Developments in Mechanical Engineering

ISSN: 2720-0639 DOI: 10.37660/dme.2022.18.10.11 **18(10)2022**

Mathematical models of the influence of cutting speed on Ra parameter

Dariusz Jędrzejczyk[1], Jarosław Robert Mikołajczyk[2]

[1] Akademia Nauk Stosowanych im. Stanisława Staszica w Pile; ORCID No.: 0000-0001-8055-9311; e-mail: darek222687@gmail.com

[2] Akademia Nauk Stosowanych im. Stanisława Staszica w Pile; ORCID No.: 0000-0001-9196-0039; e-mail: jmikolajczyk@ans.pila.pl

Summary: The paper presents the influence of the cutting speed (by changing the spindle rotational speed) on the change of Ra parameter value. For this relationship, mathematical models were built in R software [1÷4, 6÷17] and their usefulness was determined.

Key words: roughness of surface, software R

1. Introduction

Machining, despite the fact that new methods of producing machine elements are constantly being developed, still remains an important method of producing machine parts, especially metal parts. The quality of the obtained surface layer of the manufactured elements depends on the types of machining used and their parameters.

In the Polish standard PN-87/M-04250 the surface layer is defined as: "… a layer of material bounded by the actual surface of the object, including this surface and the part of the material deeper from the actual surface that shows altered physical and sometimes chemical characteristics in relation to the characteristics of this material in the depth of the object."

The surface layer of machine elements, which is formed during the manufacturing process, is called a technological surface layer. Its features, i.e. properties, qualities and structure, depend to the greatest extent on the parameters and types of the applied after-machining and type of construction material from which the co-acting elements were made.

The technological features obtained in the production process are not constant. Under the influence of operational external forces, including environmental influences, loads or, for example, temperature, they undergo changes. These changes may also occur spontaneously, i.e. without the participation of external factors. The surface layer being at this stage of the machine's existence is called the working surface layer.

The properties and qualities of the surface layer are determined to the greatest extent by the parameters describing the stereometric structure of the surface, more commonly referred to as the surface texture. The group of parameters describing the surface texture includes the following parameters:

- roughness;
- lay direction;
- defects of surface texture;
- wavy finish.

As defined in the Polish standard PN-EN ISO 4287: 1999, the surface roughness is defined as a set of actual surface irregularities, conventionally defined as deviations of the profile measured from the reference line within the segment where waviness and shape deviations are not taken into account. The roughness parameters that define the surface texture are classified into the following groups:

- vertical parameters (amplitude);
- horizontal parameters (distance, horizontal);
- hybrid parameters (mixed);
- characteristic curves.

The vertical roughness parameters include, among others, Ra parameter, i.e. the arithmetic mean of the profile ordinates. In this paper, this parameter was analyzed depending on the speed of cutting with a right turning tool during longitudinal turning.

In the right turning tool (Fig. 1) the main cutting edge is on the right when viewed from the top. The names and symbols of general purpose gripper turning tools are included, among others, in the PN-75 / M-58352 standard.

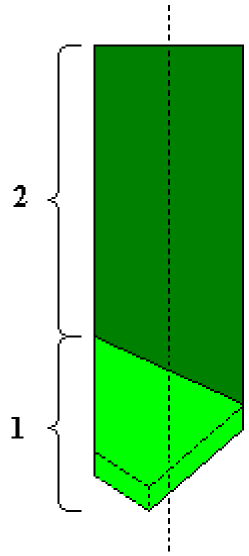

Fig. 1. Straight turning tool, right: 1 – working part; 2 – handle

2. Test conditions

In these tests, a steel shaft of S235JR grade was used as the material subjected to machining [Table 1] with a diameter of Ø30 mm and length of 100 mm. The machining was performed on a conventional TOS S32 lathe, Czech production, using a right gripper turning tool NNB 20x20 S10W (Fig. 1). Ra parameter was measured with the Mahr MarSurf PS10 profilographometer, five times for each sample.

The constant parameters of the cutting process were:

- type of coolant;
- coolant flow rate;
- cutting depth a = 0.8 mm;
- feed p = 0.14 [mm/rev]

Rotational speed was a variable parameter of the cutting process. In this paper, the following rotational speeds were used: 250; 400; 800 and 1250 [rpm]. Ten (10) samples were made for each rotational speed.

The spindle speed n_1 = 250 rpm corresponds to the cutting speed v_1 = 23.56 [m/min].

The spindle speed n_2 = 400 rpm corresponds to the cutting speed v_2 = 37.69 [m/min].

The spindle speed n_3 = 800 rpm corresponds to the cutting speed v_3 = 75.39 [m/min].

The spindle speed n_4 = 1250 rpm corresponds to the cutting speed v_4 = 117.81 [m/min].

Table 1. Chemical composition of steel S235JR [www.thesteelprice.com]

Element	Content [%]
P	≤ 0.035
S	≤ 0.035
Mn	≤ 1.40
N	≤ 0.012
C	0.13
Si	0.18
Cu	≤ 0.55
Cr	0.07
Mo	0.024

where:

S – sulfur;
Mn – manganese;
C – carbon;
N – nitrogen;
Cr – chrome;
P – phosphorus;
Cu – copper;
Mo – molybdenum;
Si – silicone.

