Inertial Position Determination Under Vibration

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Abstract

The main purpose of navigation systems is a position determination different moving objects. For standard inertial navigation systems algorithm, the initial position data are required. Normally these data may be determined by astronomical techniques or satellite and radio navigation systems. However, astronomical techniques depend on climate conditions and satellite and radio systems can be disturbed by electromagnetic countermeasures. It is proposed to use an Inertial Measurement Unit (IMU) and a navigation computer for autonomous determination of an initial position. IMU should be composed of three accelerometers, three gyroscopes and a signal processing circuit. The method proposed the latitude determination uses projections of the Earth's rate measured by orthogonal IMU gyroscopes and projections of the gravity acceleration measured by orthogonal IMU accelerometers. Experimental tests of the IMU with a ring laser gyros triad and precision pendulum accelerometers confirmed the efficiency of the method on a fixed base. The behavior of the latitude determination under vibration is researched. The latitude determination under vibration was considered analytically by Power Spectral Density theory. It was estimated a variance of the latitude under the broad-band vibration. The method operability under harmonic and random vibration of the base at the real time is considered.

Keywords: latitude determination, initial alignment, random vibration

1. Introduction

The main part of modern navigation equipment is strapdown inertial navigation system (SINS). It can provide a full range of attitude, speed and navigation information [1]. Common SINS consists of navigation computer and inertial measurement unit (IMU) which consists of three-axis gyroscopes and accelerometers. They are mounted orthogonal to object's body frame [2]. SINS are commonly used for aircrafts, ships, and missiles measure and control systems. This navigation tasks can be made by global navigation satellite system (GNSS) and astronomical navigation system as well [3], but only SINS provides navigation solution with high-frequency and autonomously.

SINS initial alignment algorithm consists of two phases: coarse alignment and fine alignment [4]. The accurate geographic latitude setting is an extremely important for initial alignment precision. Without geographic latitude information, SINS cannot provide any navigation solutions [5]. Modern SINS are often complex with GNSS, so geographic latitude can be received. However, GNSS satellite signals can be lost in tunnels, underwater, forests or even drown down by special equipment. Thus, inertial geographic latitude and longitude determination become crucial prerequisite for modern SINS.

Besides, there is a scalar checking problem, which achieves by redundancy of information of two frames orientation by measurement of two ones vectors projections on this frames [6], [7].

Analytical SINS coarse alignment method was described in [8–10]. It based on Earth's rate vector measurement by gyros triads and local gravity acceleration vector measurement by accelerometers triads in body frame while SINS is static [11]. The nonlinear initial alignment method for SINS is described in [12]. The SINS dynamic model based on nonlinear error equations with the consideration of the mismatch between the calculated navigation frame and the real frame. In [13], the derivation of latitude determination was presented. The author presented the latitude estimation for initial alignment, but did not explore latitude determination errors. The problem on SINS functioning without latitude data described in [14]. The method based on quaternion calculation Earth's rate vector measurement to synthesize the initial algorithm for stationary and moving base. Ref. [15, 16] described the method for SINS latitude determination and coarse alignment on vibrating base, but errors was not analyzed. A new method for determining the latitude of a fixed base was presented in [17–19]. It was developed for precision IMU consist of ring laser gyros and pendulum accelerometers. The method error values were presented. The different latitude determination methods, including magnitude method, geometric method, and analytical methods were introduced in [20]. Authors researched real IMU data signals and defined method errors.

The paper considered a latitude determination method in case of vibration base. It was simulated in several different body states: fixed-state object, linear and angular harmonic and random vibration. Method error is presented as well.

2. Latitude determination on fixed-state body

We will use the geographic navigation frame $O\xi\eta\zeta$ and body frame $Oxyz$ as shown on Fig.1. Where presented φ – geographic latitude, φ_o – geocentric latitude, $\vec{g}$ – gravity vector, $\vec{\Omega}$ – Earth's rate vector.

