Application the GPS Code Observations in BSSD Method for Recovery the Position of the Aircraft

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Abstract:
In this paper, the results of aircraft positioning based on GPS code observations in air aviation are presented. The aircraft position was recovery using Between Satellite Single Difference (BSSD) method in GPS system. The BSSD method was applied for designation the precise position of Cessna 172 plane in flight test in air navigation. The coordinates of Cessna 172 plane were determinated using least square estimation in XYZ geocentric frame. The average accuracy of aircraft position equals to 0.797 m for X axis, 0.496 m for Y axis and 0.966 m for Z axis, respectively. In addition, the protection level of HPL parameter amounts to 4.991 m and 5.749 m for VPL term. In paper, the XYZ coordinates of Cessna 172 plane were also compared with PPP solution from GAPS software. The value of RMS bias is about 1.642 m for X axis, 0.902 m for Y axis and 0.892 m for Z axis, respectively.

Keywords: *GPS, BSSD method, air navigation, least square estimation, air transport*

1. Introduction

In the 21st century the GNSS satellite technique became a universal method of the positioning of the aircraft in air transport area. The special use of the GNSS satellite technique in the air transport is possible due to application of the assist systems ABAS, SBAS and GBAS. The ABAS system enables to locate the aircraft based on registered satellite observations by the onboard GNSS receiver installed in the aircraft. Moreover, also essential role in the ABAS system fulfils module and a mechanism of appointing the position of the aircraft. The algorithm of appointing the position of the aircraft is usually based on mathematical model of the RAIM system for GNSS observations. System RAIM guarantees continuity, monitoring, integrity and the accuracy of determination of position the aircraft. Moreover a detection of blunder errors from GNSS observations is important in RAIM system component [12].

The system of SBAS support enables to use differencing corrections for locating the aircraft. Differencing corrections are being sent from satellites of geostationary systems circulating among the Earth. Universally used EGNOS system is an example of such a geostationary system in Europe. Ultimately the system of SBAS support has to improve the position of the aircraft for the range of the accuracy from 1 m to 5 m. Moreover, the system of SBAS support is intended for applying in the civil aviation for approach the landing of the type SBAS APV-I [6].

The system of GBAS support requires the installation of the expensive technical infrastructure for the airport. By design the GBAS system is supposed to provide the radio navigation aids during the approach of aircraft to the landing with the application for the reference RTK GNSS station. The position of the aircraft is determined based on differencing corrections sent from the network of GNSS receivers, installed at the airport. System GBAS in practice has to be applied for a precise approach of aircraft to landing PA of category I [10].

Specific methods or techniques of the precise positioning of the aircraft for air transport are implemented in each supporting systems ABAS, SBAS and GBAS. In the case of ABAS system the most universal methods of positioning of the aircraft can be:

- Single Point Positioning method (positioning method with L1-C/A code) [15], [18], [20];
- Precise Point Positioning method (method of the precise positioning for undifference code observations P1/P2 and phase L1/L2 in GNSS system) [5], [7], [21];
- Between Satellite Single Difference method (differential the GNSS observations between the satellites) [16];
- Doppler method (Use Doppler effect to determine the speed of the aircraft) [17], [25], [31].

The SBAS support system mainly uses the Single Point Positioning (SPP) method for code observation L1-C/A [2], [3]. As part of the SBAS system, satellites position, satellite clock bias, ionosphere and troposphere delays are corrected. SBAS differential corrections in "EMS" format are used in calculations [13].

There are two methods of positioning an aircraft in the GBAS support system:

- DGPS method (correction of aircraft position for the use of code measurements) [9], [23];
- RTK-OTF method (correction of aircraft position for the use of phase measurements) [2], [9], [30], [32].

