JAROSŁAW JOOSTBERENS
Silesian University of Technology in Gliwice

Selected issues of identifying dynamic properties of hard coal flotation process

The article features selected issules related to the identification of dynamic properties of the hard coal flotation process, as an object with one control input and one output. The content of ash in flotation tailings, available for measurements, was adopted as an output signal. The author presented the theoretical basis to use the selected methods for estimating the parameters of the flotation process dynamic models. In addition, the article provides a calculation example.

key words: hard coal flotation, automatic control, ash content monitoring

1. INTRODUCTION

Flotation is a physical and chemical method for processing mineral resources. In the case of hard coal, the method is used for the type of feed which is composed of 0.5 mm grains (max <1 mm). From the point of view of control, the hard coal flotation process is an unlimited dynamic object. It can be described as an object with many inputs and outputs, however in industrial processes only a part of input and output signals are subject to measurements. The basic input quantities of the flotation process are: intensity of the feed flow Q_n with the ash content A_n and the concentration of solids in the feed K_{cs}, intesity of the flotation reagent flow V_o, intensity of air flow for the aeration of slurry Q_a , and the level of suspension in the flotation chamber h.

The output quantities are: volume of concentrate W_K, ash content in the concentrate A_k, volume of waste W_o, ash content in the waste A_o. Control signals are: intensity of the flotation reagent flow, suspension level in the flotation chamber, intensity of air flow for aeration. The parameters of the feed, on the other hand, should be treated as disturbance due to their random changes. In Polish industrial systems, the content of ash in the feed as well as quantity and quality parameters of the concentrate are unavailable for measurements. They are measured periodically in quality control laboratories in mines based on selected samples. The only quality parameter which is available for measurements is the content of ash in flotation tailings – the measurements are conducted with the MPOF sensor [11]. Usually, it is also possible to measure the intensity of the feed flow and the concentration of solids in the feed.

From the point of view of automatic control of the coal flotation process, it is important to be familiar with its dynamic properties which are presented as input-output dependencies with the use of a certain mathematical model in the form of differential equations, finite-difference equations, transfer functions and discrete transfer functions. Making simulation models of the coal flotation process, as well as automatic control systems and regulation systems for this process, should be based on the knowledge of the process static characteristics and dynamic properties. Therefore it is of the utmost importance to improve estimation methods and, at the same time, to determine dynamic models which can copy dynamic properties of industrial coal flotation processes.

2. MEASURING DYNAMIC PROPERTIES OF COAL FLOTATION PROCESS

In order to carry out an identification experiment it is necessary to be familiar with the observation of input quantities and corresponding output quantities. In the case of hard coal flotation, the object in question can be treated as an object with one output which is available for measurements (A_o). Then the identification objective lies in the determination of a mathematical model which describes dynamic properties of the flotation process with the selected input and output in the form of ash content in flotation tailings. Thus it is vital to select a method to measure dynamic properties and a method to determine the model parameters.

According to [3] there are three types of methods for measuring dynamic properties:

- active methods: the object operations are disturbed by determined input signals and then output signals are observed,
- passive methods: output and input signals of the disturbed object are observed during a certain period of time and the obtained information is processed statistically,
- mixed methods: a passive method is supplemented with deliberately introduced disturbances of the object operations.

The identification of the dynamic model of the coal flotation process is related to the selection of a method for measuring dynamic properties of this process. If it is possible to conduct the measurements with the use of an active method in industrial conditions, then it is advisable to acquire measurement data with the use of this method. In such a case, it is justifiable to excite the object with a signal whose form in time is known and which is introduced to the input on purpose. Then the identification of the dynamic model includes the following [8]:

- introducing a special identification signal to the input of the object,
- registering the value of the input and output quantity in a discrete manner during the transient state of the process.

As far as active methods of measuring dynamic properties are concerned, the excitation signal can be a single impulse, unit step, speed step, acceleration step, and a sinusoidal signal. When the signal is step excitation, the method is called step response. It is not possible to perform a unit step in a thorough manner. In fact, each real step signal has a certain time of step accumulation. It is important that this time should be scant in comparison with the time constant of the object.