According to scalar product of two vectors:

$$\vec{\Omega}\cdot\vec{g} = \Omega\cdot g\cdot\cos\left(\frac{\pi}{2}+\varphi\right). \tag{1}$$

Scalar product can be submitted by their projections:

$$\vec{\Omega}\cdot\vec{g} = \Omega_x\cdot g_x + \Omega_y\cdot g_y + \Omega_z\cdot g_z\,. \tag{2}$$

where $\Omega_x, \Omega_y, \Omega_z$ – Earth's rate vector projections on body frame, g_x, g_y, g_z – gravity vector projections on the same frame.

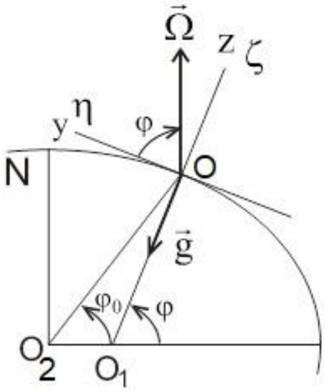

Figure 1. IMU in meridian plane

Compare right parts (1) and (2) we get

$$\sin\varphi = -\frac{1}{\Omega\cdot g}\left(\Omega_x\cdot g_x + \Omega_y\cdot g_y + \Omega_z\cdot g_z\right). \tag{3}$$

Here $\Omega = \sqrt{\Omega_x^2 + \Omega_y^2 + \Omega_z^2}$, $g = \sqrt{g_x^2 + g_y^2 + g_z^2}$.

Geographic latitude can be easily received from equation (3). This method requires gyroscopes and accelerometers measurements only.

3. Latitude determination on harmonic vibration base

More general case of body movement is vibration. It was sated as linear $\vec{w}_v\left(w_x, w_y, w_z\right)$ and angular $\vec{\omega}_v\left(\omega_x, \omega_y, \omega_z\right)$ harmonic components.

In this case the IMU angular rate will be

$$\vec{\omega} = \vec{\omega}_v + \vec{\Omega}. \tag{4}$$

At the same time accelerometers will measure apparent gravitational acceleration

$$\vec{a} = \vec{w}_v - \vec{g}. \tag{5}$$

Applying new formulas (4) and (5) movement for general method (1) we can get

$$\vec{\omega}\cdot\vec{a} = \omega\cdot a\cdot\cos\beta, \tag{6}$$

where β – angle between IMU angular rate and apparent acceleration;

$$\omega = \sqrt{\left(\omega_x + \Omega_x\right)^2 + \left(\omega_y + \Omega_y\right)^2 + \left(\omega_z + \Omega_z\right)^2};$$
$$a = \sqrt{\left(w_x - g_x\right)^2 + \left(w_y - g_y\right)^2 + \left(w_z - g_z\right)^2}.$$

On the other hand, vector product (6) can be written by its projections

$$\vec{\omega} \cdot \vec{a} = \left(\omega_x + \Omega_x\right)\left(w_x - g_x\right) + \left(\omega_y + \Omega_y\right)\left(w_y - g_y\right) + \left(\omega_z + \Omega_z\right)\left(w_z - g_z\right). \tag{7}$$

Left parts of (6) and (7) are the same, so right part must be equal, so

$$\cos\beta = \frac{1}{\omega \cdot a}\left[\left(\omega_x + \Omega_x\right)\left(w_x - g_x\right) + \left(\omega_y + \Omega_y\right)\left(w_y - g_y\right) + \left(\omega_z + \Omega_z\right)\left(w_z - g_z\right)\right]. \tag{8}$$

If the base do not move ($\vec{w}_v = \vec{\omega}_v = 0$), equation (8) will come

$$\cos\beta = -\frac{1}{\Omega \cdot g}\left(\Omega_x \cdot g_x + \Omega_y \cdot g_y + \Omega_z \cdot g_z\right).$$

The right part of last equations is equal to right part of (3). Therefore, their left parts must be equal as well – $\cos\beta = \sin\varphi$, i.e. equations transform to (3).

