OPERATIONS RESEARCH AND DECISIONS

No. 2 2014
DOI: 10.5277/ord140204

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SOLVING INVENTORY ROUTING PROBLEMS USING LOCATION BASED HEURISTICS

Inventory routing problems (IRPs) occur where vendor managed inventory replenishment strategies are implemented in supply chains. These problems are characterized by the presence of both transportation and inventory considerations, either as parameters or constraints. The research presented in this paper aims at extending IRP formulation developed on the basis of location based heuristics proposed by Bramel and Simchi-Levi and continued by Hanczar. In the first phase of proposed algorithms, mixed integer programming is used to determine the partitioning of customers as well as dates and quantities of deliveries. Then, using 2-opt algorithm for solving the traveling sales-person problem the optimal routes for each partition are determined. In the main part of research the classical formulation is extended by additional constraints (visit spacing, vehicle filling rate, driver (vehicle) consistency, and heterogeneous fleet of vehicles) as well as the additional criteria are discussed. Then the impact of using each of proposed extensions for solution possibilities is evaluated. The results of computational tests are presented and discussed. Obtained results allow to conclude that the location based heuristics should be considered when solving real life instances of IRP.

Keywords: *inventory routing problem*, *optimization*, *vendor managed inventory*

1. Introduction

The 21st century is the age of new information technologies and its rapid development had a huge impact on businesses. Current interest and rapid development in supply chain management concepts include even faster and more efficient ways of disseminating information. With increasing frequency, the implementation goals of new concepts in logistics is not, as of yet, an increase in business efficiency but moreover, an improvement in the entire supply chain. Increasing the level of management from business and performance improvements within the whole supply chain results in

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an ever increasing complexity present in the studied aspects; these complexities significantly impede detailed analysis and modeling. Furthermore, in many cases, the evaluation outcome of new solutions is achievable only in the long run, whereas their implementation requires operational level decision making.

Vendor managed inventory (VMI) is an example of a concept which can be applied to at least two links in the supply chain. This concept involves the responsibility transfer for inventory level control to move from the receiver to the supplier. Under this strategy, the receiver does not execute orders (such as in classical strategies) but only determines the maximum level of material storage for materials covered by VMI. In exchange for this commitment, the supplier has the ability to determine the size and terms of delivery. Correctly implemented VMI ensures increased efficiency for both the customer (store operating cost reduction), as well as on the supplier side (fewer constraints when planning distribution).

The supplier problem within the VMI concept has generated numerous operational research models and multiple methods of solving them. Based on operational research problems, they are defined as inventory routing problems (IRP). In the following part of the paper, the development of IRP is defined and the basic approaches in the modeling of the problem are described. Next, the possible criteria for solving problems of this group and the most common extensions in practice. This paper makes three main contributions. First, an evidence that location based framework proposed by Bramel and Simchi-Levi [11] is very useful approach for solving routing problems. Second, common applied in IRP operational constraints are introduced into location based heuristic. Finally, solutions obtained by location based heuristic to common IRP test instances with different operational constraints are compared with other algorithms.

2. Literature review

The paper by Beltrami and Bodin [8] may be deemed a pioneering work regarding IRP problems. This work, presented in the 1970's, focused on modeling and simple solution techniques. In the following papers by Fisher et al. [20] and Bell et al. [7], mixed integer programming was used first to obtain a solution for the IRP instance. Subsequently, the first approach to solve a large IRP instance was made by Golden et al. [23] and by Dror [17], they investigated the large distribution system of liquid propane to residential and industrial customers. In the former, the basic components of the IPR problem are discussed and the simulation approach with vehicle routing algorithms is proposed. The latter contains a comparison of several different computational schemes and some computation results are presented. Burns et al. [12] try an analytic approach to solving IRP. They considered the optimal trade-off between inventory and transportation costs. The two distribution strategies are taken into account: direct shipping (i.e., shipping separate loads to each customer) and peddling (i.e., dispatching trucks that deliver items to more than one customer per load). The presented results indicate that the cost trade-off in each strategy depends on the shipment size. For direct shipping, the optimal shipment size is given by the economic order quantity (EOQ) model, while for peddling; the optimal shipment size is a full truck. In the latter case, trade-off also depends on the number of customers included on a peddling route.

