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Fractals in underwater acoustics

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ABSTRACT

The concept of fractals and their applications for underwater acoustics are discussed. Waves scattering by a fractal surface is considered. Ray chaos in the case of waves propagation in inhomogeneous underwater waveguides is discussed. Results of investigation of multifractal characteristics of flow noise, cavitation noise (acoustic turbulence) and the noise emitted by elastic structures are given.

INTRODUCTION

Researchers develop various models in order to "understand" Nature. Geometry occupies one of central places in these models. Traditionally the basis for these concepts are Euclidean lines, circles, spheres, etc. However everybody understands that geometry of Nature is more complex than Euclidean concepts.

About 20 years ago V. Mandelbrot got everybody interested in fractal geometry [1]. The idea of fractals caught attention of researchers active in many fields of science. Successful application of fractal models in physics is due to the fact that fractal forms are inherent to a huge number of processes and structures. And this is not accidental. Many models of rise and growth of disordered objects of various nature are reduced finally to the models of percolation transition and diffusion-restricted aggregation. In the first case a fractal percolation cluster is formed while in the second this is a fractal aggregate. Models for many disordered processes are based on various cases of random walk or dynamic chaos which also have fractal properties. Essentially, introduction of the idea of fractals opened an opportunity for mathematical description of a very general laws of geometric properties of physical world and Nature on the whole. Now many researchers consider Nature to be fractal. There is nothing surprising in attempts to connect the idea of fractals with wave processes and problems of underwater acoustics.

FRACTALS

At the beginning of the century the concepts of the Hausdorf measure and the Hausdorf-Besikovich dimension (HBD) existed in mathematics. The theory of this dimension was developed in 1920s. It was clear according to the character of HBD that it is connected with metrics and not with topology, i. e. with the way of the set construction. The HBD dimension can be of any value. This provided an opportunity to speak about mathematical sets and space of non-integer dimension.

In mathematics a fractal represents a set of points in metric space. It is impossible to establish any traditional measure of integer dimension for such set, i. e. length, square or volume (their dimensions are the first, second and third power of length respectively). For example traditional measurements of a fractal curve length may produce an infinite result, while traditional measurements of square under it may produce zero result. The problem of measurements for such sets is solved with the help the Hausdorf measure and HBD.

Initially Mandelbrot determined fractal as a set with HBD greater than the topological dimension. Later he proposed to change for the following. The name of fractal is given to a structure consisting of parts which are similar to the whole in some sense. Fractal is an object or structure with a property of scale invariance (scaling) [1, 2]. They are characterized by fractal dimension which is not always coincide with HBD. Cellular and mass fractal dimensions are used frequently. They are easily determined experimentally.

Mathematical, physical and statistical fractals are considered. The Koch curve, the Serpinsky carpet and the curve describing the Brown walk may be an example of fractals of the first kind. As it is known, such curve is integer everywhere but not differentiable. The Weierstrass-Mandelbrot function is frequently used for description of a fractal. Physical fractals have the property of scale invariance in a restricted interval of scales. They may be differentiated but the values of derivatives may be rather large. For statistical physical fractals the property of scaling exists usually only within a limited range of scales. Statistical fractals properties are characterized by correlation (structural) functions and their spectra. There are mass and surface fractals and fractals arising in turbulent flow. An important feature of fractal models is power laws of correlation (structural) functions and their spectra. Lately fractal models have been used for consideration of scattering and radiation of waves by fractal objects and waves propagation in inhomogeneous and disordered media (see reviews in [3-5]).