There could be single case, then IMU body frame coincides with geographic navigation frame, so $\Omega_x = 0,\quad \Omega_y = \Omega\cos\varphi,\quad \Omega_z = \Omega\sin\varphi$ and base is vibrating about vertical axis $w_x = w_y = 0,\quad w_z \neq 0;\ \omega_x = \omega_y = 0,\quad \omega_z \neq 0$.

Substitute this values into equation (8):

$$\cos\beta = \frac{\omega_z + \Omega\sin\varphi}{\sqrt{\Omega^2 + \omega_z^2 + 2\omega_z\Omega\sin\varphi}}. \tag{9}$$

Received equation shows that linear vibration $w_z \neq 0$ do not effect on $\cos\beta$ value. At the same time $\cos\beta$ value highly depends on base angular rate motion $\omega_z \neq 0$.

If the base move only with linear vertical vibration ($w_x = w_y = 0,\quad w_z \neq 0;$ and $\omega_x = \omega_y = \omega_z = 0$), $\cos\beta = \sin\varphi$ can be got from (9). In this case latitude determination method (3) works well.

However, in the general case, body movement will lead to acceleration deviation from vertical, and angular rate deviation from Earth's rate vector and method (3) will not works.

Let's consider body space vibration case.
IMU is sated on base, which moves with linear and angular vibration:

$$\begin{aligned}
&\omega_x = \omega_{xm}\sin\left(2\pi\cdot f_{\omega x}\cdot t+\theta_{\omega x}\right); && w_x = w_{xm}\sin\left(2\pi\cdot f_{wx}\cdot t+\theta_{wx}\right);\\
&\omega_y = \omega_{ym}\sin\left(2\pi\cdot f_{\omega y}\cdot t+\theta_{\omega y}\right); && w_y = w_{ym}\sin\left(2\pi\cdot f_{wy}\cdot t+\theta_{wy}\right);\\
&\omega_z = \omega_{zm}\sin\left(2\pi\cdot f_{\omega z}\cdot t+\theta_{\omega z}\right). && w_z = w_{zm}\sin\left(2\pi\cdot f_{wz}\cdot t+\theta_{wz}\right).
\end{aligned} \quad (10)$$

where $\omega_{xm},\omega_{ym},\omega_{zm}$ – angular rate vibration amplitudes, w_{xm},w_{ym},w_{zm} – linear vibration amplitudes, $f_{\omega x}, f_{\omega y}, f_{\omega z}$ – angular rate vibration frequencies, f_{wx}, f_{wy}, f_{wz} – linear vibration frequencies, $\theta_{\omega x},\theta_{\omega y},\theta_{\omega z}$ – angular rate vibration phases, $\theta_{wx},\theta_{wy},\theta_{wz}$ – linear vibration phases.
Substitute (10) into (8)

$$\begin{aligned}
\cos\beta = \frac{1}{\omega\cdot a}\big(&\omega_x w_x+\omega_y w_y+\omega_z w_z-\omega_x g_x-\omega_y g_y-\omega_z g_z+\\
&+\Omega_x w_x+\Omega_y w_y+\Omega_z w_z-\Omega_x g_x-\Omega_x g_x-\Omega_x g_x\big).
\end{aligned} \quad (11)$$

Modeling of (11) is shown in Figure 2.

