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FAULT DIAGNOSIS FOR COMPLEX SYSTEMS BASED ON DYNAMIC EVIDENTIAL NETWORK AND MULTI-ATTRIBUTE DECISION MAKING WITH INTERVAL NUMBERS

DIAGNOSTYKA USZKODZEŃ SYSTEMU ZŁOŻONEGO OPARTA NA DYNAMICZNYCH SIECIACH DOWODOWYCH ORAZ WIELOATRYBUTOWEJ METODZIE PODEJMOWANIA DECYZJI Z WYKORZYSTANIEM LICZB INTERWAŁOWYCH

The complexity of modern system structures and failure mechanisms makes it very difficult to locate the system fault. It has characteristics of dynamics of failure, diversity of distribution and epistemic uncertainties, which increase the challenges in the fault diagnosis significantly. This paper presents a fault diagnosis framework for complex systems within which the failure rates of components are expressed in interval numbers. Specifically, it uses a dynamic fault tree (DFT) to model the dynamic fault behaviors and deals with the epistemic uncertainties using Dempster-Shafer (D-S) theory and interval numbers. Furthermore, a solution is proposed to map a DFT into a dynamic evidential network (DEN) to calculate the reliability parameters. Additionally, diagnostic importance factor (DIF), Birnbaum importance measure (BIM) and heuristic information values (HIV) are taken into account comprehensively in order to obtain the best fault search scheme using an improved VIKOR algorithm. Finally, an illustrative example is given to demonstrate the efficiency of this method.

Keywords: *diagnosis strategy, D-S theory, interval numbers, dynamic evidential network, VIKOR.*

Złożoność nowoczesnych struktur systemowych oraz mechanizmów uszkodzeń powoduje trudności w lokalizacji uszkodzeń systemu. Systemy złożone charakteryzują się cechami, takimi jak dynamika uszkodzeń, różnorodność rozkładów oraz niepewność epistemiczna, które czynią wyzwania dotyczące diagnostyki uszkodzeń znacznie trudniejszymi. W niniejszym artykule przedstawiono metodę diagnozowania uszkodzeń systemów złożonych, w której intensywność uszkodzeń poszczególnych składników wyraża się za pomocą liczb przedziałowych. W szczególności, podejście to wykorzystuje dynamiczne drzewo błędów (DFT) do modelowania dynamicznych zachowań związanych z uszkodzeniami oraz rozwiązuje problem niepewności epistemicznej przy użyciu teorii Dempstera-Shafera (DS) oraz liczb przedziałowych. W celu obliczenia parametrów niezawodności, zaproponowano rozwiązanie polegające na odwzorowaniu DFT w dynamiczną sieć dowodową (DEN). Dodatkowo, w sposób kompleksowy wykorzystano czynnik ważności diagnostycznej (DIF), miarę ważności Birnbauma (BIM) oraz wartości informacji heurystycznej (HIV), aby przy użyciu udoskonalonego algorytmu VIKOR uzyskać najlepszy system wyszukiwania błędów. Skuteczność omawianej metody zilustrowano na podstawie przykładu.

Słowa kluczowe: *strategia diagnostyczna, teoria Dempstera–Shafera, liczby przedziałowe, dynamiczna sieć dowodowa, VIKOR.*

1. Introduction

With the development of science and technology, the functional requirement and modernization level of modern equipments are increasing, which makes these systems become more and more complex and raises some challenges in fault diagnosis. These challenges are shown as follows. (1) Failure dependency of components. Modern engineering systems are becoming increasingly complex, which makes components interact with each other. So, dynamic fault behaviors should be taken into account to construct the fault model. (2) The life distributions of components are different. Modern systems include a variety of components, and they may have different life distributions. Some classical static modeling techniques, including reliability block diagram model [12], fault tree (FT) model [20], and binary decision diagrams (BDD) model [23] have been widely used to model static systems. But these models assume that all components follow the exponential distribution. However, in the practical engineering, different components may have different distributions. For complex systems, a mixed life distribution should be used to analyze these systems. (3) There are a large number of uncertain factors and uncertain information. Many complex systems have adopted a variety of fault tolerant technologies to improve their dependability. However, high reliability makes it difficult to get sufficient fault data. In the case of the small sample data, the traditional methods based on the probability theory are no longer appropriate for complex systems. Aiming at these challenges mentioned above, many efficient diagnostic methods have been proposed. In order to model the dynamic failure characteristics, DFT [6], Markov model [28] and dynamic Bayesian networks (DBN) [9, 26] have been proposed to capture the above mentioned dynamic failure behaviors. DFT is widely used to model the dynamic systems as the extensions of the traditional static fault trees with sequence- and function-dependent failure behaviors. Ge et al. present an improved sequential binary decision diagrams (SBDD) method for highly coupled DFT where different dynamic gates often coexist and interact by repeated events [7]. A new approach was proposed by Merle et al. to solve DFT with priority dynamic gate and repeated events [17]. Chiacchio et al. presented a composition algorithm based on a Weibull distribution to address the resolution of a general class of DFT [2]. However, these methods assume that all components obey to the same distribution and cannot handle the challenge (2). Furthermore, these methods, which are usually assumed that the failure rates of the components are considered as crisp values describing their reliability characteristics, have been found to be inadequate to deal with the challenge (3) mentioned above. Therefore, fuzzy sets theory has been introduced as a useful tool to handle the challenge (3). The fuzzy fault tree analysis model employs fuzzy sets and possibility theory, and deals with ambiguous, qualitatively incomplete and inaccurate information [8, 16, 18]. To deal with the challenge (1) and (3), fuzzy DFT analysis has been introduced [13-14] which employs a DFT to construct the fault model and calculates the reliability results based on the continuous-time BN under fuzzy numbers. However, these approaches cannot handle the challenge (2). For this purpose, Mi et al. proposed a new reliability assessment approach which used a DFT to model the dynamic characteristics within complex systems and estimated the parameters of different life distributions using the coefficient of variation (COV) method [19]. To a certain extent, this method can meet the above challenges. But it is confined to the reliability analysis and cannot be used for the fault diagnosis. Dugan introduced a diagnostic importance factor (DIF) to determine the diagnosis sequence using DFT analysis [1]. However, the solution for DFT is based on Markov Chain which has an apparent state space explosion problem. In the work of [3], a hybrid fault diagnosis approach was proposed based on fault tree analysis and Bayesian network. Nevertheless, it used a static fault tree model and could not capture the dynamic failure behaviors. Furthermore, diagnosis strategies of these methods are only based on DIF and usually could not do decision making when there were many attributes for consideration. In addition, these diagnostic methods are usually assumed that the failure rates of components are regarded as crisp values and cannot deal with the challenge (3). To overcome these difficulties and limitations, Duan et al. proposed a diagnosis method based on fuzzy sets theory and DFT, which used fuzzy sets theory to estimate the failure rates of basic events and solved the DFT based on discrete-time Bayesian networks [5]. However, this approach could not handle the challenge (2). In addition, all the diagnosis algorithms are based on the single attribute decision making, and usually cause minimal cut sets with a smaller DIF to be diagnosed first [24], thereby influencing the diagnosis efficiency.

