

## SLIP EFFECTS ON SQUEEZING FLOW OF NANOFUID BETWEEN TWO PARALLEL DISKS

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In this study, the influence of temperature and wall slip conditions on the unsteady flow of a viscous, incompressible and electrically conducting nanofluid squeezed between two parallel disks in the presence of an applied magnetic field is investigated numerically. Using the similarity transformation, the governing coupled partial differential equations are transformed into similarity non-linear ordinary differential equations which are solved numerically using the Nachtsheim and Swigert shooting iteration technique together with the sixth order Runge-Kutta integration scheme. The effects of various emerging parameters on the flow characteristics are determined and discussed in detail. To check the reliability of the method, the numerical results for the skin friction coefficient and Nusselt number in the absence of slip conditions are compared with the results reported by the predecessors and an excellent agreement is observed between the two sets of results.

**Key words:** nanofluid, slip effect, squeezing flow, magnetic field.

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### 1. Introduction

In recent years, the study of heat and mass transfer of viscous fluids in the squeezing flow has been increased due to their applications in many branches of science and engineering. Such flows are induced by two approaching parallel surfaces in relative motion. The use of on MHD fluid in lubrication prevents the adverse impact of temperature on the fluid viscosity when the system operates under extreme conditions. Stefan [1] first initiated pioneering work on the squeezing flow by invoking a lubrication approach. Rashidi *et al.* [2] studied the two-dimensional and axisymmetric squeezing flow between parallel plates. The effect of magnetic field in the squeezing flow between infinite parallel plates due to the normal motion of plates was discussed by Siddiqui *et al.* [3]. Domairry and Aziz [4] provided the approximate analytic solution for the squeezing flow of a viscous fluid between parallel disks with suction or blowing. Hayat *et al.* [5] extended

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the work presented in Ref. [4] to analyze the squeezing flow of a second grade fluid between parallel disks. Recently, Mustafa *et al.* [6] discussed the squeezing flow of a viscous fluid between two parallel plates.

In the recent past a new class of fluids, namely nanofluids, have attracted the attention of the engineering community. An innovative way of improving the thermal conductivities of heat transfer fluids is to suspend small solid particles in the fluids. Nanofluids are nanometer-sized particles dispersed in a base fluid such as water, ethylene glycol, toluene and oil. The addition of high thermal conductivity metallic nanoparticles increases the thermal conductivity of such mixtures; thus enhancing their overall energy transport capability. Nanofluids have properties that make them potentially useful in many heat transfer processes including microelectronics, fuel cells, pharmaceutical processes and hybrid-powered engines. They exhibit enhanced thermal conductivities and heat transfer coefficients compared to the base fluids. For this reason nanofluids are often preferred to conventional coolants like oil, water, ethylene glycol mixtures. The enhancement of thermal conductivities by nanofluids was first discussed by Choi [7]. Subsequently, Choi *et al.* [8] showed that the addition of a small amount of nanoparticles to conventional fluids (less than 1% by volume) approximately doubled the thermal conductivity of the liquid. It should be noticed that there have been published several recent papers [9-16] on the mathematical and numerical modeling of convective heat transfer in nanofluids. Kandasamy *et al.* [17] investigated an MHD boundary layer flow of a nanofluid over a stretching surface using scaling group transformation. Hashmi *et al.* [18] provided analytical solutions for the squeezing flow of nanofluids between parallel disks in the presence of a magnetic field. Recently, Das [19] studied the stagnation point nanofluid flow in the presence of chemical reaction and magnetic field using the Lie group analysis.

In these studies the fluid velocity is assumed to be zero relative to the solid boundary. But this is not true for fluid flows at the micro and nano scale. Investigation shows that a slip flow happens when the characteristic size of the flow system is small or the flow pressure is very low. A partial slip may occur on a stationary and moving boundary when the fluid is particulate such as emulsions, suspensions, foams, and polymer solutions. To describe the phenomenon of slip, Navier [20] introduced a boundary condition which states that the component of the fluid velocity tangential to the boundary walls is proportional to tangential stress. Later, several researchers [21-23] extended the Navier boundary conditions. A number of significant studies [24-28] have been done analytically and numerically regarding the slip flow regimes over different types of flow configurations. These results demonstrated that the boundary layer equation can be used to study the flow at the micro electro mechanical system (MEMS) scale and provide useful information to study the effects of rarefaction on the shear stress and structure of the flow. The magnetohydrodynamic squeezing flow of nanofluids between parallel disks was performed by Hussain *et al.* [29]. Recently, Das [30] analyzed the convective heat transfer characteristic of nanofluid over a permeable stretching surface with partial slip velocity.

The author's best knowledge, a study on an unsteady MHD squeezing flow of a nanofluid between two parallel disks with velocity slip and temperature jump has never been made till date. Therefore, the objective of the present paper is to investigate the slip effects on the squeezing flow of a nanofluid between two parallel disks in the presence of a magnetic field.

## 2. Mathematical formulation of the problem

Let us consider an axisymmetric flow of an incompressible electrically conducting viscous nanofluid between two parallel disks separated by a variable distance  $h(t)=H(1-\alpha t)^{1/2}$  where  $\alpha$  is a characteristic parameter having dimensions of time inverse. The upper disk at  $z=h(t)$  is approaching the lower stationary disk at  $z=0$  with a velocity  $v(t)=dh/dt$  until they touch. A uniform magnetic field of strength  $B(t)=B_0(1-\alpha t)^{-1/2}$  is applied perpendicular to the disks as shown in Fig.1. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. The fluid structure is everywhere in local thermodynamic equilibrium. The plate is maintained at a constant temperature.

