POLISH MARITIME RESEARCH Special Issue 2018 S1 (97) 2018 Vol. 25; pp. 166-171
10.2478/pomr-2018-0038

MODELING OF THE TWO-DIMENSIONAL FLOW CAUSED BY SEA CONDITIONS AND WIND STRESSES ON THE EXAMPLE OF DEAD VISTULA

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ABSTRACT

The article presents the results of two-dimensional modeling of flows caused by the sea conditions and wind stresses on the example of Dead Vistula. Based on the available bathymetric data, a numerical model of the river section was created, which was supplemented with data on the position of the water table depending on hydrometeorological conditions. To describe the flow field in steady conditions, a simplified model of two-dimensional flow in the form of the bi-harmonic Helmholtz equation for the current function has been adopted, taking into account additional impacts caused by wind stresses on the water surface. Then the current function was converted into the velocity vector components. This equation, supplemented with appropriate boundary conditions, has been solved numerically using the finite difference method. On the basis of the available literature, 4 variants of hydrometeorological conditions were adopted, depending on the direction and strength of wind and sea conditions. The obtained results were compared with the results of published measurements taken on the studied section of the river. These calculations were the basis for the implementation of a two-dimensional model of the spread of pollutants in the studied section of the Dead Vistula.

Keywords: two-dimensional modeling of flows, bi-harmonic Helmholtz equation, hydrodynamics of water flow

INTRODUCTION

Flow modeling in rivers is an important tool to support design work in the field of hydrotechnical construction. It is also the first stage in the work on the analysis of the spread of pollutants and their impact on the receiver's waters and the surrounding environment (Wielgat and Zima, 2016). In such a situation there is a need to reconstruct the hydrodynamics of water flow on the analyzed section of the river. In practice, different models are used, and their choice depends on the characteristics of the object and the needs for which the analysis is performed (Szymkiewicz, 2010). For long sections of rivers, one-dimensional models are used, importing the tested section into a linear object, averaging all parameters across the entire cross-section. In the case of wide rivers, two-dimensional models are used, averaged vertically (along the depth). This applies to cases when the analysis requires evaluation of variable conditions along the river's width - for example, any reproduction of flow in the vicinity of the hydro-technical building and in the case of inflow of water from the shore or discharge of additional contaminated water. Three-dimensional models are used very rarely, mainly in scientific works and relate to research on specific objects (Jasińska, 2002).

In the cases of modeling two-dimensional flows, the often used mathematical model are the so-called shallow water equations (SWE), or the Saint-Venant equations for a two-dimensional area (Tan 1992, Szydłowski et al., 2013). They are a proven tool, but from a formal point of view they constitute an initial-boundary problem and require to define the boundary conditions, i.e. the initial condition and boundary conditions. In the case of a flow simulation in the so-called characteristic flow conditions, hydrological calculations give information on flows and characteristic states. In the case of many analyzes (including analysis regarding the spread of pollutants - Wielgat and Zima, 2016, Winter 20017, and Winter 2018), this information is insufficient. It is then necessary to determine the distribution of the speed field throughout the analyzed section of the river. The SWE solution first requires a solution in accordance with the adopted conditions regarding the position of the water table for a given flow. In many cases, this involves calculations that allow the identification of resistance to movement until the position of a water table similar to the characteristic states is obtained. The SWE solution also requires time integration, which in the case of fixed (permanent) motion requires simulation until the flow conditions are established, which is not always unambiguous. Therefore, in the case of calculations for characteristic conditions (for flow and condition), the natural way will be to look for simplified models. In computational hydraulics, such models are often used. This article presents one such model that allows to obtain velocity distribution in two-dimensional area with variable depth solving the modified Helmholtz equation (Anderson, 1995). Additionally, due to the nature of external interactions resulting from the location of the analyzed section of the river, the influence of wind stresses was taken into account in the equation. The calculations were made for the section of Dead Vistula, whose hydrodynamics are caused by sea conditions in the Gulf of Gdańsk and wind accumulation. Obtained results of calculations carried out in specific weather conditions were compared with the results of carried out measurements presented in this work (Jasińska, 2002).