Motivated by the problems mentioned above, this paper presents a novel diagnosis strategy for complex systems based on DEN and an improved VIKOR algorithm shown in Fig. 1. It pays close attention to meeting above three challenges. In view of the challenge (1), it uses a DFT to capture the dynamic failure mechanisms. For the challenges (2) and (3), a mixed life distribution is used to analyze complex systems, and the COV method is employed to estimate the parameters of life distributions for components with interval numbers. Furthermore, relevant reliability parameters can be calculated by mapping a DFT into a DEN in order to avoid the aforementioned problems. At last, components' DIF, BIM and HIV are taken into account comprehensively to design a novel diagnosis strategy using an improved VIKOR algorithm. The proposed method takes full advantages of DFT, interval numbers for handling uncertainty, DEN for inference and VIKOR for the best fault search scheme, which is especially suitable for fault location of complex systems.

The remaining of this paper is organized as follows: In section 2, a DEN modeling is introduced and the conversion process from a DFT to a DEN is also provided; Section 3 presents a new fault diagnosis method based on an improved VIKOR algorithm; An illustrative example is provided to demonstrate the proposed method in Section 4; Finally, conclusions are made in Section 5.

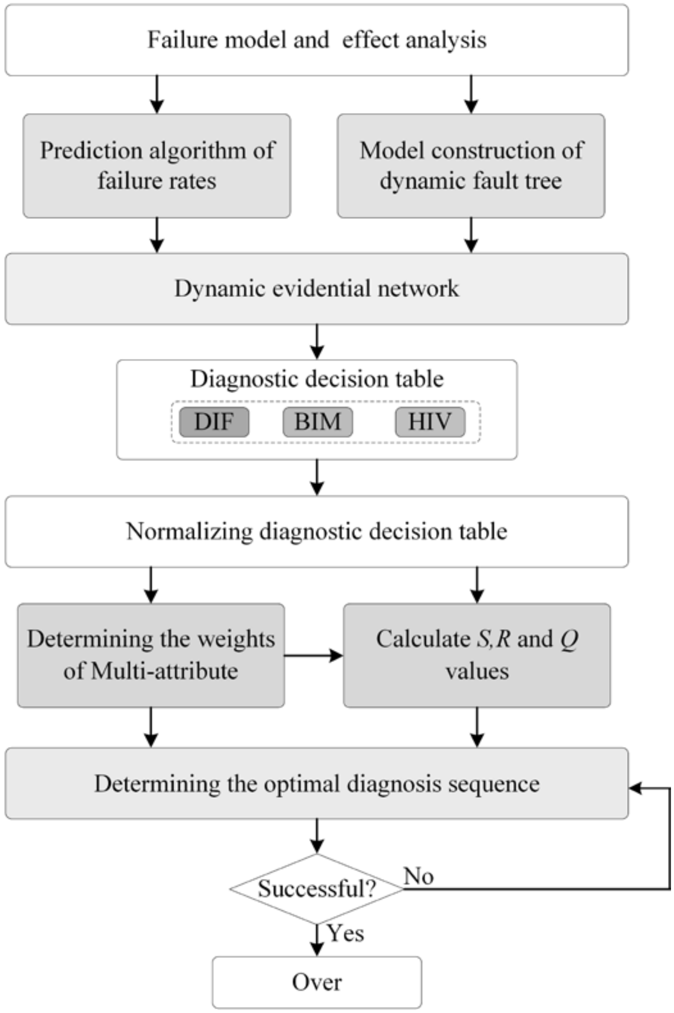

Fig. 1. A novel fault diagnosis framework for complex systems

2. DEN

D-S evidence theory has a unique ability in the expression of epistemic uncertainties. The evidence theory can be well compatible with the theory of probability. This section will describe how to compute the reliability parameters using DEN. The following simply introduces the relevant definitions and theorems in this paper, and more information can be referred to literatures [4,10,22]. Evidential Network is based on graph theory and D-S theory. It is a promising graphical tool for representing and managing uncertainties. Each node represents a variable, and arcs indicate direct conditional relations between the connected nodes. DEN, an extension of evidential network, takes into account the time by defining different nodes to model variables with respect to different time slices [21]. It includes the initial network and the temporal transition network. Each time slice corresponds to a static evidential network, and the time slices are a directed acyclic graph $G_T = < V_T, E_T >$ corresponding to the conditional probabilities. The V_T and E_T are respectively nodes of time T and directed arcs. A directed arc linked two variables belonging to different time slices.

In evidence theory, $\Theta = \{W_i, F_i\}$ is the knowledge framework of the component i and the focal elements are given by:

$$2^{\Theta} = \{\{\varnothing\}, \{W_i\}, \{F_i\}, \{W_i, F_i\}\} \tag{1}$$

where $\{W_i\}$ and $\{F_i\}$ denote the working and the failure state respectively. The state of $\{W_i, F_i\}$ corresponds to the epistemic uncertainty.

Belief measure (Bel) defines the lower bound of the probabilities that the focal element exists, and plausibility measure (Pl) defines the upper bound of the probabilities that the focal element exists. The basic belief assignment in the system state expresses an epistemic uncertainty, where *Bel* and *Pl* measures are not equal and bound the system reliability. Therefore, the basic probability assignment (BPA) of component *i* can be computed as:

$$\begin{aligned} & m(\{W_i\}) = Bel(\{W_i\}) \\ & m(\{F_i\}) = 1 - Pl(\{W_i\}) \\ & m(\{W_i, F_i\}) = Pl(\{W_i\}) - Bel(\{F_i\}) \end{aligned} \tag{2}$$

Presumably, the upper and lower bounds of the component reliability [$\underline{P(x)}, \overline{P(x)}$] is equivalent to the BPA in the DEN:

$$\begin{aligned} & m(\{W_i\}) = 1 - \overline{P(x)} \\ & m(\{F_i\}) = \underline{P(x)} \\ & m(\{W_i, F_i\}) = \overline{P(x)} - \underline{P(x)} \end{aligned} \tag{3}$$

where $Bel(\{F_i\}) = \underline{P(x)}, Pl(\{F_i\}) = \overline{P(x)}$.