In industrial conditions it is possible to have a step change of the control signal in the flotation process, particularly with respect to the intensity of the flow of the flotation reagent, as the flotation process is characterized with significant delays and time constants. Real step signals, in general, have accumulation values different from unity. It is vital that the step change of the excitation signal should have sufficient value to ensure a marked change of the input signal. On the other hand, the step change cannot be too big due to the non-linearity of the object.

Most frequently, the step method provides enough accuracy to avoid long analyses [8]. In simple cases the shape of the step characteristics informs about the type of the model. However, in more complex cases (multi-inertial objects, inertial and oscillation objects, etc.) the reasoning becomes uncertain [10].

As far as the coal flotation process is concerned, the step response method, which is aimed at achieving the course of the ash content in flotation tailings due to the step change of the flotation reagent, can be used after the process is driven to its fixed state – stability of all input and output parameters in time. Then it is necessary to step change the volume of the flotation reagent provided to the object. At the same time we have to register the course of the input (V_o) and output signals and monitor on-line the state of the remaining quantities. The principle of measuring dynamic properties of the coal flotation process is presented in Fig. 1

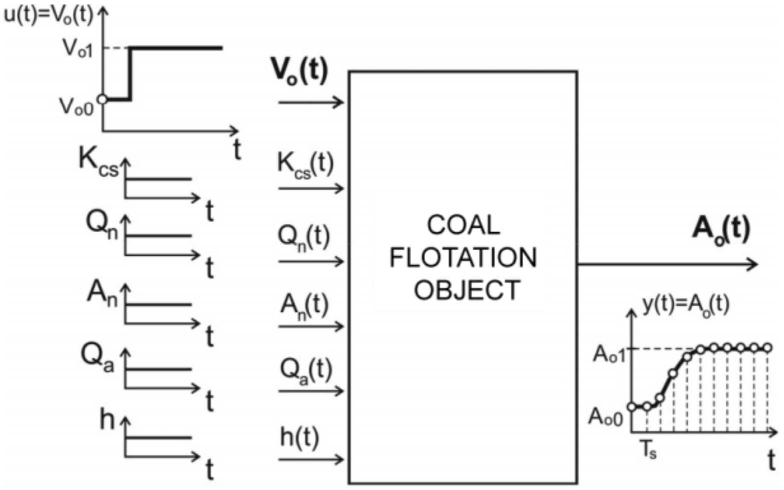

Fig. 1. Measuring dynamic properties of the flotation process with the use of the step response method

As it was mentioned before, the experiment to determine step characteristics can be conducted with the use of measuring apparatus installed on the object. An alternative method of identification is to collect a series of samples at the determined sampling times T_s. The laboratory results of ash contents in flotation tailings of successive samples will make the step characteristics. In both cases an important issue is accurate selection of the sampling period duration T_s.

The collected series of samples should enable to have an accurate picture of the whole analogue signal wave. In the case of computer registration of measurement data (on-line) from particular sensors installed in the system for monitoring the coal flotation process, the sampling period is usually enough. The determination of the sampling period during the experiment, based on collecting samples from flotation tailings, becomes particularly important due to organizational difficulties encountered while conducting such an experiment. The sampling period of input and output signals of the identified object should enable to recapture dynamic properties of the object.

Another important issue related to the identification experiment is the selection of the model structure and a method to estimate the parameters of this model baed on the measurement data. During the identification experiment the parameters of the dynamic model can be estimated on line (based on recursive methods) or can be determined after certain measurement data are acquired (batch methods).

Let us focus on direct methods (batch methods). In the case of objects which are observed in a discrete manner, the parameters of the model can be determined with the use of the least squares method (*LS*), maximum likelihood method (*ML*) or instrumental variable method (*IV*).

It is worth to employ several methods to estimate the parameters of the dynamic model. In such a case the final result can be the model which suits the best the empirical data with respect to the assumed criterion. Thus the identification experiment lies in collecting measurement data (excitation and output signals), determining the model structure, and determining a criterion to adapt the model to the empirical data. Finally, one has to make suitable calculations to determine the parameters of the model.