MATHEMATICAL MODEL

To describe the river water flow field, a model was adopted in this work in which, along with the fulfillment of the mass behavior equation, the variability of rotation of the fluid element is also assumed (Anderson, 1995). In the case of a two-dimensional speed vector: u=[u_x,u_y] this condition can be written as follows:

$$\mathrm{rot}\,\mathbf{u} = \Omega(x, y) \tag{1}$$

where: $\Omega(x, y)$ is the assumed function of the vorticity distribution. If the Stokes assumption (neglecting the forces of inertia) is applied for fixed flow conditions and constant water temperature and relation (1) is taken into account, the result is:

$$\Delta\Omega = 0 \tag{2}$$

This is the equation for the harmonic of the Ω function, called the Helmholtz equation. If the current function $\psi(x, y)$ for vertically averaged plane motion is defined in the following way:

$$\frac{\partial\psi}{\partial y} = h \cdot u_x, \quad -\frac{\partial\psi}{\partial x} = h \cdot u_y \tag{3}$$

(where h is the depth), finally, equation (2) (the biharmonic equation) can be written in the following form:

$$\Delta\Omega = \Delta\Delta\psi = \Delta\left(\frac{1}{h}\Delta\psi - \frac{1}{h^2}\nabla\psi\nabla h\right) = 0 \tag{4}$$

Equation (4) can be obtained by transforming the equation of momentum conservation in stationary motion conditions, taking into account the Stokes assumption. In this equation, speed rotation is a harmonic function described by the Laplace equation. Taking into account additional forces affecting the water movement caused by stress on the surface from the wind, causes equation (4) to take the following form:

$$\Delta\left(\frac{1}{h}\Delta\psi - \frac{1}{h^2}\nabla\psi\nabla h\right) = -\frac{1}{\kappa}\, rot_z \frac{\mathbf{T}}{h} \tag{5}$$

where: κ coefficient of turbulent viscosity, **T** stress vector of tangential wind in tow-dimensional space (x,y) T=[T_x,T_y]. The coordinates of the wind stress vector are defined as follows (Tan, 1992):

$$T_x = \frac{\rho_\alpha}{\rho} \cdot \alpha\sqrt{w_x^2 + w_y^2} \cdot w_x \tag{6}$$

$$T_y = \frac{\rho_\alpha}{\rho} \cdot \alpha\sqrt{w_x^2 + w_y^2} \cdot w_y \tag{7}$$

where: α coefficient of wind stresses, $\alpha = 2.5 \times 10^{-3}$, ρ and ρ_α density of water and air, w_x, w_y components of the wind velocity vector.

In general, the Dirichlet problem should be solved for the biharmonic operator. A 4th order equation is obtained, which requires to solve simultaneously the determination of two conditions at the edge of the Γ area: the Dirichlet $\psi\big|_\Gamma = const$ type condition and the Neumann $\frac{\partial\psi}{\partial n}\Big|_\Gamma = 0$ type. The solution to this problem in the two-dimensional space *(x, y)* is the current function. Using the relation (3), you can calculate the components of the velocity vector in the whole area. To solve the equation (5) the finite difference method (FDM) was used (Anderson 1995, Potter 1980), covering the solution area with a square mesh of $\Delta x = \Delta y$. The adoption of a square mesh with fixed dimensions has its advantages and disadvantages. The advantage is that such an approach significantly simplifies the final formula of the model (especially considering additional forces at the level of the basic equation). However, the disadvantage is the inability to differentiate it in some parts of the computing area. This requires, unfortunately, the adoption of a suitable dense grid in the entire computational area. This approach also improves the stability of the solution.

The use of differential diagrams leads to the final solution of algebraic linear equations. This system is characterized by the lack of dominant elements on the main diagonal matrix of ratios, which unfortunately negatively affects the accuracy of the solution for most known numerical methods (Szymkiewicz, 2010). In this work, the simple iteration method (Jacobi's method) was used. The properties of the created system of equations cause that the iterative process is slowly convergent and there is a danger of the solution being under-iterated. Therefore, during the solution of each task, the iteration termination test was individually selected, analyzing its impact on the final solution.

