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# Positive electrical circuits with zero transfer matrices and their discretization

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Positive continuous–time and discrete–time linear electrical circuits with zero transfer matrices are addressed. It is shown that there exists a large class of positive electrical circuits with zero transfer matrices. The electrical circuits are unreachable, unobservable and unstable for all values of the resistances, inductances and capacitances. The discrete–time linear positive electrical circuits are introduced. It is shown that: 1) the discrete–time electrical circuit is asymptotically stable for all values of the discretization step if and only if the corresponding continuous–time electrical circuit is asymptotically stable; 2) the discretization of the continuous–time electrical circuit does not change their reachability, observability and transfer matrices.

KEYWORDS: positive electrical circuit, reachability, observability, stability, discretization, transfer matrix

## 1. Introduction

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive systems theory is given in the monographs [5, 12]. Variety of models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc.

The notion of controllability and observability and the decomposition of linear systems have been introduced by Kalman [25, 26]. These notions are the basic concepts of the modern control theory [1, 23, 24, 27]. They have been also extended to positive linear systems [5, 12]. The decomposition of the pair ($A$, $B$) and ($A$, $C$) of the positive discrete–time linear system has been addressed in [9]. The positive circuits and their reachability has been investigated in [13, 23] and controllability and observability of electrical circuits in [8, 23].

The reachability of linear systems is closely related to the controllability of the systems. Specially for positive linear systems, the conditions for the controllability are much stronger than for the reachability [12, 23]. Tests for the reachability and controllability of standard and positive linear systems are given in [12, 17, 23]. The positivity and reachability of fractional continuous–time linear systems and electrical circuits have been addressed in [11, 13, 15, 19, 23].

Decoupling zeros of positive discrete–time linear systems have been introduced in [10].

Stability of fractional linear 1D discrete–time and continuous–time systems has been investigated in the papers [2, 3, 21], and of 2D fractional positive linear systems in [6]. The notion of practical stability of positive fractional discrete–time linear systems has been introduced in [16], and the positive linear systems consisting of $n$ subsystems with different fractional orders has been analyzed in [14]. Some recent interesting results in the fractional systems theory and its applications can be found in [4, 20]. The reachability and observability of fractional positive continuous–time linear systems have been addressed in [18] and constructability and observability of standard and positive electrical circuits in [7].

In this paper the positive continuous–time and discrete–time electrical circuits with zero transfer matrices will be addressed.

The paper is organized as follows. In section 2 the basic definitions and theorems concerning the positivity, reachability and observability of electrical circuits are recalled. Positive electrical circuits with zero transfer matrices are presented in section 3. Positive discrete–time electrical circuits are analysed in section 4. The reachability, observability and transfer matrices of discrete–time linear systems are addressed in section 5. Concluding remarks are given in section 6.

The following notation will be used: $\Re$ is the set of real numbers, $\Re^{n\times m}$ represents the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ denotes the set of $n\times m$ matrices with nonnegative and $\Re_+^n = \Re_+^{n\times 1}$, $M_n$ stand for the set of $n\times n$ Metzler matrices (real matrices with nonnegative off–diagonal entries), $I_n$ is the $n\times n$ identity matrix.

## 2. Positivity, reachability and observability of electrical circuits

Consider linear electrical circuits composed of resistors, capacitors, coils and voltage (current) sources. As the state variables (the components of the state vector $x(t)$) we choose the voltages on the capacitors and the currents in the coils. Using Kirchhoff's laws we may describe the linear circuits in transient states by the state equations

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1a}$$

$$y(t) = Cx(t), \tag{1b}$$

where $x(t)\in\Re^n$, $u(t)\in\Re^m$, $y(t)\in\Re^p$ are the state, input and output vectors and $A\in\Re^{n\times n}$, $B\in\Re^{n\times m}$, $C\in\Re^{p\times n}$.

**Definition 1.** [12, 23] The linear electrical circuit (1) is called (internally) positive if the state vector $x(t) \in \Re_+^n$ and output vector $y(t) \in \Re_+^p$, $t \geq 0$ for any initial conditions $x(0) \in \Re_+^n$ and all inputs $u(t) \in \Re_+^m$, $t \geq 0$.