3. Test results

For the analysis of the measurement results of the roughness parameter Ra taken were data obtained on the basis of the performed tests. The obtained values of Ra parameter were implemented into R software, for further analysis (Table 2).

Table 2. Measurement results of Ra [μm] parameter for the spindle rotational speed n_1 = 250 rpm (own study)

Sample No.	Meas. 1	Meas. 2	Meas. 3	Meas. 4	Meas. 5
1	8.125	8.937	7.466	8.129	8.164
2	8.018	8.494	8.330	8.179	8.255
3	8.286	8.133	8.882	8.628	8.482
4	7.382	8.919	6.299	7.019	7.405
5	8.764	7.024	8.006	7.037	7.708
6	8.189	7.811	8.817	7.671	8.122
7	8.732	8.592	7.494	7.809	8.157
8	6.367	9.063	8.809	9.165	8.351
9	9.072	7.254	8.494	7.391	8.053
10	9.266	8.384	7.672	7.395	8.179

Record of the data implemented in R software is presented below.

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10
1	8.125	8.018	8.286	7.382	8.764	8.189	8.732	6.367	9.072	9.266
2	8.937	8.494	8.133	8.919	7.024	7.811	8.592	9.063	7.254	8.384
3	7.466	8.330	8.882	6.299	8.006	8.817	7.494	8.809	8.494	7.672
4	8.129	8.179	8.628	7.019	7.037	7.671	7.809	9.165	7.391	7.395
5	8.164	8.255	8.482	7.405	7.708	8.122	8.157	8.351	8.053	8.179

where:

V1 – measurement of the Ra parameter for sample No. 1;
V2 – measurement of the Ra parameter for sample No. 2;
V3 – measurement of the Ra parameter for sample No. 3;
V4 – measurement of the Ra parameter for sample No. 4;
V5 – measurement of the Ra parameter for sample No. 5;
V6 – measurement of the Ra parameter for sample No. 6;
V7 – measurement of the Ra parameter for sample No. 7;
V8 – measurement of the Ra parameter for sample No. 8;
V9 – measurement of the Ra parameter for sample No. 9;
V10 – measurement of the Ra parameter for sample No. 10

For the surface roughness results of Ra parameter for the rotational speed n_1 = 250 rpm, the correlation values were calculated using the Pearson's method and Spearman's method for individual samples (Table 3). Although individual samples were made in the same way, with the same technical parameters, there is a certain set of samples for which there is practically no correlation. This is the case, for example, for samples 2 and 6, also for 7-9, 2-3. This may come as a surprise as the correlations should be strong. For samples (samples) that have extreme values that differ from other samples, there is a suspicion for example that the surface was scratched during improper storage or transport. This can significantly affect the value of correlation.

The concept of correlation concerns the relationship between tested variables, with correlation dealing with the strength of this relationship. The correlation value is between -1 and +1. If there is a strict dependence, a functional dependence even, between the variables X and Y, then the correlation is -1 or +1 (extreme cases). For example, when the correlation is zero, then the tested variables are not correlated. The closer the correlation value is to +1 or -1, the stronger the correlation.

Table 3. The correlations calculated in R software between the roughness results of Ra parameter for individual samples (own study)

Samples	Pearson correlation	Spearman correlation
1-2	0.3918665	0.4
1-3	-0.8966406	-0.7
1-4	0.980129	0.9
1-5	-0.4966322	-0.8
1-6	-0.7696881	-0.7
1-7	0.7276639	0.4
1-8	0.1162835	0.3

Table 3 cont.

1-9	-0.5944154	-0.8
1-10	0.3295114	0.2
2-3	-0.08749211	-0.1
2-4	0.4640231	0.3
2-5	-0.6617565	-0.6
2-6	-0.01214795	0
2-7	-0.1514389	-0.3
2-8	0.752785	0.4
2-9	-0.6504981	-0.6
2-10	-0.3659762	-0.1
3-4	-0.9140491	-0.9
3-5	0.06222598	0.3
3-6	0.5929967	0.3
3-7	-0.9556219	-0.9
3-8	0.3283553	0.2
3-9	0.1788884	0.3
3-10	-0.7114709	-0.8
4-5	-0.4053582	-0.5
4-6	-0.6279986	-0.4
4-7	0.7602898	0.7
4-8	0.06861448	-0.1
4-9	-0.5036692	-0.5
4-10	0.4145363	0.6
5-6	0.6135462	0.8
5-7	0.2282299	0.1
5-8	-0.8908094	-0.8
5-9	0.9929628	1
5-10	0.6557021	0.3
6-7	-0.4342381	-0.1
6-8	-0.1970355	-0.7
6-9	0.6834789	0.8
6-10	0.008012282	0.3
7-8	-0.5916796	-0.5
7-9	0.1116369	0.1
7-10	0.8793803	0.9
8-9	-0.8355276	-0.8
8-10	-0.8632772	-0.7
9-10	0.5633632	0.3

Figure 2 shows the roughness values of Ra parameter for sample No.1. For the obtained measurements of the roughness value of Ra parameter for the rotational speed n_1 = 250 rpm, a box plot was generated in R (Fig. 3). There are two outliers visible. These are measurements No. 2 and No. 3.

