Krzysztof Magnucki
Łukasiewicz – Rail Vehicles Institute "TABOR"
Dennis Alsdorf
Havel Metal Foam GmbH
Jerzy Lewiński
Michał Kowalski
Łukasiewicz – Rail Vehicles Institute "TABOR"
Alexander Richter
Havel Metal Foam GmbH
Andrzej Zbierski
Łukasiewicz – Rail Vehicles Institute "TABOR"

Analytical, FEM-numerical and experimental studies of bending of a sandwich plate-strip with metal foam core

The subject of the study is a sandwich plate-strip subjected to a four-point load. An analytical model of the strip was developed, taking into account the classical zig-zag theory, namely the broken line hypothesis. Three parts of the plate-strip are distinguished: two of them are the edge parts, where bending and the shear effect is considered, the third one is the middle part subjected to pure bending. The total maximum deflection of the plate-strip and the maximum deflection of the selected middle part of the plate-strip are calculated. The FEM-numerical study is carried out similarly to the analytical approach. The experimental study was carried out on the test stand in the Institute of Rail Vehicles TABOR. The analytical, numerical and experimental results are compared each with other. The sandwich panels can be used as parts of the floor or rail vehicle paneling.

1. Introduction

The basis of analytical modelling of sandwich structures was initiated in the middle of the 20th century. Vinson [16] presented a review of the works written in the 20th century, dealing with the problems of structural mechanics applied to the sandwich structures. Reddy [14] provided an extensive monograph devoted to laminated composite plates and shells. The author described the methods of modeling and analysis of these structures, inclusive of various theories used for this purpose. Magnucki and Szyc [8] presented a co-author monograph on the strength and stability problems of sandwich beams and plates with aluminium foam cores. Kozak [5] described the use of the steel sandwich panels in ship structures.

Sayyad and Ghugal [15] submitted a critical review of the literature devoted to laminated composite and sandwich structures, published mainly in the 21st century. The work quotes various theories and hypotheses applicable in this field and highlights a possible scope for the future research. Banhart [1] considered the problems of manufacturing metal foams and other porous metallic structures. The author quotes various innovatory production methods and the ways for characterizing the properties of cellular metals. The paper also presents the fields of application of these materials in various industrial branches. Icardi [2] developed a model designed for analysis of laminated and sandwich beams. The displacements are so formulated as to satisfy the continuity conditions of the transverse shear, transverse normal stress and stress gradient.

The zig-zag hypothesis improves accuracy of the approach that is compared with other 3-D elasticity solutions and with various models available in the literature. Jasion and Magnucki [3] dealt with the sandwich beams subjected to four-point bending. The principle of stationary total potential energy enabled to derive the formulae determining the critical stresses in the faces of the beam, related to upper face wrinkling. The results obtained based on the analytical model and FEM analysis are shown for several sandwich beams with various thicknesses and core properties. Jasion *et al*. [4] investigated the global and local buckling of the face sheets of sandwich beams and sandwich circular plates. A mathematical model of the displacements, with consideration of the shear effect, is developed.

The analytical solutions are compared to the results obtained with the use of Finite Element Method and with experimental methods. Magnucka-Blandzi and Magnucki [6] analyzed a simply supported sandwich beam with a metal foam core. Nonlinear hypothesis of deformation of the beam plane cross