**Theorem 1.** [5, 12, 23] The linear electrical circuit is positive if and only if

$$A \in M_n, \; B \in \Re_+^{n \times m}, \; C \in \Re_+^{p \times n}. \tag{2}$$

**Definition 2.** [5, 12, 23] The positive electrical circuit (1) is called reachable in time $t \in [0, t_f]$ if for every given final state $x_f \in \Re_+^n$ there exists an input $u(t) \in \Re_+^m$, $t \in [0, t_f]$ which steers the state of the electrical circuit from zero initial conditions $x(0) = 0$ to the final state $x_f$.

**Definition 3.** [12] A matrix $A \in \Re_+^{n \times n}$ is called monomial if each its row and each its column contains only one positive entry and the remaining entries are zero.

**Theorem 2.** [12, 23] The positive electrical circuit (1) is reachable if and only if the reachability matrix

$$R_n = [B \quad AB \quad \cdots \quad A^{n-1}B] \in \Re_+^{n \times nm} \tag{3}$$

contains a monomial matrix.

**Definition 4.** [12, 23] The positive electrical circuit (1) is called observable in time $t \in [0, t_f]$ if knowing its input $u(t) \in \Re_+^m$ and its input $y(t) \in \Re_+^p$ for $t \in [0, t_f]$ it is possible to find its unique initial condition $x_0 = x(0) \in \Re_+^n$.

**Theorem 3.** [12, 23] The positive electrical circuit (1) is observable in time $t \in [0, t_f]$ if and only if the matrix $A \in M_n$ is diagonal and the matrix

$$O_n = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \in \Re_+^{pn \times n} \tag{4}$$

contains a monomial matrix.

The transfer matrix of the positive electrical circuit (1) is given by

$$T(s) = C[I_n s - A]^{-1} B \in \Re^{p \times m}(s), \tag{5}$$

where $\Re^{p \times m}(s)$ is the set of $p \times m$ rational matrices in $s$.

**Theorem 4.** [10, 12] If the pair ($A$,$B$) of the standard electrical circuit (1) is not reachable then some pole–zero cancellations occur in

$$\frac{\operatorname{adj}[I_n s - A]B}{\det[I_n s - A]}. \tag{6}$$

If the pair ($A$,$C$) of the standard electrical circuit (1) is not observable then some pole–zero cancellations occur in

$$\frac{C\,\mathrm{adj}[I_n s - A]}{\det[I_n s - A]}, \tag{7}$$

where $\mathrm{adj}[I_n s - A]$ denotes the adjoint matrix of $[I_n s - A]$.

**Theorem 5.** If for the standard electrical circuit (1)

$$T(s) = C[I_n s - A]^{-1} B = 0 \tag{8}$$

then

$$O_n R_n = 0, \tag{9}$$

where $O_n$ and $R_n$ are defined by (4) and (3), respectively.

**Proof.** Proof is given in [22].

**Theorem 6.** Let for the standard electrical circuit (1) the condition (8) be satisfied. Then

1) the pair (*A*,*B*) is unreachable if $C \neq 0$,
2) the pair (*A*,*C*) is unobservable if $B \neq 0$.

**Proof.** Proof is given in [22].

## 3. Linear electrical circuits with zero transfer matrices

**Example 1.** Consider the electrical circuit shown in Fig. 1 with given resistances $R_1$, $R_2$, $R_3$ inductance *L*, capacitance *C* and voltage source *e*.

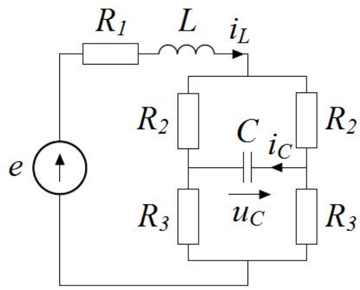


Fig. 1. Electrical circuit of Example 1

Using Kirchhoff's laws we may write the equations

$$\begin{aligned} e &= R i_L + L\frac{di_L}{dt}, \quad R = R_1 + \frac{R_2 + R_3}{2}, \\ i_C &= C\frac{du_C}{dt} = 0. \end{aligned} \tag{10}$$

