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THE USE OF TWO PROJECTIVE PARTLY COMPOSED REPRESENTATION FOR CONSTRUCTING CONICAL PANORAMAS

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Abstract. The paper refers to the author's earlier considerations dealing with the construction of cylindrical panoramas. It presents the geometrical bases of the direct mapping of the panorama on the unreeled rotary conical backgrounds by means of two projective partly composed representation.

Keywords: panorama image, conical perspective, engineering graphics

1. Introduction

The analysis of the available literature discussing theoretical principles of the construction of panoramic images, permits to state that the examples of panorama constructions presented there are indirect since they require support mappings of imagined objects carried out by Monge method. However, a way of the construction of conical panoramas presented in the paper, contrary to the known methods, is direct and realized by means of the so-called two projective partly composite representation. In general two projective partly composite representation is composed of two projectings : the *main* one and the *supplemental* one. The given figure is represented in the main projecting as well as the image of it received in the supplemental projecting [1],[2].

2. Basic information

Two projective partly composite representation, which is used to the mapping of panoramas is the representation where the main projecting is a panoramic projecting onto $\hat{\tau}$ background, being a rotary conical surface or the segment of that surface. The S center of that projecting is a proper point of the l axis. However the supplemental projecting which enables restitution is such as in a vertical perspective the rectangular projecting onto the π plane perpendicular to the l axis of the $\hat{\tau}$ background (in the case of vertical panorama) or the normal hyper-bundle projecting onto the $\hat{\tau}$ background (in the case of horizontal panorama). Defining the apparatus of the main projecting in this way, a projection of any proper $X \neq S$ point is a X^S point which is a common element of the $\hat{\tau}$ background and a $\overrightarrow{SX}$ half-line beginning with the S point and passing the X one (Fig.1a). However, the image of the $Y_\infty \notin l$ point in infinity in general is a sum of two X_∞^S and $\overline{X}_\infty^S$ points which the straight line punctures the $\hat{\tau}$ background in (Fig.1b).

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a) b)

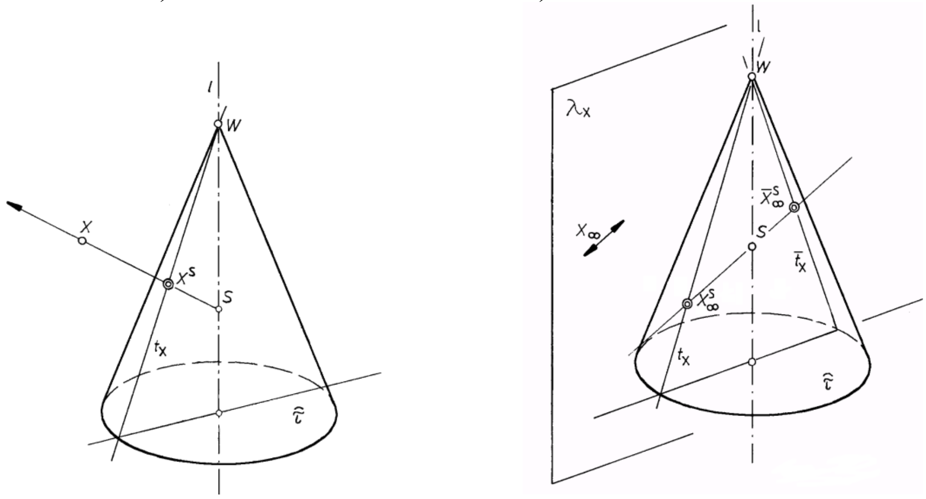

Fig.1: The image of the point in panoramic conical projecting: a) $X \neq S$, b) $X_\infty \notin l$

It is worth noticing that two *A* and *B* variants of the apparatuses of the panoramic conical projecting can be conspicuous depending on the way of the situating the top of the surface towards the χ plane: lower down or higher up (Fig. 2a,b).

a) b)

