsilon}_f = \dot{\boldsymbol{\omega}} = [0, \dot{q}, \dot{r}]^T$. There are also transformation matrices taking into account that some vectors are represented in other coordinate systems.

[1]To maintain compact formulas, when using transformation matrix elements, the subscript *f/b* is omitted, for example, instead of $(\mathbf{L}_{f/b})_{23}$ is L_{23}.

2.10. Equations of the bomb angular motion

The equation of the bomb angular motion about its mass centre B has been obtained by balancing the moments of external forces and inertia forces acting on the bomb:

$$\mathbf{M}_{b\_b} + \mathbf{M}_{b\_a} + \mathbf{M}_{b\_R} = 0 \tag{14}$$

where: $\mathbf{M}_{b\_b} = -\dfrac{d\mathbf{K}_b}{dt}$ - the moment of inertia forces,

$\mathbf{M}_{b\_a}$ – the moment of aerodynamic forces,

$\mathbf{M}_{b\_R}$ – the moment of reaction forces from the WSS.

The final form of this equation is written in the $Bx_b y_b z_b$ system fixed to the bomb. Considering that the bomb angular momentum $\boldsymbol{K}_b$ is equal to the product of the matrix of the bomb inertia moments $\boldsymbol{I}_b$ and the angular velocity $\boldsymbol{\Omega}$:

$$\mathbf{K}_b = \mathbf{I}_b \mathbf{\Omega} = \begin{bmatrix} I_{b\_x} & -I_{b\_xy} & -I_{b\_xz} \\ -I_{b\_yx} & I_{b\_y} & -I_{b\_yz} \\ -I_{b\_zx} & -I_{b\_zy} & I_{b\_z} \end{bmatrix} \mathbf{\Omega} \tag{15}$$

the equation of the bomb angular motion takes the form:

$$\mathbf{I}_b \dot{\mathbf{\Omega}} = \mathbf{M}_{b\_a} + \mathbf{M}_{b\_R} - \mathbf{\Omega} \times \left(\mathbf{I}_b \mathbf{\Omega} \right) \tag{16}$$

The reaction force $\mathbf{F}_{b\_R}$ from the WSS is applied at point M. Therefore, the reaction moment is equal to:

$$\mathbf{M}_{b\_R} = \mathbf{l}_M \times \mathbf{F}_{b\_R} = -\mathbf{l}_M \times \mathbf{F}_{f\_R} = \mathbf{l}_M \times \left(\mathbf{F}_{f\_b} + \mathbf{F}_{f\_a} + \mathbf{F}_{f\_g} \right) \tag{17}$$

Taking into account the above relationship in (16), the final form of the equation of bomb angular motion is obtained:

$$\begin{aligned} \mathbf{I}_{Vb} \dot{\mathbf{V}} + \left(\mathbf{I}_b + \mathbf{I}_{\Omega b} \right) \dot{\mathbf{\Omega}} + \mathbf{I}_{\omega b} \dot{\boldsymbol{\omega}} = \left(\mathbf{M}_{b\_a} + \mathbf{l}_M \times (\mathbf{L}_{b/f} \mathbf{F}_{f\_a}) \right) - \mathbf{\Omega} \times \left(\mathbf{I}_b \mathbf{\Omega} \right) + m_f \mathbf{l}_M \times \left(\mathbf{L}_{b/g} \mathbf{g} \right) + \\ - m_f \mathbf{l}_M \times \left(\mathbf{\Omega} \times \mathbf{V} + \mathbf{\Omega} \times \left(\mathbf{\Omega} \times \left(\mathbf{l}_M + \mathbf{L}_{b/f} \mathbf{l}_F \right) \right) + 2 \mathbf{\Omega} \times \left(\mathbf{L}_{b/f} \left(\boldsymbol{\omega} \times \mathbf{l}_F \right) \right) \right) \end{aligned} \tag{18}$$

where matrices are marked:

$$\mathbf{I}_{Vb} = m_f l_M \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \mathbf{I}_{\Omega b} = m_f l_M \begin{bmatrix} 0 & 0 & 0 \\ -L_{12} l_F & l_M + L_{11} l_F & 0 \\ -L_{13} l_F & 0 & l_M + L_{11} l_F \end{bmatrix}$$

$$\mathbf{I}_{\omega b} = m_f l_M l_F \begin{bmatrix} 0 & 0 & 0 \\ 0 & L_{33} & -L_{23} \\ 0 & -L_{32} & L_{22} \end{bmatrix}$$

2.11. Equations of the WSS angular motion

The equation of the WSS angular motion about the point M is the balance of the moments acting relative to it:

$$\mathbf{M}_{f\_b} + \mathbf{M}_{f\_g} + \mathbf{M}_{f\_a} + \mathbf{M}_{f\_a*} + \mathbf{M}_{f\_R} = 0 \tag{19}$$

The following moments are included:

- the moment of gravitational forces $\mathbf{M}_{f\_g} = \mathbf{l}_F \times m_f \mathbf{g}$,
- the moment of the aerodynamic forces applied at the WSS mass centre $\mathbf{M}_{f\_a} = \mathbf{l}_F \times \mathbf{F}_{f\_a}$,
- the aerodynamic moment about the WSS mass centre $\mathbf{M}_{f\_a*}$,
- the moment of inertia forces $\mathbf{M}_{f\_b} = \iiint \mathbf{r}'_f \times d\mathbf{F}_{f\_b}$,
- the moment of the bomb reaction force $\mathbf{M}_{f\_R}$, which is equal to zero due to the zero arm of the force $\mathbf{M}_{f\_R} = \mathbf{0}$.