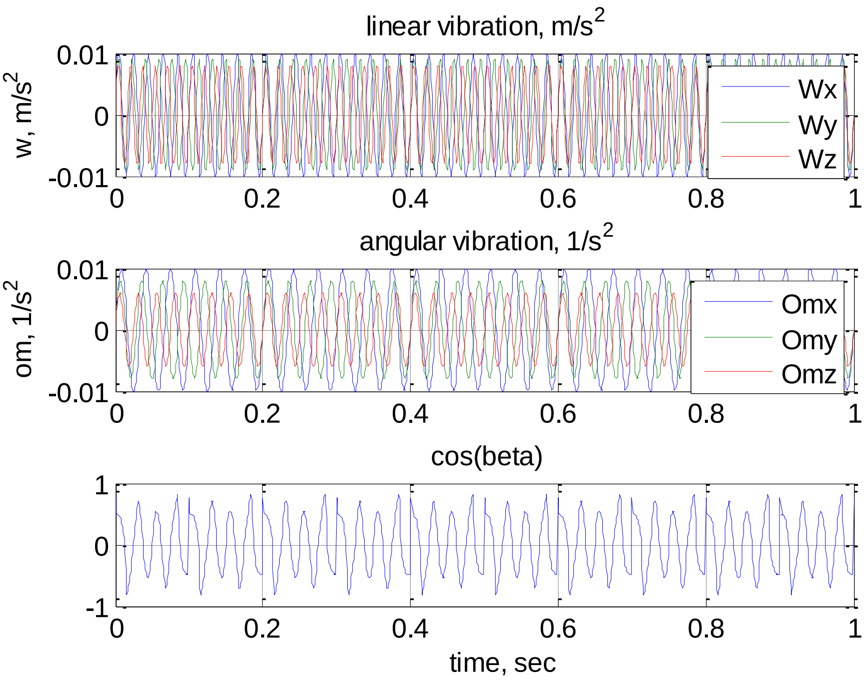

Figure 2. Linear and angular vibration with $\cos\beta$ value

The linear and angular vibration values are:
w_{xm}=0,01 m/s^2, w_{ym}=0,009 m/s^2, w_{zm}=0,008 m/s^2, $f_{wx}=50$ Hz, $f_{wy}=55$ Hz, $f_{wz}=60$ Hz, $\theta_{wx}=\theta_{wy}=\theta_{wz}=0$; ω_{xm}=0,01 $1/\text{s}$, ω_{ym}=0,008 $1/\text{s}$, ω_{zm}=0,006 $1/\text{s}$, $f_{\omega x}=30$ Hz, $f_{\omega y}=35$ Hz, $f_{\omega z}=40$ Hz, $\theta_{\omega x}=\theta_{\omega y}=\theta_{\omega z}=0$; In case when $\Omega_x=0, \Omega_y=\Omega\cos\varphi, \Omega_z=\Omega\sin\varphi$, $g_x=g_y=0, g_z=-g$.

Latitude was sated with value φ=50°, $\sin\varphi$=0,766. Mean value of equation (11) amounted to $\langle\cos\beta\rangle$=1,314·10^{-2}.

The results of equation (11) modeling, with the same linear and angular vibration frequencies, are shown on Fig. 3.