As the output *y* we choose

$$y = u_C. \tag{11}$$

The equations (10) and (11) can be rewritten in the form

$$\frac{d}{dt}\begin{bmatrix} u_C \\ i_L \end{bmatrix} = A_1 \begin{bmatrix} u_C \\ i_L \end{bmatrix} + B_1 e, \quad y = C_1 \begin{bmatrix} u_C \\ i_L \end{bmatrix}, \tag{12a}$$

where

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{R}{L} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}, \quad C_1 = [1 \quad 0]. \tag{12b}$$

By Theorem 1 the electrical circuit is positive for all values of $R_1$, $R_2$, $R_3$, $L$ and $C$ since from (12b) we have

$$A_1 \in M_2, \; B_1 \in \Re_+^2, \; C_1 \in \Re_+^{1\times 2}. \tag{13}$$

The transfer function of the electrical circuit is

$$T(s) = C_1[I_2 s - A_1]^{-1} B_1 = [1 \quad 0] \begin{bmatrix} s & 0 \\ 0 & s + \frac{R}{L} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} = 0 \tag{14}$$

for all values of $R_1$, $R_2$, $R_3$, $L$ and $C$.

Note that

$$\det[I_n s - A_1] = \begin{vmatrix} s & 0 \\ 0 & s + \frac{R}{L} \end{vmatrix} = s\left(s + \frac{R}{L}\right), \; s_1 = 0, \; s_2 = -\frac{R}{L} \tag{15}$$

and the electrical circuit is unstable for all values of $R_1$, $R_2$, $R_3$, $L$ and $C$.

By Theorems 2 and 3 the positive electrical circuit with (12b) is unreachable and unobservable since the matrices

$$R_2 = [B_1 \quad A_1 B_1] = \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & -\frac{R}{L^2} \end{bmatrix}, \quad O_2 = \begin{bmatrix} C_1 \\ C_1 A_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{16}$$

have only one monomial column and one monomial row, respectively. From (16) we have

$$O_2 R_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ \frac{1}{L} & -\frac{R}{L^2} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{17}$$

In general case the class of positive electrical circuits with zero transfer matrix can be presented in the form shown in Fig. 2.

**Theorem 7.** The class of electrical circuits shown in Fig. 2 are positive electrical circuits with zero transfer functions if and only if their common parts are positive electrical circuits.

An example of positive electrical circuit with zero transfer matrix for $n_1 = n_2 = 1$ is given in the following example.

**Example 2.** Consider the electrical circuit shown in Fig. 3 with given resistances $R_k$, $k = 1,...,9$, inductances $L_i$, $i = 1,...,4$, capacitances $C_1$, $C_2$ and voltage sources $e_j$, $j = 1,2,3$.