2.1. Mapping a static fault tree into a DEN

The conditional probabilities of each node in the static evidential network have been discussed in detail in [25]. Fig. 2 shows an AND gate and its equivalent DEN. Equation 4 and 5 give the conditional probability of each node.

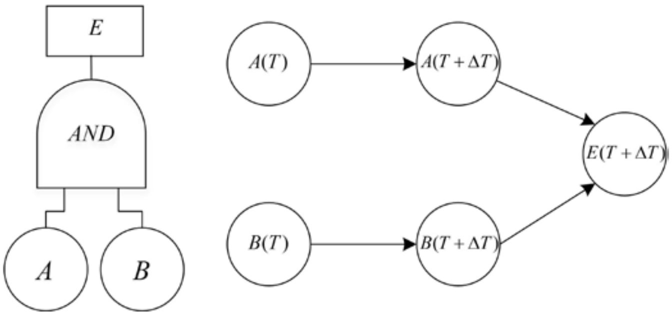

Fig. 2. An AND gate and its equivalent DEN

$$\begin{cases} P(A(T+\Delta T)=1 \mid A(T)=0) = 1-\overline{P(x)} \\ P(A(T+\Delta T)=\{0,1\} \mid A(T)=0) = \overline{P(x)}-\underline{P(x)} \\ P(A(T+\Delta T)=1 \mid A(T)=1) = 1 \\ P(A(T+\Delta T)=1 \mid A(T)=\{0,1\}) = 1-\overline{P(x)} \end{cases} \tag{4}$$

$$\begin{cases} P(E=1 \mid A(T+\Delta T)=1, B(T+\Delta T)=1) = 1 \\ P(E=\{0,1\} \mid A(T+\Delta T)=1, B(T+\Delta T)=\{0,1\}) = 1 \\ P(E=\{0,1\} \mid A(T+\Delta T)=\{0,1\}, B(T+\Delta T)=1) = 1 \\ P(E=\{0,1\} \mid A(T+\Delta T)=\{0,1\}, B(T+\Delta T)=\{0,1\}) = 1 \\ P(E=1 \mid else) = 0 \\ P(E=\{0,1\} \mid else) = 0 \end{cases} \tag{5}$$

2.2. Mapping a DFT into a DEN

DFT extended the traditional fault tree by defining some dynamic gates to capture the sequential and functional dependencies. Usually, there are six types of dynamic gates defined: the Functional Dependency Gates (FDEP), the Cold Spare Gates (CSP), the Hot Spare Gates (HSP), the Warm Spare Gates (WSP), the Priority AND Gates (PAND), and the Sequence Enforcing Gates (SEQ).

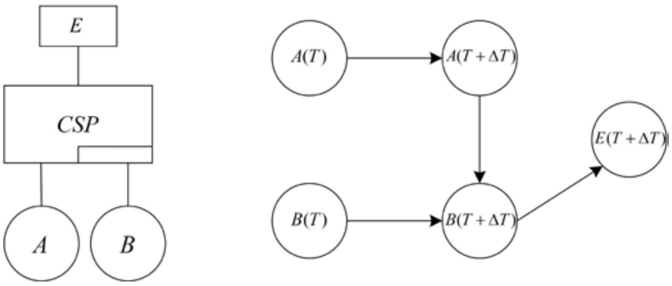

Fig. 3. CSP gate and its equivalent DEN

The following section briefly discusses a CSP gate as it is used later in the example. The CSP gate includes one primary input and one or more alternate inputs. Fig. 3 shows a CSP gate and its equivalent DEN. Suppose that *A* and *B* follow the same distribution, then $\underline{P(x)}$ and $\overline{P(x)}$ denote the lower probability and upper probability of the nodes respectively. At this point, node *A* has the same conditional probability with the AND gate of the node *A* and the conditional probability of other node *B* can be calculated by the following equations:

$$\begin{cases} P(B(T+\Delta T)=0 \mid A(T+\Delta T)=0) = 0 \\ P(B(T+\Delta T)=1 \mid A(T+\Delta T)=1, B(T)=0) = \underline{P(x)} \\ P(B(T+\Delta T)=1 \mid A(T+\Delta T)=1, B(T)=1) = 1 \\ P(B(T+\Delta T)=1 \mid A(T+\Delta T)=1, B(T)=\{0,1\}) = \underline{P(x)} \\ P(B(T+\Delta T)=\{0,1\} \mid A(T+\Delta T)=\{0,1\}) = \overline{P(x)}\text{-}\underline{P(x)} \end{cases} \tag{6}$$

$$\begin{cases} P(E=0 \mid B(T+\Delta T)=0) = 1 \\ P(E=1 \mid B(T+\Delta T)=1) = 1 \\ P(E=\{0,1\} \mid B(T+\Delta T)=\{0,1\}) = 1 \end{cases} \tag{7}$$

2.3. Calculating reliability parameters

After a DFT model is built, The DFT is converted into an equivalent DEN using the proposed method. Once the structure of the DEN is known and the probability tables are filled, the reliability parameters of the system can be calculated using the DEN inference algorithm. These reliability parameters mainly include system unreliability, DIF, BIM and HIV, which are used for fault diagnosis in the proposed method.

2.3.1. System unreliability

Calculating the system unreliability is very simple using the following equation:

$$P_S = [\underline{P_S}, \overline{P_S}] = [Bel(\{F_S\}), Pl(\{F_S\})] \tag{8}$$

where $[Bel(\{F_S\}), Pl(\{F_S\})]$ represents the failure probability of system.

2.3.2. DIF

DIF is defined conceptually as the probability that an event has occurred given that the top event has also occurred. DIF is the cornerstone of reliability based diagnosis methodology [1]. DIF can be used to locate the faulty components in order to minimize the system checks and diagnostic cost. It is given by:

$$DIF_i = P(i \mid S) = [Bel(\{F_{i|S}\}), Pl(\{F_{i|S}\})] \tag{9}$$

where i is a component of system S; $P(i \mid S)$ is the probability that the basic event i has occurred given the top event has occurred.

Suppose the system has failed at the mission time, we input the evidence that system has failed into DEN and get the DIF of components using the inference algorithm.