WAVES SCATTERING BY AN UNEVEN FRACTAL SURFACE

Angular dependence of intensity of waves scattered by a large- scale smooth (Euclidean) surface is determined by the distribution of surface slopes. A fractal surface cannot be differentiated and it suggests the presence of unevennesses of all scales. It does not have determined slopes. Thus, strictly speaking, neither the Kirchhof approximation (tangent plane) nor the perturbation method may be applied for determination of scattered field. Probably this is the reason why diffraction of a plain wave coming through a phase screen was considered in one of the first papers [6] instead the problem of scattering by a rough surface. Wave thickness of the screen was determined by a product of surface height and wave number of an incident wave. A surface with the structure of generalized Gaussian fractal with the structural function of a surface random fractal. It is demonstrated that angular dependence of wave intensity after the screen coincides with the density of stable according to Levi distribution. The Levi statistics („the Levi flights") are characterized by the property of scale invariance and plays an important role in the fractal theory. Statistical characteristics in the case of waves scattering by screens with the Gaussian random fractal phase have been considered also in [7]. In particular, a subfractal surface with a fractal structural function of slopes was used as a model. Such surface may be differentiated and has slopes changing from point to point continuously.

Real surfaces are scale invariant within a limited range of scales and in the majority of cases they may be considered as random fractal surfaces. Sea waves have fractal properties in both cases of determined [8] and random waves [9, 10]. Wind waves create random fractal sea surface within the scale of 0.1 - 100 m with the fractal dimension D=2.25 [9]. The ocean floor is fractal [9].

Authors of some papers considered waves scattering by a large-scale fractal surface which was described by the "shortened" Weierstrass-Mandelbrot function. Unevennesses were considered to be slope and large in comparison with the wavelength. Such surface cannot be differentiated only in a limited number of points. The Kirchhof approximation and surface fractal (power) structural function were used. For example, it was demonstrated in [11] if the fractal dimension of the surface was increased, angular characteristics of fluctuations intensity of scattered field were broadened and the amplitudes of peaks of the angular dependence of intensity decreased. The slope angle of the tangent to these peaks increases with the increase of fractal dimension. However the average intensity of field fluctuations and average field in the mirror direction do not depend on the fractal dimension and are determined by the average square value of surface shifts. The field of a laser harmonic sound source at a random large- scale fractal statistically homogeneous isotropic surface with slope unevennesses was considered in [12]. It was demonstrated that the angular dependence of average intensity of field fluctuations was broadened and the value of field fluctuations intensity decreased with the increase of fractal dimension of the surface.

PROPAGATION OF WAVES IN INHOMOGENEOUS AND DISORDERED MEDIA

Fractal structures may be observed in the ray pattern in a longitudinally inhomogeneous waveguide. The study of these effects is based on the representation of ray equations in the Hamiltonian form and the analogy with the results of nonlinear Hamiltonian dynamics [13].

A model of a waveguide filled with a homogeneous media and with completely reflecting walls is has been considered in [14]. One wall is periodically uneven. Rays in such waveguide propagate being alternatingly reflected by walls. Rays propagation is described by a nonlinear image determining the angle and longitudinal coordinate of ray reflection from a plane wall through the angle and longitudinal coordinate of the preceding reflection from the wall. If unevennesses are absent, the length of ray cycle is constant. If one of the walls is periodically uneven then rays oscillate. An uneven wall influences most strongly the rays which are in nonlinear resonance with the period of unevennesses. Ray dynamics becomes chaotic with inherent to chaos fractal properties of phase portrait in the case of overlapping resonances. Ray chaos in an acoustic waveguide in shallow sea with periodically uneven floor is considered in [14]. Ray chaos arises in the case of sufficiently small angles of outcoming rays. Apparently, conditions for appearance of chaotic (fractal) dynamics of rays in deep-water ocean waveguides were considered for the first time in [15]. Consideration of chaotic ray dynamics in the case of acoustic signals propagation in the ocean is given in [16]. The study is based on the eikonal equation in the Hamiltonian form. The necessary condition for chaotic behavior of rays is local instability of these equations solutions. It is demonstrated that the condition for local instability is satisfied in the case of small longitudinal disturbance (which may be caused for example by internal waves in the ocean) in a double-axis underwater acoustic channel. Numeric solution of the equations for a typical sound channel in the Northern Athlantics confirmed this result. Thus, ray chaos and fractal properties of signals may turn out to be typical for ocean acoustics.