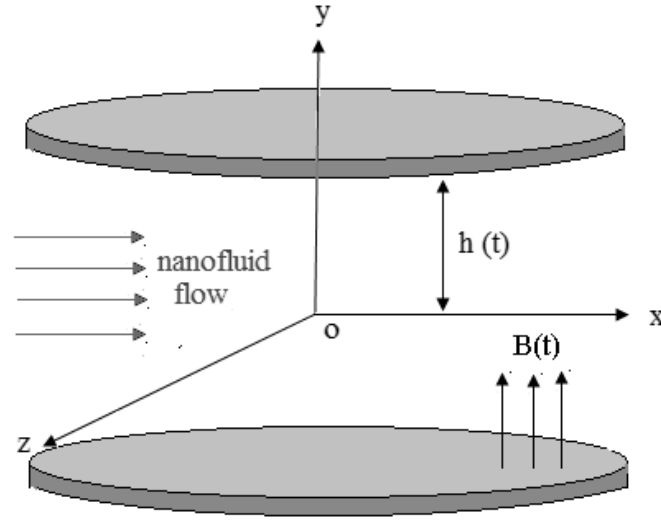


Fig.1. Physical model of the problem.

Under the stated assumptions, the governing conservation equations at unsteady state can be expressed as

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (2.1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho_f} \frac{\partial p}{\partial r} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right) - \frac{\sigma B^2(t)}{\rho_f} u, \quad (2.2)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho_f} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right), \quad (2.3)$$

$$\begin{aligned} \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = & \alpha \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \\ & + \tau \left[ D_B \left( \frac{\partial C}{\partial r} \frac{\partial T}{\partial r} + \frac{\partial C}{\partial z} \frac{\partial T}{\partial z} \right) + \frac{D_T}{T_m} \left\{ \left( \frac{\partial T}{\partial r} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right\} \right], \end{aligned} \quad (2.4)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} = D_B \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right) + \frac{D_T}{T_m} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) \quad (2.5)$$

where  $u$  and  $w$  are radial and axial velocities along  $r$  and  $z$ -axis, respectively,  $\rho_f$  is the density of the nanofluid,  $\sigma$  is the electrical conductivity,  $\nu$  is the kinematic viscosity,  $\tau = (\rho c)_p / (\rho c)_f$  is the ratio of the effective heat capacity of the nanoparticle material and the nanofluid,  $\alpha$  is the thermal diffusivity,  $D_B$  is the Brownian motion coefficient,  $D_T$  is the thermophoretic diffusion coefficient,  $T$  is the temperature of the fluid,  $T_m$  is the mean fluid temperature and  $C$  is the nanoparticles concentration.

The appropriate boundary conditions for the problem are

$$u = -\beta_l \frac{\partial u}{\partial z}, \quad w = \frac{w_0}{\sqrt{1-\alpha t}}, \quad T = -\gamma_l \frac{\partial T}{\partial z} + T_w, \quad C = C_w \quad \text{at} \quad z = 0, \quad (2.6)$$

$$u = \beta_l \frac{\partial u}{\partial z}, \quad w = w_h \left( = \frac{dh}{dt} \right), \quad T = \gamma_l \frac{\partial T}{\partial z} + T_h, \quad C = C_h \quad \text{at} \quad z = h(t)$$

where  $\beta_l$  and  $\gamma_l$  are the velocity and thermal slip parameters, respectively.  $T_w$  and  $C_w$  denote the temperature and nanoparticles concentration at the lower disk while the temperature and nanoparticles concentration at the upper disk are  $T_h$  and  $C_h$ , respectively.

Let us introduce the following dimensionless quantities [4]

$$\eta = \frac{z}{H\sqrt{1-\alpha t}}, \quad u = \frac{\alpha r}{2(1-\alpha t)} f'(\eta), \quad w = -\frac{\alpha H}{2\sqrt{1-\alpha t}} f(\eta), \quad (2.7)$$

$$\theta = \frac{T - T_h}{T_w - T_h}, \quad \phi = \frac{C - C_h}{C_w - C_h}.$$

In view of Eqs (2.7), the continuity Eq.(2.1) is automatically satisfied while Eqs (2.2)-(2.5) after some mathematical simplification reduce to

$$f'''' - S(\eta f'''' + 3f'' + ff' - 2ff''') - M^2 f'' = 0, \quad (2.8)$$

$$\theta'' + \text{Pr} S(2f\theta' - \eta\theta') + \text{PrNb}\theta'\phi' + \text{Pr} Nt\theta'^2 = 0, \quad (2.9)$$

$$\phi'' + \text{Le}S(2f\phi' - \eta\phi') + \frac{Nt}{\text{Nb}}\theta'' = 0. \quad (2.10)$$

Also the boundary conditions (2.6) become

$$f = A, \quad f' + \beta f'' = 0, \quad \theta + \gamma \theta' = 1, \quad \phi = 0 \quad \text{for} \quad \eta = 0, \quad (2.11)$$

$$f = \frac{1}{2}, \quad f' - \beta f'' = 0, \quad \theta - \gamma \theta' = 0, \quad \phi = 0 \quad \text{for} \quad \eta = 1.$$

Here  $S = \frac{\alpha H^2}{2\nu}$  is the squeeze number where  $S > 0$  corresponds to the plates moving apart while  $S < 0$

corresponds to the plates moving together (the so called squeezing flow),  $M = HB_0 \sqrt{\frac{\sigma}{\mu}}$  is the Hartmann

parameter,  $\text{Pr} = \frac{\nu}{\alpha}$  is the Prandtl number,  $\text{Le} = \frac{\nu}{D_B}$  is the Lewis number,  $\text{Nb} = \frac{\tau D_B (C_w - C_h)}{\nu}$  is the

Brownian motion parameter,  $\text{Nt} = \frac{\tau D_T (T_w - T_h)}{\nu T_m}$  is the thermophoresis parameter,  $A = \frac{w_0}{\alpha H}$  is the

suction/injection parameter,  $\beta = \frac{\beta_l}{H\sqrt{1-\alpha t}}$  is the dimensionless velocity slip parameter and  $\gamma = \frac{\gamma_l}{H\sqrt{1-\alpha t}}$  is the dimensionless temperature slip parameter.

