10.2478/v10170-011-0043-x

Archives of Control Sciences
Volume 23(LIX), 2013
No. 1, *pages 79–92*

An exponential observer and a controller for a class of nonlinear systems

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In this paper, we study the observer design problem for a class of nonlinear systems. Specifically, we design an exponential observer for a separately excited DC motor. Moreover, a stabilizing controller is designed for the system to ensure the exponential stability of the solutions toward their desired values. Simulations results show that proposed observer is able to reconstruct the states of the system. In addition, the simulation results indicate that the designed controller works well.

Key words: nonlinear system, observer, stability, controller, stabilization

1. Introduction

The design of observer for dynamical systems modeled by nonlinear differential equations is generally a difficult task due to the presence of nonlinearities. The usual requirements are the global Lipschitz condition ([1], [2], [3], [4], [5]) or the existence of change of coordinates which transforms the system into the desired observer form ([6]). Several researchers have studied for the control of DC motors by using different techniques ([7], [8], [9]). The stability problem of such systems is also considered in terms of ultimate bounded solutions ([10], [11], [12]). This approach is very general and powerful although there is an inherent difficulty associated with the selection of a suitable Lyapunov function.

The objective of this paper is to present an observer design for the separately excited DC motor. Furthermore, stability of the system is studied where a controller which guarantees the global and exponential stability of the trajectories of the system is designed.

The paper is organized as follows. The problem under consideration is formulated in section 2. The observer design for the DC motor is detailed in section 3. The speed controller design for the motor is presented in section 4. The simulation results are presented and discussed in section 5. Finally, some concluding remarks are given in section 6.

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Received 22.05.2012.

2. Problem formulation

A separately excited DC motor can be described using the following set of ordinary differential equations:

$$\begin{cases} \dfrac{di_a}{dt} = \dfrac{1}{L_a}(v_a - R_a i_a - K_m i_f w) \\ \\ \dfrac{di_f}{dt} = \dfrac{1}{L_f}(v_f - R_f i_f) \\ \\ \dfrac{dw}{dt} = \dfrac{1}{J_m}(K_m i_a i_f - B_m w - T_l) \end{cases}$$

where i_a and i_f are the armature and field currents; w is the rotor speed. The voltages v_a and v_f are the armature and field voltages. The resistances R_a and R_f are the armature and field resistances; L_a and L_f are the armature and field inductances. The constant K_m is the motor torque constant, J_m is the inertia of the motor, and B_m is the damping coefficient. The load torque is T_l.

For ease of presentation, we define the following constants:

$$K_1 = -\frac{R_a}{L_a},\ K_2 = -\frac{K_m}{L_a},\ K_3 = -\frac{R_f}{L_f},\ K_4 = \frac{K_m}{J_m},\ K_5 = -\frac{B_m}{J_m}.$$

Note that, $K_i < 0$ for $i = 1,2,3,5$. Also, denote the state variables x_1, x_2 and x_3 such that:

$$x_1 = i_a \quad x_2 = i_f \quad x_3 = w.$$

Therefore, the equations describing a separately excited DC motor can be written as:

$$\begin{cases} \dot{x}_1 = K_1 x_1 + K_2 x_2 x_3 + \dfrac{1}{L_a} v_a \\ \\ \dot{x}_2 = K_3 x_2 + \dfrac{1}{L_f} v_f \\ \\ \dot{x}_3 = K_4 x_1 x_2 + K_5 x_3 - \dfrac{1}{J_m} T_l \end{cases} \tag{1}$$

For this class of control systems, we can design an observer to estimate the states of the system. Also, we can design a stabilizing controller that rends the system globally and exponentially stable. It should be noted that the voltage v_a and v_f in (1) are the inputs of the system. Also, note that the design of the controller will be performed without taking into account the load torque; a load observer can be designed to estimate the load of the system [8].