section and the theorem of minimum total potential energy enabled to derive the differential equations of equilibrium. Their solution allowed to determine optimal dimensionless parameters of the beam.
Magnucka-Blandzi [7] considered a simply supported rectangular sandwich plate compressed in plane. Assumption of the field of displacements and geometric relationships allowed to derive a system of differential equations and, in consequence, to determine the critical loads for a family of the sandwich plates. Magnucki *et al*. [9], [10] and [11] analyzed the structures provided with an aluminum foam core. The studies have been devoted to a five-layer sandwich beam under axial compression or bending and to sandwich circular plate subjected to pure bending. The principle of stationary total potential energy enabled to derive the system of partial differential equations of equilibrium. The analytical solutions obtained for these structures were compared to the theoretical, numerical and experimental results.
Paczos *et al*. [13] considered short sandwich beams with special honeycomb structure of the core. Assumption of the "zig-zag" hypothesis of deformation of the beam plane cross section allowed to develop an analytical model. The analytical results so calculated were compared to those obtained experimentally.
Magnucki [12] presented a study on simply supported sandwich beams and I-beams of symmetrical structure, subjected to three-point bending and uniformly distributed load. Two variants of deformation of planar cross sections of the beams were taken into account: the classical "broken line" hypothesis and the nonlinear polynomial hypothesis. The differential equations of equilibrium of these structures enabled calculating their deflections with consideration of the shear effect.
The subject of the study is a simply supported sandwich plate-strip of length L and width b carrying the four-point bending (Fig. 1).

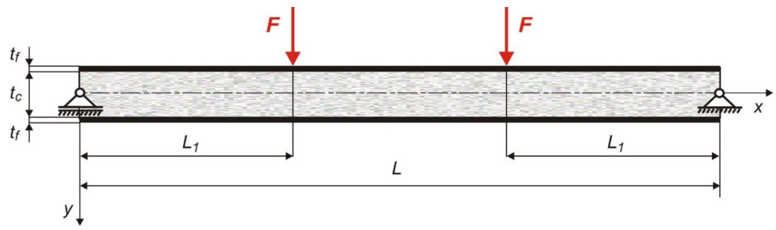
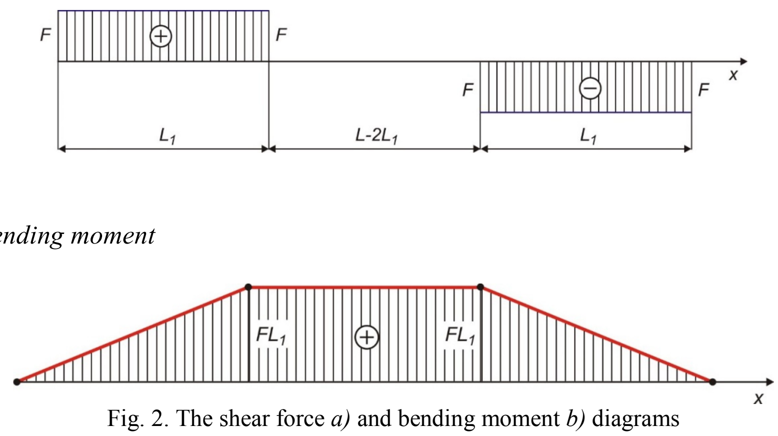


Fig. 1. Scheme of the sandwich plate-strip under four-point bending

The shear force and bending moment diagrams are shown in Fig. 2.

a) The shear force



b) The bending moment

Fig. 2. The shear force *a)* and bending moment *b)* diagrams

2. Analytical study

The analytical model of the sandwich plate-strip is formulated based on the classical "broken-line" theory-hypothesis (Fig. 3).

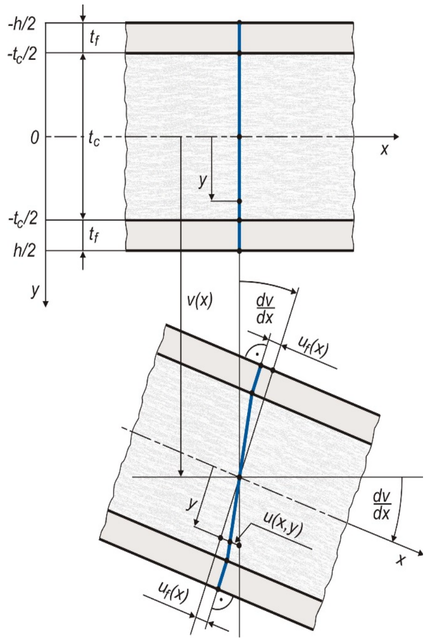


Fig. 3. Scheme of the deformation of planar cross section of the sandwich plate-strip