To determine the moment $\mathbf{M}_{f\_b}$, one has to calculate series of integrals. Finally, the equation in $Mx_f y_f z_f$ coordinate system is obtained:

$$\begin{aligned}\mathbf{I}_{Vf}\dot{\mathbf{V}} + \left(\mathbf{I}_{\Omega f} + \mathbf{I}_f \mathbf{L}_{f/b}\right)\dot{\mathbf{\Omega}} + \left(\mathbf{I}_{\omega f} + \mathbf{I}_f\right)\dot{\boldsymbol{\omega}} = \mathbf{M}_{f\_g} + \mathbf{M}_{f\_a} + \mathbf{M}_{f\_a*} + \\ - m_f \mathbf{l}_F \times \left[\left(\mathbf{L}_{f/b}\mathbf{\Omega}\right) \times \left(\mathbf{L}_{f/b}\mathbf{V}\right)\right] - m_f \mathbf{l}_F \times \left[\left(\mathbf{L}_{f/b}\mathbf{\Omega}\right) \times \left(\left(\mathbf{L}_{f/b}\mathbf{\Omega}\right) \times \left(\mathbf{l}_F + \mathbf{L}_{f/b}\mathbf{l}_M\right)\right)\right] + \\ - \left(\mathbf{L}_{f/b}\mathbf{\Omega}\right) \times \left(\mathbf{I}_f\left(\mathbf{L}_{f/b}\mathbf{\Omega}\right)\right) - 2\left(\mathbf{I}_{cor1} + \mathbf{I}_{cor2}\mathbf{L}_{f/b}\right)\mathbf{\Omega}\end{aligned} \tag{20}$$

where:

$$\mathbf{I}_{Vf} = m_f l_F \begin{bmatrix} 0 & 0 & 0 \\ -L_{31} & -L_{32} & -L_{33} \\ L_{21} & L_{22} & L_{23} \end{bmatrix}, \qquad \mathbf{I}_f = \begin{bmatrix} I_{f\_x} & -I_{f\_xy} & -I_{f\_xz} \\ -I_{f\_yx} & I_{f\_y} & -I_{f\_yz} \\ -I_{f\_zx} & -I_{f\_zy} & I_{f\_z} \end{bmatrix},$$

$$\mathbf{I}_{\omega f} = m_f l_F^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{I}_{\Omega f} = m_f l_F \begin{bmatrix} 0 & 0 & 0 \\ \left(L_{32}L_{31} - L_{33}L_{21}\right)l_M & L_{33}\left(l_F + L_{11}l_M\right) - L_{31}^2 l_M & L_{21}L_{31}l_M - L_{32}\left(l_F + L_{11}l_M\right) \\ \left(L_{23}L_{21} - L_{22}L_{31}\right)l_M & L_{21}L_{31}l_M - L_{23}\left(l_F + L_{11}l_M\right) & L_{22}\left(l_F + L_{11}l_M\right) - L_{21}^2 l_M \end{bmatrix},$$

$$\mathbf{I}_{cor1} = m_f l_F^2 \begin{bmatrix} 0 & 0 & 0 \\ -rL_{11} & -rL_{12} & -rL_{13} \\ qL_{11} & qL_{12} & qL_{13} \end{bmatrix}$$

$$\mathbf{I}_{cor2} = \begin{bmatrix} rI_{f\_xy} - qI_{f\_xz} & rI_{f\_y^2} - qI_{f\_yz} & rI_{f\_yz} - qI_{f\_z^2} \\ -rI_{f\_x^2} & -rI_{f\_xy} & -rI_{f\_xz} \\ qI_{f\_x^2} & qI_{f\_xy} & qI_{f\_xz} \end{bmatrix},$$

$$\begin{aligned} I_{f\_x^2} &= 0.5\left(-I_{f\_x} + I_{f\_y} + I_{f\_z}\right) \\ I_{f\_y^2} &= 0.5\left(I_{f\_x} - I_{f\_y} + I_{f\_z}\right) \\ I_{f\_z^2} &= 0.5\left(I_{f\_x} + I_{f\_y} - I_{f\_z}\right) \end{aligned}$$

2.12. Kinematic relations

Equations (13), (18), and (20) describe the translational motion of the WSS-bomb system, the angular motion of the bomb, and the angular motion of the WSS about its attachment point *M*, respectively. They are supplemented with kinematic relations [20] that allow you to calculate:

- angles determining the spatial configuration of the bomb:

$$\begin{aligned} \dot{\Phi}_b &= P + (Q\sin\Phi_b + R\cos\Phi_b)\tan\Theta_b \\ \dot{\Theta}_b &= Q\cos\Phi_b - R\sin\Phi_b \\ \dot{\Psi}_b &= (R\cos\Phi_b + Q\sin\Phi_b)/\cos\Theta_b \end{aligned} \tag{21}$$

- angles defining the position of the WSS relative to the bomb:

$$\begin{aligned} \dot{\Theta}_{b/f} &= q \\ \dot{\Psi}_{b/f} &= r \end{aligned} \tag{22}$$

- trajectory of the bomb:

$$\begin{bmatrix} \dot{x}_{Cg} \\ \dot{y}_{Cg} \\ \dot{z}_{Cg} \end{bmatrix} = \mathbf{L}_{b/g}^{-1} \begin{bmatrix} U \\ V \\ W \end{bmatrix} \tag{23}$$

2.13. Final set of equations

The WSS-bomb system is the system of two rigid bodies, which are influenced by the gravity force and aerodynamic forces. A complete description of the motion of this system is obtained by solving the system of equations (13), (18), (20)–(23). This is a system of 16 ordinary differential equations that can be written in the form:

$$\mathbf{A}\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \tag{24}$$

where $\boldsymbol{x}$ is the vector of flight parameters:
$\mathbf{x} = [U, V, W, P, Q, R, q, r, \Phi_b, \Theta_b, \Psi_b, \Theta_{b/f}, \Psi_{b/f}, x_g, y_g, z_g]^T$

3. AERODYNAMIC FORCES AND MOMENTS

Aerodynamic forces and moments acting on the bomb ($\mathbf{F}_{b\_a}$, $\mathbf{M}_{b\_a}$) and on the WSS ($\mathbf{F}_{f\_a}$, $\mathbf{M}_{f\_a*}$) can be determined by knowing their aerodynamic characteristics and flow conditions. Various methods are used to obtain these characteristics. Engineering methods, based on theoretical and experimental formulas, are of great practical importance [21-25].