3. Observer design

In this section of the paper, we are interested in designing an observer to the following class of input-output systems:

$$\begin{cases} \dot{x}_1 = K_1 x_1 + K_2 x_2 x_3 + \dfrac{1}{L_a} v_a \\ \dot{x}_2 = K_3 x_2 + \dfrac{1}{L_f} v_f \\ \dot{x}_3 = K_4 x_1 x_2 + K_5 x_3 - \dfrac{1}{J_m} T_l \\ y = x_1 \end{cases} \tag{2}$$

It is well known that, generally, not all the states of a system are available for direct measurement. Therefore, the unmeasurable states must be estimated since they are needed for feedback. The observer is a dynamical system which is expected to reconstruct the states of the system. Our objective is to design a state reconstructor for the system (2) such that the global exponential convergence to zero of the resulting error system can be guaranteed.

Before presenting the result, we introduce the following definition.

Definition 1 *The system* (2) *is exponentially state reconstructible if there exists a state observer* $\dot{\hat{x}} = G(\hat{x}, y, u)$*, where* u *is the input of the system, and* λ *and* γ *are positive numbers such that:*

$$\|e(t)\| = \|\hat{x}(t) - x(t)\| \leqslant \lambda(\|e(0)\|)e^{-\gamma t}, \ \forall t \geqslant 0. \tag{3}$$

Here, $\hat{x}(t) = [\hat{x}_1(t)\ \hat{x}_2(t)\ \hat{x}_3(t)]^T$ are the estimates of the states $x(t) = [x_1(t)\ x_2(t)\ x_3(t)]^T$.

For design purposes, we assume that the load torque is zero. Therefore, the equations of the system can be written as:

$$\begin{aligned} \dot{x}_1 &= K_1 x_1 + K_2 x_2 x_3 + \frac{1}{L_a} v_a \\ \dot{x}_2 &= K_3 x_2 + \frac{1}{L_f} v_f \\ \dot{x}_3 &= K_4 x_1 x_2 + K_5 x_3 \\ y &= x_1 \end{aligned} \tag{4}$$

We propose to use the following state observer:

$$\begin{aligned} \dot{\hat{x}}_1 &= K_1 \hat{x}_1 + K_2 \hat{x}_2 \hat{x}_3 + L_1(x_1 - \hat{x}_1) + \frac{1}{L_a} v_a \\ \dot{\hat{x}}_2 &= K_3 \hat{x}_2 + \frac{1}{L_f} v_f \\ \dot{\hat{x}}_3 &= K_4 x_1 \hat{x}_2 + K_5 \hat{x}_3 \end{aligned} \tag{5}$$

where L_1 is a positive scalar which will be chosen later. The system (5) will produce $\hat{x}(t)$ which is the estimate of the state $x(t)$.

We present the following theorem for the state reconstruction of system (4).

Theorem 1 *The system* (5) *is an exponential observer for* (4).

Proof Define the following estimation errors:

$$\begin{aligned} e_1(t) &= \hat{x}_1 - x_1 \\ e_2(t) &= \hat{x}_2 - x_2 \\ e_3(t) &= \hat{x}_3 - x_3 \end{aligned} \tag{6}$$

It is clear from (4) - (6) that $e_2(t)$ satisfies the following differential equation:

$$\dot{e}_2(t) = K_3 e_2(t). \tag{7}$$

Thus, by integrating the above equation from zero to t and denoting the initial condition of $e_2(t)$ by $e_2(0)$, one gets:

$$e_2(t) = e_2(0)e^{K_3 t}. \tag{8}$$

Therefore, it can be concluded that $e_2(t)$ converges exponentially to zero as t tends to infinity since K_3 is negative.

Also, it is clear from (4) - (6) that the error $e_3(t)$ satisfies the following differential equation:

$$\dot{e}_3(t) = K_5 e_3(t) + K_4 y e_2(t). \tag{9}$$

Using equation (8), the above equation can be written as:

$$\dot{e}_3(t) = K_5 e_3(t) + K_4 y e_2(0) e^{K_3 t}. \tag{10}$$

By integrating the above equation from zero to t and denoting the initial condition of $e_3(t)$ by $e_3(0)$, one gets:

$$e_3(t) = e_3(0)e^{K_5 t} + K_4 e_2(0) \int_0^t y(\tau) e^{K_5(t-\tau)} e^{K_3 \tau} d\tau. \tag{11}$$

The constants K_3 and K_5 are parameters of the motor and they are such that $K_3 \neq K_5$.