The basic purpose of this research paper is the accuracy assessment of aircraft positioning in air transport based on solution of Between Satellite Single

Difference (BSSD) method. For this purpose, the aircraft coordinates, the standard deviations of aircraft coordinates, the HPL and VPL integrity parameters, the DOP coefficients were designated and analyzed in the paper. All obtained parameters were determined with a probability of 95% for the global Chi-square test. Numerical calculations used navigation data from the Topcon HiperPro receiver placed onboard the Cessna 172, which performed a test flight around Dęblin aerodrome. The position of the aircraft was reproduced in the author's APS software (Aircraft Positioning Software) in the Between Satellite Single Difference module. The calculations were made in post-processing mode for GPS code observations.

2. The Mathematical Model of BSSD Positioning Method

The basic observational equations in the BSSD positioning method are based on the application of a single difference operation for code observations in GPS system as follows [1]:

$$\Delta P1_{ij} = \Delta\rho_{ij} + c\cdot\Delta dts_{ji} + \Delta I_{ij} + \Delta T_{ij} + \Delta SDCB_{ij,L1} + \Delta M_{ij,L1} \quad (1)$$

where:

Δ – the single difference operator for code measurements allows to determine the difference in code measurements from two satellites tracked by one receiver in the same measurement period,

$P1_{ij} = P1_i — P1_j$ – the value of a single code difference between satellites i and j on L1 frequency in GPS (expressed in meters),

i – satellite index,

j – satellite index,

cc – light speed (expressed in m/s),

$\rho_{ij} = \rho_i - \rho_j$ – difference in geometry distance between satellites i and j on L1 frequency in GPS (expressed in meters),

$$\rho_i = \sqrt{\left(x - X_i\right)^2 + \left(y - Y_i\right)^2 + \left(z - Z_i\right)^2},$$

$$\rho_j = \sqrt{\left(x - X_j\right)^2 + \left(y - Y_j\right)^2 + \left(z - Z_j\right)^2},$$

(x, y, z) – coordinates of the aircraft,

(X_i, Y_i, Z_i) – i-th satellite coordinates,

(X_j, Y_j, Z_j) – j-th satellite coordinates,

dts_{ji}– difference in satellites tracking between i and j satellites on L1 frequency in GPS system (expressed in seconds),

I_{ij} – difference in ionosphere delay between i and j satellites on L1 frequency in GPS (expressed in meters),

T_{ij} – difference between the values of the troposphere delay between satellites i and j on L1 frequency in GPS (expressed in meters),

$SDCB_{ij,L1}$ – difference between hardware delay values between satellites i and j on L1 frequency in GPS (expressed in meters),

$M_{ij,L1}$ – difference in value of multipath effect and measurement noise between satellites i and j on L1 frequency in GPS (expressed in meters).

The observational equation (1) was recorded for code observations P1 on the carrier frequency L1 in the GPS navigation system. In equation (1) the unknown parameters are the coordinates of the aircraft, entangled in the geometric distance factor. The GPS satellite coordinates are derived from Kepler orbital motion model or interpolated from precision ephemeris. The satellite clock bias corrections are based on onboard ephemeris navigation data or interpolated from precision ephemeris. In addition, the precision values of the satellite clock correction can be determined using a universal „CLK" format. The ionospheric delay parameter in the BSSD method is based on the Klobuchar model, and the tropospheric correction is based on the tropospheric deterministic model. The values of multipath and measuring noise are determined on the basis of empirical models or omitted from the observation equation (1). Equation (1) also includes parameters of hardware delays for SDCB satellites, referenced to L1 frequency. It is worth noting that in the math equation (1) there is no parameter of the receiver clock correction, which is eliminated from the observation equations using the single difference operator. The coordinates of the aircraft are determined in a stochastic process for applying the least squares method as follows [26]:

$$\begin{cases} \mathbf{Q_X} = \mathbf{N^{-1}} \cdot \mathbf{L} \\ \mathbf{v} = \mathbf{A} \cdot \mathbf{Q_X} - \mathbf{dl} \\ m0_{post} = \sqrt{\dfrac{[\mathbf{pvv}]}{n-k}} \\ \mathbf{C_{Qx}} = m0^2_{post} \cdot \mathbf{N^{-1}} \\ \mathbf{m_{Qx}} = diag\left(\sqrt{\mathbf{C_{Qx}}}\right) \end{cases} \quad (2)$$

where:

Qx – vector with unknown parameters,

$\mathbf{N} = \mathbf{A^T} \cdot \mathbf{p} \cdot \mathbf{A}$ – matrix of the normal equations frame,

A – matrix of coefficients, matrix is full rank,

p – matrix of weights,

$$\mathbf{p} = \frac{1}{m0^2_{priori} \cdot \mathbf{ml}^2},$$

$m0_{priori}$ – standard error of unit weight a priori,

$m0_{priori} = 1$,

ml – matrix with mean errors of pseudoranges,

$$\mathbf{ml} = \sqrt{\left(\frac{ml_0}{\sin(El_i)}\right)^2 + \left(\frac{ml_0}{\sin(El_j)}\right)^2},$$

ml_0 – standard deviation of code measurement P1 in GPS system, ml_0 = 1 m,

El_i – elevation angle for satellite i-th,

El_j – elevation angle for satellite j-th,

$\mathbf{L} = \mathbf{A^T} \cdot \mathbf{p} \cdot \mathbf{dl}$ – vector of absolute terms,

dl – vector with difference between measurements and modeled parameters,

$m0_{post}$ – standard error of unit weight a posteriori,

n – number of observations,

k – number of designated parameters,

k = 3 for each measurement epoch,
v – vector of residuals,
$\mathbf{C}_{\mathbf{Qx}}$ – variance-covariance matrix of parameters designates in the XYZ geocentric frame,
$\mathbf{m}_{\mathbf{Qx}}$ – standard deviations of the designated parameters referenced to the XYZ geocentric frame.
The stochastic process of developing kinematic GPS observations is carried out sequentially for all recorded measured epochs by a satellite receiver mounted on an aircraft. In addition, the standard deviations of the coordinates of the aircraft are also determined in the stochastic process of GPS observations. It is worth noting that the determined coordinates of the aircraft and their accuracy are related to the geocentric coordinate XYZ frame.

3. The Experiment and Results

Verification and reliability of the presented mathematical aircraft positioning model has been tested for kinematic GPS observations obtained from the Topcon HiperPro dual-frequency geodetic receiver. The Topcon HiperPro receiver was placed in the cockpit of the Cessna 172. The geodetic receiver recorded GNSS observations during the flight test for air navigation. Registered GNSS observations were used to reconstruct the trajectory of the Cessna 172 aircraft and to assess the accuracy of the designated position [4].

The coordinates of the Cessna 172 aircraft in the XYZ geocentric frame were determined by the APS software (Aircraft Positioning Software), which operates in the Scilab 5.4.1 language environment. The APS software is a free software tool for developing kinematic post-processing GNSS observations. The APS software enables the implementation of GNSS code observations for GPS and GLONASS. In addition, the APS software has 3 basic computing modules, as:

- SPP (Single Point Positioning) module,
- IF LC (Ionosphere-Free linear combination) module,
- BSSD (Between satellite Single Difference) module [18].
- For calculation purposes, the BSSD module has been configured as follows:
- GNSS system: GPS system,
- type of observations: P code at 1st frequency,
- type of RINEX file: 2.10,
- source of satellite ephemeris data: precise ephemeris from the CODE Analysis Center [33],
- source of satellite clock data: precise ephemeris from the CODE Analysis Center [33],
- method of satellite position computation: 9-degrees Lagrange polynomial,
- method of satellite clock bias computation: 9-degrees Lagrange polynomial,
- satellite clock bias correction: satellite clock bias from precise ephemeris is corrected using Differential Code Biases for code observations in BSSD method [27],
- effect of Earth rotation and time of pseudorange travelling through atmosphere: applied,
- relativistic effect: applied,
- ionosphere source: Klobuchar model,
- troposphere source: Simple model,
- Time Group Delays (TGD): not applied,
- Satellite Differential Code Biases (SDCB): applied,
- instrumental bias for receiver: eliminated,
- receiver clock bias: eliminated,
- multipath and measurement noise: not applied,
- satellite and receiver phase center offset: based on ANTEX file from IGS service,
- Sagnac effect: applied,
- -cutoff elevation: 5^0,
- positioning mode: kinematic,
- computation mode: post-processing,
- mathematical model of solution: least square estimation in iterative scheme,
- adjustment processing: applied,
- maximum number of iteration in single measurement epoch: N=10,
- number of unknown parameters: k=3, for each measurement epoch,
- number of observations: n>3, for each measurement epoch,
- interval of computations: 1 s,
- initial coordinates of aircraft position: based on header of RINEX file,
- time of GNSS system: GPS Time,
- reference frame: IGS'08,
- format of output coordinates: geocentric XYZ and ellipsoidal BLh,
- local test of residuals: applied,
- global statistical test: test Chi-square,
- value of $m0_{post}$ after adjustment processing: $m0_{post} \in (0,9:1,1)$,
- significance level: $(1-\alpha) = 0.95$,
- DOP coefficients: estimated,
- coefficients value for HPL and VPL level: $k_{HPL} = 6$ and $k_{VPL} = 5.33$ [8].