The IRP research directions can be divided into three main streams. In the first stream, the vehicle routing formulation is extended to take into consideration time horizon and inventory issues. This field was initiated in research by Fisher et al. [20] and Bell et al. [7], where the set partitioning formulation is incorporated into a discrete time inventory planning problem. This approach is described as the application of time-discretized integer programming models to determine the set of customers to be visited, as well as the amount of product to deliver. Actually, such problem formulation enables a route covering timetable to be generated. Recent studies in this area include research by Bertazzi et al. [10] and Campbell and Savelsberg [13]. The first paper covers the analysis of interrelations between transportation costs and inventory costs for a simple task with five customers. The second branch of research analyzes the use of similarly defined mixed integer programming models to determine suggested delivery quantities over a long time horizon. Subsequently, the values thus determined are used in the second phase to compute the exact quantities of delivery over a short time horizon. The most promising formulation in this field was proposed by Archetti et al. [4], and Archetti et al. [5]. In the first paper, the exact procedure is presented while the second describes the hybrid heuristic where the MIP formulations and tabu search are combined into an iterative algorithm. The average error of hybrid heuristics for small instances (up to 50 customers and up to 6 periods) is lower than 0.1% whereas the maximum error is lower than 2%. In comparison, the heuristic proposed by Bertazzi et al. [9] guarantees an average error below 3% and a maximum error lower than 15%.

Campbell and Savelsberg's results may also be included in the second stream of research where the planning horizon is shortened through the computation of suggested replenishment quantities over a long time horizon and, based on the results obtained, the subsequent determination of a supply timetable and routes for the next few days. The first papers in this branch were: Dror et al. [17] and Dror and Ball [18]. This approach was extended upon and improved by Dror and Trudeau [19]. The last paper in this stream, by Bard et al. [6], works with a rolling horizon of IRP where a short term planning problem is defined for a two-week period. According to the rolling horizon approach, only decisions for the first week are implemented.

The third research stream considers the division of a customer base into delivery groups based on their respective demands and other method-specific parameters. Then, each delivery is performed to all customers in a given group with routes determined with use of the classical VRP or TSP algorithms. Such approach is used, among others, in the papers by Anily and, Federgruen [1–3]. In these approaches, it was assumed that each customer belongs to one or more delivery groups. If such situation occurs a specific fraction of its demand is allocated to each of appropriate delivery group. The class of low complexity heuristics is proposed and it is shown that the obtained solutions are asymptotically optimal. In the paper by Gallego and Simchi-Levi [21], a lower bound on the long run average cost over all inventory-routing strategies is proposed. This lower bound is used to point out that the effectiveness of direct shipping over all inventory-routing strategies is at least 94% whenever the economic lot size of each of the retailers is at least 71% of vehicle capacity. These results provide a useful approach regarding how to decide when the much more difficult task of finding cost-effective routes should be applied. Bramel and Simchi-Levi [11] proposed a general framework for modeling routing problems which is called location based heuristic. In this approach, the routing problem is approximated by another combinatorial problem called capacitated concentrator location problem (CCLP). Because of the seed sets which represents customers served together, this heuristic allows us to implement this variant of solving IRP in an easy way. The most recent use of these ideas was used in the paper by Chan et al. [14] and Gaur and Fisher [22]. In the former, the fixed partitioning and zero inventory ordering are considered and the asymptotic effectiveness is characterized. The latter, describes the implementation of inventory routing algorithms in a supermarket chain. The implementation provided savings of 4% of distribution costs in its first year of implementation and is expected to yield 12–20% savings as the firm expands its usage.

3. Inventory routing problem

An inventory routing problem concerns a repeated distribution of a product from a vendor to set customers over a given periods (planning horizon). For each customer, the daily consumption is given and the state of the warehouse at period 0. The vendor operates a homogenous fleet of vehicles with a given capacity per vehicle, used to deliver the supplies.

The objective is to minimize the total distribution and inventory cost over the planned horizon under the assumption that none of the customers reports stockout. A solution to an IRP is a detailed way of distributing products (delivery quantities and dates, as well as the delivery routes for each vehicle in every period). Such a solution is known as a distribution and stock replenishment policy.