Also, note that since the motor is properly controlled, the output $y(t)$ is bounded; this means that there exist a positive scalar B_y such that $|y| < B_y$. Thus,

$$|e_3(t)| \leqslant |e_3(0)|e^{K_5 t} + B_y K_4 |e_2(0)| \frac{1}{|K_3 - K_5|} |e^{K_3 t} - e^{K_5 t}|. \tag{12}$$

Since $e^{K_3 t}$ together with $e^{K_5 t}$ tend to zero as t tends to infinity because $K_3 < 0$ and $K_5 < 0$, then it can be concluded that the error $e_3(t)$ tends exponentially to zero as t tends to infinity. Therefore, there exist a time $T > 0$ and a positive constant λ which depends on T, B_y, K_4 $|e_2(0)|$ and $|e_3(0)|$, such that:

$$|e_3(t)| \leqslant \lambda_3 e^{\frac{1}{2}\sup(K_3,K_5)t}. \tag{13}$$

Therefore, it can be concluded that the error $e_3(t)$ converges exponentially to zero as t tends to infinity.

Using (4) - (6), one can deduce that the error $e_1(t)$ satisfies the following differential equation:

$$\begin{aligned} \dot{e}_1(t) &= (K_1 - L_1)e_1 + K_2(\hat{x}_2\hat{x}_3 - x_2x_3) \\ &= (K_1 - L_1)e_1 - K_2e_2e_3 + K_2\hat{x}_2e_3 + K_2\hat{x}_3e_2. \end{aligned} \tag{14}$$

By integrating the above equation from zero to t and denoting the initial condition of $e_1(t)$ by $e_1(0)$, one can write:

$$\begin{aligned} e_1(t) &= e_1(0)e^{(K_1-L_1)t} \\ &\quad + K_2\int_0^t (\hat{x}_2(s)e_3(s) + \hat{x}_3(s)e_2(s) - e_2(s)e_3(s))\, e^{(K_1-L_1)(t-s)}ds. \end{aligned} \tag{15}$$

Note that, we shall choose L_1 such that $K_1 - L_1 < 0$. Moreover, note that from (5) the components $\hat{x}_2$ and $\hat{x}_3$ are bounded since the errors $e_2(t)$ and $e_3(t)$ converge to zero exponentially and since the system is properly controlled. Hence, there exist positive constants M_2 and M_3 such that,

$$|\hat{x}_2| \leqslant M_2 \quad \text{and} \quad |\hat{x}_3| \leqslant M_3. \tag{16}$$

Therefore, since the errors $e_2(t)$ and $e_3(t)$ converge to zero exponentially, it can be concluded that $e_1(t)$ will also converge to zero as t tends to infinity. Indeed, taking into account the above expressions and the estimations (8) and (13), it follows that:

$$\begin{aligned} |e_1(t)| &\leqslant |e_1(0)|e^{(K_1-L_1)t} \\ &\quad + |K_2|M_2\lambda_3 e^{(K_1-L_1)t}\int_0^t e^{(\frac{1}{2}\sup(K_3,K_5)-(K_1-L_1))s}ds \\ &\quad + |K_2|M_3|e_2(0)|e^{(K_1-L_1)t}\int_0^t e^{(K_3-(K_1-L_1))s}ds \\ &\quad + |K_2||e_2(0)|\lambda_3 e^{(K_1-L_1)t}\int_0^t e^{(K_3+\frac{1}{2}\sup(K_3,K_5)-(K_1-L_1))s}ds. \end{aligned} \tag{17}$$