3. DETERMINING PARAMETERS OF DYNAMIC PROPERTIES MODEL OF COAL FLOTATION PROCESS

3.1. Selected methods of estimating model parameters

The coal flotation process is a dynamic non-linear object due to its non-linear static characteristics [1, 2, 5, 7]. This has a direct impact on the way to identify dynamic properties of this process.

In order to determine the parameters of the model in the form of a transfer function or finite-difference equation, it is necessary to reduce the model to a linear range with small deviations from the given working point or while the process moves from one working point to another. To get proper calculations for the parameters determination, we have to reduce initial values of input and output signals to zero, as it is shown in Fig. 2. The range of static characteristics between the working points *P1* and *P2* should be close to a linear one.

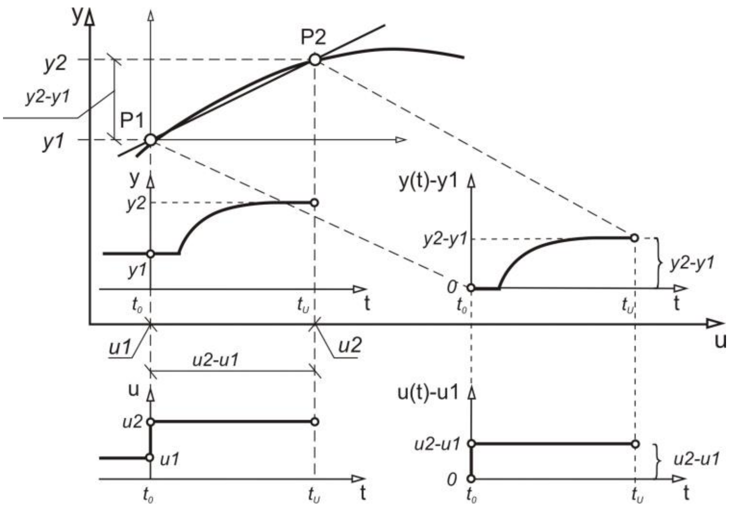

Fig. 2. Measuring dynamic properties of the flotation process with the use of the step response method

An extremely important issue is to select the method for estimating the model parameters. An essential range of the identification experiment is to solve the issues related to the preparation and performance of industrial tests leading to the collection of interesting measurement data (section 2). Another important issue is to select the structure of the model. According to the results of many industrial tests [2, 4, 5, 6], dynamic properties of the coal flotation process with the control input V_o and output A_o can be described thorougly enough by a model which has a structure of an inertial first-order element with time delay.

Knowing the structure of the model, it is possible to select the method for determining the model parameters.

The following symbols were adopted for further discussions:

- $u(t)$ – step change of the flotation reagent volume $u(t) = V_o(t)–V_o(0)$,
- $y(t)$ – change of the output signal, i.e. ash content in flotation tailings $y(t) = A_o(t)–A_o(0)$,
- $A_o(0)$, $V_o(0)$ – initial values of input and output signals respectively at the moment $t(0)^{(-)}$ (before the identification experiment starts).

The response of the object with the structure of an inertial first-order element with time delay to the step signal excitation is expressed by the equation $u(t) = u_0 1(t)$. This is presented in Fig. 3.

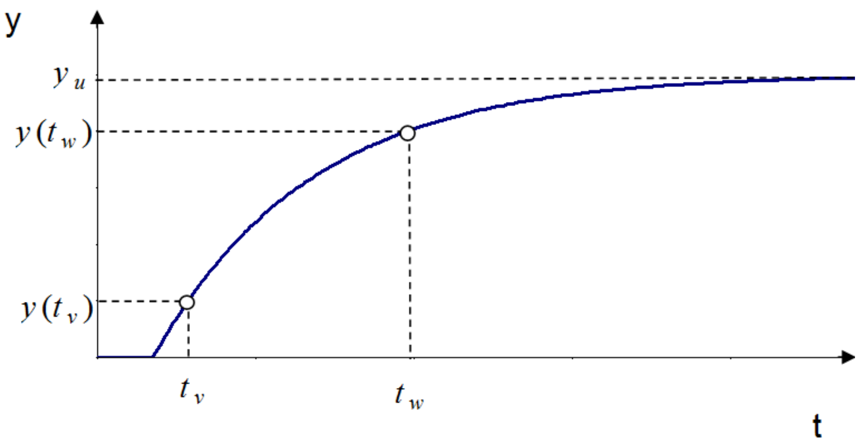

Fig. 3. Step response of the object with the structure of an inertial first-order element with time delay