IRP models from the group concerning the flow models of vehicles is formed by extending the formulation used in VRP models. For an example of such formulations and discussion regarding possible ways of determining solutions, see the works of Coelho et al. [15]. Because the model in the cited work allows the transfer of goods between buyers and also takes into account the cost of supplier storage, the presented formulation has been simplified to match the basic variant of IRP.

In the formulation labeled as M1, there are four decision variables. The first v^B_{nmkt} represents the connections used in solution. When this variable is set to 1, this means that the connection between the nodes (n, m) is utilized by the vehicle k during the period t. Otherwise, the variable is set to 0. The second variable y^H_{nt} corresponds to the level in the customer warehouse n over the period t. The next decision variable v^C_{nkt} determines the size of delivery to the recipient n over period t via vehicle k. The binary variable v^D_{nkt} takes the value 1 if in period t by vehicle k delivered goods to the recipient n. Otherwise; the variable is set to 0. In order to ensure consistency of routes, a constraint is used for the elimination of inconsistent paths Miller–Tucker–Zemlin used by Desroches and Laporte [16], requiring the use of a continuous decision variable z^A_{nkt}, reflects the volume of deliveries made by the vehicle whilst serving the recipient. The model utilizes parameters which determine the value of storage costs for customer n and travel the section (n, m), denoted by c^H_n and c^V_{nm}. The last four parameters that define the characteristics of the facilities used in the delivery such as minimum and maximum stock levels at the customer n, customer demands n over period t and vehicle load k is denoted by the symbols $q^{Y\max}_n$, $q^{Y\min}_n$, d_{nt}, q^V_k.

Model M1

Minimize

$$\sum_{n\in N}\sum_{t\in T} c^H_n y^H_{nt} + \sum_{(n,m)\in A}\sum_{k\in K}\sum_{t\in T} c^V_{nm} v^B_{nmkt} \tag{1}$$

with constraints:

$$y^H_{nt} = y^H_{n,t-1} - d_{nt} + \sum_{k\in K} v^C_{nkt}, \qquad \forall n\in N, \qquad t\in T \tag{2}$$

$$q^{Y\min}_n \le y^H_{nt} \le q^{Y\max}_n, \qquad \forall n\in N, \qquad t\in T \tag{3}$$

$$\sum_{k\in K} v^C_{nkt} \le q^{Y\max}_n - y^H_{n,t-1}, \qquad \forall n\in N, \qquad t\in T \tag{4}$$

$$\sum_{k\in K} v^C_{nkt} \le q^{Y\max}_n \sum_{m\in N^0}\sum_{k\in K} v^B_{nmkt}, \qquad \forall n\in N, \qquad t\in T \tag{5}$$

$$\sum_{n\in N} v_{nkt}^{C} \le q_{k}^{V}, \qquad \forall k\in K, \qquad t\in T \tag{6}$$

$$v_{nkt}^{C} \le q_{n}^{Y\max} v_{nkt}^{D}, \qquad \forall n\in N, \qquad k\in K, \qquad t\in T \tag{7}$$

$$\sum_{m\in N} v_{nmkt}^{B} = \sum_{m\in N} v_{mnkt}^{B} = v_{nkt}^{D}, \qquad \forall n\in N, \qquad k\in K, \qquad t\in T \tag{8}$$

$$\sum_{m\in N} v_{nmkt}^{B} \le 1, \qquad \forall k\in K, \qquad t\in T \tag{9}$$

$$\sum_{k\in K} v_{nkt}^{D} \le 1, \qquad \forall n\in N, \qquad t\in T \tag{10}$$

$$z_{nkt}^{A} - z_{mkt}^{A} + q_{k}^{V} v_{nkt}^{D} \le q_{k}^{V} - v_{mkt}^{C}, \qquad \forall n\in N, \qquad m\in N, \qquad k\in K, \qquad t\in T \tag{11}$$

$$v_{nkt}^{C} \le z_{nkt}^{A} \le q_{k}^{V}, \qquad \forall n\in N, \qquad k\in K, \qquad t\in T \tag{12}$$

$$v_{nkt}^{D}, v_{nmkt}^{B} \in \{0,1\}, \qquad \forall n\in N, \qquad m\in N, \qquad k\in K, \qquad t\in T \tag{13}$$

$$v_{nkt}^{C} \ge 0, \qquad \forall n\in N, \qquad k\in K, \qquad t\in T \tag{14}$$