Hence,

$$\begin{aligned}
|e_1(t)| \leqslant\ & |e_1(0)|e^{(K_1-L_1)t} \\
&+M_2\lambda_3|K_2|\frac{1}{|\frac{1}{2}\sup(K_3,K_5)-(K_1-L_1)|}|e^{(\frac{1}{2}\sup(K_3,K_5)t}-e^{(K_1-L_1))t}| \\
&+M_3|K_2||e_2(0)|\frac{1}{|K_3-(K_1-L_1)|}|e^{K_3t}-e^{(K_1-L_1))t}| \\
&+\lambda_3|K_2||e_2(0)|\frac{|e^{(K_3+\frac{1}{2}\sup(K_3,K_5)t}-e^{(K_1-L_1))t}|}{|K_3+\frac{1}{2}\sup(K_3,K_5)-(K_1-L_1)|}.
\end{aligned} \tag{18}$$

Notice that L_1 is chosen such that $K_1 - L_1 < 0$, $K_3 - (K_1 - L_1) \neq 0$ and $K_3 + \frac{1}{2}\sup(K_3,K_5) - (K_1 - L_1) \neq 0$. This is possible for L_1 small enough. Hence, since $K_3 < 0$, $\sup(K_3,K_5) < 0$ and $K_1 - L_1 < 0$, then there exist a time $\widetilde{T} > 0$, $\lambda_1 > 0$ and $K < 0$, such that

$$|e_1(t)| \leqslant \lambda_1 e^{-Kt} \quad \text{for all} \quad t \geqslant \widetilde{T}. \tag{19}$$

Therefore, it can be concluded that the error $e_1(t)$ converges exponentially to zero as t tends to infinity.

Consequently, we conclude that the system (5) is a suitable state observer which can be used to reconstruct the states of the system (4).

4. Controller Design

This section deals with the design of a controller for the DC motor such that the shaft of the motor rotates to its desired speed.

Let x_{1d}, x_{2d} and x_{3d} be the desired constant values of the states x_1, x_2 and x_3 respectively. Clearly, the following condition should be satisfied at steady state:

$$K_4 x_{1d} x_{2d} + K_5 x_{3d} = 0.$$

In the following, we will design a state feedback controller for the motor while assuming that all the states of the system are available for feedback.

Define the regulation errors $\bar{e}_1$, $\bar{e}_2$ and $\bar{e}_3$ as follows:

$$\begin{aligned}
\bar{e}_1(t) &= x_1 - x_{1d} \\
\bar{e}_2(t) &= x_2 - x_{2d} \\
\bar{e}_3(t) &= x_3 - x_{3d}
\end{aligned} \tag{20}$$

Also, let g_1 and g_2 be positive scalars.

Theorem 2 *The control law:*

$$\begin{aligned} v_a &= L_a(-K_1 x_{1d} - K_2 x_2 x_3 - g_1 \bar{e}_1) \\ v_f &= L_f(-K_3 x_{2d} - g_2 \bar{e}_2) \end{aligned} \tag{21}$$

when applied to the the DC motor model given by (4) *guarantees the exponential convergence of the states of the motor to their desired values as t tends to infinity.*

Proof The application of the control law given by (22) into the dynamics of the DC motor given by (4) leads to the following closed loop dynamics:

$$\begin{aligned} \dot{x}_1 &= K_1 x_1 + K_2 x_2 x_3 - K_1 x_{1d} - K_2 x_2 x_3 - g_1 \bar{e}_1 \\ \dot{x}_2 &= K_3 x_2 - K_3 x_{2d} - g_2 \bar{e}_2 \\ \dot{x}_3 &= K_4 x_1 x_2 + K_5 x_3 \end{aligned} \tag{22}$$

Taking the time derivative of (20) and using equations (22) and the fact that

$$K_4 x_{1d} x_{2d} + K_5 x_{3d} = 0,$$

one obtains:

$$\begin{aligned} \dot{\bar{e}}_1 &= (K_1 - g_1)\bar{e}_1 \\ \dot{\bar{e}}_2 &= (K_3 - g_2)\bar{e}_2 \\ \dot{\bar{e}}_3 &= K_5 \bar{e}_3 + K_4 \bar{e}_1 \bar{e}_2 + K_4 x_{2d} \bar{e}_1 + K_4 x_{1d} \bar{e}_2 \end{aligned} \tag{23}$$