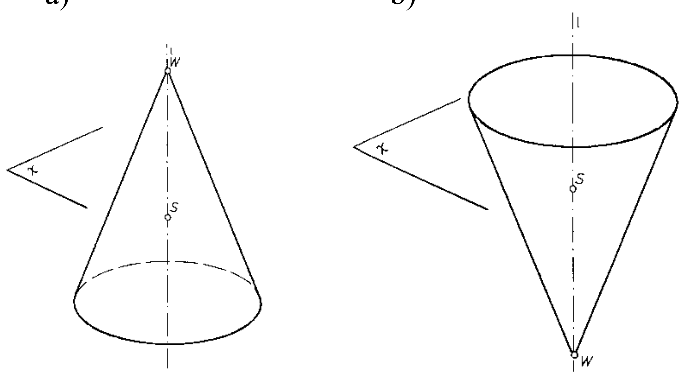

Fig.2: The possible variants of the apparatuses of panoramic conical projectings : a) A variant, b) B variant

Taking that above into consideration four variants of the conical panorama are distinguished:

- the *vertical conical panorama* $\mathbf{Z}_1$ of A type
- the *horizontal conical panorama* $\mathbf{Z}_2$ of A type
- the *vertical conical panorama* $\mathbf{Z}_3$ of B type
- the *horizontal conical panorama* $\mathbf{Z}_4$ of B type.

This diversity is caused by different possibilities of situating the conical surface towards χ plane as well as both kind of the supplemental projecting .

3.The construction of the conical panorama image on the unreeled background

The description of the action causing the construction of panoramic image in the unreeled background will be shown on the example of the so-called vertical conical panorama - $\mathbf{Z}_1$ of A type.

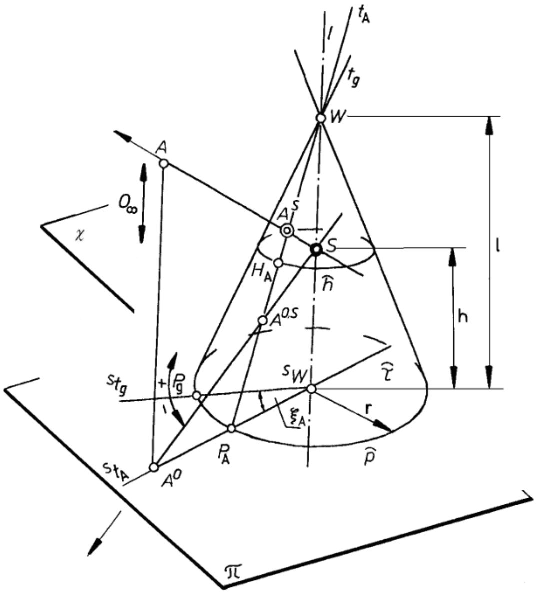

Fig.3: Defined apparatus of the vertical conical panorama $\mathbf{Z}_1$ for the needs of converting panorama mappings into their equivalent on the unreeled $\hat{\tau}$ background

The apparatus of the vertical conical panorama considered at present is made up from (Fig.3):
- the apparatus of the main projecting which is the central projecting from the S center onto rotary conical $\hat{\tau}$ background,
- the χ and π plains perpendicular to the l axis, called the horizon plane and the base plane,
- the apparatus of the supplemental projecting which is the projecting from the $O_\infty \in l$ centre onto the π plane.

The image of any $X \neq S$ point is received on the $\hat{\tau}$ background, subjecting the X point and X^O point to the main projecting, where the X^O point is the result of the supplemental rectangular projecting of the X point onto π base plane. Obtained $(X^O, X^{O,S})$ pair of projections is included in one generatrix of the conical surface (Fig.3).

The knowledge of the mapping of points in panorama representation gives theoretical possibilities of creating panoramas of different figures on the non-deformed conical background. However, for the graphical mapping effects of projecting on the flat surface it is necessary to transform images contained in the $\hat{\tau}$ background to their counterparts in the unreeled background. Such aim can be achieved by projecting cylindrical background and images contained in it from the S center onto the π base plane. It enables establishing projective relations between points on generatrices in this flat surface obtained in that projecting and their counterparts in unreeled background.