Aerodynamic characteristics can also be computed using numerical methods of fluid dynamics or other commercial software. Where possible, tests may be carried out in wind tunnels. In the calculations presented in this paper, the aerodynamic characteristics obtained with the use of the Prodas software [19] were applied, which were modified on the basis of tests in the wind tunnel.

Figure 4 shows the forces acting on the bomb. Their calculation requires knowledge of the bomb velocity relative to air $\boldsymbol{V}_{\mathrm{aer}} = [U_{\mathrm{aer}}, V_{\mathrm{aer}}, W_{\mathrm{aer}}]^{\mathrm{T}}$. This velocity is equal to the difference between the velocity relative to the inertial system $\boldsymbol{V}$ and the wind velocity $\boldsymbol{V}_{\mathrm{wind}} = [U_{\mathrm{wind}}, V_{\mathrm{wind}}, W_{\mathrm{wind}}]^{\mathrm{T}}$ relative to the same system:

$$\mathbf{V}_{aer} = \mathbf{V} - \mathbf{V}_{wind} \tag{25}$$

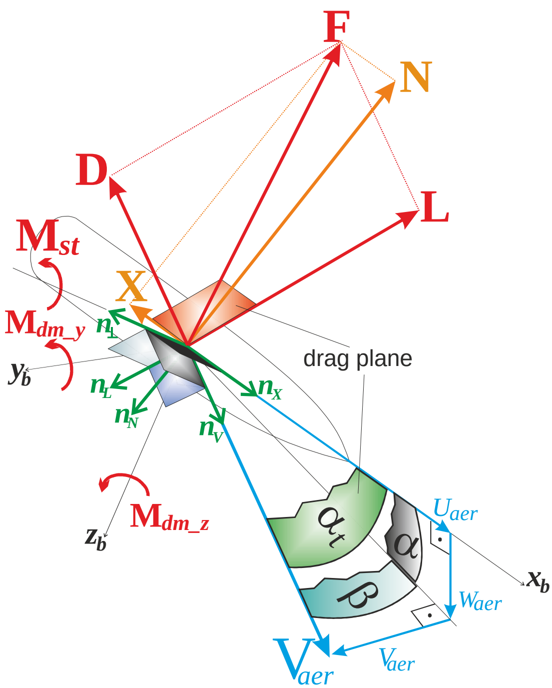

Fig. 4. Aerodynamic forces and moment

3.1. Aerodynamic angles

The following aerodynamic angles affect aerodynamic forces and moments acting on the bomb: the angle of attack α, the sideslip angle β, and the nutation angle α_t. They are shown in Fig. 4. They can be calculated as follows:

$$\alpha = \arctan \frac{W_{aer}}{U_{aer}}, \quad \beta = \arcsin \frac{V_{aer}}{|\mathbf{V}_{aer}|}, \tag{26}$$
$$\alpha_t \approx \sqrt{\beta^2 + \alpha^2}$$

3.2. Unit vector

TABLE 1
DEFINITIONS OF UNIT VECTORS

Symbol and description	CALCULATION/ COMPONENTS
$\boldsymbol{n}_X$ Coinciding with the Bx_b axis	$\boldsymbol{n}_X=[1,0,0]^T_C$ in $Bx_by_bz_b$ system
$\boldsymbol{n}_V=[n_{Vx},n_{Vy},n_{Vz}]^T$ Coinciding with the bomb velocity relative to air $\boldsymbol{V}_{aer}$.	$n_{Vx}=\frac{U_{aer}}{\lvert\mathbf{V}_{aer}\rvert}$, $n_{Vy}=\frac{V_{aer}}{\lvert\mathbf{V}_{aer}\rvert}$, $n_{Vz}=\frac{W_{aer}}{\lvert\mathbf{V}_{aer}\rvert}$
$\mathbf{n}_\perp=[n_{\perp x},n_{\perp y},n_{\perp z}]^T$ Perpendicular to the drag plane formed by the Bx_b and Bx_a axes.	$\mathbf{n}_\perp=\frac{\mathbf{n}_V\times\mathbf{n}_X}{\lvert\mathbf{n}_V\times\mathbf{n}_X\rvert}=\frac{1}{\sqrt{(n_{Vz})^2+(n_{Vy})^2}}\begin{bmatrix}0\\ n_{Vz}\\ -n_{Vy}\end{bmatrix}$
$\boldsymbol{n}_L=[n_{Lx},n_{Ly},n_{Lz}]^T$ Perpendicular to the plane formed by $\mathbf{n}_V$ and $\mathbf{n}_\perp$.	$\mathbf{n}_L=\frac{\mathbf{n}_V\times\mathbf{n}_\perp}{\lvert\mathbf{n}_V\times\mathbf{n}_\perp\rvert}$
$\boldsymbol{n}_N=[n_{Nx},n_{Ny},n_{Nz}]^T$ Perpendicular to the plane formed by $\boldsymbol{n}_X$ and $\mathbf{n}_\perp$	$\mathbf{n}_N=\frac{\mathbf{n}_X\times\mathbf{n}_\perp}{\lvert\mathbf{n}_X\times\mathbf{n}_\perp\rvert}$