The Cessna 172 coordinates (x, y, z) of the IGS'08 frame were determined in the course of the studies (see Fig. 1). Dispersion of the resulting coordinates along the X axis are between 3687676.312 m and 3707691.906 m; respectively along the Y axis are between 1456985.549 and do 1493528.844 m; while along the Z axis are between 4955328.731 m and 4974526.934 m.

For the designated (x, y, z) coordinates of the Cessna 172, their accuracy was also determined (see Fig. 2). The average accuracy of the X coordinate is 0.797 m and the median is 0.782 m. The X coordinate accuracy is between 0.253 m and 1.505 m. The average accuracy of the Y coordinate equals 0.496 m and the median is 0.489 m. In addition, the accuracy dispersion for the Y coordinate is between 0.115 m and 0.839m. The average accuracy of the Z coordinate equals 0.966 m and the median is 0.951 m. On the other hand, the dispersion of the obtained Z coordinate accuracy is between 0.494 m and 1.877 m.

Figure 3 presents the positioning accuracy of the Cessna 172 aircraft with respect to the ellipsoid coordinate BLh frame (B – Latitude, L – Longitude, h –ellipsoidal height). The accuracy values of the aircraft

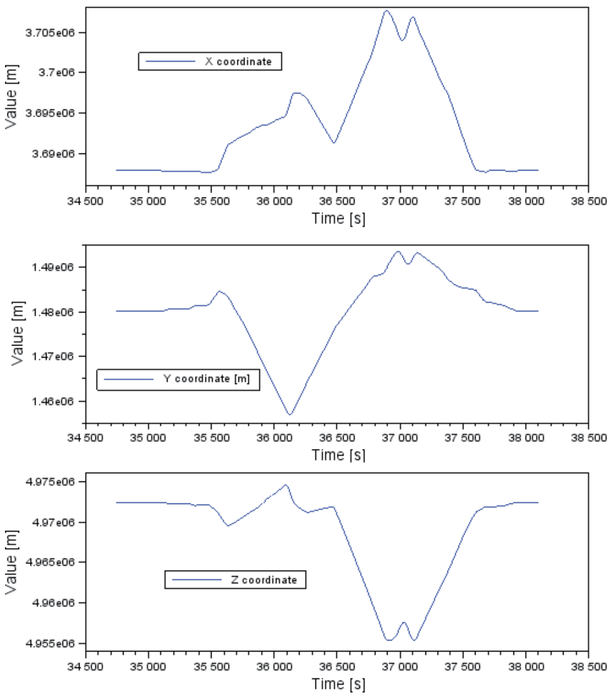

Fig. 1. The trajectory of Cessna 172 aircraft along XYZ axis

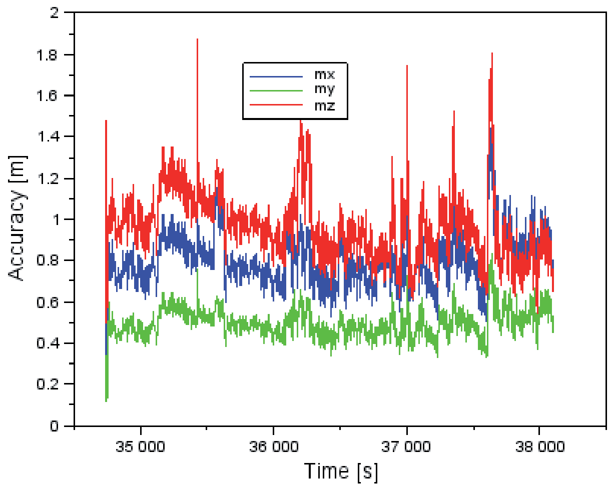

Fig. 2. The accuracy of aircraft position in XYZ geocentric frame

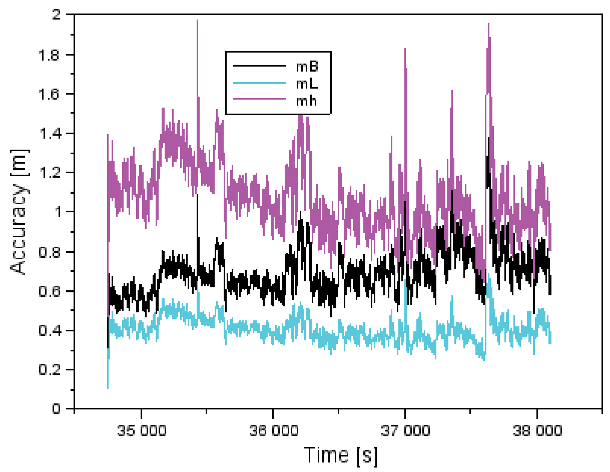

Fig. 3. The accuracy of aircraft position in BLh ellipsoidal frame