The longitudinal displacements in accordance with Fig. 3 are as follows:

- the upper face $\langle -h/2 \le y \le -t_c/2 \rangle$

$$u^{(u-f)}(x,y) = -\left[y\frac{dv}{dx} + u_f(x) \right], \tag{1}$$

- the core $\langle -t_c/2 \le y \le t_c/2 \rangle$

$$u^{(c)}(x,y) = -y\left[\frac{dv}{dx} - 2\frac{u_f(x)}{t_c} \right], \tag{2}$$

- the lower face $\langle t_c/2 \le y \le h/2 \rangle$

$$u^{(l-f)}(x,y) = -\left[y\frac{dv}{dx} - u_f(x) \right], \tag{3}$$

where: $u_f(x)$ – longitudinal displacement of the faces, $v(x)$ - deflection, x, y – coordinates, t_f, t_c – thicknesses of the faces and core, $h = 2t_f + t_c$ – total thickness of the plate-strip.

Therefore, the strains and stresses

- the upper face

$$\varepsilon_x^{(u-f)}(x,y) = \frac{\partial u}{\partial x} = -\left(y\frac{d^2v}{dx^2} + \frac{du_f}{dx} \right), \gamma_{xy}^{(u-f)}(x,y) = \frac{\partial u}{\partial y} + \frac{dv}{dx} = 0, \tag{4}$$

$$\sigma_x^{(u-f)}(x,y) = E_f \varepsilon_x^{(u-f)}(x,y), \qquad \tau_{xy}^{(u-f)}(x,y) = 0, \tag{5}$$

- the core

$$\varepsilon_x^{(c)}(x,y) = \frac{\partial u}{\partial x} = -y\left(\frac{d^2v}{dx^2} - \frac{2}{t_c}\frac{du_f}{dx} \right), \qquad \gamma_{xy}^{(c)} = \frac{\partial u}{\partial y} + \frac{dv}{dx} = 2\frac{u_f(x)}{t_c}, \tag{6}$$

$$\sigma_x^{(c)}(x,y) = E_c \varepsilon_x^{(c)}(x,y), \qquad \tau_{xy}^{(c)} = \frac{E_c}{1+\nu_c}\frac{u_f(x)}{t_c}, \tag{7}$$

- the lower face

$$\varepsilon_x^{(l-f)}(x,y) = \frac{\partial u}{\partial x} = -\left(y\frac{d^2v}{dx^2} - \frac{du_f}{dx} \right), \qquad \gamma_{xy}^{(l-f)}(x,y) = \frac{\partial u}{\partial y} + \frac{dv}{dx} = 0, \tag{8}$$

$$\sigma_x^{(l-f)}(x,y) = E_f \varepsilon_x^{(l-f)}(x,y), \qquad \tau_{xy}^{(l-f)}(x,y) = 0, \tag{9}$$

where: E_f, E_c, ν_c – material constants (Young's moduli and Poisson ratio of the core).

The bending moment

$$M_b(x) = b\left\{ \int_{-h/2}^{-t_c/2} y\sigma_x^{(u-f)}(x,y)dy + \int_{-t_c/2}^{t_c/2} y\sigma_x^{(c)}(x,y)dy + \int_{t_c/2}^{h/2} y\sigma_x^{(l-f)}(x,y)dy \right\}, \tag{10}$$

where: b – width.

Substituting the expressions for normal stresses (5), (7), (9) into the equation (10) and integrating, one obtains

$$C_{vv}\frac{d^2v}{dx^2} - 2C_{vu}\frac{1}{t_c}\frac{du_f}{dx} = -12\frac{M_b(x)}{E_c b t_c^3}, \tag{11}$$

where: $C_{vv} = 1 + 2e_f\left(3 + 6\chi_f + 4\chi_f^2\right)\chi_f$, $\quad C_{vu} = 1 + 6e_f\left(1 + \chi_f\right)\chi_f$ – coefficients,

$e_f = E_f/E_c$, $\chi_f = t_f/t_c$ - dimensionless parameters.