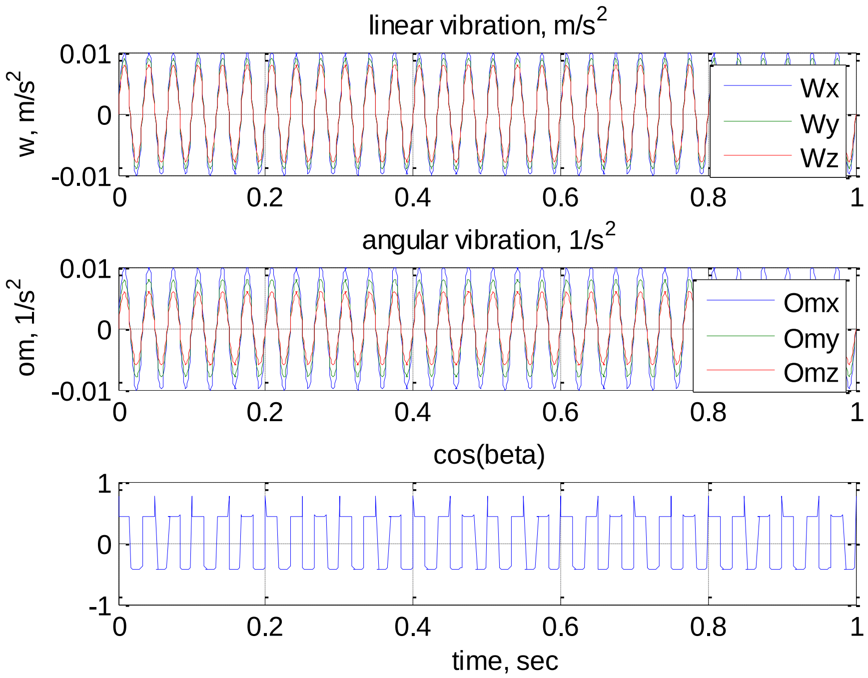

Figure 3. Same frequencies linear and angular vibration with $\cos\beta$ value

The linear and angular vibration values are:
w_{xm}=0,01 m/s^2, w_{ym}=0,009 m/s^2, w_{zm}=0,008 m/s^2, $f_{wx}=f_{wy}=f_{wz}=30$ Hz, $\theta_{wx}=\theta_{wy}=\theta_{wz}=0$; ω_{xm}=0,01 1/s, ω_{ym}=0,008 1/s, ω_{zm}=0,006 1/s, $f_{\omega x}=f_{\omega y}=f_{\omega z}=30$ Hz, $\theta_{\omega x}=\theta_{\omega y}=\theta_{\omega z}=0$.

Equation (11) mean value almost doubled: $\langle\cos\beta\rangle$=2,313·10^{-2}.

Thus, vertical linear vibration does not influence on latitude determination, but space vibration makes this method unworkable.

4. Latitude determination on random vibration base

Let write the formula (8) for latitude determination on vibration base in the form:

$$\sin(\varphi_* + \Delta\varphi) \approx \frac{\left[(\omega_x + \Omega_x)(w_x - g_x) + (\omega_y + \Omega_y)(w_y - g_y) + (\omega_z + \Omega_z)(w_z - g_z)\right]}{\Omega \cdot g}, \quad (12)$$

where φ_* – latitude without vibrations, $\Delta\varphi$ – error of latitude determination.

Error mean value and error dispersion of latitude determination are

$$\langle \Delta\varphi \rangle \approx \frac{1}{\cos\varphi_* \Omega\, g} \langle \omega_x w_x + \omega_y w_x + \omega_z w_z \rangle \ . \quad (13)$$

We assume that the random vibrations are interconnected along three IMU axes. Then the accelerometer vibrations can be set as projections of the acceleration vibration vector

$$w_x = n_{ax} w_v;\ w_y = n_{ay} w_v;\ w_z = n_{az} w_v , \quad (14)$$

where n_{ax}, n_{ay}, n_{az} – directional cosines of acceleration vibration vector relative to IMU's axes.

Similarly, gyroscope vibration can be set – as projections of the angular vibrational velocity vector

$$\omega_x = n_{\varepsilon x} \omega_v;\ \omega_y = n_{\varepsilon y} \omega_v;\ \omega_z = n_{\varepsilon z} \omega_v , \quad (15)$$

where $n_{\varepsilon x}$, $n_{\varepsilon y}$, $n_{\varepsilon z}$ – directional cosines of angular vibrational velocity vector relative to IMU's axes.

Then, we can get

$$\langle \Delta\varphi \rangle \approx \frac{1}{\cos\varphi_* \Omega g} \int_{-\infty}^{\infty} \left(n_{ax} n_{\varepsilon x} + n_{ay} n_{\varepsilon y} + n_{az} n_{\varepsilon z} \right) S_{w\omega}(\omega) d\omega , \quad (16)$$

where $S_{w\omega}(\omega)$ – vibration cross-spectral density.