CHARACTERISTICS OF THE STUDIED AREA

The Dead Vistula is part of the hydrographic complex delta that has developed over the last centuries. This area is characterized by the interaction of land and sea waters, it has changed in a natural way and as a result of human activity. The current hydrographic state of the Vistula delta cutoff was significantly influenced by the dug-thru at the end of the 19th century, draining the Vistula waters directly to the sea and the sluice in Przegalin. This changed significantly the hydrodynamic conditions on the section of the Vistula delta, creating a dead branch – Dead Vistula. The Vistula river ceases to be powered by the waters of the Vistula, it is connected with the sea by two arms: the port channel in Nowy Port and the Vistula Śmiała river. Only small amounts of Vistula waters get to the Dead Vistula thru the ship sluices in Przegalina. This fact has a significant impact on the hydrodynamic conditions prevailing in the Dead Vistula, in which the water movement is determined primarily by sea conditions and blowing winds. Works on modeling hydrodynamics of this section of the Vistula River were carried out at the IBW PAS in Gdańsk. Modeling results are presented in the paper (Jasińska, 2002). As part of this study, in the assumed hydrodynamic model, the results included in this work were assumed as boundary conditions (adopted speed distributions in cross-sections limiting the assumed mathematical model - near the Siennicki Bridge, in the estuary cross-section of the Vistula Śmiała and cross-section at Sobieszewo height).

Fig.1. Dead Vistula with available cross-sections (red lines) and the adopted computational area (area shaded in read) (source: ortofotomap from http://mapy.geoportal.gov.pl)

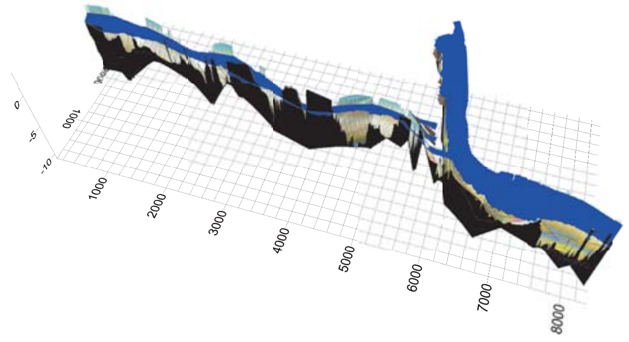

Fig.2. Numerical model of the bottom with the location of the water table

In the latter part of the work, a numerical model of the Dead Vistula section was made from the cross-section at the height of the Siennicki Bridge to the cross-section at Sobieszewo (about 8.5 km), including the 2.5-kilometer section of the Vistula Śmiała, to its mouth at the Gulf of Gdańsk. Using the available data on the measured computational cross-sections, a numerical model of the bottom of the Dead Vistula section was generated. The bathymetry between the measured cross-sections was generated by linear interpolation (Fig. 2).

Due to the assumed hydrodynamic model, the water table was also required to be assumed. Information on the location of the water table was adopted adequately to the conditions for which calculations of the velocity field distribution were made (Jasińska, 2002).

NUMERICAL SIMULATIONS

The hydrodynamic calculations were made in four variants presented in the study (Jasińska, 2002): for the eastern wind of 5 m/s, for the western wind of 5 m/s and for extreme conditions, i.e. for the western wind of 15 m/s and 25 m/s. When determining boundary conditions in the calculations performed, averaged vertical velocity values were assumed (despite the often occurring diversity of flows in the near bottom and near surface layers), assuming in the majority of values presented in the work (Jasińska, 2002) obtained in the near surface layer.

Regardless of the computational variant adopted, the velocity field distribution on the section of the Dead Vistula concerned was calculated using equation (5). The calculations were made using a square grid with sides $\Delta x=\Delta y=5{,}0$m. The size of the grid was based on the dimensions of the calculation area in the Isthmus region. The obtained results are shown in Figures 3 to 6. Due to the density of the calculation grid, it was presented by every 65th vector of speed (Fig. b) and by every 10th vector (Fig. c).

Fig. 3 presents the results of the distribution of the current line and the corresponding velocity distribution for the variant of weak eastern wind with the strength E 5 m/s. This wind causes the inflow from the Gulf of Gdańsk section of the Vistula Śmiała, inflow from the Przegalin side, and flow in the branch of the Dead Vistula towards the west and outflow to the Gulf of Gdańsk near Nowy Port. The calculations assume: an influx of the Vistula Śmiała River due to the average sea state with an average speed of 0.05 m/s, inflow in the section from Świbno with the speed of 0.02 m/s and outflow through the section near the Siennicki Bridge at the speed of 0,04 m/s. As a result of the calculations carried out, speed distribution was achieved that reproduced the basic flow directions.

The maximum values of the velocity vector module occurred at the level of the Isthmus and were at the level of 0.25 m/s, which is consistent with the results presented in the work (Jasińska, 2002).