The transverse-shear force with consideration of the shear stress (7) is as follows

$$T(x) = b\int_{-t_c/2}^{t_c/2}\tau_{xy}^{(c)}(x,y)dy = \frac{E_c b}{1+\nu_c}u_f(x). \tag{12}$$

Hence, the unknown function of the longitudinal displacement

$$u_f(x) = (1+\nu_c)\frac{T(x)}{E_c b}. \tag{13}$$

The bending problem of the sandwich plate-strip is described in two intervals:

1) $0 \le x \le L_1$ – the edge part (the bending with shear effect)

The transverse-shear force is assumed in the following form

$$T(x) = \frac{1}{\tanh(k)}\tanh\left[k\left(1 - \frac{x}{L_1}\right)\right]F, \tag{14}$$

where: k – coefficient ($k \to \infty$).

The bending moment $M_b(x) = Fx$.

The equation (11), after first integration with consideration of the above expression (14), is as follows

$$C_{vv}\frac{dv}{dx} = C_1 + 2(1+\nu_c)\frac{C_{vu}}{\tanh(k)}\tanh\left[k\left(1 - \frac{x}{L_1}\right)\right]\frac{F}{E_c b t_c} - 6x^2\frac{F}{E_c b t_c^3}, \tag{15}$$

where: C_1 – integration constant.

The condition for $x = L_1$:

$$C_{vv}\left.\frac{dv}{dx}\right|_{L_1} = C_1 - 6\frac{FL_1^2}{E_c b t_c^3}. \tag{16}$$

The equation (15) after integration takes the following form

$$C_{vv}v(x) = C_2 + C_1 x - 2(1+\nu_c)\frac{C_{vu}}{k\tanh(k)}\ln\left\{\cosh\left[k\left(1 - \frac{x}{L_1}\right)\right]\right\}\frac{FL_1}{E_c b t_c} - 2x^3\frac{F}{E_c b t_c^3}, \tag{17}$$

where: C_2 – integration constant.

The condition for $x = 0$, $\quad v(0) = 0$, from which

$C_2 = 2(1+\nu_c)\dfrac{\ln[\cosh(k)]}{k\tanh(k)}C_{vu}\dfrac{FL_1}{E_c b t_c}$, taking into account that $\lim\limits_{k\to\infty}\dfrac{\ln[\cosh(k)]}{k\tanh(k)} = 1$.

Therefore

$$C_2 = 2(1+\nu_c)C_{vu}\frac{FL_1}{E_c b t_c}. \tag{18}$$

Thus, the deflection of the first part–i.e. edge part for $x = L_1$ is as follows

$$C_{vv}v(L_1) = 2(1+\nu_c)C_{vu}\frac{FL_1}{E_c b t_c} + C_1 L_1 - 2\frac{FL_1^3}{E_c b t_c^3}. \tag{19}$$

1) $L_1 \le x \le L/2$ – the middle part (the pure bending, without shear effect)

The transverse-shear force $T(x) = 0$, the bending moment $M_b(x) = FL_1$. Therefore, the equation (11) after integration takes the following form

$$C_{vv}\frac{dv}{dx} = C_3 - 12x\frac{FL_1}{E_c b t_c^3}, \tag{20}$$

where: C_3 – integration constant.

Two conditions for the slope of the deflection curve:

- for $x = L/2$, $C_{vv}\left.\dfrac{dv}{dx}\right|_{L/2} = 0$, from which $C_3 = 6\dfrac{FL_1 L}{E_c b t_c^3}$,
- for $x = L_1$, $C_{vv}\left.\dfrac{dv}{dx}\right|_{L_1} = C_3 - 12\dfrac{FL_1^2}{E_c b t_c^3}$,

therefore

$$C_{vv}\left.\frac{dv}{dx}\right|_{L_1} = 6(L-2L_1)\frac{FL_1}{E_c b t_c^3}. \tag{21}$$

The equation (20) after integration takes the following form

$$C_{vv}v(x) = C_4 + 6x\frac{FL_1 L}{E_c b t_c^3} - 6x^2\frac{FL_1}{E_c b t_c^3}, \tag{22}$$

where: C_4 – integration constant.