TABLE 2
DEFINITIONS OF AERODYNAMIC FORCES AND MOMENTS

General formula Force/moment value Force/moment coefficient	Force/moment components in the $Bx_by_bz_b$ frame
$\mathbf{D}=-D\mathbf{n}_V$ $D=C_D\frac{\rho\lvert\mathbf{V}_{aer}\rvert^2}{2}S$	$D_x=-D\cdot n_{Vx}$ $D_y=-D\cdot n_{Vy}$ $D_z=-D\cdot n_{Vz}$
$\mathbf{L}=-L\mathbf{n}_L$ $L=C_L\frac{\rho\lvert\mathbf{V}_{aer}\rvert^2}{2}S$	$L_x=-L\cdot n_{Lx}$ $L_y=-L\cdot n_{Ly}$ $L_z=-L\cdot n_{Lz}$
$C_L=C_{L\alpha}\sin\alpha_t+C_{L\alpha3}(\sin\alpha_t)^3$	
$\mathbf{X}=-X\mathbf{n}_X$ $X=C_X\frac{\rho\lvert\mathbf{V}_{aer}\rvert^2}{2}S$	$X_x=-X$ $X_y=0$ $X_z=0$
$C_X=C_{X0}+C_{X\alpha2}(\sin\alpha_t)^2$	
$\mathbf{N}=-N\mathbf{n}_N$ $N=C_N\frac{\rho\lvert\mathbf{V}_{aer}\rvert^2}{2}S$	$N_x=0$ $N_y=-N\cdot n_{Ny}$ $N_z=-N\cdot n_{Nz}$
$C_N=C_{N\alpha}\sin\alpha_t+C_{N\alpha3}(\sin\alpha_t)^3$	
$\mathbf{M}_{st}=M_{st}\mathbf{n}_\perp$ $M_{st}=C_m\frac{\rho\lvert\mathbf{V}_{aer}\rvert^2}{2}Sd$	$M_{st\_x}=0$ $M_{st\_y}=M_{st}\cdot n_{2y}$ $M_{st\_z}=M_{st}\cdot n_{2z}$
$C_m=C_{m\alpha}\sin\alpha_t+C_{m\alpha3}(\sin\alpha_t)^3$	
$\mathbf{M}_{dm}=M_{dm}\cdot\left(\mathbf{n}_X\times\frac{d\mathbf{n}_X}{dt}\right)$ $M_{dm}=C_{m\_q}\frac{\rho\lvert\mathbf{V}_{aer}\rvert}{2}Sd^2$	$M_{dm\_x}=0$ $M_{dm\_y}=C_{m\_q}\left(\frac{Qd}{\lvert\mathbf{V}_{aer}\rvert}\right)\frac{\rho\lvert\mathbf{V}_{aer}\rvert^2}{2}Sd$ $M_{dm\_z}=C_{m\_q}\left(\frac{Rd}{\lvert\mathbf{V}_{aer}\rvert}\right)\frac{\rho\lvert\mathbf{V}_{aer}\rvert^2}{2}Sd$
$C_{m\_q}=C_{mq0}+C_{mq2}(\sin\alpha_t)^2$	

S is the reference cross-sectional area of the bomb; d is its diameter.

Figure 4 indicates also unit vectors that are used to calculate aerodynamic forces. They are defined in Table 1 below. Based on the unit vector $\boldsymbol{n}_V$, the value of the nutation angle α_t can also be determined, i.e., the angle between the velocity vector and the longitudinal axis of the bomb:

$$\alpha_t = \arccos(n_{Vx}) \tag{27}$$

3.3. The aerodynamic forces

The aerodynamic forces acting on the bomb can be divided into two groups - static and dynamic ones [26-28]. The static forces depend on the nutation angle α_t and the Mach number. The dynamic forces are the result of bomb rotation. They are small and can be omitted. Finally, the aerodynamic forces can be presented in two ways:

- $\boldsymbol{D}$ – the drag force coinciding with $\boldsymbol{V}_{aer}$ (opposite direction), as well as the lift force $\boldsymbol{L}$ perpendicular to $\boldsymbol{V}_{aer}$;
- the axial force $\boldsymbol{X}$ coinciding with Bx_b axis (opposite direction) as well as the normal force $\boldsymbol{N}$ perpendicular to Bx_b axis.

The resultant aerodynamic force $\mathbf{F}_{b\_a}$ is equal to the sum of all the forces described above and it can be written as:

$$\mathbf{F}_{b\_a} = \mathbf{D} + \mathbf{L} \quad \text{or} \quad \mathbf{F}_{b\_a} = \mathbf{X} + \mathbf{N} \tag{28}$$

depending on which pair of forces is included.

3.4. The aerodynamic moments

The aerodynamic moments acting on the bomb are also divided into static and dynamic ones. Because the normal force $\boldsymbol{N}$ is applied at the centre of pressure that does not coincide with the centre of mass, the static pitching moment $\boldsymbol{M}_{st}$ is created, aiming at the rotation of the bomb in the drag plane. The direction of this moment is the same as the unit vector $\mathbf{n}_{\perp}$. The dynamic moment has a damping character - it damps any rotation of the bomb and depends on angular velocities. For bombs, only the pitching Q and yawing R angular velocities are important. These velocities produce the damping moment $\boldsymbol{M}_{dm}$. The rolling velocity P is usually small, and the rolling damping moment and the Magnus moment can be omitted.

All these forces and moments are calculated using expressions given in Table 2 above.