The goal function (1) minimizes the total cost of storage and transportation in the analyzed planning horizon. Constraints (2) and (3) respectively, ensure the consistency of stocks in subsequent periods and the inventory level under the given parameters $q_n^{Y\max}$ and $q_n^{Y\min}$. A further constraint (4) means that no delivery exceeds the acceptable supply level. The task of constraint (5) is to link the decision variables and, resulting from the fact that the supply is only possible in the period in which the target location is visited. Using this observation that supply cannot exceed acceptable stock levels, instead of the technical value big*M* which is equal to a very large positive number, the parameter value $q_n^{Y\max}$ was used. Doing so is intended to accelerate the process. Although a modified form of the constraint does not affect the solution, it defines significantly the value of lower limit which is determined in the process of solving a mixed programming model. The more accurate an estimate of the lower limit, the more potential solutions can be rejected, allowing a higher resolution process. The subsequent constraint (6) ensures that the maximum vehicle load capacity is not exceeded. The constraint (7) is used to bind the variable v_{nkt}^{C} with the binary variable v_{nkt}^{D} – delivery can be executed $v_{nkt}^{C} > 0$ if and only if $v_{nkt}^{C} = 1$. As with the constraint (5) instead of the symbol Big$M$, the parameter $q_n^{Y\max}$ is used. Constraint (8) guarantees the continuity of vehicle movement and connects the values of the decision variables v_{nmkt}^B and v_{nkt}^D. Condition (9) limits the number of vehicles used up to one, while the constraint (10) ensures that delivery to the customer will be executed no more than once in any period. The following constraints (11) and (12) are used to ensure consistency of routes over the respective periods. The first of these ensures that for all pairs (m, n) of customer location, where the vehicle delivers goods to the recipient m, and immediately afterwards to n, the difference in delivery volume undertaken from the beginning of the route to the recipient n and from the beginning of the route to recipient m must be equal to the size of delivery to recipient n. If such delivery is not undertaken, namely $q_k^V v_{nkt}^D = 0$ and $v_{nkt}^C = 0$, then constraint (11) will always be true. Constraint (12) ensures that the total capacity does not exceed that of the vehicle.

4. Location based heuristics for routing problems

The basic idea used in the proposed approach is the use of integer programming in simplifying the form of the objective function. The main advantages of this approach is the ease of taking into account any additional obligations resulting from specific practical problems, and the ability to apply advanced optimization tools such as the CPLEX or Gurobi packages. Furthermore, model simplification concerns the objective function meaning that the obtained solution will satisfy all the conditions of the problem. In this case, the decision maker accepting suboptimal solution guarantees obtainment within an acceptable time.

The objective function in the basic version of the IRP has two components, the cost of supply and the cost of storage. The majority of problems during solution cause decision-makers to take into account the cost of supply, because the problem, even for one period is an NP-hard. In the proposed framework, the simplification of representation is taken into account. In the classical routing model, the route length is presented as the sum of route arcs. These arcs have to create a coherent cycle (Fig. 1a), which causes the most problems when solving this task.

As an alternative, the method of determining the route length was presented by splitting the route into two components. The first is the length of the direct route from the depot location to the furthest customer on the route. The second component is the increase in the direct route from the depot to the furthest customer after adding to it an additional customer. Each additional customer is an additional element of the second component. The length of the route from the warehouse (node 1) to 5 customers presented in the figure (Fig. 1b) in the proposed approach will consist of a direct route length to the furthest customer (customer 5) and the sum of increases in that route after adding ever single customer from 4 additional customers in this route (i.e., customers 6, 5, 3, 2). It should be noted,that while in Fig. 1a the weights of the arcs of the graph correspond to the distance between the customers, in Fig. 1b, the weight of arcs (1, 5) is equal to the length of the direct route to customer 5 (i.e. 1–5–1), and the weights of the remaining arcs, i.e. (5, n) are equal to the increase in length of the route 1–5–1 after adding to it customer n (i.e. 1–5–n–1).