The deflection for $x = L_1$ is as follows

$$C_{vv}v(L_1) = C_4 + 6(L-L_1)\frac{FL_1^2}{E_c b t_c^3}. \tag{23}$$

Taking into account the compatibility condition of the deflection curve slope (the expressions (16) and (21)), one obtains the integration constant

$$C_1 = 6(L-L_1)\frac{FL_1}{E_c b t_c^3}. \tag{24}$$

Similarly, the compatibility condition of the deflection (the expressions (19) and (23)) provides the integration constant

$$C_4 = 2(1+\nu_c)C_{vu}\frac{FL_1}{E_c b t_c} - 2\frac{FL_1^3}{E_c b t_c^3}. \tag{25}$$

Therefore, the deflection of the sandwich plate-strip for $x = L_1$ (23) is as follows

$$v(L_1) = 2\left[(1+\nu_c)C_{vu} + (3-4\alpha_1)\alpha_1\lambda^2\right]\alpha_1\frac{\lambda}{C_{vv}}\frac{F}{E_c b}, \tag{26}$$

where: $\alpha_1 = L_1/L$, $\lambda = L/t_c$ – dimensionless coefficients.

Thus, the maximum total deflection of the sandwich plate-strip based on the expression (22) takes the following form

$$v_{\max}^{(total)} = v\left(\frac{1}{2}L\right) = \left[2(1+\nu_c)C_{vu} + \frac{1}{2}\left(3-4\alpha_1^2\right)\lambda^2\right]\alpha_1 \frac{\lambda}{C_{vv}}\frac{F}{E_c b}. \tag{27}$$

The maximum deflection of the middle part (Fig. 4)

$$v_{\max}^{(m)} = v_{\max}^{(total)} - v(L_1) = \frac{3}{2}(1-2\alpha_1)^2\alpha_1 \frac{\lambda^3}{C_{vv}}\frac{F}{E_c b}. \tag{28}$$

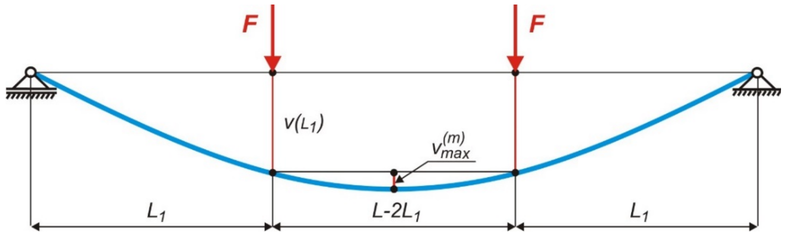


Fig. 4. Scheme of the deflection of the sandwich plate-strip under the four-point load

Example data: $L = 900\,\text{mm}$, $L_1 = 300\,\text{mm}$, $b = 250\,\text{mm}$, $t_f = 2\,\text{mm}$, $t_c = 16\,\text{mm}$, $E_f = 70000\,\text{MPa}$, $\nu_c = 0.3$, $E_c = 3640\,\text{MPa}$, and force $F = F_0/2 = 1500\,\text{N}$ ($F_0 = 3000\,\text{N}$ – the total load). Therefore, the maximum deflection of the middle part (28) $v_{\max}^{(m)}$=0.82 mm, the maximum total deflection (27) $v_{\max}^{(total)}$=6.37 mm, and (26) $v(L_1)$=5.55 mm in the point of force *F* application.

3. FEM-numerical study

The FEM model of the sandwich plate-strip is developed with the use of the SolidWorks software package. Symmetry of the plate-strip allows to confine the model to a quarter of the whole structure (Fig. 5). Its proper behavior is ensured by the boundary conditions imposed on it. The plate-strip model is divided into about 942 thousand 3D tetrahedral finite elements with 4 Jacobian points. Number of the FEM nodes amounted nearly to 1 400 000. Example of a part of the mesh is shown in Fig. 6.