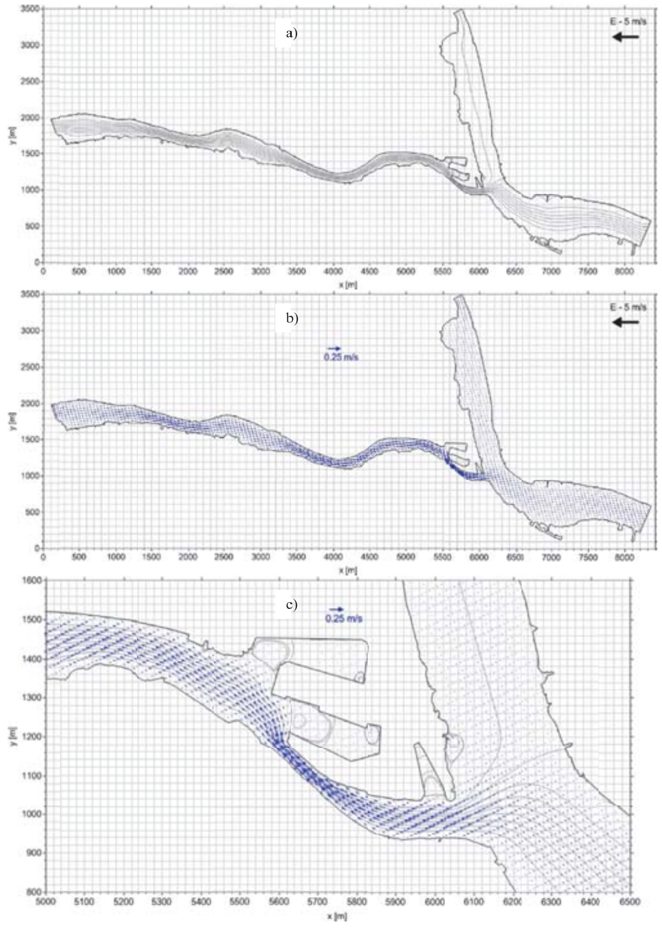

Fig.3. Results of hydrodynamic calculations: a) distribution of current lines, b) their corresponding speed field, and c) detailed results near Isthmus, in eastern wind conditions E - 5 m/s

Weak wind from the west (W) with a force of 5 m/s, causes an almost completely different current circulation in Dead Vistula. The outflow in section at the mouth of the Vistula Śmiała towards the Gulf of Gdańsk with a speed of about 0.05 m/s is registered. In the Sobieszewo region, the water flow is consistent with the wind direction at a speed of about 0.03 m/s. From the direction of Siennicki Bridge, the water inflow is visible with the speed of about 0.06 m/s. As a result, a directed flow in the branch of the Dead Vistula towards the east can be observed, assuming the maximum values of the velocity vector module in the vicinity of the Isthmus at the level of 0.28 m/s. The results obtained are shown in Fig. 4.