Aerodynamic forces and moments acting on the WSS ($\mathbf{F}_{f\_a}$, $\mathbf{M}_{f\_a*}$) can be calculated in a similar way knowing its aerodynamic characteristics and the flow around the WSS.

4. RESULTS OF SIMULATIONS

The mathematical model of the WSS-bomb system, described above, has been used to simulate the spatial motion of this system. This model is represented by the set of ordinary differential equations (24). It allows you to analyse the impact of various factors on the dynamics of motion. This is particularly important because the WSS is a part of the control system, and its incorrect operation may result in inefficient guidance. Sample results, obtained with the use of the developed simulation software written in Fortran, are shown below. The most important geometric factors are shown in Fig. 5. They are:

- l_M – the distance between the bomb mass centre *B* and the WSS attachment point *M*,
- l_F – the distance between the WSS mass centre *F* and the WSS attachment point *M*,
- l_A – the distance between the WSS aerodynamic centre *A* and the WSS attachment point *M*. l_A decides the value of the WSS moment $\mathbf{M}_{f\_a*}$.

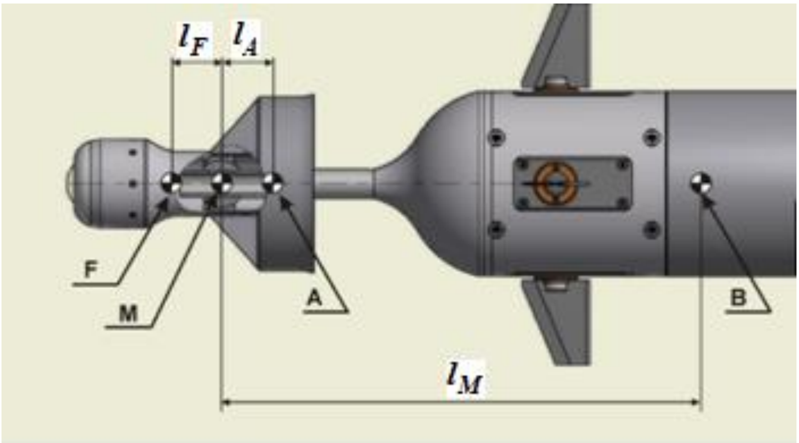

Fig. 5. Tested distances l_M, l_F, and l_A

As stated earlier, the aerodynamics characteristics both for the bomb and for the WSS were obtained using Prodas commercial software [19] and results of the wind tunnel tests. Exemplary aerodynamic characteristics are presented in Figs. 6 and 7.

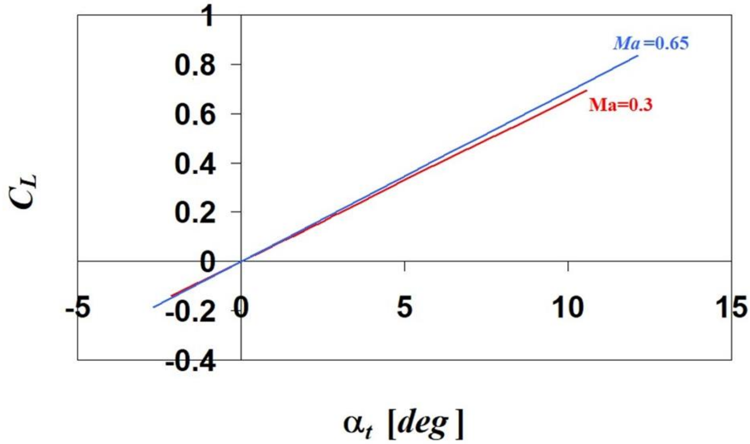

Fig. 6. Lift force coefficient

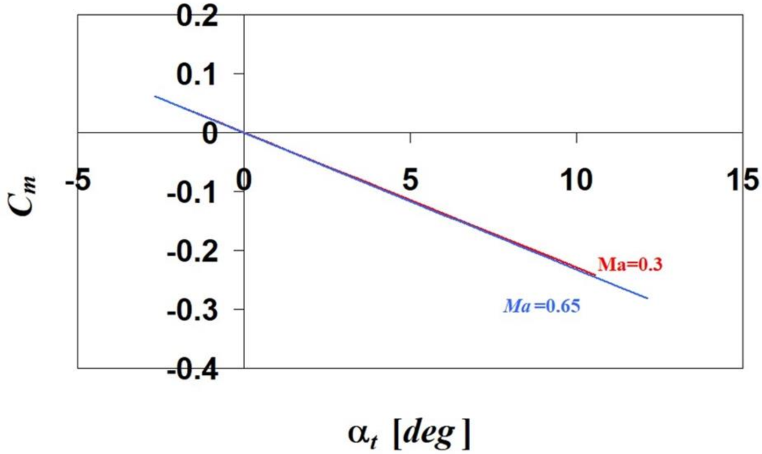

Fig. 7. Aerodynamic forces and moment

The prototype of the guided bomb (Fig. 2b) was the object of investigation. It was assumed that initial conditions are as follows: a level flight at the altitude of 3000 meters with the velocity of 55 m/s.

4.1. General description of analysis

As a result of the simulation, the time courses of flight parameters were obtained. It was assumed that the angular pitching velocity of the seeker was the most reliable for assessing the correctness of its operation. This course is a superposition of the long period motion (LPM) associated with the oscillation of the bomb pitch angle and the motion of the seeker's own (short period motion - SPM). Each of these motions has different frequency and damping.