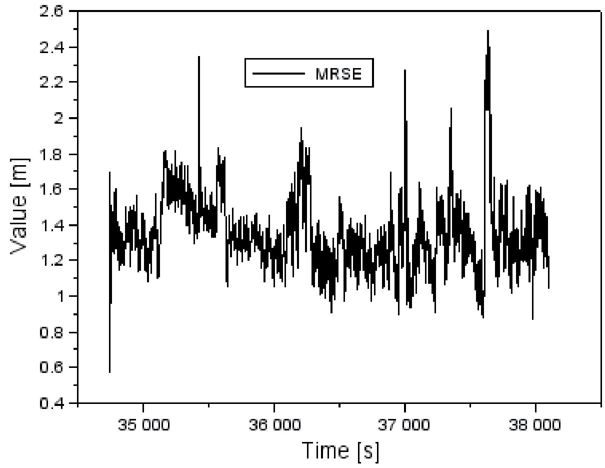

Fig. 4. The MRSE parameter for aircraft position

position of Figure 3 were determined on the basis of the transformation from the geocentric coordinate XYZ frame to the ellipsoidal BLh frame [22]. The average accuracy of B coordinate equals 0.696 m and the median is 0.680 m. The accuracy of the coordinate B is between 0.306 m and 1.383 m. The average accuracy of the L coordinate is 0.407 m and the median is 0.401 m. In addition, the dispersion of accuracy results for the L coordinate is between 0.103 m and 0.692 m. The average accuracy of the h-coordinate equals 1.079 m and the median is 1.056 m. The dispersion of the obtained h coordinate accuracy is between 0.466 m and 1.973 m. The Annex 10 to the Chicago Convention specifies the technical standards for the admission of a GPS system for use in air transport [11]. The typical accuracy of aircraft positioning for horizontal navigation must not be less than 17 m. From the obtained results of the accuracy of the determination of horizontal coordinates B and L it follows that the maximum values of the standard deviation of the coordinate B are better than 1.4 m and for the coordinate L better than 0.7 m. In addition, the vertical navigation accuracy of the ICAO instruction equals 37 m. The accuracy of the ellipsoidal height determination from the APS program is better than 2 m. Therefore, the developed aircraft positioning system for horizontal and vertical navigation meets the ICAO accuracy criteria.