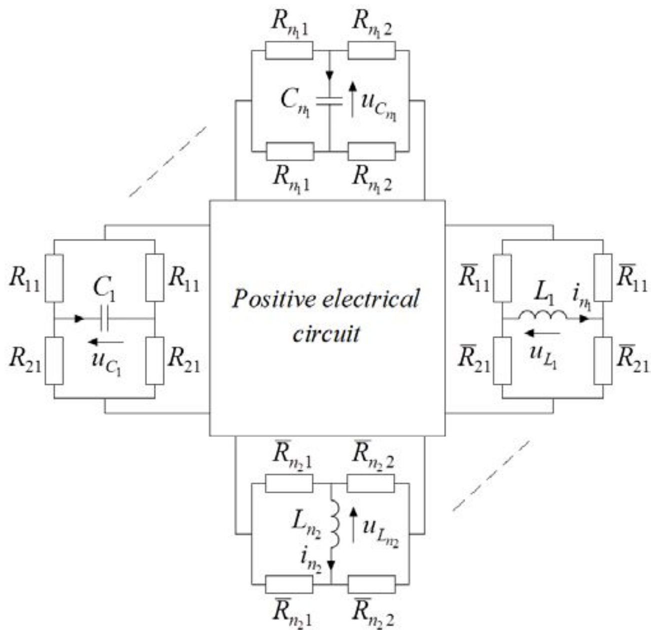


Fig. 2. Positive electrical circuit with zero transfer matrix

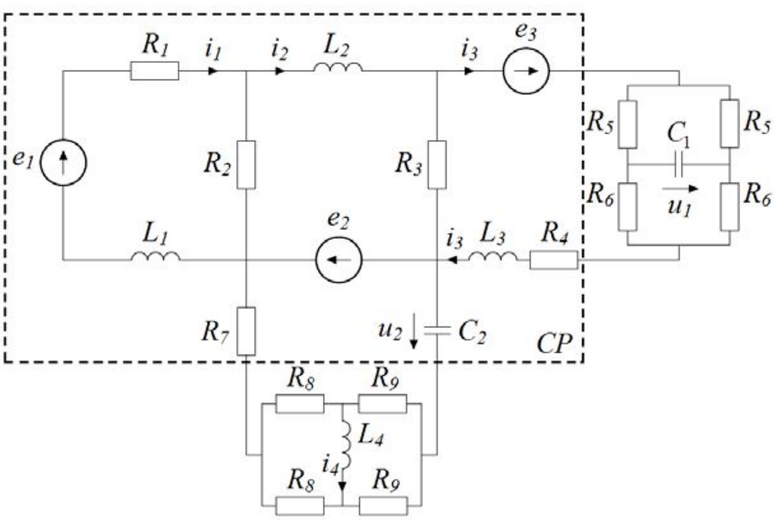


Fig. 3. Positive electrical circuit

Using Kirchhoff's laws we may write the equations

$$
\begin{aligned}
e_1 &= (R_1+R_2)i_1 - R_2 i_2 + L_1 \frac{di_1}{dt},\\
e_2 &= -R_1 i_1 + (R_2+R_3)i_2 - R_3 i_3 + L_2 \frac{di_2}{dt},\\
e_3 &= -R_2 i_2 + \overline{R}_1 i_3 + L_3 \frac{di_3}{dt},\ \overline{R}_1 = R_3 + R_4 + \frac{R_5+R_6}{2},\\
e_2 &= \overline{R}_2 C_2 \frac{du_2}{dt} + u_2,\ \overline{R}_2 = R_7 + \frac{R_8+R_9}{2},\\
i_C &= C_1 \frac{du_1}{dt} = 0,\\
u_{L_4} &= L_4 \frac{di_4}{dt} = 0.
\end{aligned}
\tag{18a}
$$

As the output we choose

$$
y = \begin{bmatrix} u_1 \\ i_4 \end{bmatrix}. \tag{18b}
$$

The equations (18) can be written in the form

$$
\begin{aligned}
\frac{d}{dt}\begin{bmatrix} u_1 \\ u_2 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} &= A \begin{bmatrix} u_1 \\ u_2 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} + B \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix},\\
y &= C \begin{bmatrix} u_1 \\ u_2 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix},
\end{aligned}
\tag{19a}
$$

where

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\overline{R}_2 C_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{R_1+R_2}{L_1} & \frac{R_2}{L_1} & 0 & 0 \\ 0 & 0 & \frac{R_1}{L_2} & -\frac{R_2+R_3}{L_2} & \frac{R_3}{L_2} & 0 \\ 0 & 0 & 0 & \frac{R_2}{L_3} & -\frac{R_1}{L_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B=\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{\overline{R}_2 C_2} & 0 \\ \frac{1}{L_1} & 0 & 0 \\ 0 & \frac{1}{L_2} & 0 \\ 0 & 0 & \frac{1}{L_3} \\ 0 & 0 & 0 \end{bmatrix},\ C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{19b}$$

Note that the electrical circuit is positive for all values of the resistances, inductances and capacitances since $A \in M_6$, $B \in \Re_+^{6\times 3}$ and $C \in \Re_+^{2\times 6}$. It is easy to check that the transfer matrix of the positive electrical circuit

$$T(s)=C[I_6 s-A]^{-1}B=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\times$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s+\frac{1}{\overline{R}_2 C_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & s+\frac{R_1+R_2}{L_1} & -\frac{R_2}{L_1} & 0 & 0 \\ 0 & 0 & -\frac{R_1}{L_2} & s+\frac{R_2+R_3}{L_2} & -\frac{R_3}{L_2} & 0 \\ 0 & 0 & 0 & -\frac{R_2}{L_3} & s+\frac{R_1}{L_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{-1}\times$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{\overline{R}_2 C_2} & 0 \\ \frac{1}{L_1} & 0 & 0 \\ 0 & \frac{1}{L_2} & 0 \\ 0 & 0 & \frac{1}{L_3} \\ 0 & 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{20}$$

and the circuit is unstable for all values of the resistances, inductances and capacitances since the matrix $A$ has two zero rows. In general case we have the following conclusion.