The most important characteristics of flow, heat and mass transfer are the reduced skin friction coefficient  $C_{fr}$ , the reduced Nusselt number  $Nu$  and the reduced Sherwood  $Sh$  that are defined as

$$C_{fr} = f''(I) \quad \text{where} \quad C_{fr} = \frac{H^2 \sqrt{I - \alpha t} Re_r}{r^2} C_f, \quad (2.12)$$

$$Nur = -\theta'(I) \quad \text{where} \quad Nur = \sqrt{I - \alpha t} Nu, \quad (2.13)$$

and  $Shr = -\phi'(I) \quad \text{where} \quad Shr = \sqrt{I - \alpha t} Sh \quad (2.14)$

where  $Re_r = \frac{r\alpha H}{2\nu}$  is the local squeeze Reynolds number.

It is worth mentioning that in the absence of slip velocity (i.e.,  $\beta = 0$ ) and slip temperature (i.e.,  $\gamma = 0$ ), the boundary conditions (2.11) reduce to (2.10) of Hashmi *et al.* [18].

### 3. Method of solution

The set of Eqs (2.8), (2.9) and (2.10) under the boundary conditions (2.11) have been solved numerically by applying the shooting iteration technique together with the sixth-order Runge-Kutta integration scheme. The unspecified initial conditions are assumed and then integrated numerically as an initial valued problem to a given terminal point. Improvement is made on the values of assumed missing initial conditions by iteratively comparing the calculated value of the dependent variable at the terminal point with its given value there. The computations are done by a written program which uses a symbolic and computational computer language MAPLE.

In order to ascertain the accuracy of our numerical results, the present study (in the absence of slip parameter) is compared with those of Hashmi *et al.* [18]. The values of  $Nur$  and  $Shr$  have been calculated for various values of  $Nb$  and  $Nt$ . An excellent agreement is found between the two set of results as shown in Tab.1. Thus the use of the present numerical code for the current model is justified.

Table 1. Comparison of the values of the local Nusselt number and local Sherwood number for various values of  $Nb$  and  $Nt$  with  $\beta = \gamma = 0$ .

Nb	Nt	Nur		Shr	
		Hashmi <i>et al.</i> [18]	Present method	Hashmi <i>et al.</i> [18].	Present method
0.1	0.1	0.52628540	0.526285397692707	0.86604666	0.866046665141329
0.5		0.63433253	0.634332530012476	0.53012814	0.530128143896286
1.0		0.78636385	0.786363850155776	0.48603919	0.486039186120291
1.5		0.95569955	0.955699547716439	0.46986157	0.469861566526085
1.5	0.5	1.17682119	1.176821184883260	0.40180718	0.401807177532398
	1.0	1.48581207	1.485811936635642	0.12619334	0.126193335885242
	1.5	1.82305276	1.823053529110100	0.39083080	0.390839865635371
	2.0	2.17915991	2.179227931099257	1.16777723	1.16800852390237

### 4. Results and discussions

In order to get an insight into the effects of different parameters of practical importance on the flow characteristics, the numerical results are presented graphically in Figs 2-16 for several sets of values of the

pertinent parameters. In the simulation the default values of the parameters are considered as  $A=2$ ,  $S=1$ ,  $Nt=Nb=0.2$ ,  $\beta=0.05$ ,  $\gamma=0.1$ ,  $M=0.5$ ,  $Le=Pr=1.0$  unless otherwise specified.

The effect of the magnetic field parameter  $M$  on the radial velocity is depicted in Fig.2. It is observed from the figure that the fluid velocity decreases with an increase in the values of  $M$  and so increases the thickness of momentum boundary layer near the wall region, i.e., for  $\eta < 0.2$  and  $\eta > 0.7$  (not precisely determined). The reason behind this phenomenon is that the application of a magnetic field to an electrically conducting nanofluid gives rise to a resistive type force called the Lorentz force. This force has the tendency to slow down the motion of the nanofluid in the boundary layer region. However, the opposite trend is observed in the central region, i.e., for  $0.2 \leq \eta \leq 0.7$  (not precisely determined) due to mass conservation. Figure 3 shows the influence of the slip parameter  $\beta$  on the radial velocity. The figure clearly indicates that in the absence of the slip velocity, i.e., when  $\beta=0$ , the radial velocity of the fluid at the disks' surfaces is the same as that of the disks' surfaces, which is zero in the present case. Also, it is observed that the magnitude of the radial velocity acquires a positive value on the disks' surface in the presence of the slip velocity which increases with the increase in the slip parameter  $\beta$  for  $\eta < 0.2$  and  $\eta > 0.8$  (not precisely determined). But the effect is totally reversed i.e., the fluid velocity decreases with an increase in the values of the slip parameter  $\beta$  for  $0.2 \leq \eta \leq 0.8$  (not precisely determined). The physics behind this is that the increased slip parameter near the wall region decreases the velocity gradient. Also, an increase in the fluid velocity near the wall will be compensated by a corresponding decrease in the fluid velocity near the central region as the mass flow rate is kept constant. With the increase in the squeeze number  $S$ , the fluid velocity decreases at first near the lower disk region, i.e., for  $\eta < 0.4$  (not precisely determined); then it starts increasing with the increase in  $S$  near the upper disk region as shown in Fig.4. These results are in a good agreement with the results obtained by Hashmi *et al.* [18] in the absence of slip parameters.