Figure 4 presents the values of the aircraft position error in the 3D space (MRSE parameter). The MRSE parameter was derived from the dependency [29]:

$$MRSE = \sqrt{mB^2 + mL^2 + mh^2} \tag{3}$$

where:
mB – accuracy of Latitude (see Figure 3),
mL – accuracy of Longitude (see Figure 3),
mh – accuracy of ellipsoidal height (see Figure 3).

The mean average of MRSE parameter equals to 1.350 m, whereas the median is about 1.324 m. The magnitude order of MRSE term is between 0.567 m and 2.497 m.

A key element in the implementation of GPS satellite technology in air transport is the determination of the positioning integrity of the aircraft. The integrity of satellite positioning in air transport is determined by HPL and VPL protection levels. A simplified formula for determining the HPL and VPL parameters can be written as follows [14]:

$$\begin{cases} HPL = k_{HPL} \cdot \sqrt{mB^2 + mL^2} \\ VPL = k_{VPL} \cdot mh \end{cases} \tag{4}$$

where:
k_{HPL} = 6, for horizontal plane,
k_{VPL} = 5.33, for vertical plane.

Figure 5 presents the values of the HPL and VPL protection levels for the presented test method. The average HPL value equals 4.991 m and the median is 4.890 m. The spread of HPL results ranges from 1.995 m to 9.554 m. The average value of the VPL parameter equals 5.749 m and the median equals 5.630 m. In addition, the dispersion of the VPL parameter results is between 2.483 m to 10.516 m.

Determining HPL and VPL protection levels using GPS satellite technology is possible for NPA GNSS non-precision approach [11]. The Annex 10 to the Chicago Convention sets the HPL and VPL protection levels for the NPA GNSS non-precision approach using the GPS navigation system. As part of the LNAV (Lateral Navigation) horizontal navigation, the limit value of satellite positioning integrity shall not exceed 556 m. Based on the results obtained, the HPL value can be found to be satisfied that the horizontal positioning reliability limit has been met. The HPL results are significantly lower than the critical HPL level for the NPA GNSS approach. For the VPL parameter, no ICAO technical standards have been introduced for navigation in the vertical plane VNAV (Vertical Navigation). Therefore, at this stage of the study it is not possible to compare the results of the VPL test with the ICAO technical standards.