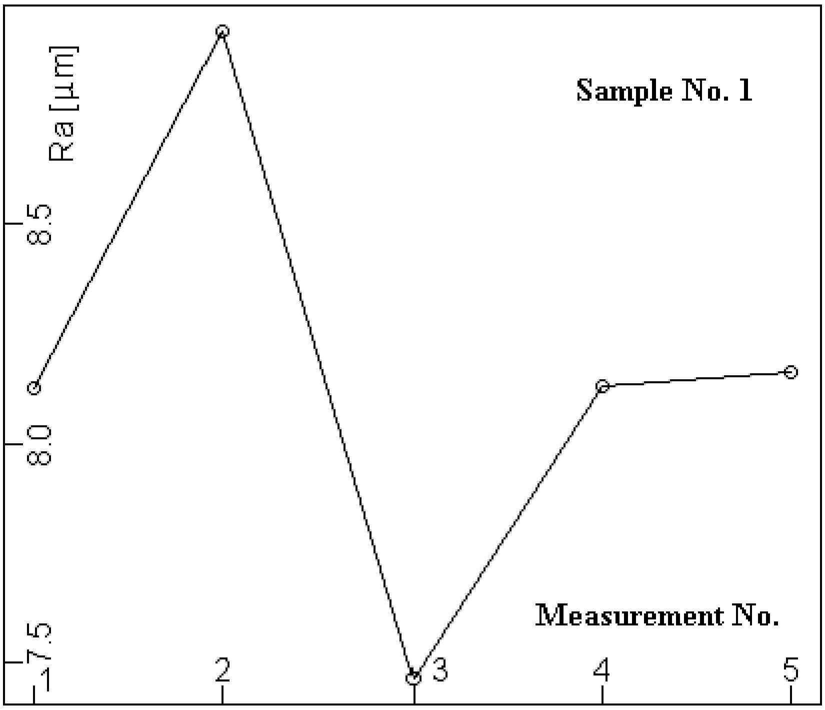

Fig. 2. Graph of the measurement values of Ra [µm] parameter for the sample No. 1 performed at the spindle speed n_1 = 250 rpm (own study)

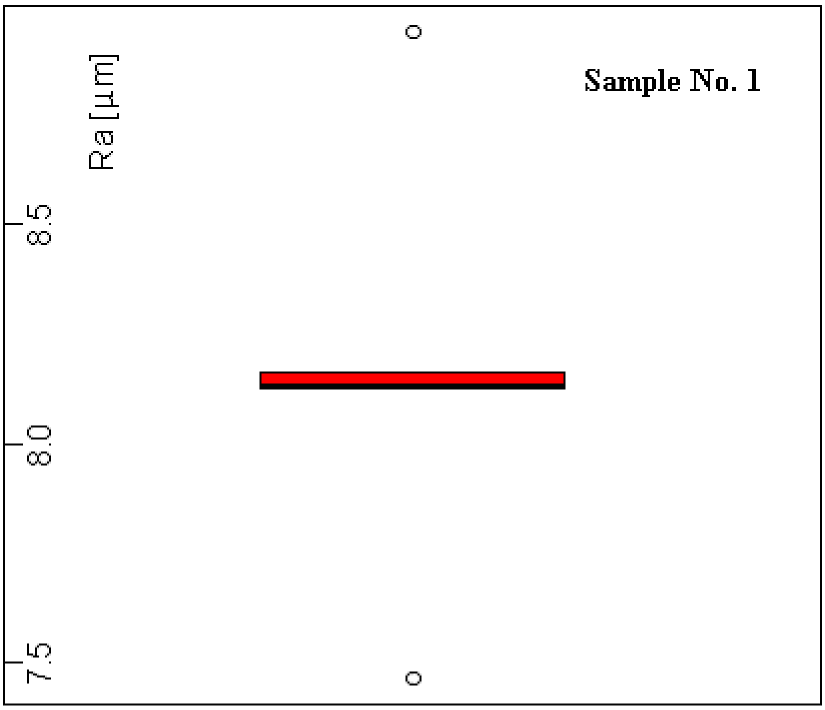

Fig. 3. Box plot of Ra [µm] parameter measurements for the sample No. 1 performed at the spindle speed n_1 = 250 rpm (own study)

The box plot looks like a box with whiskers, hence its different name: a box and whisker plot. Almost all observations are located between the whiskers (except for outliers but in our case there are none). Outliers are those that deviate from the quartiles more than 1.5 IQR (1.5 is the default value of the 'range' argument).

The value of the measured range R is significant (R = 1.471 μm). Perhaps it is the scratching of surface due to improper storage.