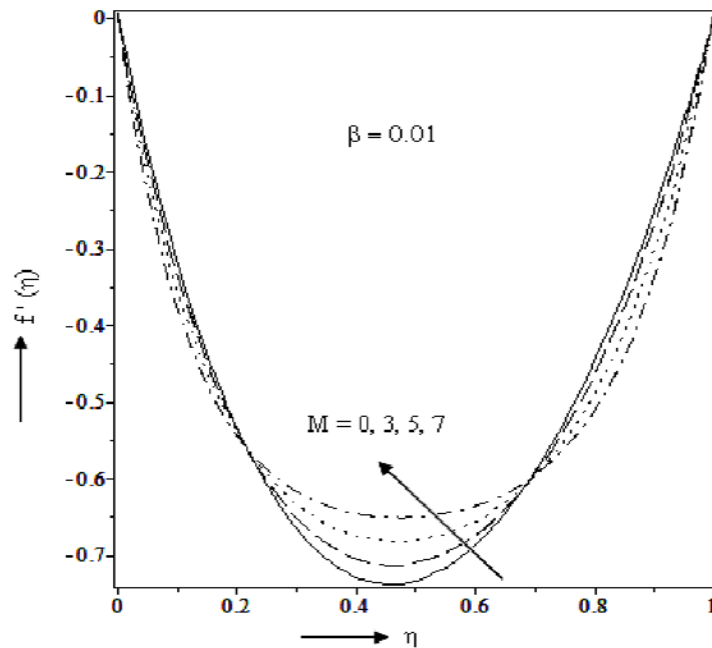


Fig.2. Velocity profiles for various values of  $M$ .

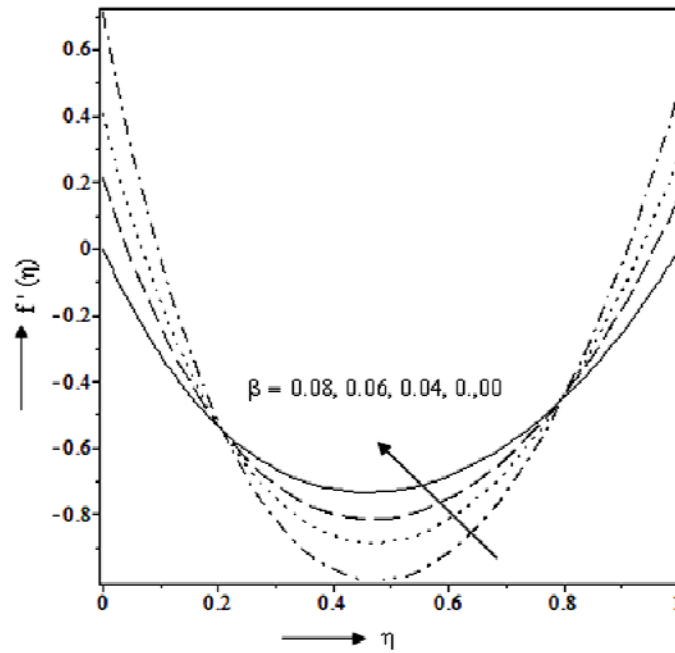


Fig.3. Velocity profiles for various values of  $\beta$ .

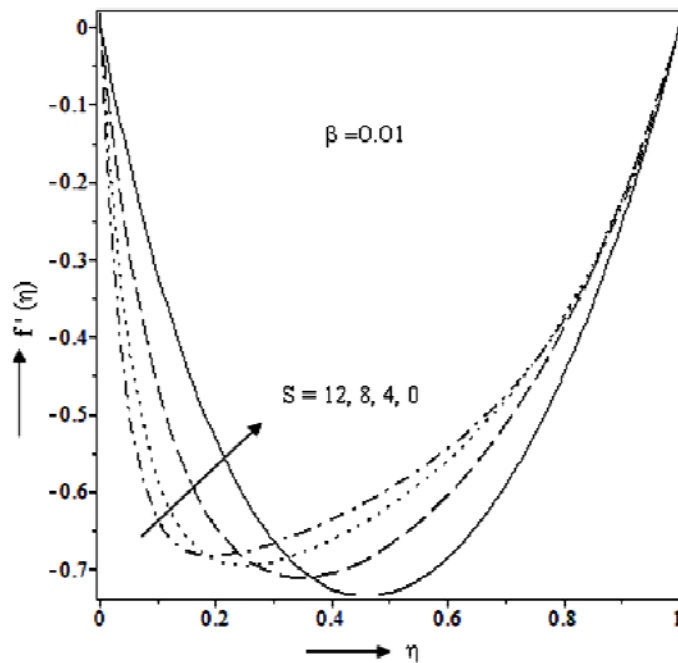


Fig.4. Velocity profiles for various values of  $S$ .