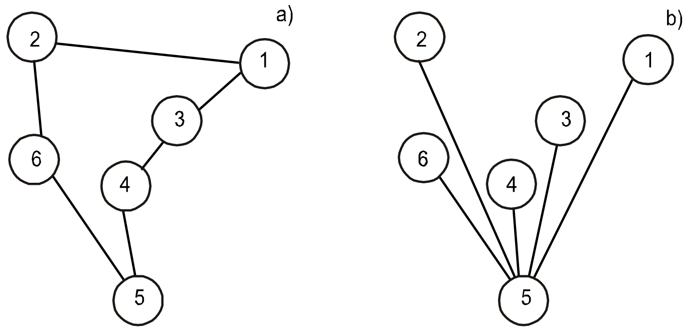

Fig. 1. Route of the vehicle: a) determining exact route length,
b) simplified way to determining route length

This approach is similar to location based heuristic framework proposed by Bramel and Simchi-Levi [11]. In the presented formulation, the sets of customers visited together which contain exactly one customer are taken into consideration. Moreover the way of cost calculation is modified. The seed points (concentrators) have to be farther from depot than each customer visited in this route (Fig. 2). Based on the empirical results, it was observed that such approach of route cost calculation ensures the best results.

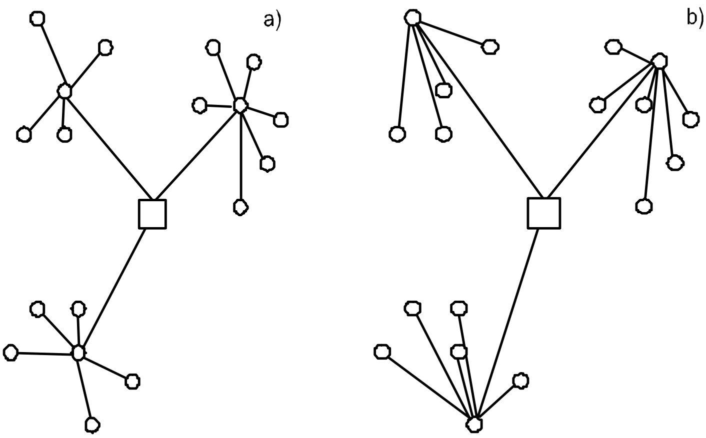

Fig. 2. The tour used to construct heuristics:
a) location based heuristics, b) proposed implementation

The length of the route consisting of one or two customers determined in this way is equal to the result using the exact method. For routes consisting of 3 or more customers, we can speak only about an approximation. It is expected that the more number of customers the worse the approximation is, however it has not been confirmed by empirical research. The selection of the main customer who is furthest away from the warehouse has been confirmed empirically as the best. Compared with the solution lengths for the test set consisting of selected VRP instances, the routes generated with this approach demonstrate that the further customer provided better solutions than nearer customer.

Another important feature of the presented approach is the ability to apply procedures that reduce the size of the problem by removing selected arcs. In this way, some constrains can be used to limit the size of the problem at the level of its generation. For instance, if the length of the route 1–5–2–1 exceeds the maximum length, the arc 2–5 is ignored. There are two approaches to reduce the size of the problem. The former is the removal of arcs which are longer than the given length. The latter is a technique according to which only the specified number of shortest arcs starting at one node are taken into account. The removal of the arcs in case of the classical models, where the given set of arcs is excluded may result in the removal of a long route even if the long arcs are omitted. In the case of the presented proposal the weight of arcs represents the increase in the length of the route, and the removal of long arcs is related to skipping long routes more the in classical approach presented above.

5. Location based formulation for inventory routing problems

The usage of a simplified method of route representation as used in the proposed inventory routing model is represented by Eqs. (15)–(25). The indices *n* and *m* represent customer location whereas *t* represents the period. The parameter w_{nm} represents the weight of the connection, determined according to procedures described in the simplified model, while the symbols d_{nt} and $q_n^{Y\max}$ denote the demand in location *n* over the period *t* and acceptable inventory levels in location *n*. In model M2, three groups of decision variables are used. Binary variable v_{nmt}^U for $n > 0$ takes the value of one, if the recipient is served during and additional period *t*; whereas $n = 0$ takes the value of one, if recipient *m* is served as the main recipient. In all other cases, the value of zero is used. Further variables are related to stock levels y_{nt}^H and v_{nmt}^C. The former denotes the level of storage in location *n* over the period *t*, while the latter represents the supply value to an additional recipient *m* in the supply to the main recipient *n* (or, if, $n = 0$ from the source location) in the period *t*. In addition, parameter a^V denotes the number of vehicles available.