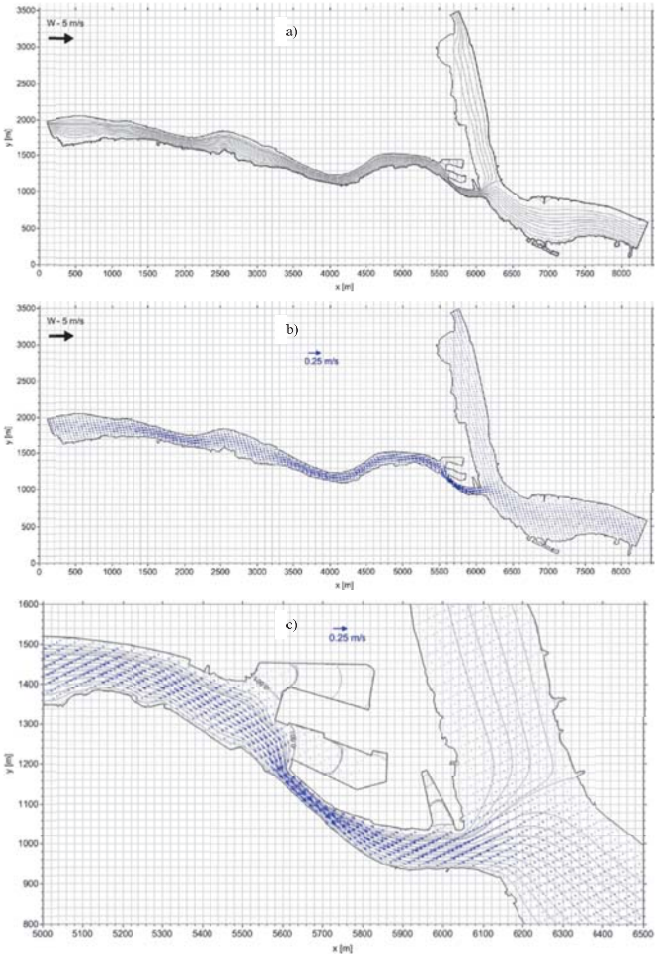

Fig.4. Results of hydrodynamic calculations: a) distribution of current lines, b) their corresponding speed field, and c) detailed results near Isthmus, in western wind conditions W - 5 m/s

The next two variants of calculations were made for extreme hydrometeorological conditions occurring in the Dead Vistula region. In the first place (variant no. 3), simulations were carried out for the case of the western wind (W) with a force of 15 m/s, assuming an increase in the sea state by approx. 0.7 m from the average state (up to 570 cm). In the area of the mouth of the Vistula Śmiała, there is an outflow to the Gulf of Gdańsk with a speed of 0.6 m/s. At the west arm of the Dead Vistula in the area of the Siennicki Bridge there is a flow towards the east (inflow) with the speed at the level of 0.3 - 0.4 m/s. At the eastern arm of the Dead Vistula (in the Sobieszewo region), a stronger flow towards the east (outflow) is visible with speeds of 0.3 m/s. As a result, a clear flow is visible in the branch of the Dead Vistula towards the east, with maximum values in the vicinity of the Isthmus at 1.5 m/s, which is in line with the results presented in the work (Jasińska, 2002). Obtained results in this calculation variant are shown in Fig. 5.

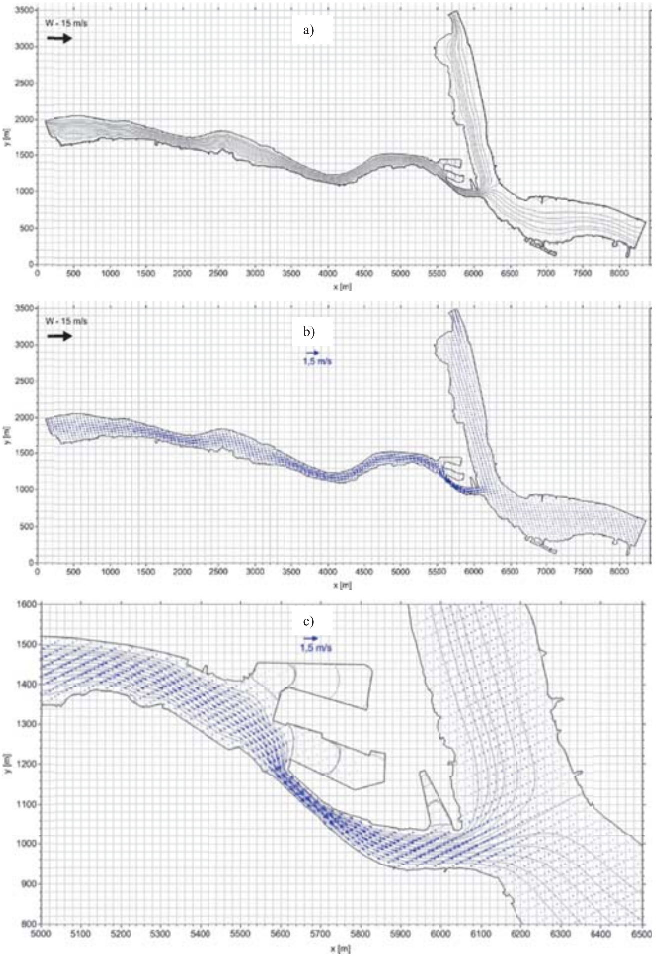

Fig.5. Results of hydrodynamic calculations: a) distribution of current lines, b) their corresponding speed field, and c) detailed results near Isthmus, in western wind conditions W - 15 m/s