This can be expressed by the following formula:

$$y(t) = y_u \cdot \left(1 - e^{-\frac{(t-\tau)}{T}}\right) \cdot 1(t+\tau) \qquad (1)$$

where:

y_u – value of the output signal in the steady state (dy/dt = 0),

T – time constant of the object,

t – time delay.

Knowing the values of the step response for the times t_v and t_w (see Fig. 2.) such that $0 < y(t_v) < y(t_w) < y_u$ and $t_v < t_w$, it is possible to make the following set of equations:

$$\begin{cases} y(t_v) = y_u \cdot \left(1 - e^{-\frac{(t_v-\tau)}{T}}\right) \\ y(t_w) = y_u \cdot \left(1 - e^{-\frac{(t_w-\tau)}{T}}\right) \end{cases} \qquad (2)$$

It is easy to notice that this is a set of equations with two unknown quantities T and τ, while $y(t_v)$, $y(t_w)$ and y_u are known. When the value y_u is transferred to the other side in both equations and both sides are divided by $-y_u$, we get the following:

$$\begin{cases} e^{\frac{(\tau-t_v)}{T}} = \left(1 - \frac{y(t_v)}{y_u}\right) \\ e^{\frac{(\tau-t_w)}{T}} = \left(1 - \frac{y(t_w)}{y_u}\right) \end{cases} \qquad (3)$$

As far as the first equation (3) is concerned, in order to determine a time constant, a logarithm was found for boths sides:

$$\frac{(\tau - t_v)}{T} = \ln\left(1 - \frac{y(t_v)}{y_u}\right) \qquad (4)$$

Transforming the equation (4) with respect to T, we obtain the following:

$$T = \frac{(\tau - t_v)}{\ln\left(1 - \frac{y(t_v)}{y_u}\right)} \tag{5}$$

Finding logarithms for both sides in the second equation (3), replacing T with the formula (5) and multiplying both sides of the equation by the value of (τ-t_v), we get the following:

$$(\tau - t_w) \cdot \ln\left(1 - \frac{y(t_v)}{y_u}\right) = (\tau - t_v) \cdot \ln\left(1 - \frac{y(t_w)}{y_u}\right) \tag{6}$$

Ordering the equation (6), we obtain the following:

$$\begin{gathered}\tau\left[\ln\left(1 - \frac{y(t_v)}{y_u}\right) - \ln\left(1 - \frac{y(t_w)}{y_u}\right)\right] = \\ t_w \cdot \ln\left(1 - \frac{y(t_v)}{y_u}\right) - t_v \cdot \ln\left(1 - \frac{y(t_w)}{y_u}\right)\end{gathered} \tag{7}$$

Determining the value τ from the equation (7), we get a formula for calculating the value of time delay:

$$\tau = \frac{t_w \cdot \ln\left(1 - \frac{y(t_v)}{y_u}\right) - t_v \cdot \ln\left(1 - \frac{y(t_w)}{y_u}\right)}{\ln\left(1 - \frac{y(t_v)}{y_u}\right) - \ln\left(1 - \frac{y(t_w)}{y_u}\right)} \tag{8}$$

The dependencies (5) and (8) are correct for the step characteristics (Fig. 3.) in the form of the output signal increase due to the step increase of the input signal, provided that the following conditions are met: $0 < t_v < t_w$ and $0 < y(t_v) < y(t_w) < y_u$. Obviosly, the known equations (5) and (8) can be applied when the disturbance which affects the object is insignificantly small.