**Conclusion 1.** The number of zero rows of the matrix $A$ is equal to the number of outputs of the system.

## 4. Discretization and positive discrete–time electrical circuits

Consider the linear electrical circuits described by (1). Applying the approximation

$$\dot{x}(t) = \frac{x(t+h) - x(t)}{h} = \frac{x_{i+1} - x_i}{h}, \; i \in Z_+ = \{0,1,...\} \tag{21}$$

to (1) we obtain the corresponding discrete–time electrical circuit described by the equations

$$x_{i+1} = A_d x_i + B_d u_i , \tag{22a}$$

$$y_i = C_d x_i , \tag{22b}$$

where $x_i = x(t) = x(hi)$, $x_{i+1} = x(t+h)$, $u_i = u(t)$, $y_i = y(t)$, $h > 0$ and

$$A_d = I_n + hA, \; B_d = hB, \; C_d = C . \tag{22c}$$

**Definition 5.** [12] The discrete–time electrical circuit (22) is called (internally) positive if $x_i \in \Re_+^n$, $y_i \in \Re_+^p$, $i \in Z_+$ for any initial conditions $x_0 \in \Re_+^n$ and all $u_i \in \Re_+^m$, $i \in Z_+$.

**Theorem 8.** The discrete–time electrical circuit (22) is positive if and only if

$$A_d \in \Re_+^{n\times n}, \; B_d \in \Re_+^{n\times m}, \; C_d \in \Re_+^{p\times n} . \tag{23}$$

**Proof.** Proof is given in [12, 20].

**Theorem 9.** The discrete–time electrical circuit (22) is positive if and only if the continuous–time electrical circuit (1) is positive and

$$h \geq \frac{1}{\max_i |a_{ii}|}, \tag{24}$$

where $a_{ii}$, $i = 1,...,n$ are the diagonal entries of the matrix $A \in M_n$.

**Proof.** From (22c) it follows that $A_d \in \Re_+^{n\times n}$ if and only if $A \in M_n$ and the condition (24) is satisfied. □

It is well–known that the eigenvalues $z_l$, $l = 1,...,n$ of the matrix $A_d \in \Re^{n\times n}$ are related with the eigenvalues $s_l$, $l = 1,...,n$ of the matrix $A \in \Re^{n\times n}$ by

$$z_l = 1 + hs_l , \; l = 1,...,n . \tag{25}$$

**Theorem 10.** The discrete–time electrical circuit (22) is asymptotically stable for $h > 0$ if and only if the continuous–time electrical circuit (1) is asymptotically stable.

**Proof.** Let $s_l = \alpha_l + j\beta_l$, $l = 1,...,n$, then the discrete–time electrical circuit (22) is asymptotically stable if and only if

$$|z_l|^2 = |1 + hs_l|^2 = (1 + h\alpha_l)^2 + (h\beta_l)^2 = 1 + 2h\alpha_l + (h\alpha_l)^2 + (h\beta_l)^2 < 1 \quad (26)$$

or

$$2\alpha_l + h(\alpha_l^2 + \beta_l^2) < 0 \quad (27)$$

and

$$h = -\frac{2\alpha_l}{\alpha_l^2 + \beta_l^2}, \ l = 1,...,n. \quad (28)$$

From (28) it follows that $h > 0$ if and only if $\alpha_l < 0$, $l = 1,...,n$, i.e. the electrical circuit (1) is asymptotically stable.

## 5. Reachability, observability and transfer matrices of discrete–time electrical circuits

In this section it will be shown that the discretization of the continuous–time electrical circuits does not change their reachability and observability.

**Definition 6.** [12] The discrete–time electrical circuit (22) is called reachable in $q$ steps if for every given final state $x_f \in \Re^n$ there exists an input sequence $u_0, u_1, ..., u_{q-1} \in \Re^m$ which steers the state from zero initial state $x_0$ to the final state $x_f$.