Table 4. Selected statistical parameters calculated in R software for the measurement of Ra parameter for the sample No. 1 and for the spindle rotational speed n1 = 250 rpm (own study)

Min	1stQu.	Median	3rdQu.	Max	IQR	R	s	d1	Mean
7.466	8.125	8.129	8.164	8.937	0.039	1.471	0.5214113	0.30912	8.164

where:

Min – minimum value;
1stQu. – lower (first) sample quartile (Q_1);
Median – median ('medial value' Q_2);
3rdQu. – upper (third) sample quartile (Q_3);
Max – maximum value;
IQR – interquartile range;
R – sample range;
s – standard deviation;
d1 – average deviation from the mean value;
Mean – arithmetic mean.

A similar procedure can be adopted for a detailed analysis of Ra parameter results for subsequent samples. However, because of enormous amount of data (four speeds, ten samples, five measurements for each sample, which makes a total of 200 measurements) which excessively increases the volume of this paper, we have focused only on determining the effect of cutting speed (by changing the spindle rotational speed) on the change of roughness parameter Ra without presenting analysis for individual samples. Such an approach to the subject makes it much easier to draw conclusions. The above mentioned data was implemented into R and is presented below.

	V1	V2	V3	V4
1	8.125	5.942	3.972	1.558
2	8.018	7.414	3.807	1.269
3	8.286	6.253	4.100	1.724
4	7.382	6.884	3.209	1.187
5	8.764	6.623	3.772	1.435
6	8.189	4.699	2.367	1.269
7	8.732	6.484	2.492	2.444
8	6.367	7.092	2.549	1.449
9	9.072	7.836	3.217	1.953
10	9.266	6.528	2.656	1.779
11	8.937	6.993	4.260	1.026

12	8.494	6.452	3.812	1.001
13	8.133	6.708	3.991	1.010
14	8.919	6.195	3.934	0.967
15	7.024	6.587	3.999	1.001
16	7.811	6.238	3.468	1.718
17	8.592	5.704	3.089	1.914
18	9.063	6.612	3.178	1.938
19	7.254	7.511	3.111	1.182
20	8.384	6.516	3.212	1.688
21	7.466	5.689	3.438	1.557
22	8.330	6.003	3.448	1.649
23	8.882	7.388	2.881	1.865
24	6.299	6.134	3.016	1.430
25	8.006	6.304	3.196	1.625
26	8.817	4.570	3.193	1.537
27	7.494	5.217	3.089	1.439
28	8.809	4.363	2.956	1.486
29	8.494	7.088	3.376	1.452
30	7.672	5.310	3.154	1.479
31	8.129	6.810	3.269	1.571
32	8.179	5.439	3.533	1.433
33	8.628	4.910	3.124	1.505
34	7.019	5.431	3.507	1.622
35	7.037	5.648	3.358	1.533
36	7.671	4.525	3.698	1.558
37	7.809	6.947	3.489	1.577
38	9.165	5.751	3.184	1.692
39	7.391	5.422	3.138	1.614
40	7.395	5.661	3.377	1.610
41	8.164	6.471	3.071	1.909
42	8.255	4.918	2.999	1.848
43	8.482	5.442	3.226	1.916
44	7.405	6.567	3.300	1.903
45	7.708	5.850	3.149	1.894
46	8.122	6.914	3.671	1.449
47	8.157	5.675	3.061	1.513
48	8.351	5.877	3.213	1.596
49	8.053	6.505	3.320	1.674
50	8.179	6.243	3.316	1.558

where:

V1 – value of the roughness parameter Ra [μm] for the speed n_1 = 250 rpm;
V2 – value of the roughness parameter Ra [μm] for the speed n_2 = 400 rpm;
V3 – value of the roughness parameter Ra [μm] for the speed n_3 = 800 rpm;
V4 – value of the roughness parameter Ra [μm] for the speed n_4 = 1250 rpm

Figure 4 presents the distribution of the numeric variable(s) (in this case the value of Ra parameter) using boxes for the tested speeds in order to compare the distribution of these variables. The following conclusions can be drawn from this figure:

- with an increase in the spindle rotational speed (and thus increase of the cutting speed) the value of the roughness parameter Ra decreases circa 4 times (from the value of approx. 8 μm to the value approx. 2 μm);
- for the spindle rotational speed n_3 = 800 rpm, three outliers were noted – it is possible that these are either measurement errors or scratches due to improper handling of samples;
- for the spindle speed n_4 = 1250 rpm, also three outliers were noted, the reason for their existence may be similar to the above mentioned ones;
- the distance between the whiskers of the box plot for the spindle speed n_4 = 1250 rpm is the smallest, i.e. the scatter of results for this case is minimal, which is beneficial from the technology point of view;
- the distance between whiskers of the box plot for the spindle speed n_1 = 250 rpm and for n_2 = 400 rpm is the largest, i.e. the scatter of results for these cases is maximum, which is very unfavorable from the point of view of technology, e.g. fits;
- the test range for the spindle speed n_4 is the smallest and amounts to 1.477 μm;
- the test range for the spindle speed n_2 is the greatest and amounts to 3.473 μm;