Model M2

Minimize

$$\sum_{n\in N^0}\sum_{m\in N^0}\sum_{t\in T} w_{nm} v^U_{nmt} \tag{15}$$

with constraints:

$$\sum_{m\in N} v^U_{nmt} \le |N| v^U_{0nt}, \qquad \forall n\in N, \qquad t\in T \tag{16}$$

$$\sum_{m\in N} v^C_{nmt} + v^C_{0nt} \le q^V, \qquad \forall n\in N, \qquad t\in T \tag{17}$$

$$v^C_{nmt} \le \left(\sum_{t'\in T} d_{nt'} + q^Y_n\right) v^U_{nmt}, \qquad \forall n\in N^0, \qquad m\in N, \qquad t\in T \tag{18}$$

$$y^H_{m,t-1} + \sum_{n\in N^0} v^C_{nmt} - d_{mt} = y^H_{mt}, \qquad \forall m\in N, \qquad t\in T \tag{19}$$

$$\sum_{n\in N} v^U_{nmt} \le 1, \qquad \forall m\in N, \qquad t\in T \tag{20}$$

$$\sum_{m\in N} v^U_{0mt} \le a^V, \qquad \forall t\in T \tag{21}$$

$$y^H_{nt} \le q^{Y\max}_n, \qquad \forall n\in N, \qquad t\in T \tag{22}$$

$$y^H_{nt} \ge 0, \qquad \forall n\in N, \qquad t\in T \tag{23}$$

$$v^U_{nmt} \in \{0,1\}, \qquad \forall (n,m)\in A, \qquad t\in T \tag{24}$$

$$v^C_{nmt} \ge 0, \qquad \forall (n,m)\in A, \qquad t\in T \tag{25}$$

The goal function given by (15) minimizes the approximate supply cost. Constraint (16) involves the delivery to the main recipient of the supply route to the additional recipient – delivery to additional recipients is possible only after supply to the main receiver. Constraint (17) limits the permissible vehicle load on any route. Constraint (18) combines the values of variables v^U_{nmt} and v^C_{nmt}. This means that delivery to the recipient will be completed only if the recipient is visited in a given period. As was observed in this example, the total volume of supplies to a particular destination must not exceed the demand value plus the storage capacity, hence, similarly to previous models, instead of noting an *M*, the term $\sum_{t' \in T} d_{nt'} + q_n^{Y\max}$ was used. The use of such provision accelerates the solution of the problem as it increases the value of the lower limit set in the optimization process. Constraint (19) ensures consistency of stock in subsequent periods. Another constraint (20) prevents combined supplies and the constraint (21) ensures that the solution will be carried out in the appropriate number of vehicles. Acceptable level of stock in each location will not be exceeded by the use of (22). The remaining three equations, (23)–(25), are the decision variable boundary limits. In order to obtain the IRP solution after the termination of the formula should be achieved by any application of determining the TSP solution. In all solutions described later in solving this popular technique were two optimal.

6. Criteria and conditions for solving IRP

M2 formulation is a basic version of the IRP. Depending on the need, model parameters can include cost, so as to achieve the minimum cost of storage and movement throughout a planning horizon, and not just as in the above – minimum executed route length. A common modification is introduced by Archetti et al. [4] in order to prevent growth in the number of deliveries undertaken is to force supply in accordance with *order-up-to level* policies (OU). The application of this policy results in the situation whereby each target delivery location is equal to the maximum inventory level. The situation where the size of delivery is only limited from above by storage capacity is determined by the maximum level (ML). OU policy can be taken into account in the model by reducing M2, which forces the delivery in any period to equal the location's maximum level *m*

$$y_{m,t-1}^{H} + \sum_{n \in N^0} v_{nmt}^{C} - d_{mt} \geq q_m^{Y\max} \sum_{n \in N^0} v_{nmt}^{U}, \qquad \forall m \in N, \qquad t \in T \tag{26}$$