Let $\bar{e}_1(0)$, $\bar{e}_2(0)$ and $\bar{e}_3(0)$ be the initial values of $\bar{e}_1(t)$, $\bar{e}_2(t)$ and $\bar{e}_3(t)$ respectively. Also, define the constants r_i $(i = 1, \ldots, 5)$ such that

$$\begin{aligned} r_1 &= K_1 - g_1 \\ r_2 &= K_3 - g_2 \\ r_3 &= K_4 \bar{e}_1(0) \bar{e}_2(0) \\ r_4 &= K_4 x_{2d} \bar{e}_1(0) \\ r_5 &= K_4 x_{1d} \bar{e}_2(0). \end{aligned}$$

Note that r_1 and r_2 are negative constants because K_3 and K_5 are negative system parameters and g_1 and g_2 are positive design parameters.

The solution of the first equation of (23) is:

$$\bar{e}_1(t) = \bar{e}_1(0) e^{r_1 t} \tag{24}$$

The solution of the second equation of (23) is:

$$\bar{e}_2(t) = \bar{e}_2(0) e^{r_2 t} \tag{25}$$

Using (24) and (25), the third equation of (23) can be written as:

$$\dot{\bar{e}}_3(t) = K_5\bar{e}_3(t) + r_3e^{(r_1+r_2)t} + r_4e^{r_1t} + r_5e^{r_2t} \tag{26}$$

By integrating the above equation from zero to t, one gets:

$$\begin{aligned}\bar{e}_3(t) &= \bar{e}_3(0)e^{K_5t} + \int_0^t e^{K_5(t-\tau)}(r_3e^{(r_1+r_2)\tau} + r_4e^{r_1\tau} + r_5e^{r_2\tau})d\tau \\ &= \bar{e}_3(0)e^{K_5t} + \frac{r_3}{-K_5+r_1+r_2}(e^{(r_1+r_2)t} - e^{K_5t}) + \frac{r_4}{-K_5+r_1}(e^{r_1t} - e^{K_5t}) \\ &\quad + \frac{r_5}{-K_5+r_2}(e^{r_2t} - e^{K_5t}) \\ &= r_6e^{K_5t} + \frac{r_3}{-K_5+r_1+r_2}e^{(r_1+r_2)t} + \frac{r_4}{-K_5+r_1}e^{r_1t} + \frac{r_5}{-K_5+r_2}e^{r_2t}\end{aligned} \tag{27}$$

where,

$$r_6 = \bar{e}_3(0) - \frac{r_3}{-K_5+r_1+r_2} - \frac{r_4}{-K_5+r_1} - \frac{r_5}{-K_5+r_2}.$$

Using equations (24), (25) and (28), it can be concluded that the errors $e_1(t)$, $e_2(t)$ and $e_3(t)$ converge to zero exponentially as t tends to infinity.

We conclude that the controller given by (22) when applied to the the DC motor model given by (4), guarantees the exponential convergence of the states of the motor to their desired values as t tends to infinity.

5. Simulation Results

A separately excited permanent magnet DC motor is used to test the proposed control scheme and the proposed observer. The used motor (an MV1042-225 motor) has the specifications given in Table 1 and Table 2.

Table 1: Values of the parameters of the DC motor

Parameter	Value
R_a	3.5 Ω
L_a	0.0432 H
R_f	233 Ω
L_f	25.5 H
K_m	1.9469
B_m	0.0025
J_m	0.0017

Table 2: Ratings of the DC motor

Parameter	Rating
Rated armature voltage	220 Volt
Rated field voltage	220 Volt
Rated power	3 KW
Rated speed	1400 RPM

We simulated the performances of the closed loop system for three different cases.