2.3.3. BIM

Birnbaum first introduced the concept of a components' reliability importance in 1969. This measure was defined as the probability that a component is critical to system failures. i.e. when component i fails it causes the system to move from a working to a failed state. BIM of a component i can be interpreted as the rate at which the system's reliability improves as the reliability of component i is improved [21]. Analytically, Birnbaum's importance interval measure of a component i can be defined using D-S theory by the following equation:

$$\begin{aligned}[I^B(i)] &= [Bel(\{W_S\} \mid \{W_i\}), Pl(\{W_S\} \mid \{W_i\})] - [Bel(\{W_S\} \mid \{F_i\}), Pl(\{W_S\} \mid \{F_i\})] \\ &= [\underline{P}_{S|Wi}, \overline{P}_{S|Wi}] - [\underline{P}_{S|Fi}, \overline{P}_{S|Fi}]\end{aligned} \tag{10}$$

where $Bel(\{W_s\} \mid \{W_i\})$ and $Pl(\{W_s\} \mid \{W_i\})$ denote respectively the belief and plausibility measures that the system is functioning when it is known that component i is in a working state. Whereas $Bel(\{W_s\} \mid \{F_i\})$ and $Pl(\{W_s\} \mid \{F_i\})$ denote respectively the belief and plausibility measures that the system is functioning when component i is in a failed state.

2.3.4. HIV

The heuristic function plays an important role in the diagnostic sequence [11]. Owing to the different complexity of components their test cost is different, a balance should be taken into account between the DIF and test cost. Therefore, a new heuristic function for complex systems, HIV is proposed. HIV represents the value of the heuristic information contained in each fault search path and the influence degree of the fault search on the next optimal fault search. With the combination of DIF and the test cost, HIV is defined by the following expression:

$$HIV_i = \frac{DIF_i}{T_i} = \frac{P(i \mid s)}{T_i} \tag{11}$$

Table 1. Evaluation standards of the test cost

Linguistic expression for test cost	Interval values
Very High	[0.9 1.0]
High	[0.7 0.9]
Moderate	[0.5 0.7]
Low	[0.3 0.5]
Very Low	[0.1 0.3]

The test cost of the components is usually very difficult to express as crisp values because of uncertainties. So the linguistic assessments are used for generating criteria and alternative ratings, which are transformed into interval numbers to describe test cost of the components for treatment by VIKOR. Table 1 shows the evaluation criteria and alternative ratings of the test cost.

3. Fault diagnosis strategy based on an improved VIKOR algorithm

The basic information provided by reliability analysis can be used to construct the diagnostic decision table for fault diagnosis. Assume that the DEN has n root nodes, each root node represents a diagnostic scheme, $x_i (i = 1, 2, \cdots, n)$ represents the diagnostic scheme and each root node has k reliability parameters. DIF enables us to discriminate between components by their importance from a diagnostic point of view. BIM is used to quantify the contributions of components' reliability to the systems' reliability and HIV plays an important role in the diagnostic sequence. DIF, BIM, and HIV are treated as attribute v1, v2 and v3 respectively. These attributes can be considered comprehensively to obtain the best faulty search scheme using an improved VIKOR algorithm [27].

3.1. Normalizing diagnostic decision table

Fault diagnosis is a process to optimize multi-attribute decision making. After the search scheme for fault diagnosis is defined, we can construct the diagnostic decision table by the corresponding evaluation attributes. However, different evaluation attributes usually have different values and dimensions, which are not directly comparable, so we should normalize the diagnostic decision table. Evaluation attributes can be divided into two classes: benefit attributes and cost attributes. There are three attributes in the diagnostic decision table, DIF, BIM and HIV, which belong to the benefit attributes. For the different data, we use the following formula to normalize them.

When the attribute x_{ij} is a benefit attribute, we use the following formula to normalize them:

$$\tilde{f}_{ij}^t = [\underline{f}_{ij}^t, \overline{f}_{ij}^t] = [\frac{\underline{x}_{ij}^t}{\sum_{i=1}^m \overline{x}_{ij}^t}, \frac{\overline{x}_{ij}^t}{\sum_{i=1}^m \underline{x}_{ij}^t}] \tag{12}$$

where x_{ij} is the j^{th} attribute value of the i^{th} component.

When the attribute x_{ij} is a cost attribute, we normalize them by using the following formula:

$$\tilde{f}_{ij}^t = [\underline{f}_{ij}^t, \overline{f}_{ij}^t] = [\frac{\frac{1}{\overline{x}_{ij}^t}}{\sum_{i=1}^m \frac{1}{\underline{x}_{ij}^t}}, \frac{\frac{1}{\underline{x}_{ij}^t}}{\sum_{i=1}^m \frac{1}{\overline{x}_{ij}^t}}] \tag{13}$$

3.2. Determining the weights of attributes

Shannon Entropy is a measure of uncertainty of information formulated in terms of probability theory [15]. It is well suited for measuring the relative contrast intensities of criteria to represent the average intrinsic information transmitted to the decision makers. Entropy weighting is a multi-attribute decision making (MADM) method used to determine the important weights of decision attributes by directly relating a criterion's importance weighting relative to the information transmitted by that criterion. However, because the elements of the decision matrix are interval numbers, the Entropy method cannot be used directly. Therefore, before the entropy method is put into use, the decision matrix needs to be quantized.

The diagnosis decision table needs to be normalized before the positive and negative ideal solutions are being calculated. The positive ideal solutions are made of all the best performance scores, and the negative solutions are made of all the worst performance scores at these measures in the diagnostic decision table. To compute the positive and negative ideals, by the relations:

$$f_j^+ = \max_{i\in M} \bar{f}_{ij}, f_j^- = \min_{i\in M} \underline{f}_{ij} \quad j \in N \tag{14}$$

Suppose that $a=[a^-,a^+]$ and $b=[b^-,b^+]$ are two interval numbers, the interval deviation degree distance $D(a,b)$ between $a=[a^-,a^+]$ and $b=[b^-,b^+]$ is :

$$D(a,b) = \sqrt{(a^- - b^-)^2 + (a^+ - b^+)^2} \tag{15}$$

The larger the interval deviation degree distance $D(a,b)$, the greater the degree of phase separation will be. In particular, when $D(a,b)=0$, then $a = b$, which means that a and b are equal.

The diagnostic decision table is the interval numbers, which are difficult to directly compare. In order to determine the weight of attributes, the concept of the interval deviation degree distance is used. The objective weights of attributes can be calculated based on the Entropy concept through the following steps:

Step 1: Transform the normalization matrix $\tilde{f}_{ij}^t = [\underline{f}_{ij}^t, \bar{f}_{ij}^t]$ into the interval deviation degree distance matrix $D=(d_{ij})_{n\times m}$, where $d_{ij} = D(f_{ij}, f_j^*)$; $f_j^* = [f_j^-, f_j^+]$.