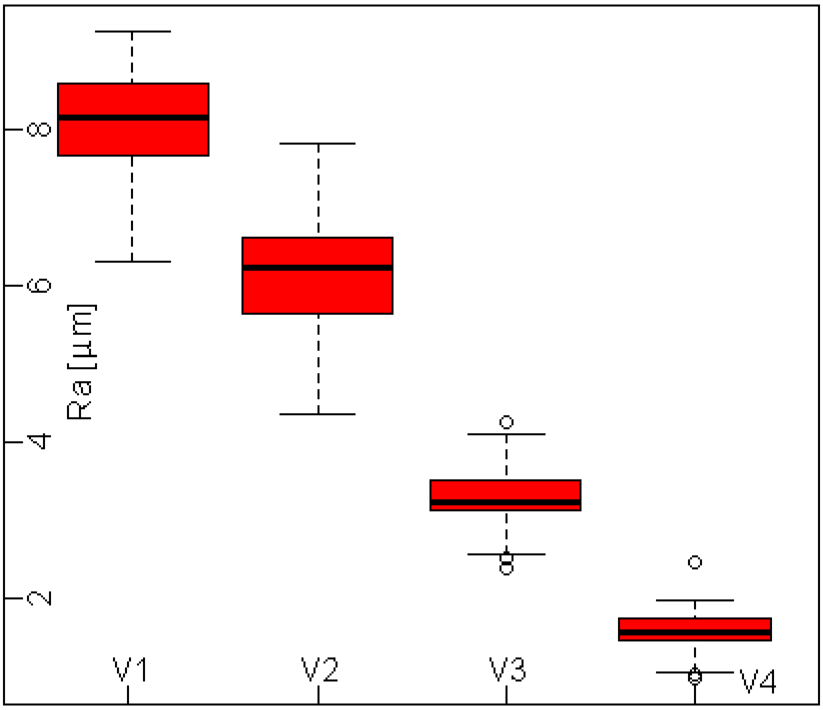

Fig. 4. Box plot of Ra [μm] parameter measurements: V1 – for speed n_1 = 250 rpm; V2 – for speed n_2 = 400 rpm; V3 – for speed n_3 = 800 rpm; V4 – for speed n_4 = 1250 (own study)

Table 5. Calculated selected statistical parameters for Ra [μm] parameter (own study)

Speed	Min	1stQu.	Median	3rdQu.	Max	IQR	R	s	d_1	Mean
v_1	6.299	7.671	8.161	8.568	9.266	0.89625	2.967	0.6903802	0.53884	9.266
v_2	4.363	5.651	6.240	6.620	7.836	0.969	3.473	0.8204341	0.6697968	6.127
v_3	2.367	3.114	3.221	3.502	4.260	0.38825	1.893	0.4043026	0.305324	3.319
v_4	0.967	1.442	1.558	1.712	2.444	0.27	1.477	0.2914197	0.2112048	1.560

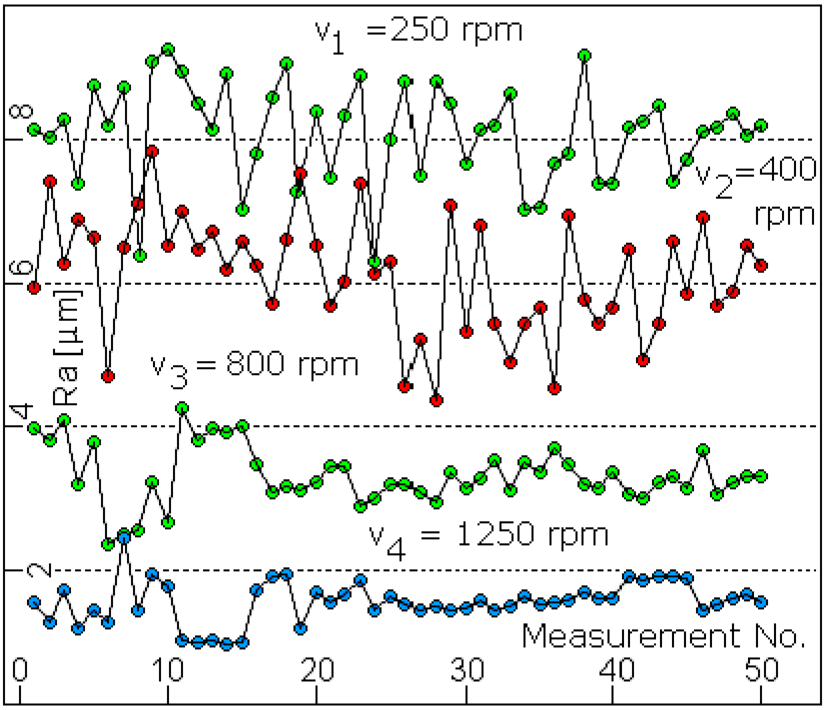

Fig. 5. Surface roughness values of Ra [μm] parameter for individual spindle rotational speeds: V1 – for speed n_1 = 250 rpm; V2 – for speed n_2 = 400 rpm; V3 – for speed n_3 = 800 rpm; V4 – for speed n_4 = 1250 rpm (own study)