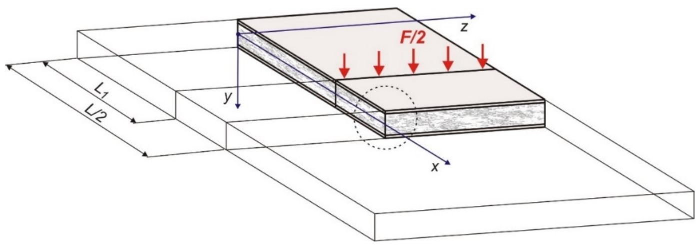


Fig. 5. A model of the sandwich plate-strip used for FEM computation

The plate-strip is located in a Cartesian coordinate system *x*,*y*,*z*. The *xz* plane is the middle plane of the strip, equivalent to its neutral plane. The *y*-axis points downward (Fig. 5).

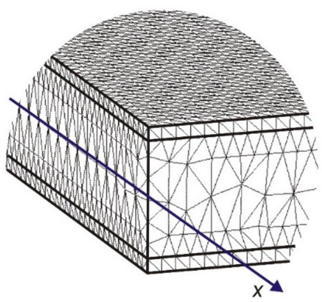


Fig. 6. A part of the FEM mesh (limited approximately to the area marked with the dotted circle in Fig. 5)

The boundary conditions listed below, applied to the surfaces of the plate-strip model, guarantee that it behaves as one fourth of the whole strip:

- The strip model is simply supported at its edge for x=0. Hence, the y displacements of the wall coplanar with the yz – plane are zero.
- The x displacements of the middle wall of the strip, parallel to the yz – plane (for x=L/2), are equal to zero.
- The z displacements of the model wall coplanar with the xy – plane (for z=0) are zeroed.

Calculation of the bending process with the data equal to those adopted in case of the former analytical approach gave the results $v_{\max}^{(total)}$ =6.27 mm and $v(L_1)$ =5.46 mm. Thus, the maximum deflection of the middle part amounted to $v_{\max}^{(m)}$ =0.81 mm.

4. Experimental study

The experimental tests have been made with the samples manufactured and delivered by Havel Metal Foam GmbH (Germany). The sample is shown in Fig. 7.

Fig. 7. Photo of the sandwich plate-strip sample

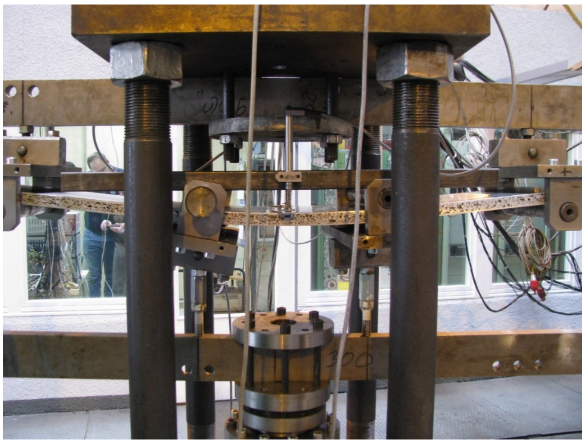


Fig. 8. The bent sample mounted on test stand (the the Łukasiewicz Research Network – RVI "TABOR" Laboratory)

The sample is mounted on the test stand and loaded with the force F_0=2F, as shown in Fig. 1. Actual sample length amounts to 1000 mm. It is supported on the rollers the span of which is equal to L=900mm. The load is applied by two another pressure rollers spaced at 300mm, symmetrically with respect to the sample. In result the sample part between these two pressure rollers is subjected to pure bending (Fig. 8).