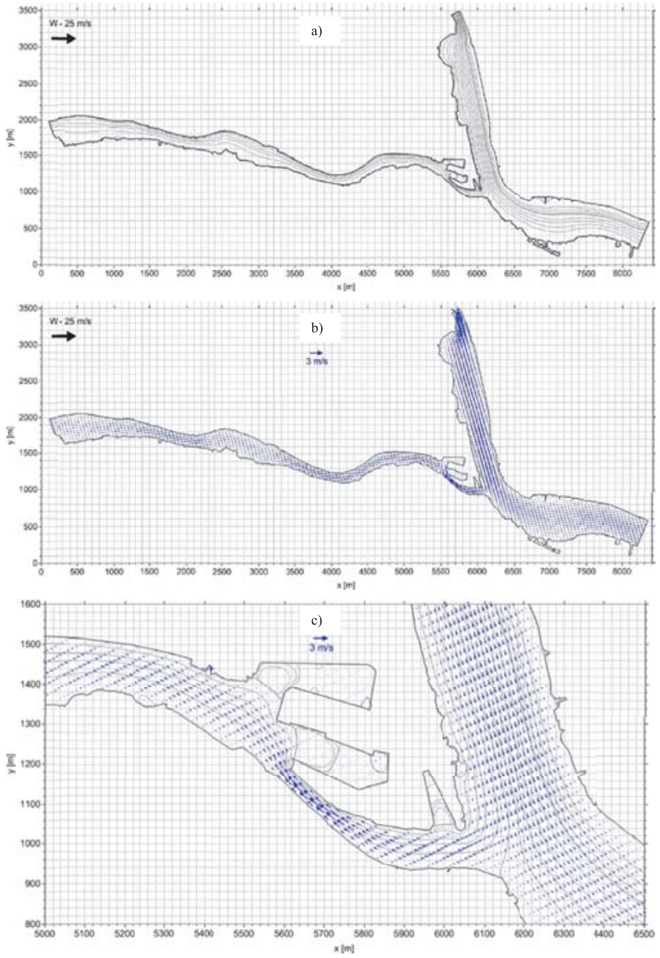

Fig.6. Results of hydrodynamic calculations: a) distribution of current lines, b) their corresponding speed field, and c) detailed results near Isthmus, in western wind conditions W - 25 m/s

The last variant concerned the case of the western wind (W) with a force of 25 m/s, assuming the sea rise from a high level of 570 cm to a level of 625 cm. In such extreme conditions of very high sea state and very strong western wind, the conditions of the flow in the estuary change. In the area of the estuary of the Wisła Śmiała, an inflow in the entire section from the Gulf of Gdańsk occurs at very high speeds of 3.0 m/s. On the west branch of the Dead Vistula in the area of the Siennicki Bridge there is a flow towards the west (outflow) at a speed of 0.5 - 0.6 m/s. At the eastern branch of the Dead Vistula (in the Sobieszewo region) a stronger flow towards the east (outflow) is visible with speeds of 0.5 m/s. As a result, a clear flow is visible in the branch of the Dead Vistula towards the west, with maximum velocities around the Isthmus at the level of over 1.0 m/s. Obtained results in this calculation variant are presented in Fig. 6.

DISCUSSION OF THE RESULTS OBTAINED AND CONCLUSIONS

The conducted computer calculations made it possible to plot the field of current function and calculate the values of the components and velocity field in the section of Dead Vistula from Sobieszewo to Siennicki Bridge (about 18.5 km) together with the section of Vistula Śmiała that flows into the Gulf of Gdańsk (about 2.5 km). On the basis of available cartographic materials, the boundary of the shoreline was marked out, and on the basis of the data on cross-sections a numerical model of the bottom was created of the considered section of the river. The model was supplemented with data on the position of the water table depending on hydrometeorological conditions. On the basis of the available literature, 4 variants of these conditions were adopted, depending on the wind direction and strength and sea conditions (for the eastern wind with a force of 5 m/s, for the western wind with a force of 5 m/s and for extreme conditions, i.e. for the western wind o strength 15 m/s and 25 m/s). Then, calculations of the velocity distribution in the traffic conditions set for each of the adopted variants were made. The obtained results of computer simulations were consistent with the averaged results of velocity distribution measurements in characteristic cross-sections of Dead Visutla presented in the work (Jasińska, 2002). This comparison was the final criterion confirming that the use of a two-dimensional model to describe hydrodynamics in the studied section of the Dead Vistula gave correct results. These calculations were the basis for the implementation of a two-dimensional model of pollution spread on the analyzed section of the Dead Vistula, which requires a solution to the transport equation in a two-dimensional case (Sawicki and Zima, 1997).

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