Figure 5 demonstrates the effects of the thermal slip parameter  $\gamma$  on the fluid temperature in the presence of suction. It is observed from the figure that in the absence of the thermal slip parameter, i.e., when  $\gamma=0$ , the temperature of the fluid and that of the disks' surfaces is same, which in the present problem is 1 and 0 for lower and upper disks, respectively. As the slip parameter  $\gamma$  increases, the temperature of the fluid increases near the lower disk surface while a reverse behaviour occurs near the upper disk surface. The effects of the parameters controlling the Brownian motion and thermophoresis on the temperature distribution are well presented in Figs 6, 7 in the presence of thermal slip. It is observed from the figures that the temperature of the fluid decreases at first with an increase in both the values of the Brownian motion parameter  $Nb$  and thermophoresis parameter  $Nt$  for  $\eta < 0.2$  (not precisely determined); then it starts increasing

near the central region, i.e., for  $0.2 \leq \eta \leq 0.8$  (not precisely determined) and then it again decreases with increasing both the values of  $Nb$  and  $Nt$  near the upper disk region i.e., for  $\eta > 0.8$  (not precisely determined). An effect of varying the value of the squeeze number  $S$  on the temperature distribution is illustrated in Fig.8. With the increase in the squeeze number  $S$ , the temperature of the fluid decreases near the lower disk region i.e., for  $\eta < 0.8$  (not precisely determined) but a reverse effect occurs near the upper disk region. It is worth mentioning that an increase in  $S$  can be associated with a decrease in the kinematic viscosity, an increase in the distance between the plates and an increase in the speed at which the plates move. These results are found to be identical to the results of Hashmi *et al.* [18] in the absence of slip parameters.

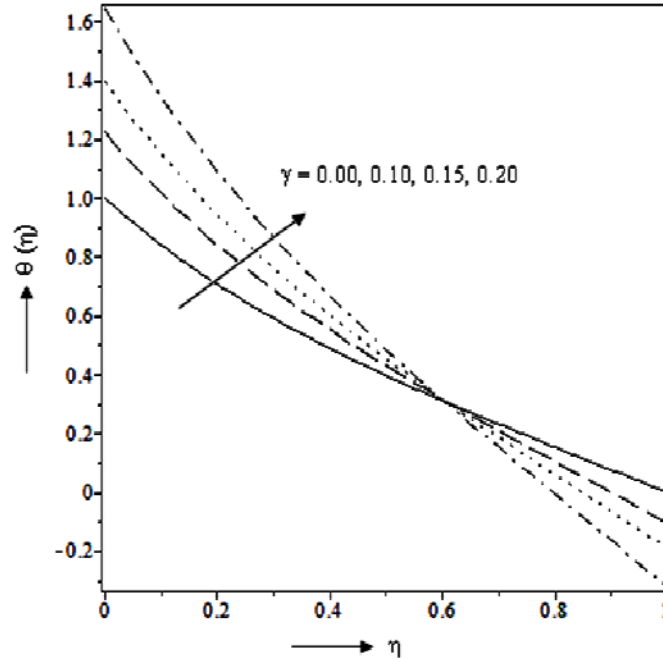


Fig.5. Temperature profiles for various values of  $\gamma$ .

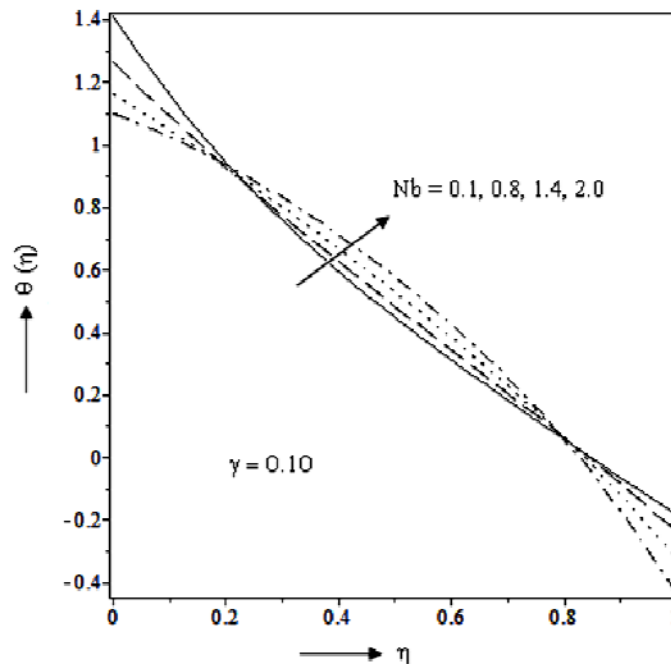


Fig.6. Temperature profiles for various values of  $Nb$ .



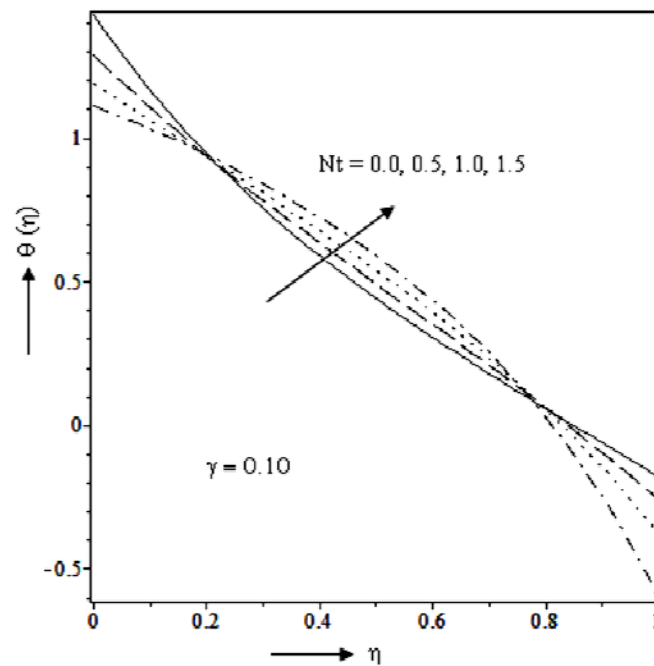
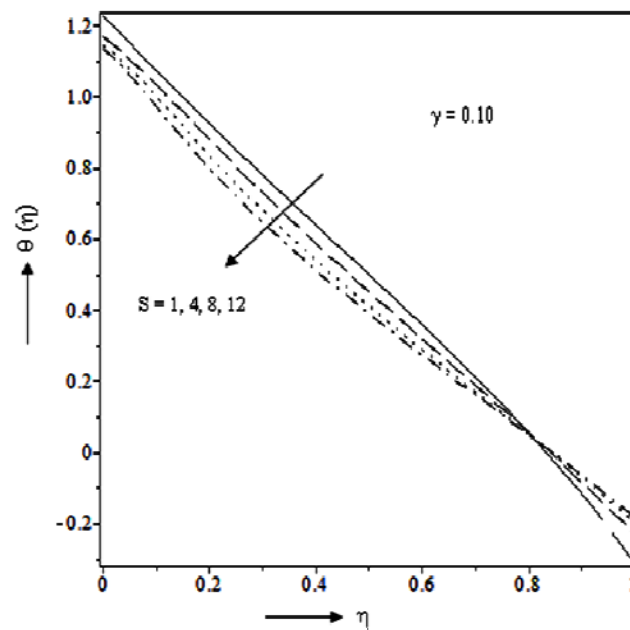
Fig.7. Temperature profiles for various values of  $Nt$ .Fig.8. Temperature profiles for various values of  $S$ .