**Theorem 11.** The discrete–time electrical circuit (22) is reachable in $q$ steps if and only if

$$\operatorname{rank} R_q = \operatorname{rank}[B_d \quad A_d B_d \quad \cdots \quad A_d^{q-1} B_d] = n. \quad (29)$$

**Proof.** Proof is similar to the proof given in [22].

**Definition 7.** [12] The discrete–time electrical circuit (22) is called observable in $q$ steps if knowing its input sequence $u_0, u_1, ..., u_{q-1}$ and output sequence $y_0, y_1, ..., y_{q-1}$ it is possible to find its unique initial condition $x_0$.

**Theorem 12.** The discrete–time electrical circuit (22) is observable in $q$ steps if and only if

$$\operatorname{rank} O_q = \operatorname{rank}\begin{bmatrix} C_d \\ C_d A_d \\ \vdots \\ C_d A_d^{q-1} \end{bmatrix} = n. \quad (30)$$

**Proof.** Proof is given in [22].

**Theorem 13.** The discrete–time electrical circuit (22) is reachable in $q$ steps if and only if the continuous–time electrical circuit (1) is reachable in $q$ steps.

**Proof.** By Theorem 11 the discrete–time electrical circuit (22) is reachable in $q$ steps if and only if

$$\operatorname{rank}[B_d \quad A_d B_d \quad \cdots \quad A_d^{q-1} B_d] = n . \tag{31}$$

Substitution of (22c) into (31) yields

$$\begin{aligned} &\operatorname{rank}[hB \quad (I_n + hA)hB \quad \cdots \quad (I_n + hA)^{q-1} hB] \\ &= \operatorname{rank}\left\{ [B \quad AB \quad \cdots \quad A^{q-1}B] \begin{bmatrix} I_n h & I_n h & \cdots & I_n h \\ 0 & I_n h^2 & \cdots & (q-1) I_n h^2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_n h^q \end{bmatrix} \right\} \\ &= \operatorname{rank}[B \quad AB \quad \cdots \quad A^{q-1}B] \end{aligned} \tag{32}$$

since the matrix

$$\begin{bmatrix} I_n h & I_n h & \cdots & I_n h \\ 0 & I_n h^2 & \cdots & (q-1) I_n h^2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_n h^q \end{bmatrix} \tag{33}$$

is nonsingular for any $h > 0$ . □

**Theorem 14.** The discrete–time electrical circuit (22) is observable in $q$ steps if and only if the continuous–time electrical circuit (1) is observable in $q$ steps.

**Proof.** Proof is similar to the proof of Theorem 13.

**Theorem 15.** The transfer matrix $T_d(z)$ of the discrete–time electrical circuit (22) is zero if and only if the transfer matrix $T(s)$ of the continuous–time electrical circuit (1) is zero.

**Proof.** The transfer matrix $T_d(z)$ of (22) is given by

$$T_d(z) = C_d [I_n z - A_d]^{-1} B_d . \tag{34}$$

Substituting (22c) and (25) into (34) we obtain

$$\begin{aligned} T_d(z) &= C[I_n + shI_n - I_n - hA]^{-1} hB = C[h(I_n s - A)]^{-1} B \\ &= C[I_n s - A]^{-1} B = T(s). \end{aligned} \tag{35}$$

Therefore, $T_d(z) = 0$ if and only if $T(s) = 0$ . □

The considerations can be easily extended to the positive electrical circuits.

## 6. Concluding remarks

Positive continuous–time and discrete–time linear systems with zero transfer matrices have been addressed. It has been shown that there exists a large class of positive electrical circuits with zero transfer matrices (Theorem 7). The electrical circuits are unreachable, unobservable and unstable for all values of the resistances, inductances and capacitances (Theorems 5 and 6). The discrete–time linear positive electrical circuits have been introduced and their positivity and asymptotic stability have been investigated (Theorem 9). It has been shown that: 1) the discrete–time electrical circuit is asymptotically stable if and only if the continuous–time electrical circuit is asymptotically stable (Theorem 10); 2) the discrete–time electrical circuit is reachable (observable) if and only if the continuous–time if and only if the continuous–time electrical circuit is reachable (observable) (Theorems 13 and 14); 3) the transfer matrix of discrete–time electrical circuit is zero if and only if the transfer matrix of continuous–time electrical circuit is zero (Theorem 15).

The considerations can be extended to the fractional positive electrical circuits with zero transfer matrices.

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