Geometrical action is the following: first of all the apparatus of the representation should be defined within an accuracy of isometry. Next, the so-called t_g base generatrix and t_A generatrix which contains a pair of projections of the certain $A \notin l$ point is distinguished in the conical background (Fig.3). Moreover, the positive circulation of measurement of ξ_i angles of $\mathbf{O}_i$ rotations transforming t_g generatrix into t_i generatrices of the $\hat{\tau}$ background is established. Nevertheless the fragment of the unreeled $\hat{\tau}$ background with distinguished the t_g base generatrix and the t_A one as well as the $\hat{p}$ base circle and the $\hat{h}$ horizon circle are drawn in the π base plane (Fig.4). The $\hat{\tau}^R$ image of the unreeled $\hat{\tau}$ background is placed itself towards the base $\hat{p}$ circle in the way shown in Fig.4.

Series of points $t_A(P_A, H_A, W, A^S, A^{O,S}, ...)$ belonging to the t_A generatrix is distinguished; where P_A point is included in the $\hat{p}$ base circle whereas H_A point is included in the $\hat{h}$ horizon circle and the W point is the top of the surface (Fig.3). Next the $\widehat{\tau}$ background and particularily t_g and t_A generatrices with established series of points are projected from the S center onto the π base plane. As a result of that projecting the ${}^{S}t_A({}^{S}P_A, {}^{S}H_A, {}^{S}W, {}^{S}A^{S}, {}^{S}A^{O,S}, ...)$ series of points are obtained in the ${}^{S}t_A$ generatrix. Then ${}^{S}t_A$ straight line with ${}^{S}t_A({}^{S}P_A, {}^{S}H_A, {}^{S}W, {}^{S}A^{S}, {}^{S}A^{O,S}, ...)$ series of points included in it is turned around ${}^{S}W$ point to the position where it is tangent to the $\widehat{p}^{R}$ base line (Fig.4). After this transformation the ${}^{O}t_A$ line with ${}^{O}t_A({}^{O}P_A, {}^{O}H_A, {}^{O}W_\infty = {}^{S}W_\infty, {}^{O}A^{S}, {}^{O}A^{O,S}, ...)$ series of points is achieved. In a row $t_A{}^{R}$ straight line matching the t_A one is located on the $\widehat{\tau}^{R}$ unreeled background as well as $P_A{}^{R}$, $H_A{}^{R}$, $W_\infty{}^{R}$ points included in it matching P_A, H_A, W_∞ ones. (A^{SR}, $A^{O,SR}$) pair of points matching (A^{S}, $A^{O,S}$) one is also included in $t_A{}^{R}$ straight line. Next $t_A{}^{R}$ straight line with distinguished $t_A{}^{R}(P_A{}^{R}, H_A{}^{R}, W_\infty{}^{R}, ...)$ series of points is turned around W^{R} point to the position where it is perpendicular to the ${}^{O}t_A$ straight line. Then ${}^{\hat{O}}t_A^{R}$ straight line is achieved with matching series of points. After that ${}^{\hat{O}}t_A^{R}$ straight line is submitted to the translation for the $\overrightarrow{{}^{\hat{O}}P_A^{R}\,{}^{O}P_A}$ vector. That geometrical action points out that ${}^{O}t_A({}^{O}P_A, {}^{O}H_A, {}^{O}W_\infty, ...)$ series of points obtained after the rotation and ${}^{T}t$ (${}^{T}P_A = {}^{O}P_A$, ${}^{T}H_A$, ${}^{T}W$, ...) series of points obtained as a result of translation are projective. As they have also united homologous ${}^{T}P_A = {}^{O}P_A$ points they are perspective ones. It gives graphical connections of the image of the point get in the additional projecting from the S centre onto the π base plane with its counterpart get in the unreeled $\widehat{\tau}$ background.