If we determine average values of the roughness parameter Ra for these data, their position will be analogous to the position of boxes in Fig. 4. If then for these few average values (four to be exact, as there were as many variable speeds) we define a model of the dependence of the roughness value Ra on the spindle speed, then a change in the value of this roughness parameter can be predicted. For average roughness values of Ra parameter, a first-order linear model was determined in R software in the following form:

$$\mathbf{y = -\,0{,}007155 \cdot x + 9{,}897863}$$

For the above mentioned model, the value of Ra parameter was calculated for individual spindle speeds (Table 6). For the developed linear model, Figure 6 presents the change in the roughness parameter Ra depending on the spindle rotational speed. According to this model, for the speed n = 1383.33 rpm, Ra parameter reaches the value 0.00013685 μm. Diagnostic charts for this linear model are shown in Fig. 7.

Table 6. Values of Ra [µm] parameter for the linear model No. 1 (own study)

Spindle rotational [rpm]	**Ra [µm] calculated average**	**Ra [µm] acc. to model No. 1**
250	9,266	8,109113
400	6,127	7.035863
800	3,319	4,173863
1250	1,560	0,954113

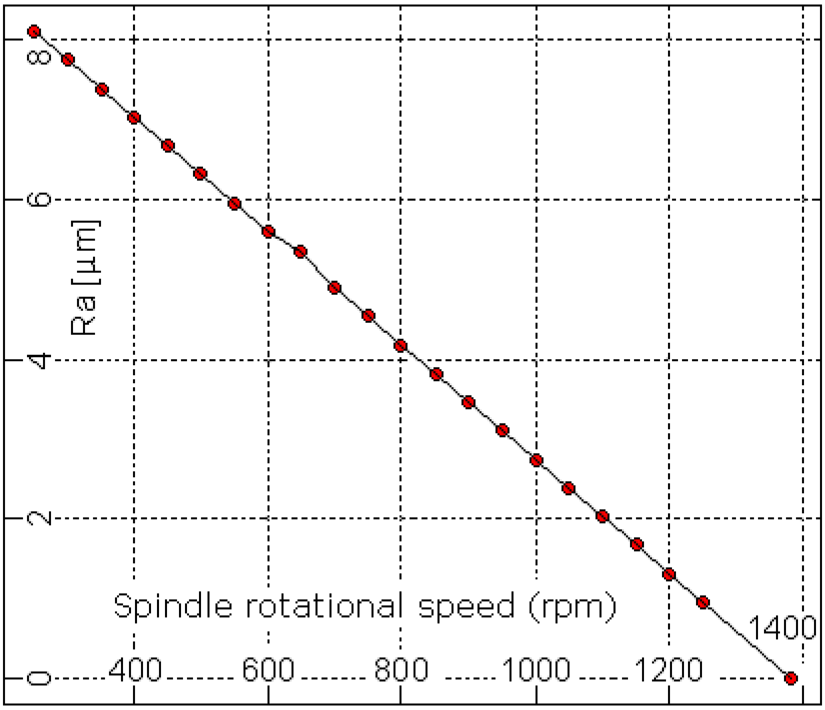

Fig. 6. Dependence of the surface roughness value of Ra [µm] parameter on the spindle rotational speed (rpm) for the linear model No. 1 (own study)

In the graph headed "Residuals vs Fitted", y_i values, fitted by the model, are represented on the axis of abscissae and ε_i residual values are shown on the axis of ordinates. For an adequate model, the residuals should not functionally depend on the dependent variable; they should have a conditional mean equal to zero regardless of the value of y_i. On this diagnostic graph we can assess whether the mean value of residuals depends on y_i (this is bad) or not (this is good).

In the graph with the heading "Normal Q-Q" (quantile graph for the normal distribution) on the axis of abscissae presented are values of the quantiles of normal distribution corresponding to the residuals, and the empirical (experimental) quantiles for the standardized residuals are presented on the vertical axis. For an adequate model, the residuals have a normal distribution, so the points on the graph should be arranged along a straight line (marked with a dashed line). Deviations from this line indicate an abnormality.

In the graph headed "Scale Location", the axis of abscissae shows the values fitted by the y_i model, and the axis of ordinates shows the roots of standardized residuals. For an adequate model, the variance of residuals should be homogeneous and, in particular, it should not be functionally dependent on the values fitted by the model. The presence of any trend line suggests a deviation from the assumption of homogeneous variance. Heterogeneous variance can be reduced by using an appropriate data transformation which stabilizes the variance.

The graph headed "Residuals vs Leverage" allows to identify atypical values (outliers). The standardized residuals are presented on the axis of ordinates, and the so-called h_i leverages (measures of the influence of this observation on the assessment of model coefficients, called leverage) are shown on the axis of abscissae. The leverage determines the influence of the observation of y_i on the valuation of the explained variable. In an adequate model, a single observation should not have a significantly greater impact on the values of coefficient ratings than other observations. This graph is based on the rule of thumb.