Figure 9 shows the variation of the nanoparticles concentration distribution for different values of the Lewis number  $Le$  in the presence of suction. It is seen that the concentration decreases with the increase in the values of  $Le$ . Also, it is observed that the concentration of the fluid decreases drastically for higher values of  $Le$  near the lower disk region. The impact of the thermophoresis on the concentration distribution is presented in Fig.10. It is seen that the effect of increasing the values of the thermophoresis parameter  $Nt$  results in increasing concentration distribution rapidly near the wall region of the lower disk whereas the

concentration decreases steadily from the lower disk region to the upper disk region in the absence of thermophoresis. Figure 11 reveals that the nanoparticles concentration decreases uniformly with increasing the values of the Brownian motion parameter  $Nb$ . These results fall in line with the results obtained by Hashmi *et al.* [18] in the absence of the slip parameter. The concentration distribution decreases significantly with increasing the values of squeeze number  $S$  as depicted in Fig.12 in the presence of suction.

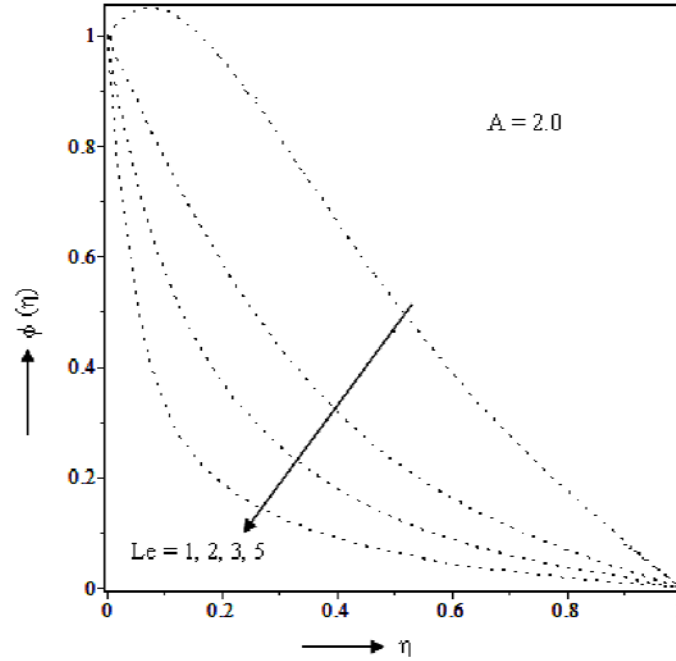


Fig.9. Concentration profiles for various values of  $Le$ .

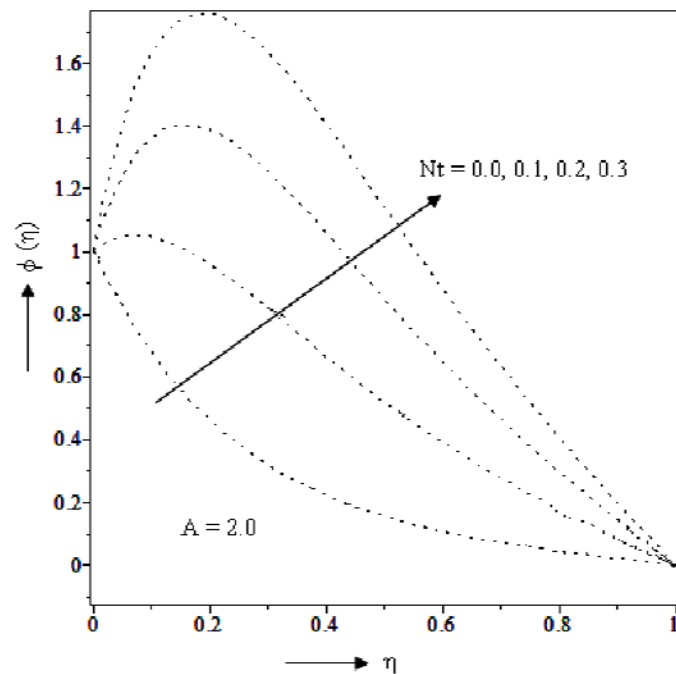


Fig.10. Concentration profiles for various values of  $Nt$ .