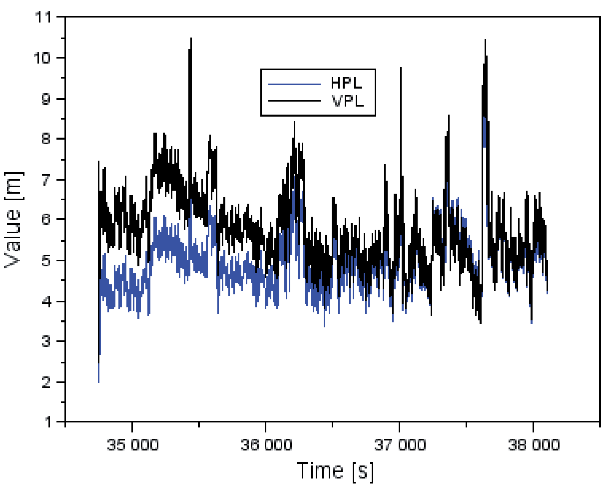

Fig. 5. The value of HPL/VPL parameters in flight test

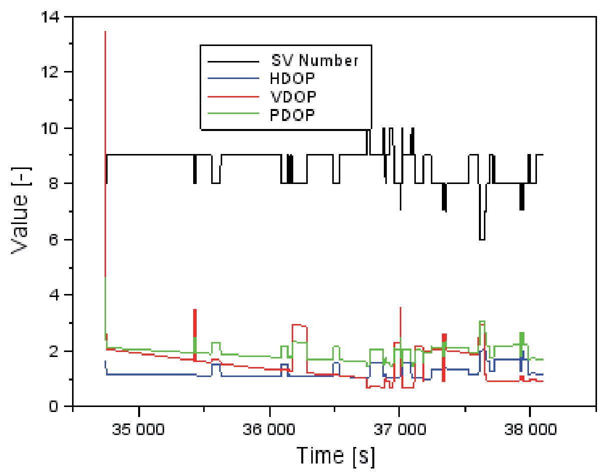

Fig. 6. The value of DOP coefficients

Figure 6 shows the results of DOP precision factors. The values of the HDOP, VDOP and PDOP coefficients were determined in the tests performed [18], [19]. The PDOP scores are from 1.1 to 2.1, with an average of 1.2. The HDOP parameter dispersion is between 0.6 and 13.5 and the mean is 1.5. It should be noted that the results of the VDOP above 13 were observed for the first 5 measuring epochs, i.e. when the number of GPS tracking satellites was 5. The PDOP results is between 1.3 and 4.1, and the average equals 1.8.

4. Discussion

The discussion evaluated the reliability parameters of the developed calculation system for the BSSD positioning method. In the first stage a statistical analysis of the obtained results was performed. During the calculations, a global statistical test Chi-square $\chi^2_{f,1-\alpha}$ was performed. The criterion for verification, control and monitoring of the Chi-square $\chi^2_{f,1-\alpha}$ test results is based on dependence [28]:

$$\sum \mathbf{pvv} = m0^2_{post} \cdot (n-k) < \chi^2_{f,1-\alpha} \tag{5}$$

where:
$\sum \mathbf{pvv}$ – sum of weighted squares of residuals,
f –number of freedom degrees,
$f = n - k$,
$(1 - \alpha)$ – significance level,
$\alpha = 0.05$.

Figure 7 presents the results of the statistical test Chi-square $\chi^2_{f,1-\alpha}$. The average value of the parameter $\sum \mathbf{pvv}$ is equal to 30.3, with a dispersion of the results obtained from 7 to 42. In turn the mean value of the statistical parameter $\chi^2_{f,1-\alpha}$ equals 44.1, for the dispersion of results from 14.1 to 58.1. It should be noted that throughout the duration of the air test parameter $\sum \mathbf{pvv}$ does not exceed the statistical value $\chi^2_{f,1-\alpha}$. Therefore, it can be inferred that the global statistical test Chi-square $\chi^2_{f,1-\alpha}$ has been completed.

In the second test, the designated (x, y, z) coordinates of aircraft were verified. For this purpose, the (x, y, z) coordinates obtained from the APS program and the GAPS program were compared. The GAPS program makes it possible to determine the position of an aircraft in the global reference IGS'08. The mathematical model for determining coordinates in kinematic mode in GAPS is based on the PPP positioning

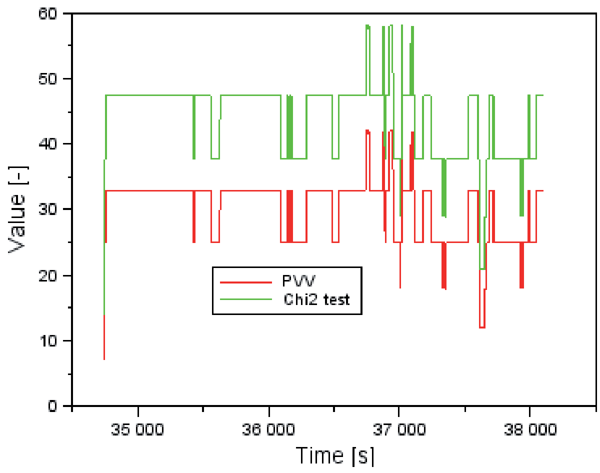

Fig. 7. The values of statistical test Chi-square

method. The GAPS program allows you to recreate the position of an aircraft with high accuracy [19]. The study examines the difference in aircraft coordinates (x, y, z) between the APS and GAPS solutions.