Step 2: Normalize the evaluation criterion for the interval deviation degree distance matrix through:

$$p_{ij} = \frac{d_{ij}}{\sum_{i=1}^{n} d_{ij}} \tag{16}$$

where $\sum_{i=1}^{n} p_{ij} = 1, j = 1,2,\cdots,m$.

Step 3: Obtain the entropy $H_j H_j$ value of the attributes j as follows:

$$H_j = -K\sum_{i=1}^{n} p_{ij} \ln p_{ij} (j=1,2,\cdots,m) \tag{17}$$

where $K = 1/\ln n$ $(K > 0, 0 \le p_{ij} \le 1)$ and assume $p_{ij} = 0$, then $p_{ij} \ln p_{ij} = 0$.

Step 4: Define the value of α_j through:

$$\alpha_j = 1 - H_j \tag{18}$$

Where α_j is the divergence degree of the intrinsic information of the attributes j. The greater the value of α_j, the more important the attribute is in the decision making process.

Step 5: Calculate the weights of attributes using the following equation:

$$\omega_j = \frac{\alpha_j}{\sum_{j=1}^{m} \alpha_j} \tag{19}$$

3.3. Calculating the values $S_i = [\underline{S}_i, \bar{S}_i]$ and $R_i = [\underline{R}_i, \bar{R}_i]$

The value $S_i = [\underline{S}_i, \bar{S}_i]$ of all the decision-making program group is calculated by the linear programming mothed:

$$\min S_i^{'} = \sum_{j=1}^{n} \omega_j^{'} \left(\frac{f_j^+ - \bar{f}_{ij}}{f_j^+ - f_j^-} \right)$$
$$s.t. \begin{cases} \underline{\omega}_j \le \omega_j^{'} \le \bar{\omega}_j, j \in N \\ \sum_{j=1}^{n} \omega_j^{'} = 1 \end{cases} \tag{20}$$

$$\max S_i^{"} = \sum_{j=1}^{n} \omega_j^{"} \left(\frac{f_j^+ - \underline{f}_{ij}}{f_j^+ - f_j^-} \right)$$
$$s.t. \begin{cases} \underline{\omega}_j \le \omega_j^{"} \le \bar{\omega}_j, j \in N \\ \sum_{j=1}^{n} \omega_j^{"} = 1 \end{cases} \tag{21}$$

Suppose the optimal solutions of model (20) and (21) are $\omega^{'} = (\omega_1^{'}, \omega_2^{'}, \cdots, \omega_n^{'})$ and $\omega^{"} = (\omega_1^{"}, \omega_2^{"}, \cdots, \omega_n^{"})$ $S_i = [\underline{S}_i, \bar{S}_i]$ respectively, then we can compute the interval values $S_i = [\underline{S}_i, \bar{S}_i]$ by the linear programming method. where $\underline{S}_i$ and $\bar{S}_i$ are defined by:

$$\underline{S}_i = \sum_{j=1}^{n} \omega_j^{'} \left(\frac{f_j^+ - \bar{f}_{ij}}{f_j^+ - f_j^-} \right) \tag{22}$$

$$\bar{S}_i = \sum_{j=1}^{n} \omega_j^{"} \left(\frac{f_j^+ - \underline{f}_{ij}}{f_j^+ - f_j^-} \right) \tag{23}$$

Similarly, the interval values $R_i = [\underline{R}_i, \bar{R}_i], i \in M$ can also be computed by the linear programming method. where $\underline{R}_i$ and $\bar{R}_i$ are given by:

$$\underline{R}_i = \max_{j\in N} \left\{ \omega_j^{'} \left(\frac{f_j^+ - \bar{f}_{ij}}{f_j^+ - f_j^-} \right) \right\} \tag{24}$$

$$\bar{R}_i = \max_{j\in N} \left\{ \omega_j^{"} \left(\frac{f_j^+ - \underline{f}_{ij}}{f_j^+ - f_j^-} \right) \right\}, i \in M \tag{25}$$

3.4. Calculating the values $Q_i = [\underline{Q}_i, \bar{Q}_i]$

We can calculate the values of $Q_i = [\underline{Q}_i, \bar{Q}_i]$ by the relations:

$$\underline{Q}_i = v\frac{\underline{S}_i - S^-}{S^+ - S^-} + (1-v)\frac{\underline{R}_i - R^-}{R^+ - R^-} \quad (26)$$

$$\overline{Q}_i = v\frac{\overline{S}_i - S^-}{S^+ - S^-} + (1-v)\frac{\overline{R}_i - R^-}{R^+ - R^-} \quad (27)$$

where $S^- = \min_i \underline{S}_i, S^+ = \max_i \overline{S}_i, R^- = \min_i \underline{R}_i, R^+ = \max_i \overline{R}_i$ and v is introduced as the weight for the strategy of maximum group utility, whereas $1-v$ is the weight of the individual regret. Usually, v can take any value from 0 to 1 and the value of v is set to 0.5 in the paper.

3.5. Determining the optimal diagnosis sequence

After the value of $Q_i = [\underline{Q}_i, \overline{Q}_i]$ expressed in interval numbers is obtained, the possibility matrix should be built to rank the alternatives. The possibility matrix can be defined as:

$$p = \begin{bmatrix} 0.5 & p(Q_1 \geq Q_2) & \cdots & p(Q_1 \geq Q_m) \\ p(Q_2 \geq Q_1) & 0.5 & \cdots & p(Q_2 \geq Q_m) \\ \cdots & \cdots & \ddots & \cdots \\ p(Q_m \geq Q_1) & p(Q_m \geq Q_2) & & 0.5 \end{bmatrix} \quad (28)$$

Then the corresponding possibility $p(x_i)$ can be obtained using the following equation.

$$p(x_i) = \sum_{j=1}^{m} p(Q_k \geq Q_i), i,k \in M \quad (29)$$

Obviously, the smaller the value $p(x_i)$, the better the diagnostic scheme. Therefore, we can determine the optimal ranking order by the value $p(x_i)$ and choose the diagnostic scheme with the minimum value $p(x_i)$.

4. Numerical Application

An illustrative example is given to illustrate how the proposed method can be used to perform the diagnosis strategy analysis for the braking system using multi-attribute decision making with interval numbers. Suppose all components follow the exponential distribution or two-parameter Weibull distribution. For the components with an exponential distribution, the interval failure rates of the basic events for the braking system can be calculated using the expert elicitation and the fuzzy sets theory. For the components with a two-parameter Weibull distribution, the interval failure rates are calculated using the COV method [19]. DFT of the braking system is shown in Fig. 4. The interval failure rates of basic events are shown in Table 2. We can map the DFT into the equivalent DEN shown in Fig. 5.