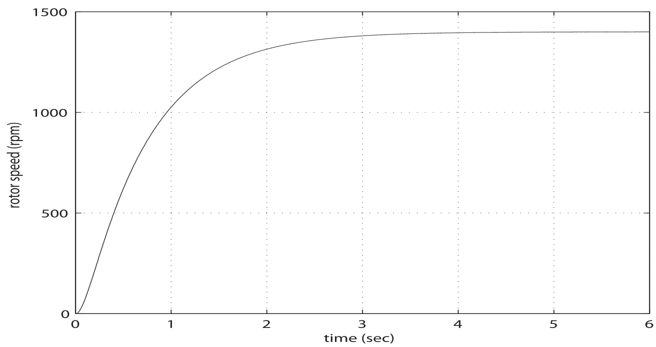

Figure 1. The plot of the rotor speed w versus time (case 1)

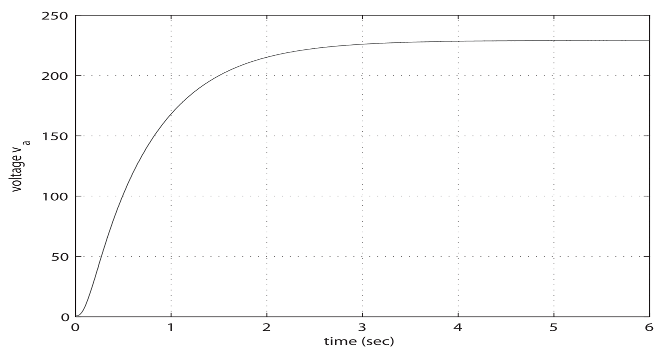

Figure 2. The plot of the armature voltage v_a versus time (case 1)

Case 1.

At first, we simulated the performance of the system when the controller given by (21) is used and assuming that all the states of the system are available for feedback. The simulation results are shown in Figure 1-3. Figure 1 shows the plot of the rotor speed w

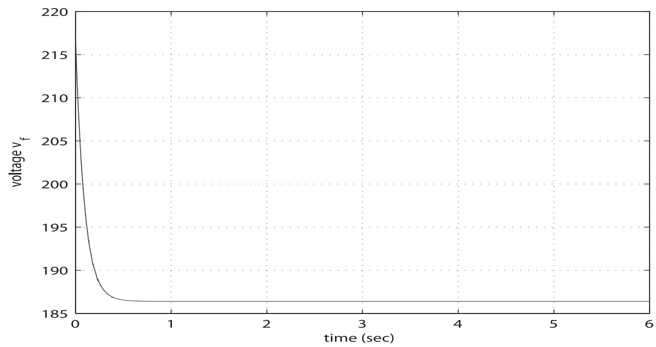

Figure 3. The plot of the field voltage v_f versus time (case 1)

versus time. It is clear from this figure that the motor rotates to its desired speed (1400 rpm) within 5 seconds. Also, Figure 2 and Figure 3 indicates that the armature voltage and the field voltage stay within a reasonable range. Therefore it can be concluded that the proposed control scheme works well.

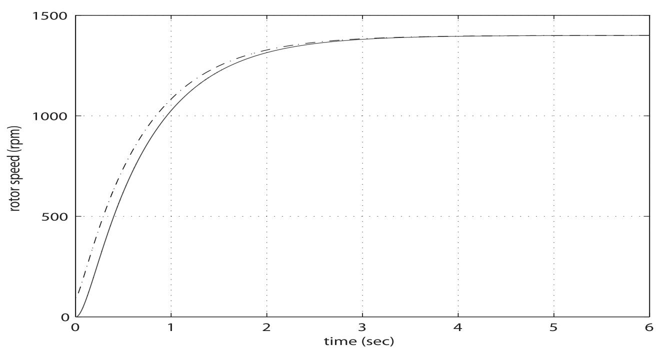

Figure 4. The plots of the rotor speed versus time (case 2)

Case 2.