Let random vibration vectors are stationary ergodic random process with spectral density

$$S_{w\omega}(\omega) = \frac{\sigma_{w\omega}^2 \cdot 2\alpha\left(\alpha^2 + \omega_0^2\right)}{\pi\left[\alpha^2 + (\omega - \omega_0)^2\right]\left[\alpha^2 + (\omega + \omega_0)^2\right]}, \quad (17)$$

where $\sigma_{w\omega}^2$ – dispersion of the random vibration, α – attenuation rate, ω_0 – prevailing frequency of vibration.

The results of method modeling in case of random vibration (17) are shown on Fig. 4. The linear and angular vibration values are:
σ_{wv} =0.01 m/s^2, $\sigma_{\omega v}$ =0.006 1/s, α =1.25, ω_0 =30*2π 1/s, φ=50.44º.
After numerical modeling we can obtain error value $\langle \Delta\varphi \rangle = 47.8$º.

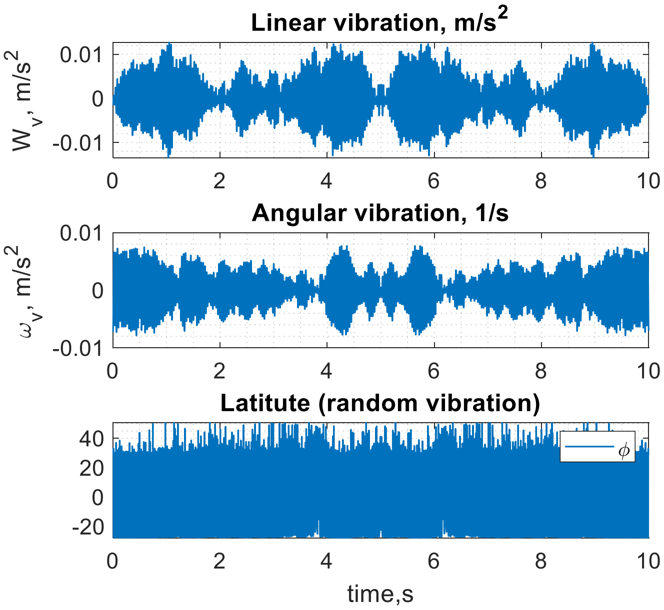

Figure 4. Latitude under random angular and linear vibration

Let look at value of $n_{ax}n_{\varepsilon x} + n_{ay}n_{\varepsilon y} + n_{az}n_{\varepsilon z}$. It is sum of directional cosines products and its value gets maximum when angular vibration and linear acceleration axis coincides. Thus means $n_{ax} = n_{\varepsilon x}, n_{ay} = n_{\varepsilon y}, n_{az} = n_{\varepsilon z}$ and we can get

$$\langle \Delta\varphi \rangle = \frac{1}{\cos\varphi_* 2\pi\Omega g} \int_{-\infty}^{\infty} \left(n_{ax}^2 + n_{ay}^2 + n_{az}^2 \right) S_{w\omega}(\omega) d\omega \ . \tag{18}$$

If use well known expression for dispersion

$$\sigma_{w\omega}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{w\omega}(\omega) d\omega,$$

from equation (18) we will get

$$\langle \Delta\varphi \rangle = \left(n_{ax}^2 + n_{ay}^2 + n_{az}^2 \right) \frac{\sigma_{w\omega}^2}{\Omega g \cos \varphi_*} . \qquad (19)$$

If we assume that $n_{ax} = n_{ay} = n_{az} = 1$ and Ω =7,29· 10^{-5} m/s, g=9,81 m/s^2, $\sigma_{w\omega}$=0.01 m/s^2, φ_*=50.44°, the latitude error is $\langle \Delta\varphi \rangle = 36{,}86$°.

5. Conclusions

The operability of the latitude determination method on vibration base is considered. The method was tested on both harmonic and random vibration. It has shown insensitivity to vertical axial harmonic base vibration, but low accuracy in three axial harmonic vibration.

Random vibration research showed big sensitivity of latitude determination to magnitudes of angular and linear vibration.

IMU must be installed on the vibroinsulated base which could decrease vibration level until maximum sensors random walk errors.

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