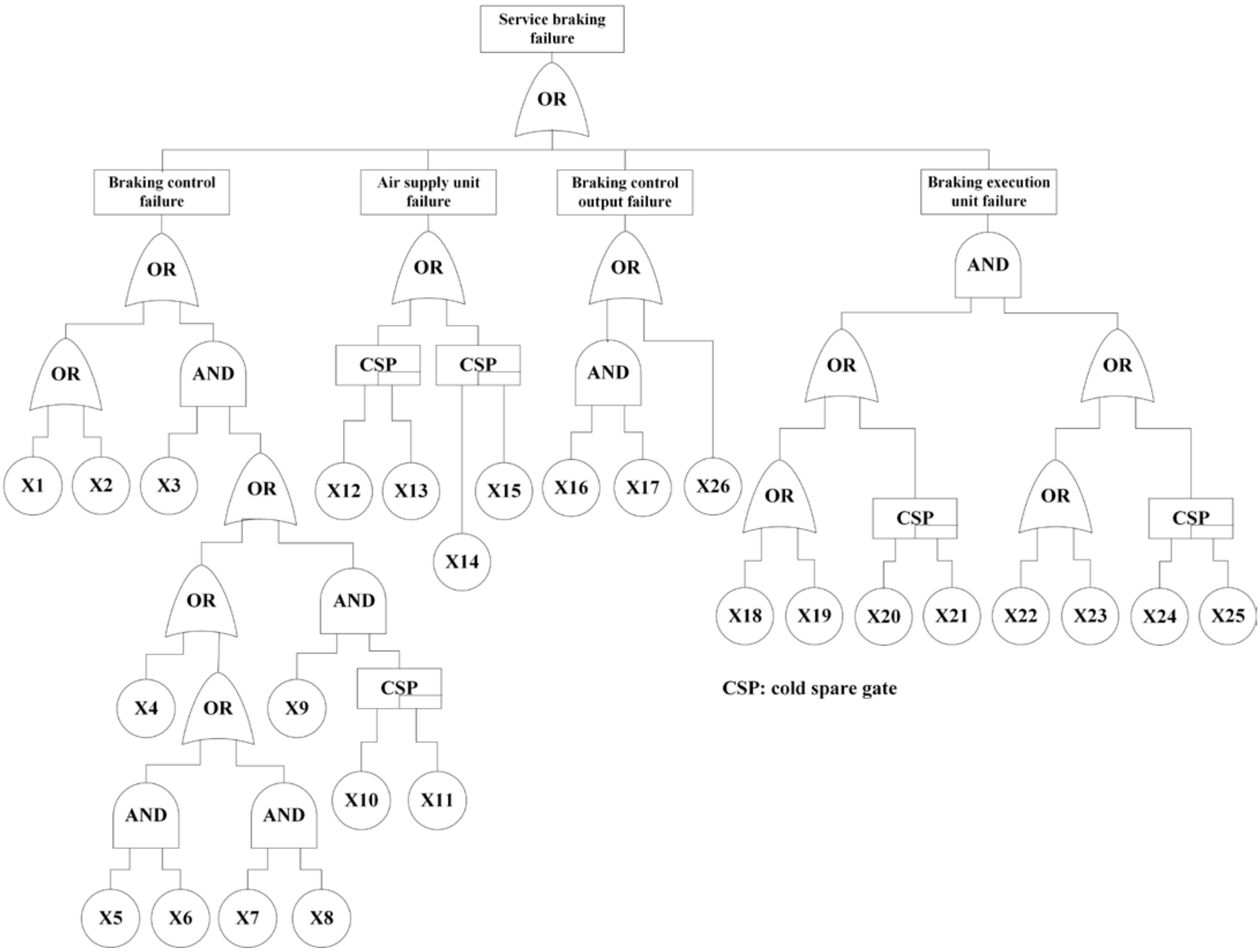

Fig. 4. A DFT for service braking failure of braking syste

Table 2. The interval failure rates of basic events

Components	Failure rate/ hour	Components	Failure rate/hour
X1	[2.88e-6 4.20e-6]	X12, X13	[6.96e-6 1.04e-5]
X3, X9	[6.08e-7 9.12e-7]	X14, X15	[5.68e-6 8.52e-6]
X10, X11	[6.08e-7 9.12e-7]	X16, X17	[5.44e-7 8.16e-7]
X4	[3.28e-7 4.92e-7]	X18, X19	[3.84e-5 5.76e-5]
X5	[1.12e-5 1.68e-5]	X20, X21	[3.84e-5 5.76e-5]
X6	[0.80e-6 1.20e-6]	X22, X23	[3.04e-5 4.56e-5]
X7	[0.88e-5 1.32e-5]	X24, X25	[3.04e-5 4.56e-5]
X8	[7.12e-6 1.07e-5]	*X26*	[6.24e-6 9.36e-6]

Table 3. Occurrence probabilities of failure at the different mission time

Mission time (h)	Interval occurrence probability
500	[0.00644 0.01321]
1000	[0.01550 0.03322]
1500	[0.02690 0.05857]
2000	[0.04035 0.08805]

In this numerical example, component *X2* follows a two-parameter Weibull distribution with parameters η and β, and the distribution function is calculated as follows:

$$F(t) = P(T \le t) = \begin{cases} 1 - exp\{-(\frac{\Delta t}{\eta})^{\beta}\}, t > 0 \\ 0 \qquad\qquad\qquad t \le 0 \end{cases} \tag{30}$$

Table 4. Components' test cost and HIV of the components

Components	Test cost T_i	HIV
X1	[0.9 1.0]	[0.14233 0.16066]
X2	[0.5 0.7]	[0.06333 0.08992]
X3	[0.1 0.3]	[0.00413 0.01850]
X4	[0.7 0.9]	[0.00074 0.00143]
X5	[0.7 0.9]	[0.02461 0.04720]
X6	[0.7 0.9]	[0.00178 0.00343]
X7	[0.7 0.9]	[0.01939 0.03723]
X8	[0.7 0.9]	[0.01572 0.03020]
X9	[0.1 0.3]	[0.00407 0.01820]
X10	[0.1 0.3]	[0.00407 0.01820]
X11	[0.1 0.3]	[3.08e-06 0.00610]
X12	[0.9 1.0]	[0.01664 0.02607]
X13	[0.9 1.0]	[0.00297 0.01099]
X14	[0.9 1.0]	[0.00111 0.00183]
X15	[0.9 1.0]	[1.83e-5 0.00062]
X16	[0.1 0.3]	[0.00373 0.01660]
X17	[0.1 0.3]	[0.00373 0.01660]
X18	[0.5 0.7]	[0.42227 0.64426]
X19	[0.5 0.7]	[0.42227 0.64426]
X20	[0.3 0.5]	[0.13046 0.31073]
X21	[0.3 0.5]	[0.01766 0.01284]
X22	[0.5 0.7]	[0.42227 0.64426]
X23	[0.5 0.7]	[0.42227 0.64426]
X24	[0.3 0.5]	[0.13046 0.31073]
X25	[0.3 0.5]	[0.01766 0.12837]
X26	[0.5 0.7]	[0.43907 0.62332]