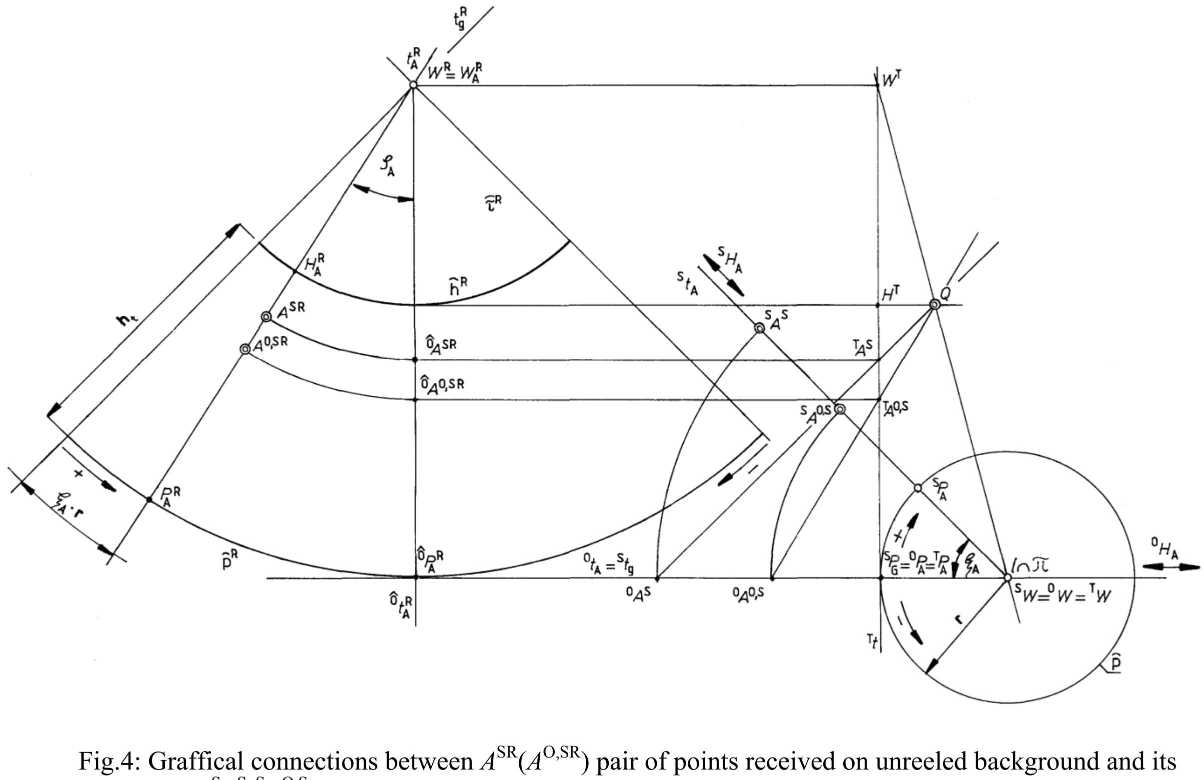

Fig.4: Graffical connections between $A^{SR}(A^{O,SR})$ pair of points received on unreeled background and its ${}^{S}A^{S}({}^{S}A^{O,S})$ mapping received by projecting from S center onto the π base plane

The skill of finding the image of the point enables the graphical mapping of the $k(A,B)$ straight line not particularly situated towards any elements of projecting apparatus and running across (A^{SR},$A^{O,SR}$) and (B^{SR},$B^{O,SR}$) points set in the unreeled background (Fig.5).The image of any k straight line not particularly situated towards any elements of projecting apparatus is composed of two k^{S} and $k^{O,S}$ curves contained in the $\widehat{\tau}$ background. In that case counterparts of these curves are $k^{S,R}$ and $k^{O,S,R}$ curves of the sine wave running across appropriately $A^{S,R}$, $B^{S,R}$ and $A^{O,S,R}$, $B^{O,S,R}$ points on the unreeled background.