It is not possible to apply the equations (5) and (8) to estimate the parameters of the model which describes the dynamics of the flotation process as an object with one control input and one output available for measurements. This is due to the noise which occurs in the measurement signal. The equations (5) and (8) can be used only in association with a filter with properly selected parameters. In this case the filter has to smooth the course of the signal at the output of the process in such a way that important changes in the process signal could not be dampened (recovering the dynamics at the output of the filter while the noise is dampened). When the filter is properly selected, the parameters of the model, in the form of time delay and time constant, can be calculated from the equations (5) and (8) based on the signal from the filter output – as it was demonstrated in Fig. 4.

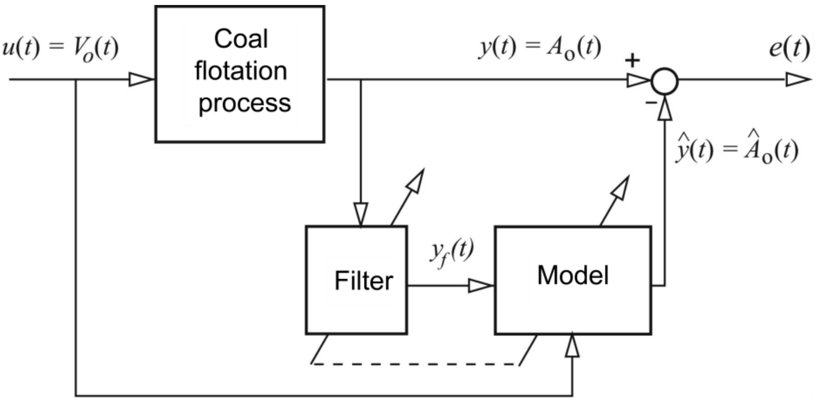

Fig. 4. Measuring dynamic properties of the flotation process with the use of a filter

Then the equations (5) and (8), which are indispensable to determine the parameters of the inertial first-order model with time delay, can be put in the following way:

$$\tau = \frac{t_w \cdot \ln\left(1 - \frac{y_f(t_v)}{y_{f_u}}\right) - t_v \cdot \ln\left(1 - \frac{y_f(t_w)}{y_{f_u}}\right)}{\ln\left(1 - \frac{y_f(t_v)}{y_{f_u}}\right) - \ln\left(1 - \frac{y_f(t_w)}{y_{f_u}}\right)} \tag{9}$$

$$T = \frac{(\tau - t_v)}{\ln\left(1 - \frac{y_f(t_v)}{y_{f_u}}\right)} \tag{10}$$

In the presented example we used a filter with the moving average principle. The filter performs symmetry averaging with respect to the calculated sample. Such a filter dampens well the random noise in the signal and can provide significant steepness of the course at the filter output when there are step changes of the filtered signal [9]. The equation of the filter has the following form:

$$y_f(k) = \frac{1}{M+1}\sum_{i=-M/2}^{M/2} y(k+i) \tag{11}$$

where:
$M+1$ – averaged number of samples (M is a non-zero natural even number),
$y_f(k)$ – signal at the output of the filter at the moment kTs.
k – 1,2…N.

What is left then is the issue how to select the parameter of the filter. When the main objective of the calculations is to estimate the parameter of the model with a structure of an inertial element with time delay, it is possible to associate the selection of the parameter of the M filter with the estimation of the model parameters. In such a case the calculations must have an iterational character. The algorithm can be presented in the following way:

1) calculate the signal at the output of the filter, at the given M,
2) set the y_{fu} value assuming that $\hat{y}_u = y_f(N-n)$, where n is a natural number from the adopted range $n=(0, n_{max})$,
3) for the given course of the output signal of the filter $y_f(k)$, calculate the parameters of the model τ and T for different pairs of points $y_f(t_v)$ i $y_f(t_w)$ in compliance with the equations (9) and (10); then select these parameters of the model that would be the best from the point of view of the adopted criterion,
4) increase M by the successive value from the adopted range $M=(0, M_{max})$ and repeat the calculations.