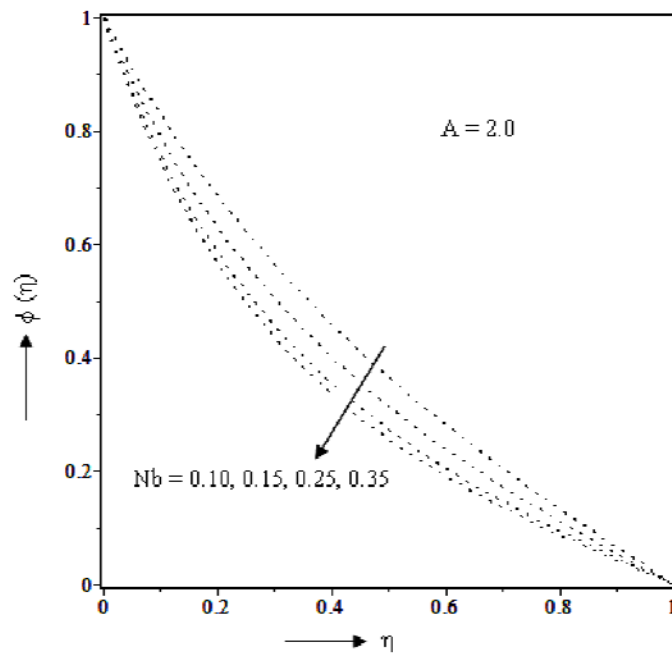


Fig.11. Concentration profiles for various values of Nb.

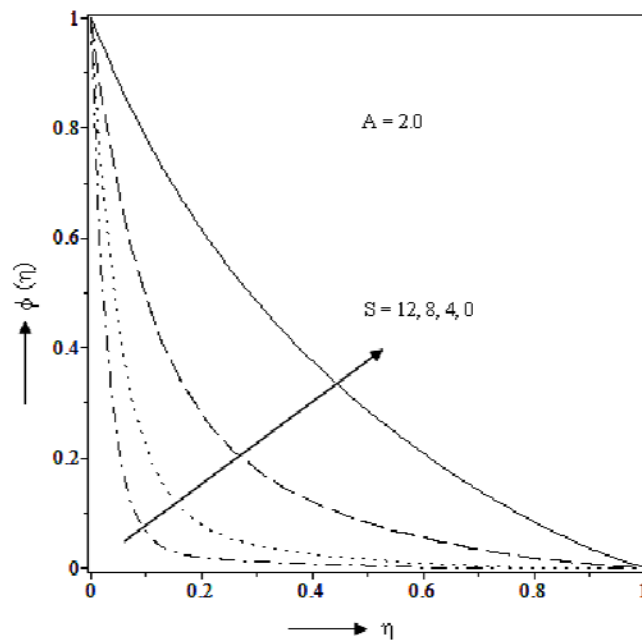


Fig.12. Concentration profiles for various values of S.

Figures 13, 14 show variation of the skin friction coefficient on the wall of the upper disk versus the squeeze number  $S$  for different values of the magnetic field parameter  $M$  and slip parameter  $\beta$ , respectively, in the presence of suction. A combination of different applications which appear in these figures guides us to understand carefully how the skin friction coefficient on the wall of the upper disk change. Increasing  $S$

tends to decrease the skin friction coefficient for different values of  $M$  and  $\beta$  but the effect is not significant for higher values of  $\beta$ . On the other hand, the magnetic field parameter  $M$  increases the skin friction coefficient on the upper disk wall whereas the slip parameter  $\beta$  moves it down to reach a lower value. Figure 15 illustrates the variation of the Nusselt number at the upper disk wall versus the thermophoresis parameter  $Nt$  for various values of the surface thermal slip parameter  $\gamma$ . This figure indicates that the Nusselt number increases with the increasing thermophoresis parameter  $Nt$ . But for a large temperature slip parameter  $\gamma$ , the increase in  $Nt$  has a prominent effect on the dimensionless heat transfer rate. It is also observed that the increase of  $\gamma$  results in an increase in the Nusselt number. Thus the temperature slip parameter has a significant effect on heat transfer due to an increased temperature gradient on the upper disk wall. Figure 16 displays that the rate of mass transfer, i.e., the Sherwood number decreases considerably as the Lewis number  $Le$  increases whereas an opposite effect is observed for the thermophoresis parameter  $Nt$ . Thus the rate of mass transfer increases actively due to the presence of thermophoresis.

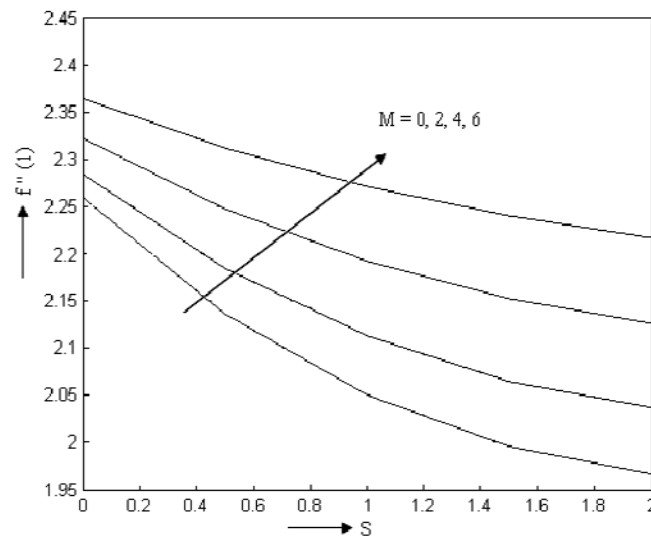


Fig.13. Effects of  $M$  and  $S$  on the skin friction coefficient.