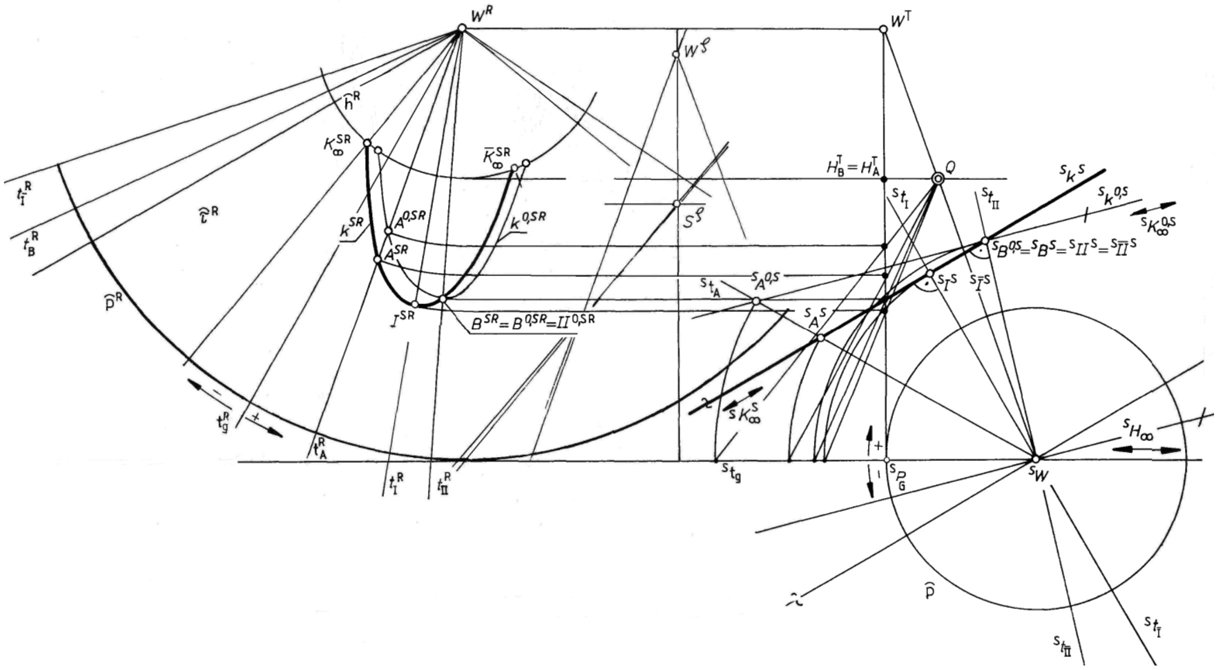

Fig.5: The mapping of the straight $u(A,B)$ line on the unreeled $\hat{\tau}$ background in the $\mathbf{Z}_1$ panorama

To obtain the data which enable drawing the curves of sine wave first of all it is necessary to find (${}^{S}A^{SR}$,${}^{S}A^{O,SR}$) and (${}^{S}B^{SR}$,${}^{S}B^{O,SR}$) projections in additional projecting from the S center onto the base plain. It is made with the help of series of points set on the $t_A{}^R$ and $t_B{}^R$ generatrices (${}^{S}A^{SR}$,${}^{S}A^{O,SR}$) and (${}^{S}B^{SR}$,${}^{S}B^{O,SR}$) projections outline ${}^{S}k^{S}$ and ${}^{S}k^{O,S}$ projections of the given straight line.

It is easy to notice that ${}^{S}I^{S}$ and ${}^{S}II^{O,S}$ points belonging appropriately to ${}^{S}k^{S}$ and ${}^{S}k^{O,S}$ straight lines and being situated the closest to the centre of a $\hat{p}$ circle match the I^{SR} and $II^{O,SR}$ points which are extremely distanced from the $\hat{h}^{R}$ horizon line on unreeled background.

Extreme points and points belonging to the $\hat{h}^{R}$ line define these sine waves explicitly since they establish amplitudes and periods. It permits applying known construction to find any points belonging to these defined curves. It is tantamount to a possibility of drawing the counterpart of the panoramic image of the straight line in the unreeled background. Of course in order to find the panoramic image of the straight line not running across given points on the unreeled background the support projection of it from the S center onto the π base plane is required. That projection is tantamount to the given straight line in the case of straight lines contained in the base.

However, at the realization of drawings of panoramic images of figures defined by their measuring properties similar as in the method of the vertical perspective for the flat background the regulation of the mapping of :

a) pairs of parallel straight lines,

b) pairs of perpendicular horizontal straight lines,

are defined.

It enables drawing conical panorama of any figure shaped by means of lines. Fig.6 shows the image of the pyramid placed on the π base in conical $\mathbf{Z}_1$ panorama. For the sake of presenting all graphical actions both visible and not visible edges of presented figure are drawn.