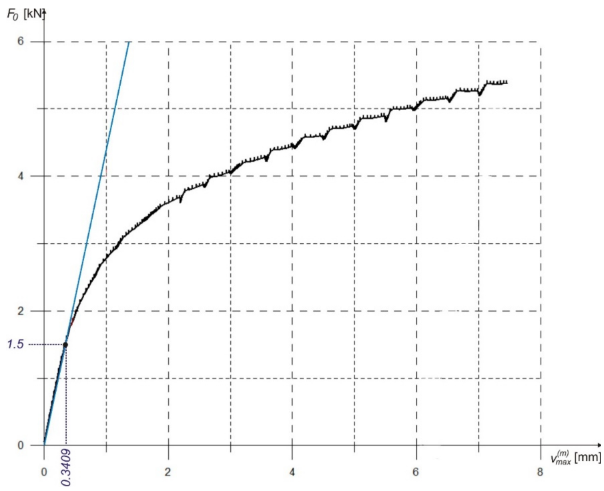


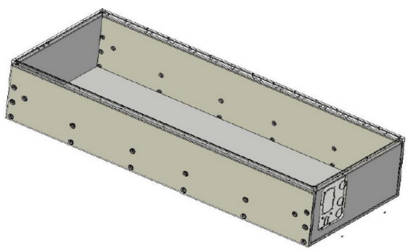
Fig. 8. The bent sample mounted on test stand (the the Łukasiewicz Research Network – RVI "TABOR" Laboratory)

The test allowed to measure the relationship between the load F_0 and maximum deflection $v_{\max}^{(m)}$ of the sample middle point (shown in Fig. 9).
It may be noticed, that the elastic (proportionality) range is observed for the load up to 1.5kN. Such a force causes the maximum deflection of the sample middle point $v_{\max}^{(m)}$ =0.3409 mm. Further growth of the load results in evident loss of proportionality. Irregularities of the curve suggest that the material undergoes gradual destruction.
For purposes of analytical and FEM numerical tests, the total load value F_0=3000 kN was adopted. During the bench tests it was found that the elastic range of the strip sample bending reaches the value $F_0 \leq 1.5$ kN. Thus, the analytically and numerically determined maximum deflection values of the sample middle part FEM in case of this load are equal and amount to 0.41 mm. They exceed the values measured experimentally by 20%. It should be noticed that deflection of the sample central part is equal to the difference between the total deflection and the one arising in the point of application of the force-load (Fig. 4). In consequence, its value is small as compared to the deflection in the load point (~ 15%).
The Havel Metal Foam GmbH uses the manufacturing technology of the three-layer structures with an aluminum foam core for production of many various structural components. The above mentioned sandwich panels could be used as parts of the floor or rail vehicle paneling. Several examples of applications of these elements manufactured by Havel Metal Foam GmbH are shown in Figs. 10÷12.

Total thickness: 20 mm
Cover sheet (top): 1.5 mm
Metal foam core: 17 mm
Cover sheet (bottom): 1.5 mm

Grubość całkowita: 20 mm
Okładzina górna: 1.5 mm
Rdzeń z piany metalowej: 17 mm
Okładzina dolna: 1.5 mm