The estimation of the model parameters can also be performed by means of one of direct methods (batch methods). For the purposes of comparison, the model parameters were calculated by means of the least squares method. In this case the identification is reduced to calculating the parameters of the finite-difference equation based on the pair of signals u, y. It can be assumed that there exists such a vector $\theta_{LS} = [\,a_1,\ a_2,\ ...,\ a_n,\ b_0,\ b_1,\ ...,\ b_n]^T$ that there occurs an equation which describes the dynamics the object and the equation has the following form:

$$\begin{aligned} y(k) = -a_1 \cdot y(k-1) - \ldots - a_n \cdot y(k-n) + b_0 \cdot u(k-m) + \\ \ldots + b_n \cdot u(k-n-m) \end{aligned} \tag{12}$$

where:
$m=\tau/T_s$ – represents time delay.

The estimator of the least squares θ_{LS} can be put in the vector and matrix form:

$$\theta_{LS} = \left(C^T C\right)^{-1} C^T y \tag{13}$$

While determining dynamic models, the C matrix contains both input and output quantities. The calculated parameters of the finite-difference equation (12) require that the parameters have to be recalculated into the form of a continuous model.

3.2. Assessing accuracy of the model parameters

In order to assess how the model matches the empirical data, it is necessary to adopt a certain criterion. When the model parameters are identified, the reference signal is not known, and to assess the model quality one has to use the registered measurement data and the signat at the model output (while knowing the input signal). In the case of direct methods, the $e(k)$ error, expressed as a difference between the input signal of the object (measurement data) and the output signal of the model, can be calculated according to the following dependency:

$$e(k) = y(k) - \hat{y}(k) \tag{14}$$

where:
$y(k)$ – measurement data,
$\hat{y}(k)$ – signal at the output of the model,
k – successive moments of time.

Using the equation(14) it is possible to calculate the sum square of the $e(k)$ error, which is expressed by the following formula:

$$SSE = \sum_{k=1}^{N}\left(e(k)\right)^2 = \sum_{k=1}^{N}\left(y(k) - \hat{y}(k)\right)^2 \tag{15}$$

where: N – number of samples.

The equation (15) is a classical assessment criterion of the model. The model quality can be also assessed by examining the value of the criterion in the form of residual variance, which can be put in the following way:

$$J = \sigma_e^2 = \frac{1}{N-1}\sum_{k=1}^{N}\left(e(k)\right)^2 = \frac{1}{N-1}\sum_{k=1}^{N}\left(y(k) - \hat{y}(k)\right)^2 \tag{16}$$

Thus the issue of a criterion to assess the accuracy of the model can be reduced to the analysis of residual variance values. Then the identification should lead to determining such parameters of the model and the filter which will minimize the criterion (16). This condition can be put in the following way:

$$\hat{\theta}, M = \min\{J\} \tag{17}$$

where:

$\hat{\theta}$ – model parameters of dynamic properties of the flotation process – T, τ, calculated with the use of the given estimation method.

3.3. Results of model parameters estimation

In order to determine dynamic properties of the flotation process, the measurement data were used from [4][6]. The measurement data of ash content come from the readings of ash meters for flotation tailings, while the parameters of the feed were monitored and their courses were, approximately, constant in time. The methods described in section 3.1 were used to determine the parameters of the dynamic models. The method to determine the parameters of the dynamic model with the use of equations (9) and (10) and a digital filter (11) was designated as *method* 1, while the least squares method – as *method* 2.

The results of the identification of the dynamic models parameters were presented in Table 1, while sample courses – in Fig. 5.

Table 1.

Results of the identification of the dynamic models of the coal flotation process

No	**Method**	**Δu [l/h]**	**T_s [s]**	**Parameters of finite-difference equation (12)**		**Parameters of continuous model**		**Residual variance**	**Initial values**		**M**
				a_1	**b_m**	**T [s]**	**τ [s]**		**y_0 [%]**	**u_0 [l/h]**	
1	1	3.6	60	-	-	215.4	191.9	0.73	49.7	3.0	6
	2			-0.7029	0.6886	170.2	180.0	0.94			-
2	1	6	60	-	-	415.8	157.6	1.63	53.8	1.5	4
	2			-0.8304	0.5272	322.8	180.0	2.29			-