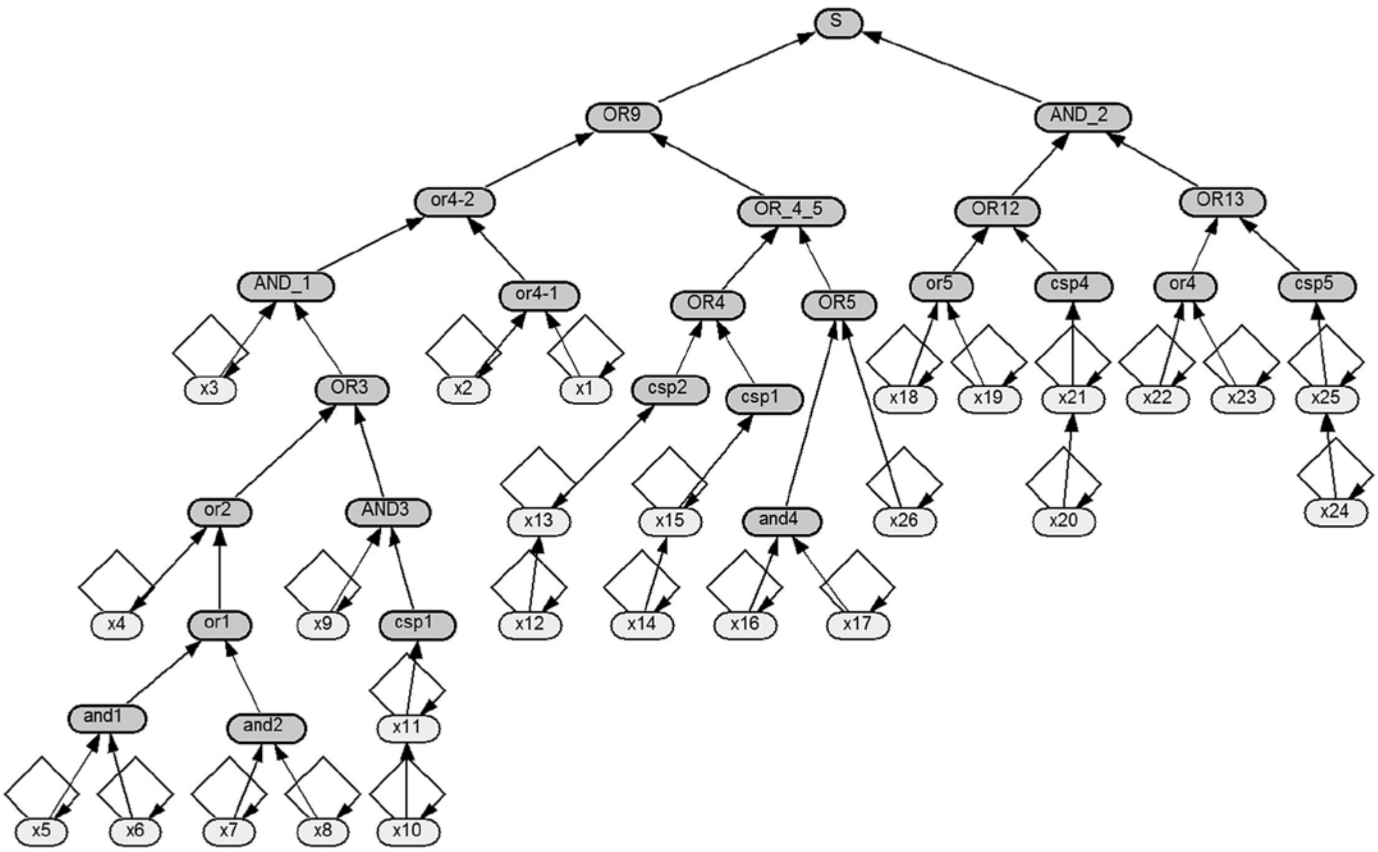

Fig. 5. A DEN of the braking system

Table 5. The diagnostic decision table for the braking system

Components	DIF	BIM	HIV
X1	[0.14233 0.14459]	[0.91965 0.965196]	[0.14233 0.16066]
X2	[0.04433 0.04496]	[0.91419 0.961368]	[0.06333 0.08992]
X3	[0.00124 0.00185]	[-0.04679 0.049173]	[0.00413 0.01850]
X4	[0.00067 0.00100]	[-0.04653 0.049354]	[0.00074 0.00143]
X5	[0.02215 0.03304]	[-0.04770 0.047695]	[0.02461 0.04720]
X6	[0.00160 0.00240]	[-0.04767 0.047745]	[0.00178 0.00343]
X7	[0.01745 0.02606]	[-0.04768 0.047724]	[0.01939 0.03723]
X8	[0.01415 0.02114]	[-0.04768 0.047734]	[0.01572 0.03020]
X9	[0.00122 0.00182]	[-0.04770 0.047695]	[0.00407 0.01820]
X10	[0.00122 0.00182]	[-0.04770 0.047695]	[0.00407 0.01820]
X11	[9.23e-7 0.00061]	[-0.04770 0.047695]	[3.08e-06 0.00610]
X12	[0.01664 0.02346]	[-0.03301 0.05561]	[0.01664 0.02607]
X13	[0.00297 0.00989]	[0.91843 0.95976]	[0.00297 0.01099]
X14	[0.00111 0.00165]	[-0.04654 0.048318]	[0.00111 0.00183]
X15	[1.83e-05 0.00056]	[0.91245 0.959648]	[1.83e-5 0.00062]
X16	[0.00112 0.00166]	[-0.04665 0.049183]	[0.00373 0.01660]
X17	[0.00112 0.00166]	[-0.04665 0.049183]	[0.00373 0.01660]
X18	[0.29559 0.32213]	[0.09422 0.228649]	[0.42227 0.64426]
X19	[0.29559 0.32213]	[0.09422 0.228649]	[0.42227 0.64426]
X20	[0.06523 0.09322]	[-0.03775 0.054004]	[0.13046 0.31073]
X21	[0.00883 0.03851]	[0.07898 0.219255]	[0.01766 0.12837]
X22	[0.29559 0.32213]	[0.09422 0.228649]	[0.42227 0.64426]
X23	[0.29559 0.32213]	[0.09422 0.228649]	[0.42227 0.64426]
X24	[0.06523 0.09322]	[-0.03775 0.054005]	[0.13046 0.31073]
X25	[0.00883 0.03851]	[0.07898 0.219255]	[0.01766 0.12837]
X26	[0.30735 0.31166]	[0.91919 0.971704]	[0.43907 0.62332]