Next, we simulated the performance of the system when the controller given by (21) is used and assumed that all the states are available for feedback. In addition and to check the workability of the designed observer, we simulated the designed observer given by (4). However, the estimates obtained from the observer were not used for feedback. The simulation results are shown in Figure 4-6. Figure 4 shows the plots of the armature current and its estimate versus time. Figure 5 shows the plots of the field current and its estimate versus time. Note that the actual currents are plotted using continuous lines while the estimated currents are plotted using dashed lines. Figure 6 shows the plots of the rotor speed and its estimate versus time.Again, the actual speed is

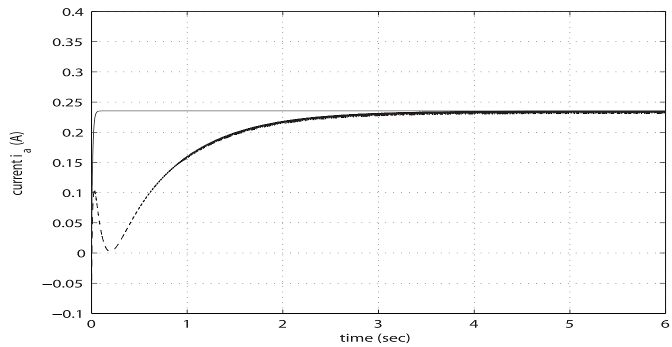

Figure 5. The plots of the armature current i_a versus time (case 2)

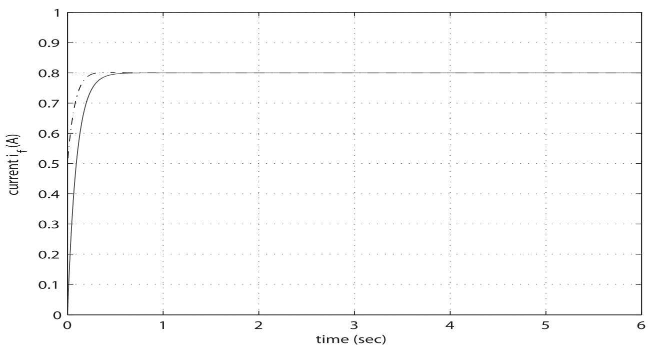

Figure 6. The plots of the field current i_f versus time (case 2)

plotted using a continuous line while the estimated speed is plotted using a dashed line. It is clear that the motor rotates to its desired speed (1400 rpm) within 5 seconds. Also, it is clear from these figures that the estimates obtained using the designed observer converge to the states of the system. Therefore it can be concluded that the proposed observer works well and it can be used to reconstruct the states of the system.

Case 3.

Finally, we simulated the performance of the system when the controller given by (21) is used in conjunction with the designed observer given by (4) (i.e., we used an observer based controller; this means that the estimates of the states are used for feedback). The simulation results are shown in Figure 7-11. It is clear that the motor rotates to its desired speed (1400 rpm) within 5 seconds. Also, the armature and field currents converge to their desired values. Figure 10 and Figure 11 indicates that the armature voltage and the field voltage stay within a reasonable range. Therefore it can be concluded that the proposed observer-based controller works well.

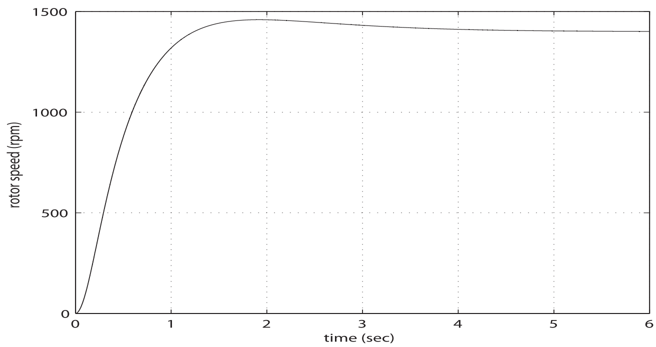

Figure 7. The plot of the rotor speed w versus time (case 3)