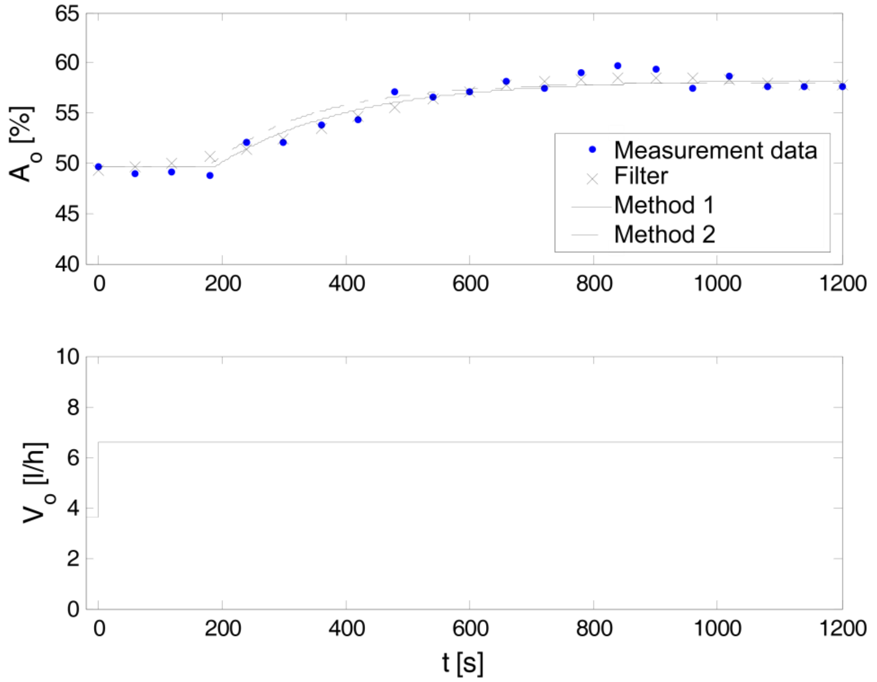

Fig. 5. Course of the signal at the output of the filter and the response of ash content in flotation tailings to the change in the flow intensity of the flotation reagent – example 1, tab. 1

Due to a non-linear character of the coal flotation process, the method based on the equations (9), (10) and data smoothing by means of a digital filter (11) makes it possible to determine the parameters of the first-order model with time delay on the basis of the step characteristics. In this case linearization takes place with the transfer of the object from one working point to another (with higher value) as a result of step increase in the control signal. The calculation results show that method 1, resulting from the equations (9), (10) and (11), can give satisfactory results as far as the minimization of the adopted criterion is concerned (16).

In both analized cases the use of the method based on the equations (9), (10) and (11) gave lower values of the adopted criterion than the least squares method. It is important to note that in comparison with the least squares method, the calculated delay time in the general case is not a multiple number of the sampling period. Therefore sometimes the achieved results can be better than those obtained with the use of the least squares method. The method based on the equations (9) and (10) requires to select the parameter of the filter and needs more calculations than the least squares method, yet the obtained results are not always satisfactory. What is more, the method is limited to the estimation of the model parameters based only on the step response of the object.

4. CONCLUSIONS

The basic stages of the identification experiment are the discussed issues concerning the measurements of dynamic properties of the coal flotation process with the use of the step response method, the selection of the method to estimate the model parameters and the criterion of the model adaptation to the empirical data. Dynamic properties of the coal flotation object with one control input (V_o) and one output available for measurements (A_o) can be presented as models with a structure of an inertial first-order element with time delay.

The article features a comparison of two methods: the method based on the equations (9), (10) and a digital filter in the equation (11) is compared with the least squares method. On the basis of the comparison we can ascertain that the use of the method based on the equations (9), (10) and the digital filter (11) can give better results (with respect to the adopted criterion) than the least squares method. However, it is important to note that this method can be used in certain conditions and only with respect to measurement data which are achieved by means of the step response method. The presented method for estimating the model parameters on the basis of the step response method (method 1) can be a useful tool to calculate the model of the coal flotation dynamics.

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