Therefore, to evaluate these parameters, the following procedure was adopted:

- the frequency of both motions was identified using the Fast Fourier Transform (FFT);
- the amplitude-phase spectrum was divided into two spectra. The separation frequency was assumed to be equal to the mean of two maximum frequencies;
- separate spectra were created for the short period motion (SPM) and the long period motion (LPM);
- for both spectra, the inverse Fourier transform was calculated, obtaining the time courses of the LPM motion forced by the bomb and the SPM motion of the own seeker;
- based on separated courses, using the Hilbert transform (HT), the damping of both motions was estimated. LPM oscillations have a time-varying period and damping. Changes in these parameters result from the increase in aerodynamic forces and moments acting on the bomb. In this case, the estimate gives only average values. Therefore, additionally - based on the analysis of the time and the amplitude of successive peaks - the temporary period and damping of this motion during the bomb's flight were calculated. This procedure is shown in Fig. 8.

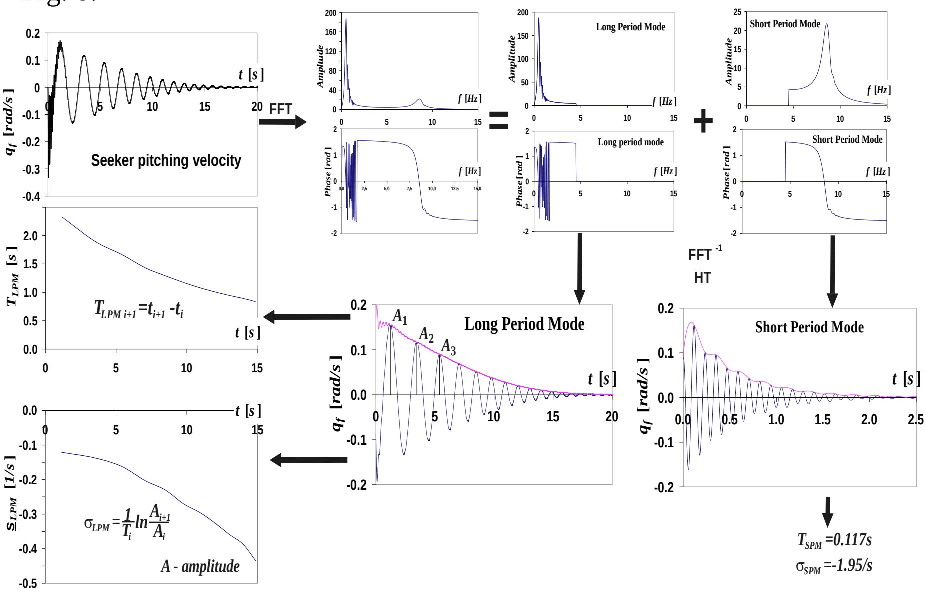

Fig. 8. Diagram of the analysis procedure

4.2. The influence of the WSS aerodynamic centre location (*A* vs. *M*)

The preliminary simulation had shown that the location of the WSS aerodynamic centre *A* is a crucial parameter. This location determines the dynamic stability of the WSS-bomb system.

Therefore, at the beginning of simulations, a position of this point was changed. It was assumed that the WSS mass centre coincidences with the attachment point ($F = M$). It means that $l_F = 0$. The extended support length was equal to 0.4 m.

The calculations show that if the aerodynamic centre *A* is at the attachment point *M*, the WSS generates lift L_f but it does not rotate about the point *M*. The WSS upward lift L_f gives a positive bomb pitching moment that is greater than that of the bomb body (negative). Therefore, it causes the bomb's nose to up. This means that the angle of the WSS relative to the bomb $\Theta_{b/f}$ is equal to the bomb pitch angle assumed with a negative sign, i.e., $\Theta_{b/f} = -\Theta_b$. This happens until the WSS leans against the lower movement limiter. This is shown in Fig. 9 (lower sketch). If the aerodynamic centre is in front of the attachment point, the WSS upward lift L_f causes the bomb nose to move up and the WSS simultaneously rotates about the attachment point *M* until it leans against the upper limiter. This is shown in the top sketch in Fig. 9.

For effective bomb control, it is imperative that the WSS axis coincides with the bomb velocity vector. WSS oscillations should be damped quickly. This is ensured when the WSS aerodynamic centre is behind the attachment point. In this case, the WSS motion is stable and consistent with the bomb motion and the axis of the WSS follows the velocity vector. Therefore, the nutation angle of the bomb α_t remains consistent with the WSS angle $\Theta_{b/f}$. This is shown in Fig. 10 for $l_A = 0.25$ cm. Since the nutation angle is calculated according to formula (27), the figure shows the absolute values of both angles. Only during the initial phase of the motion, there are slight differences resulting from the yet still undamped free oscillations of the WSS. But the control system is usually inactive during this phase of flight.

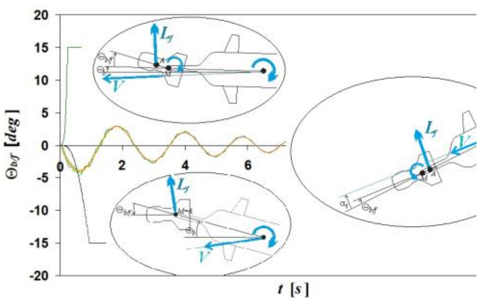

Fig. 9. Pitch angle of the WSS relative to the bomb

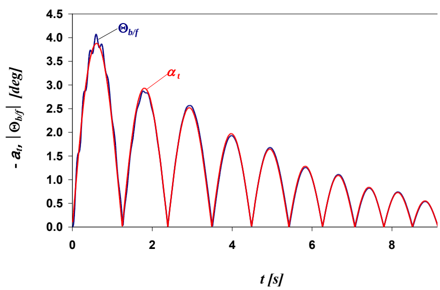

Fig. 10. WSS pitch angle $\Theta_{b/f}$ vs. nutation angle α_t

Figure 11 shows the value of the oscillation frequency of the LPM motion (average value) and the SPM motion of the seeker for various values of the length l_A. Figure 12 concerns the damping of these motions.