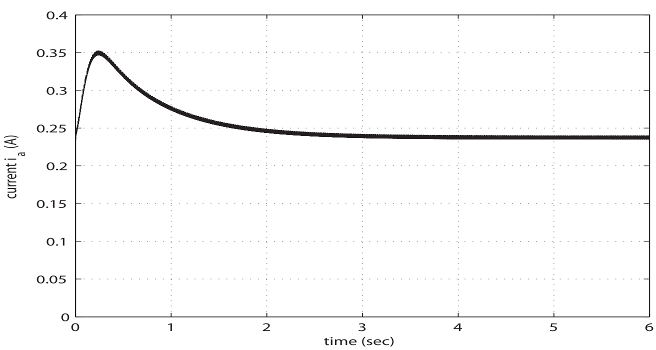

Figure 8. The plot of the armature current i_a versus time (case 3)

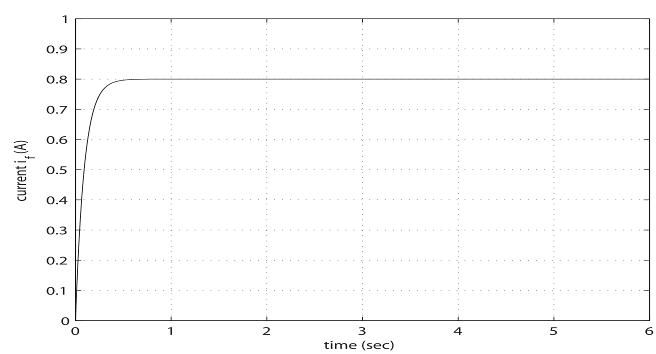

Figure 9. The plot of the field current i_f versus time (case 3)

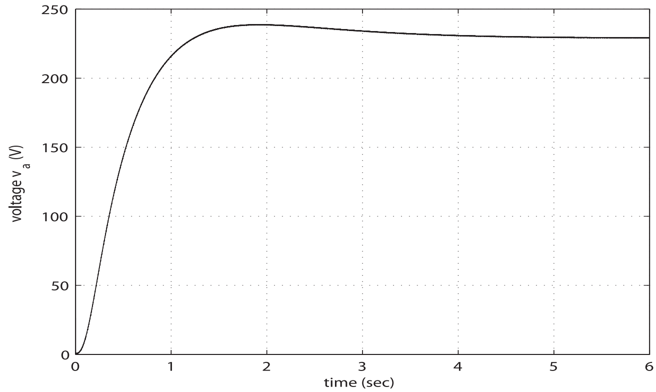

Figure 10. The plot of the armature voltage v_a versus time (case 3)

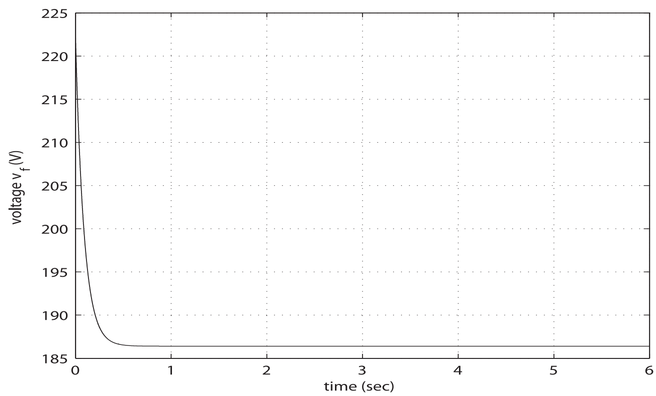

Figure 11. The plot of the field voltage v_f versus time (case 3)

6. Conclusion

This paper deals with the design of an exponential observer as well as the design of a control scheme for the speed control of a separately excited DC motor. At first, an exponential observer is designed to reconstruct the unmeasurable states of the system. Then, a controller which guarantees the exponential convergence of the states of the system to their desired values is designed. Simulation results indicate that the proposed controller and the proposed observer work well. Moreover, simulation results show that the proposed observer-based control scheme gave good results.

Future work will address the implementation of the proposed observer and the proposed controller on an experimental setup. Robustness to the uncertainties of the parameters of the systems as well as robustness to the changes in the load will also be studied.

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