The introduction of constraints (26) ensures that at the end of the planning period, in which delivery takes place, that warehouse level will be equal to the maximum level of storage. In practice, this condition can result in the maximum warehouse level, where supply will take place in the same period but before issuing the whole demand. Thus, it is possible to use a less restrictive version of the constraint:

$$y_{m,t-1}^{H} + \sum_{n \in N^0} v_{nmt}^{C} \geq q_m^{Y\max} \sum_{n \in N^0} v_{nmt}^{U}, \qquad \forall m \in N, \qquad t \in T \tag{27}$$

By applying universal approaches such as mixed linear programming, results in the fact that M2 can be extended by additional constraints. Parameters such as the minimal interval between deliveries for a single customer, the number of deliveries in a planning horizon and the capacity level of the vehicle all considered by Coelho et al. [15] for model M1, can all be easily considered in the proposed formulation. Assuming, q_n^D denotes the required distance between supplier to customer n, and set number of deliveries to recipient n, is noted as q_n^L, then taking into account the minimal access between suppliers and the appropriate number of deliveries will be obtained by introducing the constraints (28) and (29) into the model M2

$$\sum_{m\in N^0} \sum_{t'=\min(1,t-q_n^D)}^{t} v_{mnt'}^U \le 1, \qquad \forall n \in N, \qquad t\in T \tag{28}$$

$$\sum_{m\in N^0} \sum_{t\in T} v_{mnt}^U = q_n^L, \qquad \forall n \in N \tag{29}$$

However, if it is necessary to ensure an adequate level of vehicle fill; into the model M2 the constraint (30) must be introduced.

$$\sum_{m\in N} v_{nmt}^C + v_{0nt}^C \ge q^V q^W v_{0nt}^U, \qquad \forall n \in N, \qquad t\in T \tag{30}$$

The basic constraints presented in literature concerning the IRP formulation often do not consider more than one product. Inclusion of more than one product is associated with a large increase in the set of feasible model solutions, which hinders their use in the planning process. The use of a simplified route representation allows the determination of solutions in such versions of IRP, unfortunately only for small-sized tasks. How to integrate the corresponding expansion into the M1 model is discussed in the previously cited work by Coelho et al. [15].

7. Results of test calculations

In order to determine possible applications of the described hybrid model, a set of tasks was used in the evaluation of algorithms for the IRP. For comparison, results were obtained with the help of the algorithms HAIR proposed by Archetti et al. [5] and ALNS introduced by Coelho [15]. The primary objective of the conducted tests, the results of which are later presented in this section, it was possible to determine M2, in particular the size and structure of tasks to be solved and an estimation of time required to perform the optimization process. In addition to using M2, the sensitivity of the algorithm to additional restrictions were examined, such as the number of deliveries in the planning horizon, the level of vehicle fill and the spacing between deliveries. The set of test instances[2] consisted of:

- Group of 50 instances of long planning horizon through 3 periods, 10 sizes (5, 10, 15, 20, 25, 30, 35, 40, 45, 50 recipients) and the cost of storage range $[0{,}01;\, 0{,}05]$, denoted hereinafter as L3.
- Group of 50 instances of long planning horizon through 3 periods, 10 sizes (5, 10, 15, 20, 25, 30, 35, 40, 45, 50 recipients) and the cost of storage range $[0{,}1;\, 0{,}5]$, denoted hereinafter as H3.
- Group of 30 instances of long planning horizon through 6 periods, 6 sizes (5, 10, 15, 20, 25, 30 recipients) and cost of storage range $[0{,}01;\, 0{,}05]$, denoted hereinafter as L6.
- Group of 30 instances of long planning horizon through 6 periods, 6 sizes (5, 10, 15, 20, 25, 30 recipients) and the cost of storage range $[0{,}1;\, 0{,}5]$, denoted hereinafter as H6.

In the first phase of experiments, the sets L3, H3, L6 and H6 were used, and with each of these two delivery policies OU and ML. The test tasks are inclusive of storage costs (both at the recipient and the supplier throughout the planning horizon, increased by a period of 0 – initial state), because the goal of function M2 was extended by the appropriate component representing the storage costs.