We can see that moving the aerodynamic focus backward causes an increase in the frequency of SPM oscillations and an increase in their damping. LPM motion parameters do not change.

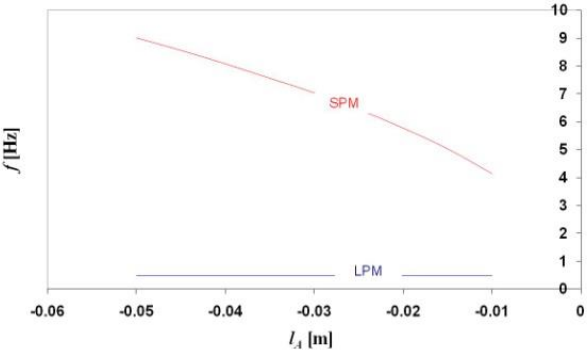

Fig. 11. LPM and SPM frequencies

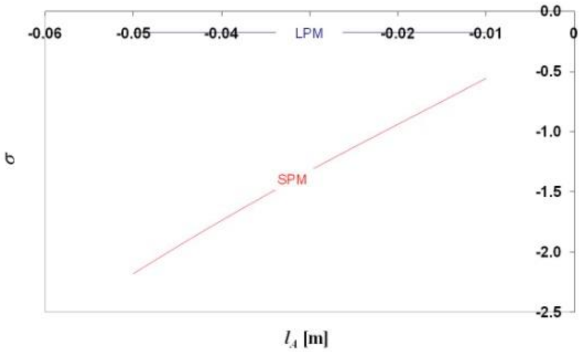

Fig. 12. LPM and SPM damping coefficients

4.3. The influence of the extended support length (*M* vs. *B*)

The influence of extended support length on the WSS-bomb dynamics was investigated for the configuration which provides the dynamic stability of the system. This means that the WSS aerodynamic centre *A* was located behind the attachment point *M* at a distance of 2 cm. Figures 13÷15 present some of the results for the extended support length l_M varied from 0.3 to 0.5 meters.

Figures 13 and 15 prove that l_M does not affect the bomb trajectory and the bomb pitch angle Θ_b. However, it does affect the oscillation of the WSS pitch angle $\Theta_{b/f}$, especially during the first phase of flight. Increasing l_M causes a slight increase in the amplitude of these oscillations, as shown in Fig. 15.

Increasing the length of l_M does not affect the frequencies of the motions of LPM and SPM, but slightly increases the damping coefficient of the SPM motion. This is shown in Fig. 16.

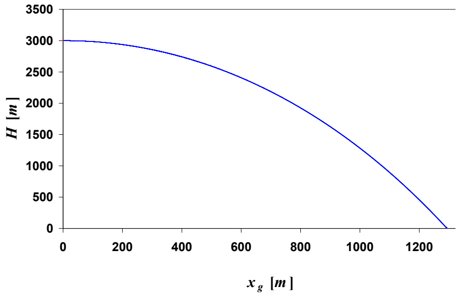

Fig. 13. Bomb trajectory

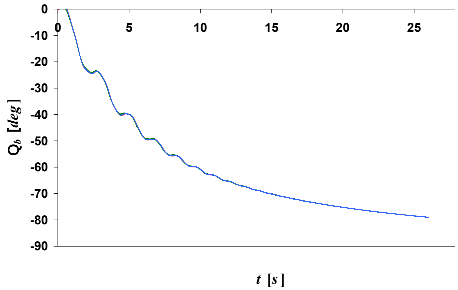

Fig. 14. Bomb pitch angle

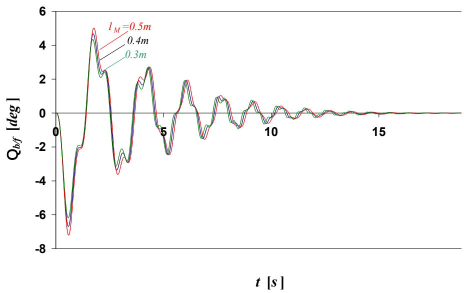

Fig. 15. WSS pitch angle

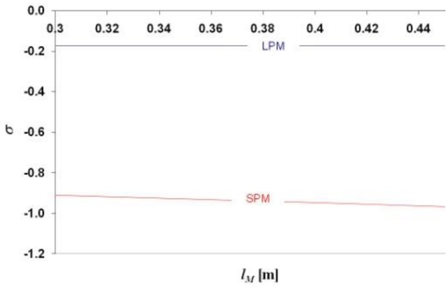

Fig. 16. LPM and SPM damping coefficients

4.4. The influence of the WSS mass centre location (*F* vs. *M*)

Just like before, it was assumed that the WSS aerodynamic centre *A* was 2 cm behind the attachment point *M*. The extended support length was equal to 0.4 m. To assess how the location of the WSS mass centre affects the motion of the WSS-bomb system, calculations were made by changing this position in the range from +25 mm (position in front of the point *M*) to -25 mm (the centre of mass at the rear of the point *M*).

The results showed that the location of the WSS centre of mass influences the WSS pitch angle $\Theta_{b/f}$. The effect on the bomb pitch angle Θ_b is also visible. This is shown in Figs. 17 and 18. The influence of l_F on the trajectory is negligible.

Figure 19 shows the value of the oscillation frequency of the LPM and of SPM motions of the seeker for various values of the length l. Figure 20 concerns the damping of these motions. We can see that moving the WSS mass centre causes a decrease in the frequency and damping of SPM oscillations. LPM motion parameters are almost constant.