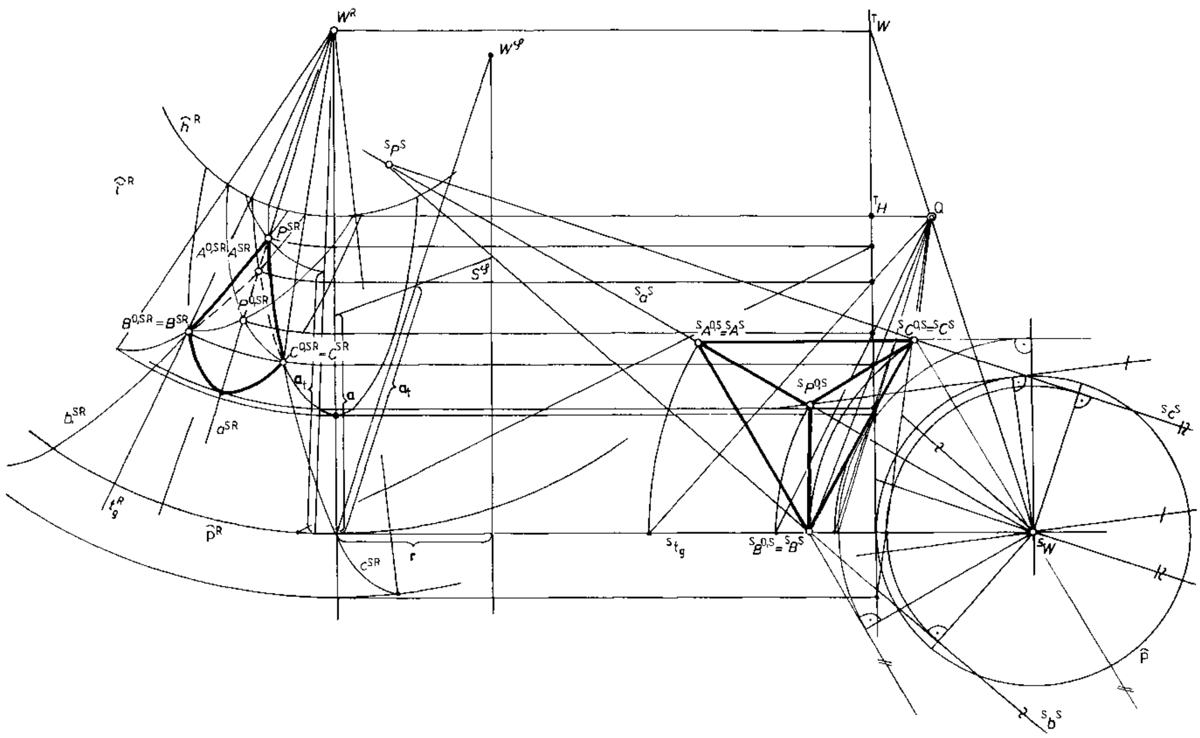

Fig. 6: The image of the pyramid drawn in the conical panorama on unreeled background

4.Conclusions

The direct mapping of the conical panorama is possible by means of the two projective partly composed representation. Projective relations between the conical panorama image and its counterpart on the unreeled background presented in the paper are the base for creating analytical algorithms for drawing panoramas with the help of computer software. It will make conical panorama drawing much more efficient.

Literature

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ZASTOSOWANIE DWURZUTOWEGO CZĘŚCIOWO ZŁOŻENIOWEGO ODWZOROWANIA DO KONSTRUKCJI PANORAMY STOŻKOWEJ

Artykuł jest nawiązaniem do wcześniejszych rozważań autora dotyczących konstrukcji panoram walcowych. Przedstawia teoretyczne podstawy do bezpośredniego zapisu panoramy stożkowej na rozwinięciu jej tła przy wykorzystaniu tzw. dwurzutowego częściowo złożeniowego odwzorowania. Wykazane w pracy zależności geometryczne są punktem wyjścia do opracowania analitycznych algorytmów do rysowania panoram przy zastosowaniu wspomagania komputerowego.