Table 1. Test task solutions for $|K| = 1$

Policy used	Test group	HAIR			MH	
		GAP [%]	CPU [s]	BPS [%]	GAP [%]	CPU [s]
OU	L3	0.07	3600	14.52	40.9	74
	H3	0.05	3600	7.81	19.3	69
	L6	0.14	3600	9.5	6.83	1051
	H6	0.07	3600	5.01	3.42	1015
ML	L3	0.07	3600	14.52	30.3	22
	H3	0.05	3600	7.81	27.6	21
	L6	0.14	3600	9.5	8.12	660
	H6	0.07	3600	5.01	3.87	613

Source: Own study based upon [4, 5] and results of calculations.

[2]Test sets are derived from the papers of work Archetti et al. [4, 5].

Table 1 shows the summarized results in terms of average quality, measured as the relative distance between the objective function value of the optimal solution or the best known solution (GAP), and the average solution time in seconds (CPU). In addition to the algorithm HAIR, which is an iterative algorithm to improve solutions, given the quality of solutions obtained after the first three minutes of the optimization process (BPS).

The proposed solution allows quick solution generation (Table 1). Unfortunately, this solution is worse than the results obtained using other methods compared. However, the main development goal was to allow the extension of a new formulation with additional components, in particular elements often associated with the production process and extension of the planning horizon. It is worth mentioning that the increase in the number of planning periods is reflected in an improvement in the quality generated solutions with the help of the proposed solutions. Most likely, this is due to the fact that during short term planning slight differences with respect to the optimum solution causes a large increase in the cost of the result solution. The presented formulation will be used with planning horizons of length 5, 12, 20 or 52, respectively, the number of working days per week, the number of months in a year, the number of days in the month and the number of weeks a year.

In the following section, the results obtained with the new formulation were compared after allowing the use of more than one vehicle. For comparison, see the results presented by Coelho et al. [15]. The results are shown in Table 2.

Table 2. Test task solutions for $|K| = 2$ and $|K| = 3$ for the policy OU

Horizon length	Test group	Number of routes per period	MH GAP [%]
6	H6	2	1.8
		3	0.5
	L6	2	2.8
		3	1.3

Source: Own work based upon *The vehicle routing problem*, [in:] P. Toth, D. Vigo (Eds.), *SIAM monographs on discrete mathematics and applications*, Philadelphia 2002, and calculation results.

The results shown in Table 2 confirm that the proposed formulation allows fast attainment of an acceptable solution. The solutions obtained using M2 are worse than comparable solutions on average of 2.3% and 0.9% for the problems with 2 and 3 routes. It is important to consider the following comparison, to determine the comparative solution matrix of computers were used, while route calculation via the M2 model is implemented on a single computer. The results confirm that the longer planning horizon the quality of solutions generated by M2 is higher.

The results in Table 3 show that the most time demanding condition to include additionally, relates to the spacing between deliveries. Least difficulty is caused by the extension of the delivery number limit. The observed relationship is true for each of the sets of test tasks. The relatively small increase in the length of resolution for reducing the number of deliveries may be due to the fact that in order to guarantee the existence of a solution, constraints introduced into the test tasks did not significantly affect the solution and hence the change in the optimization process.

Table 3. Increase in time required to solve M2 with the introduction of additional resources [%]

Test group	Spacing between supply	Vehicle fill level	Number of deliveries
H3	34.54	25.32	6.23
L3	42.84	34.27	4.56
H6	130.54	67.38	8.23
L6	183.54	120.22	12.65

Source: Own study based upon the results of calculations.

8. Conclusions

The undertaken computational experiment allowed us to confirm the effectiveness of the hybrid approach. In the presented example, all tasks solved were test tasks. The proposed solution is not the exact solution, during the optimization procedure a simplified representation of vehicle route length is used, making impossible to give an estimate concerning the quality of generating solutions. Such an estimate was obtained by comparing the results generated with a set of test tasks. However, the primary goal is to develop a hybrid approach which is necessary to solve this task by using more advanced methods than simple heuristics.

The work is the result of a number of studies on methods of planning new concepts based on the idea of collaboration in the supply chain. With regard to the concept of distribution management based on cooperation between links of the logistics network, the results presented confirm the high potential but also, and unusual for classical implementation, difficulties in procurement planning. Further work undertaken will be comparative studies using sets of test tasks. In addition, it is planned to implement approaches using edge removal technologies aimed at reducing the size of the model and accelerate the solving process.

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Received 17 September 2013
Accepted 14 January 2014