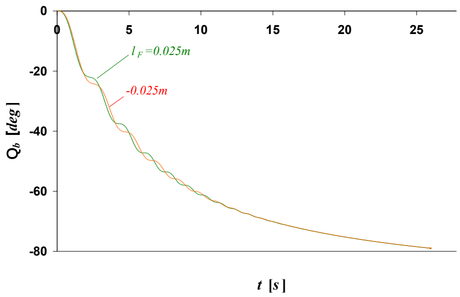

Fig. 17. Bomb pitch angle Θ_b

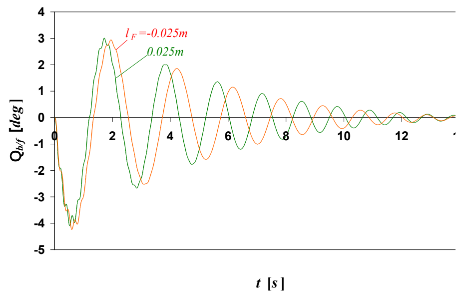

Fig. 18. WSS pitch angle $\Theta_{b/f}$

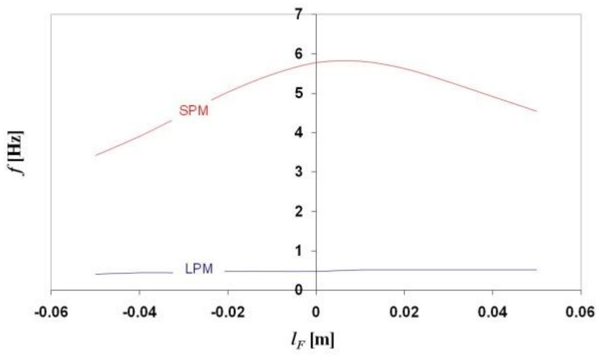

Fig. 19. LPM and SPM frequencies

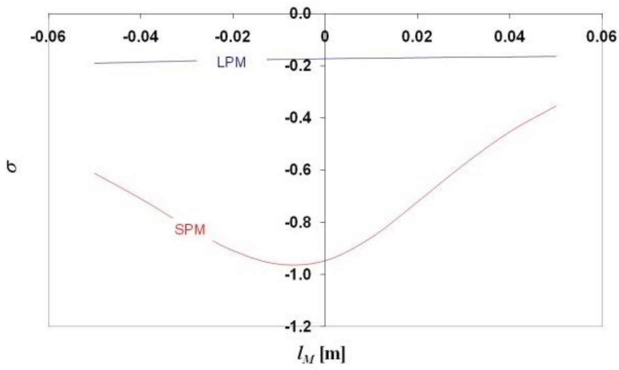

Fig. 20. LPM and SPM damping coefficients

5. CONCLUSIONS

Knowledge of the dynamics of the WSS-bomb system is important for effective bomb control. Incorrect selection of design parameters may lead to difficulties in homing or even prevent it. The presented model of the system motion is sufficient to perform simulations already at the stage of designing the WSS-bomb system. It is currently used for such purposes. The shown sample results indicate how to choose construction parameters and what to pay attention to.

A separate issue is the influence of the aerodynamic characteristics of the bomb and the WSS on the motion of the system and the effectiveness of the control. These characteristics are necessary to calculate the aerodynamic forces and moments according to the formulas given in Table 2. One should also take into account the aerodynamic interference between the bomb and the WSS. A separate problem that concerns the WSS-bomb system is the reduction of the bomb's stability by the WSS and by the control surfaces located in front of the bomb. This issue is also under investigation

FUNDING

The authors received no financial support for the research, authorship, and/or publication of this article.

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Stabilność bomby z układem śledzącym stabilizowanym przepływem

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Streszczenie. Podstawowym celem tego artykułu jest rozważenie głównych czynników wpływających na stabilność bomby z układem śledzącym stabilizowanym przepływem (WSS). Artykuł zawiera pełny opis matematyczny ruchu systemu bomba-WSS. Model ten został uzyskany z wykorzystaniem zasad mechaniki. Pozwala obliczyć ruch przestrzenny bomby i ruch WSS względem bomby. Sposób obliczania sił aerodynamicznych jest również opisany odpowiednimi wyrażeniami. Przedstawiono i omówiono przykładowe wyniki symulacji dotyczące wpływu geometrii układu na dynamikę jego ruchu. Równania ruchu układu zostały określone z wykorzystaniem praw mechaniki dotyczących układu wieloczłonowego. Stanowiły one podstawę do opracowania programu komputerowego do symulacji ruchu układu. Siły i momenty aerodynamiczne obliczono z wykorzystaniem oprogramowania Prodas oraz w oparciu o wyniki badań bomby w tunelu aerodynamicznym. Dane masowe i geometryczne układu dotyczą badanego prototypu bomby lotniczej. Wynikiem badań jest wszechstronna ocena wpływu parametrów geometrycznych ruchomego układu śledzącego na stateczność dynamiczną układu bomba-WSS. Wiedza ta jest niezbędna do właściwego zaprojektowania praw sterowania bazujących na sygnałach rejestrowanych przez WSS. Podstawowym parametrem wpływającym na stateczność dynamiczną układu jest położenie ogniska aerodynamicznego układu śledzącego. Powinien on znajdować się za punktem mocowania WSS do bomby. Położenie środka masy WSS względem punktu mocowania ma również niekorzystny wpływ na jego stateczność. Natomiast długość żerdzi łączącej WSS z bombą jest nieistotna.

Słowa kluczowe: stateczność dynamiczna, bomby sterowane laserowo, równania ruchu układu bomba-WSS