Fig. 10. Compressor housing cover



Welded construction 1515×600×298 mm
with integrated ∅8 cooling tube system

Konstrukcja spawana 1515×600×298 mm
ze zintegrowanym układem rur chłodzących
∅8

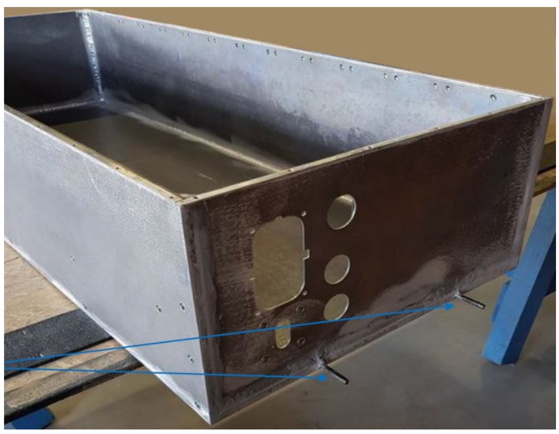
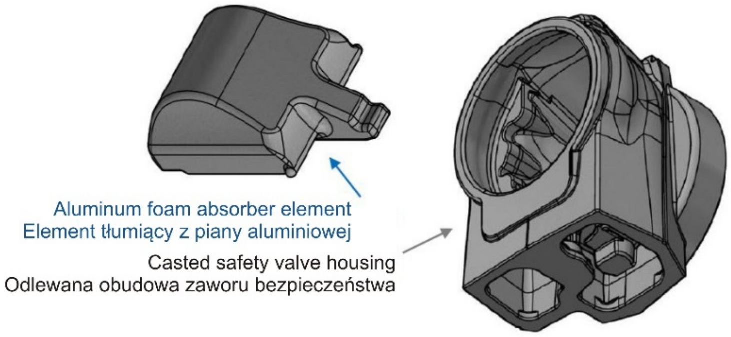
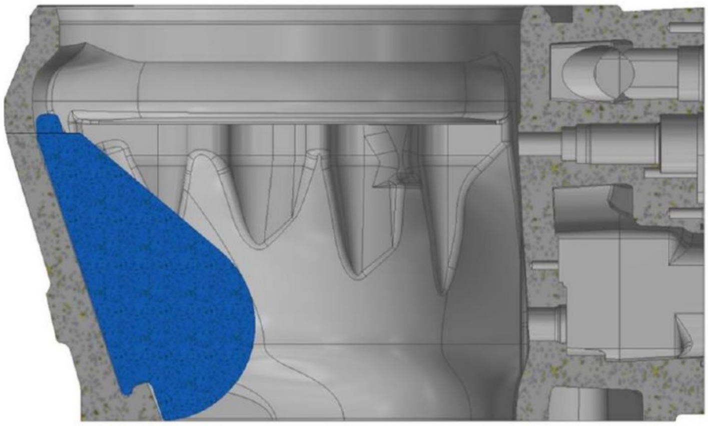


Fig. 11. A case for a 600V battery system of the electric vehicles



Przekrój poprzeczny tłumika zgniotu
Cross section of the crumple damper



The absorber element inside the casted safety valve housing
Element tłumiący wewnątrz obudowy zaworu bezpieczeństwa

Fig. 12. Crumple damper of a 5” safety valve

It may be assumed that the use of three-layer structures with an aluminum foam core should expand. Such structures have many advantages, they are distinguished by low mass, maintaining the required strength. They also have valuable properties in case of impact loads.

5. Conclusions

The deflections of the sample strip have been determined with three methods: analytically, FEM numerically and experimentally. For purposes of the first two approaches the Young’s moduli of the faces and the core have been arbitrarily adopted as E_f=72000 (aluminium) MPa, E_c=3640 MPa (equivalent value for the aluminium foam) and the Poisson ratio of the core ν_c=0.3. Comparison of these two solutions is satisfactory, the deflections well match with each other (Table 1).
Nevertheless, in the laboratory test only the maximum deflection of the sample middle point was measured and gave a result deviating by around 20% from that determined with two above solutions. Moreover, in case of the load equal to F_0=3000N the foam material enters a plastic range, that does not occur in the equivalent material having the trial Young’s modulus E_c.

Comparison of the results obtained in analytical, FEM-numerical and experimental approaches **Table 1.**

Method	Analytical	FEM-numerical	Experimental
F_0 [kN]	–	3.0	1.5
$v_{\max}^{(total)}$[mm]	6.91	7.00	–
$v(L_1)$[mm]	6.04	6.12	–
$v_{\max}^{(m)}$[mm]	0.87	0.88	0.3409
k (29) [kN/mm]	3.488	3.409	4.400

Deflection of the central part of the sample, equal to the difference between the total deflection and the maximum deflection in the load application point (Fig. 4), is small and amounts about to 15% of the deflection in the load point.

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