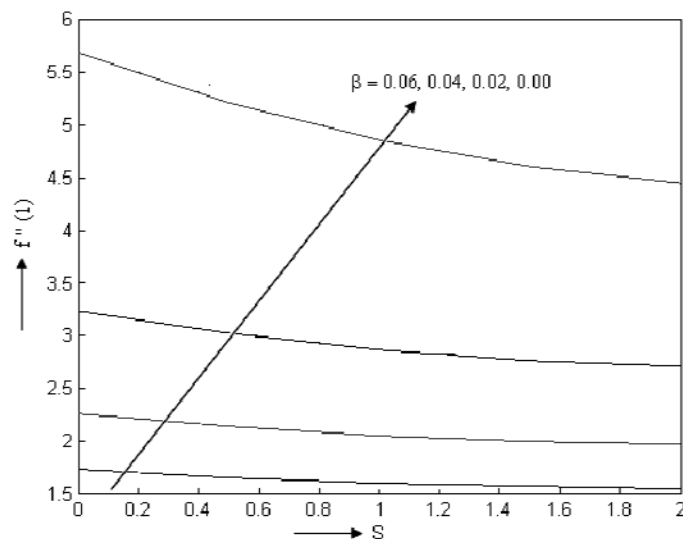


Fig.14. Effects of  $\beta$  and  $S$  on the skin friction coefficient.

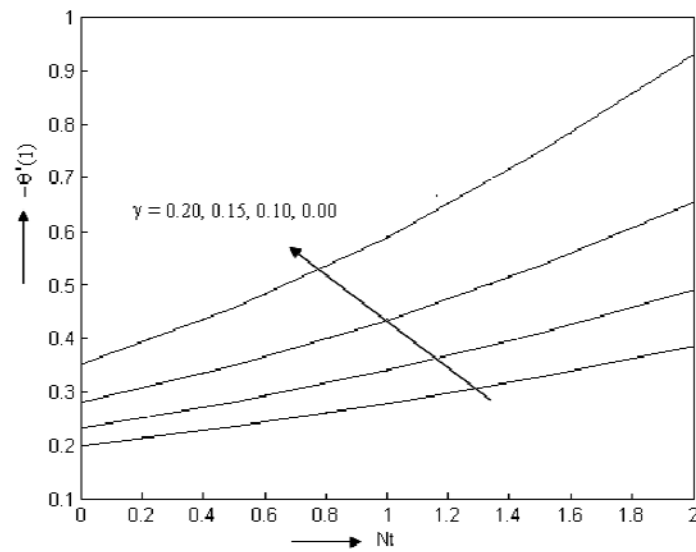


Fig.15. Effects of  $\gamma$  and  $Nt$  on the Nusselt number.

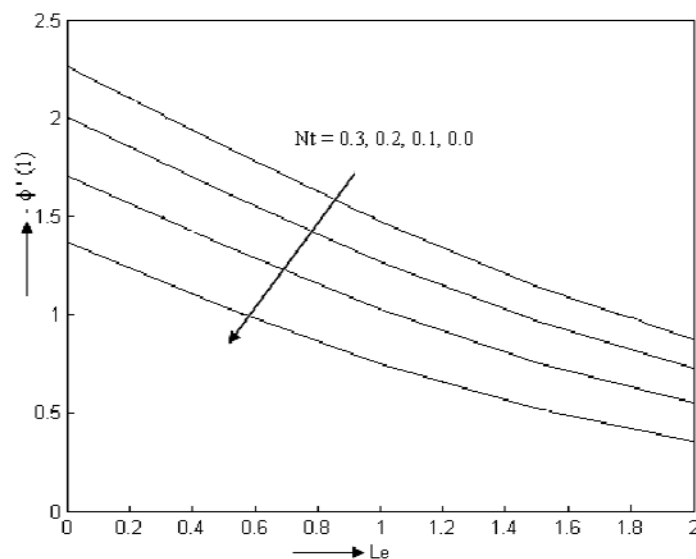


Fig.16. Effects of  $Nt$  and  $Le$  on the Sherwood number.

## 5. Conclusions

In the present study, the influence of velocity and thermal slip conditions on the unsteady MHD squeezing flow in a viscous, incompressible and electrically conducting nanofluid between two parallel disks is analyzed using a numerical technique. The Nachtsheim and Swigert shooting iteration technique together with the Runge -Kutta sixth-order integration scheme is used to obtain the numerical solution of the nonlinear flow problem. Numerical results are presented through graphs to illustrate the details of the flow characteristics and their dependence on material parameters. The results presented here are potentially in controlling wall shear stress as well as the Nusselt and the Sherwood number. Also, the present study has a direct association with nanofluidics and microscale flows where the no-slip boundary

condition usually breaks down. Some important positive effects of the slip in nanofluidics are the friction reduction, energy conservation and micromixing of biological samples under a magnetic field in microchannels.

## Nomenclature

$A$	– suction/injection parameter
$B_0$	– strength of the magnetic field $kgs^{-2}A^{-1}$
$C$	– nanoparticle volumetric fraction
$c_p$	– specific heat at constant pressure
$D_B$	– Brownian diffusion coefficient
$D_T$	– thermophoretic diffusion coefficient
$M$	– magnetic field parameter
Nu	– local Nusselt number
Pr	– Prandtl number
Sh	– local Sherwood number
$s$	– squeeze number
$T$	– temperature variable $K$
$u, v$	– radial and axial velocities $ms^{-1}$
$\beta$	– velocity slip parameter
$\gamma$	– thermal slip parameter
$\eta$	– similarity variable
$\theta$	– dimensionless temperature
$\nu$	– kinematic viscosity
$\tau$	– ratio between effective heat capacity of the nanoparticle and the fluid
$\phi$	– dimensionless nanoparticle concentration

## Subscript

Le	– Lewis number
Nb	– Brownian motion parameter
Nt	– thermophoresis parameter
w	– condition on the sheet
'	– differentiation with respect to $\eta$

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