Table 6. Interval values of S, R and Q for all components

Components	$S_i = [\underline{S}_i, \overline{S}_i]$	$R_i = [\underline{R}_i, \overline{R}_i]$	$Q_i = [\underline{Q}_i, \overline{Q}_i]$
X1	[0.4365 0.6208]	[0.2503 0.2896]	[0.5827 0.7376]
X2	[0.5777 0.7352]	[0.2873 0.3140]	[0.7120 0.8338]
X3	[0.9620 0.9974]	[0.3320 0.3328]	[0.9776 0.9969]
X4	[0.9718 0.9989]	[0.3329 0.3333]	[0.9840 0.9985]
X5	[0.9153 0.9723]	[0.3091 0.3326]	[0.9721 0.9866]
X6	[0.9697 0.9980]	[0.3317 0.3329]	[0.9183 0.9839]
X7	[0.9277 0.9782]	[0.3142 0.3326]	[0.9326 0.9868]
X8	[0.9364 0.9823]	[0.3179 0.3326]	[0.9427 0.9889]
X9	[0.9627 0.9976]	[0.3320 0.3328]	[0.9780 0.9971]
X10	[0.9627 0.9976]	[0.3320 0.3328]	[0.9780 0.9971]
X11	[0.9702 1.0000]	[0.3333 0.3339]	[0.9838 1.0000]
X12	[0.9336 0.9767]	[0.3200 0.3295]	[0.9445 0.9812]
X13	[0.6554 0.7903]	[0.3278 0.3326]	[0.8145 0.8908]
X14	[0.9971 0.9984]	[0.3326 0.3332]	[0.9832 0.9980]
X15	[0.6705 0.7952]	[0.3333 0.3339]	[0.8307 0.8953]
X16	[0.9632 0.9976]	[0.3322 0.3329]	[0.9785 0.9972]
X17	[0.9632 0.9976]	[0.3322 0.3329]	[0.9785 0.9972]
X18	[0.2466 0.5716]	[0.2466 0.3023]	[0.4797 0.7322]
X19	[0.2466 0.5716]	[0.2466 0.3023]	[0.4797 0.7322]
X20	[0.7144 0.8985]	[0.3045 0.3305]	[0.8086 0.9428]
X21	[0.8107 0.9595]	[0.2940 0.3281]	[0.8415 0.9703]
X22	[0.2466 0.5716]	[0.2466 0.3023]	[0.4797 0.7322]
X23	[0.2466 0.5716]	[0.2466 0.3023]	[0.4797 0.7322]
X24	[0.7134 0.8985]	[0.3045 0.3305]	[0.8080 0.9428]
X25	[0.8107 0.9595]	[0.2940 0.3281]	[0.8415 0.9703]
X26	[0.0217 0.3798]	[0.0109 0.1980]	[0 0.4727]

Table 7. The value of $p(x_i)$ for all components

Components	$p(x_i)$	Components	$p(x_i)$	Components	$p(x_i)$
X1	5.1472	X10	17.7954	X19	3.5006
X2	9.0748	X11	17.9431	X20	13.4062
X3	17.7862	X12	16.9198	X21	14.6798
X4	17.9213	X13	12.4142	X22	3.5006
X5	16.4790	X14	17.8992	X23	3.5006
X6	17.8562	X15	12.8610	X24	13.3949
X7	16.8035	X16	17.8063	X25	14.6798
X8	17.0278	X17	17.8063	X26	0.5000
X9	17.7954	X18	3.5006		

The interval life $[t_{R=0.95}\ \ t_{R=0.5}]$ of $X2$ is [2100 4200] using the general accelerated life test. Other components follow an exponential failure distribution with parameter λ, and the distribution function is calculated as follows:

$$F(t) = P(T \le t) = 1 - \exp(-\lambda \Delta t) \tag{31}$$

Supposing that the mission time is T=2000 hours and ΔT =500 hours, we can calculate the system unreliability using the Eq. (8). Table 3 shows the top event occurrence probabilities at the different mission time.

Considering to the different complexity of components their test cost is different. According to the evaluation standards of the test cost in Table 1, we can calculate HIV using the Eq. (11). Table 4 shows the components' test cost and HIV of the components. Solving the DEN using the inference algorithm gives the DIF and BIM of components for the braking system. The diagnostic decision table for the braking system is shown in Table 5.

Based on the entropy methodology, the weights of the three attributes, ω_1=0.3339, ω_2=0.3326, ω_3=0.3335 are obtained using the Eq. (12) - (19). Table 6 presents the interval values of S, R and Q for all components. And the values of $p(x_i)$ as shown in Table 7 are calculated using the Eq. (29). It can be seen from the results of Table 7 that the optimal diagnosis sequence of the braking system is:
X26≻ X18(X19 X22 X23) ≻ X1≻X2≻ X13≻ X15≻ X24≻ X20≻X25≻ X21 ≻ X5≻ X7≻ X12≻ X8≻ X3≻ X10≻ X9≻ X16(X17) ≻ X6≻ X14≻ X4≻ X11.

5. Conclusions

In this paper, we have discussed the use of DFT, DEN and an improved VIKOR algorithm to locate complex systems failure. Specifically, it has emphasized three important issues that arise in engineering diagnostic applications, namely the challenges of failure dependency, different life distributions and epistemic uncertainty. In terms of the challenge of epistemic uncertainty, the failure rates of the basic events for complex systems are expressed in interval numbers; In terms of the challenge of failure dependency, DFT is used to model the dynamic behaviors of system failure mechanisms. In terms of the challenge of multiple life distributions, a mixed life distribution is used to analyze complex systems. Furthermore, we calculate some reliability results by mapping a DFT into an equivalent DEN in order to avoid some disadvantages. In addition, we take DIF, BIM and HIV into account and obtain the optimal diagnostic ranking order using an improved VIKOR algorithm. The proposed method takes full advantage of DFT for modeling, interval numbers for handling uncertainty and VIKOR for the best fault search scheme, which is especially suitable for fault diagnosis of complex systems.

In the future work, we will focus on how to incorperate sensors data to optimize the diagnosis efficiency.

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