The difference in (x, y, z) coordinates is defined as follows:

$$\begin{aligned} dx &= x_{APS} - x_{GAPS} \\ dy &= y_{APS} - y_{GAPS} \\ dz &= z_{APS} - z_{GAPS} \end{aligned} \tag{6}$$

where:

x_{APS} – x coordinate of aircraft based on APS solution (see equation (1)),
x_{GAPS} – x coordinate of aircraft based on GAPS solution,
y_{APS} – y coordinate of aircraft based on APS solution (see equation (1)),
y_{GAPS} – y coordinate of aircraft based on GAPS solution,
z_{APS} – z coordinate of aircraft based on APS solution (see equation (1)),
z_{GAPS} – z coordinate of aircraft based on GAPS solution.

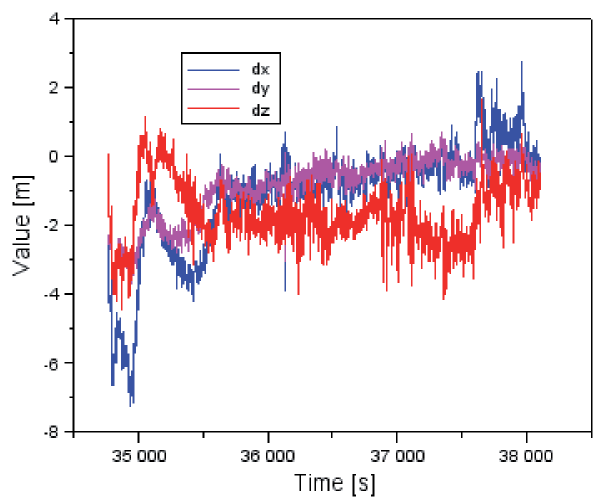

Fig. 8. The difference of XYZ coordinate of aircraft based on APS and GAPS solution

Figure 8 presents values of (dx, dy, dz) coordinates based on the comparison between APS and GAPS. The mean difference for the x-coordinate of the aircraft equals -1.141 m and the median parameter equals about -0.764 m. In addition, the dispersion of results for parameter dx is between -7.269 m and +2.778 m. It is worth noting that the RMS error [24] for the dx parameter equals 1.642 m. The mean difference for the y coordinate of the aircraft equals -0.861 m and the median parameter is -0.584 m. In addition, the dispersion of results for the dy term is between -3.693 m and +0.739 m. It should be noted that the RMS error for the dy parameter equals 0.902 m. The mean difference for the coordinate from the aircraft equals -1.656 m and the median parameter is -1.758 m. In addition, the dispersion of results for the dz term is between -4.469 m and +1.700 m. It should be noted that the RMS error for the dz parameter equals 0.892 m.

5. Conclusions

This article presents the results of research on the implementation of GPS satellite navigation technology In particular, the position of the Cessna 172 aircraft was determined using GPS code observations. The study used the BSSD method to reconstruct the aircraft position within the ABAS support system. The P1 code observations recorded by a GPS receiver aboard an aircraft were used in the calculations. Aircraft position calculations were performed in the APS program in the Scilab 5.4.1. language environment. The designation of the Cessna 172 aircraft in the APS program was expressed in geocentric coordinates XYZ in reference IGS'08 frame. Typical aircraft positioning accuracy in geocentric coordinates XYZ is better than 2 m. It should be added that the accuracy of aircraft position obtained meets the ICAO criteria for the use of GPS in air transport. The MRSE parameter is calculated in the paper, whose accuracy is higher than 3 m. In addition, the article sets the integrity parameters of the satellite positioning of HPL and VPL in air transport. The accuracy of the HPL and VPL parameters is higher than 11 m. The work was also carried out to control the calculations by conducting a global Chi-square statistical test at a confidence level of 0.95. The Chi-square test confirmed the internal consistency of the calculated results in the APS program. In the article in the external audit, the geocentric coordinates of the aircraft were compared with the results of the GAPS program. On the basis of the comparison between the APS and GAPS programs, it was found that the RMS values for Y and Z coordinates are less than 1 m, and for the X coordinate less than 2 m. In the future, the BSSD method for GLONASS code observations will be tested in kinematic mode.

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