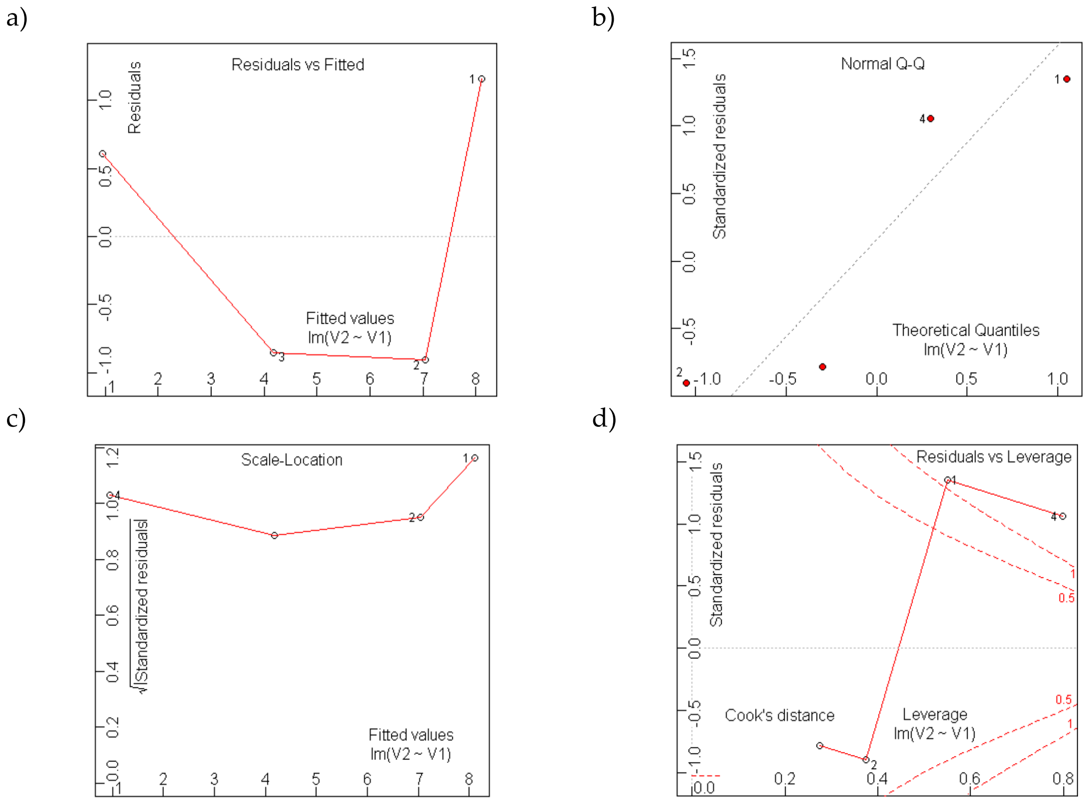

Fig. 7. Diagnostic charts for the linear model No. 1 (own study)

Then, the second degree regression model was determined for the dependence of Ra parameter on the spindle rotational speed - model No. 2.
Model No. is defined by the following relationship:

$$\mathbf{y = 1.591 \cdot x^2 - 5.554 \cdot x + 5.068}$$

The diagnostic charts for the above mentioned model are presented in Fig. 8. From these two models, model No. 1 is better suited to actual values (as observed when looking at Ra parameter values obtained for individual spindle speeds).

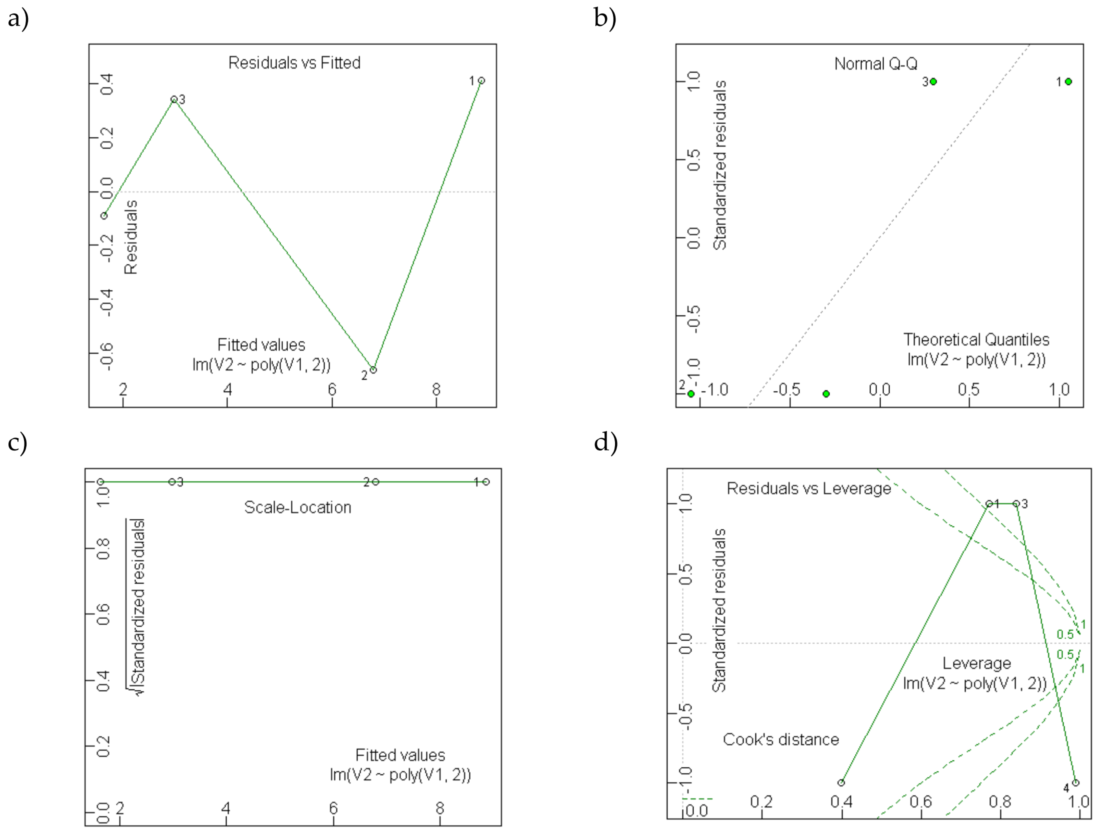

Fig. 8. Diagnostic charts for the linear model No. 2 (own study)

3. Summary

Based on the conducted tests, one may come to the following conclusions:

1. Increasing the rotational speed of the spindle with the mounted workpiece significantly reduces the value of roughness parameter Ra, with the remaining machining parameters staying constant, down to the value of ca. 1 μm (accuracy class 6÷7). This corresponds to very precise turning. The very precise turning is used most often when final machining is performed, however both high dimensional accuracy of machining and low roughness of the machined surface are required.
2. The course of these changes is relatively accurately reflected in the determined linear model of Ra parameter change, taking into account the dependence on the spindle rotational speed.
3. According to the specific linear model, for the speed n = 1383.33 rpm, the value of Ra parameter would reach only 0.00013685 μm. It is difficult to assess, without experimentally verifying it, what value this parameter would actually have for the above mentioned rotational speed. In the technical documentation of the TOS S32 lathe it can be noted that the maximum rotational speed of spindle is 3200 rpm, so it would be possible to check it for this machine. Of course, this model allows (at least theoretically) to determine the value of the spindle rotational speed for which Ra would be exactly zero.

References

[1] Arnold, S.F., "The theory of linear models and multivariate analysis", Wiley, New York 1981.

[2] Baldin, D.J., "A tutorial on statistical methods for population association studies", https://cge.mdanderson.org/~instructor/Public/Balding.pdf, 2006.

[3] Biecek, P., „Analiza danych z programem R. Modele liniowe z efektami stałymi, losowymi i mieszanymi", Wydawnictwo Naukowe PWN, Warszawa 2013.

[4] Gągolewski, M., „Programowanie w języku *R*. Analiza danych, obliczenia, symulacje", Wydawnictwo Naukowe PWN, Warszawa 2016.

[5] Górski, E., „Obróbka skrawaniem", Wydawnictwa Szkolne i Pedagogiczne, Warszawa 1972.

[6] Krzyśko, M., „Wielowymiarowa analiza statystyczna", Wydawnictwo Naukowe UAM, Poznań 2000.

[7] Scheffe, H., "The analysis of variance", Wiley, New York 1959.

[8] Searle, S.R., Casella, G., McCulloch, C.E., "Variance component"s, Wiley, New York 1992.

[9] Seber, G.A.F., Lee, A.J., "Linear regression analysis", Wiley, Hoboken, New Jersey 2003.

[10] Sheather, S.J., "A Modern approach to regression with *R*", Springer, New York 2010.

[11] Sneynin, O.B., "C.F. Gauss and the Theory of Errors", *History of Exact Sciences* Vol. 20, No. 1, 1981, s. 21-72. http://www.springerlink.com/index/U07K6T656NVL068.pdf.

[12] Stigler, S.M., "Gauss and the invention of least squares", *The Annals of Statistics,* Vol. 9, No. 3, 1981, s. 465-474. http://www.jstor.org/stable/2240811.

[13] Tibshirani, R., "Regression shrinkage and selection via the lasso", *Journal of the Royal Statistical Society*, Series B, Vol. 58, No. 1, 1996, s. 267-288.

[14] Wheeler, B., "Permutation tests for linear models in R", http://cran.r-project.org/web/packages/lmPerm.pdf, 2010.

[15] Wolffinger, R., Tobias, R., Sall, J., "Computing Gaussian Likelihoods and their Derivatives for General Linear Mixed Models", *SIAM Journal on Scientific Computing*, Vol. 15, No. 6, 1994, s. 1294-1310.

[16] Zeileis, A., Hothorn, T., "Diagnostic checking in regression relationships", http://cran.r-project.org/web/packages/lmtest/vignettes/lmtest-intro.pdf, 2010.

[17] Zou, H., Hastie, T., "Regularization and variable selection via the elastic net". *Journal of the Royal Statistical Society*, Series B. 67, 2005, s. 301-320.