The results given above are valid for the case of two-dimensional waveguides. The situation is changed qualitatively in the case of three-dimensional problems if two-dimensional inhomogeneity is taken into account. Diffusion in phase space (the Arnold diffusion) becomes possible from the point of view of dynamic ray chaos. Three-dimensional effects were considered recently in [17] for the case of bottom ocean waveguide with uneven floor. Rays diffusion consists qualitatively of random changes of rays propagation direction along the route in the horizontal plane. It is noted that any direction is possible, including the direction opposite to the initial one.

Let us stress that fractal ray dynamics arises not in a randomly inhomogeneous medium. Longitudinal disturbances may have quite a regular character. There is nothing surprising in this. It is well known now that chaotic oscillations may arise in nonlinear regular dynamic systems under the effect of non- regular forces [18].

Investigation of fractal characteristics of acoustic waves propagation in disordered statistically inhomogeneous media is based on the analysis of solutions of the Helmholz equation first of all. The basis for consideration is the Born approximation as a rule, though multiple scattering of waves by fractals have been considered too. Models of media with mass and surface fractals as well as with fractals caused by medium turbulence have been used. Medium fractal properties have been characterized by correlation (structure) functions and their spectra. One of major goals for this research has been clearing out of spectral (frequency) laws of waves attenuation caused by sound scattering by medium fractal structures. It is known that sound waves attenuation is governed by an exponential law with the index depending on frequency according to a power law. This index has whole values for classical mechanisms of attenuation. For example, attenuation caused by the Rayleigh scattering has the index equal to 4. It is demonstrated in [19] that the index value may serve as a measure for density of scattering objects in an inhomogeneous medium, and in the case of a disordered medium the index value coincides with the fractal dimension of the medium. These ideas have been discussed in [20] in connection with seismic acoustic waves propagation in the Earth lithosphere. Analogous estimations of acoustic signals attenuation in the ocean are unknown to us.

Transversal shift and broadening of a wave beam in an acoustic medium with small-scale and fractal unevennesses have been studied numerically in [21]. A parabolic equation has been used. The medium is characterized by inhomogeneities changing smoothly along the direction of waves propagation and short-correlated in the transversal direction. Fractal broadening and shift of the beam have been considered.

FLOW NOISE

Application of fractal concept to the problem of noise and vibration was discussed in [22]. Flow noise is caused by nonstationary turbulent motion of liquid particles in the boundary layer, wake flow and in general around a body when it moves in a liquid. Flow noise increases rapidly with the increase of motion velocity and becomes dominating fast. Turbulence is a classic example of chaotic oscillations arising in regular nonlinear systems. Mandelbrot was apparently the first who connected turbulence with the idea of fractal [1, 23]. However it is necessary to note that as long ago as in 1926 Richardson suggested to use the Weierstrass function as a model for description of turbulence [24]. Fractal properties of turbulence are evident already from the fact that the famous Kolmogorov law characterizing the spectrum of turbulence in the inertial interval has power dependence with a non-integer exponent. An important property of turbulence is intermittence. This phenomenon is strongly connected with the idea of multifractal and singularities spectrum. A multifractal is a composition of fractal sets of various dimensions [2, 25]. Singularities spectrum carries information on the local structure of a process in time and space. Multifractal analysis gives sensible "squeeze" from information on different degree moments of a two-point probabilities distribution.

Noise of turbulent boundary layer (TBL) is usually connected with wall pressure pulsations. The author of [26] studied experimentally multifractality and universality of intermittence of wall pressure pulsations in TBL in the case of liquid flow in a pipe. It was demonstrated the pulsations intermittence was caused by intermittent character of turbulent energy production in TBL in the process of development of coherent structures, i. e. turbulent "splashes".

The rise of cavitation bubbles and developed cavitation is possible in TBL under certain conditions. Bubbles which oscillate and collapse, create cavitation noise. This noise is sometimes called acoustic turbulence. This indicates the chaotic nature of cavitation noise and its fractal properties. Really, a nonlinearly oscillating bubble in a liquid is a typical example of a nonlinear dynamic system with chaotic behavior [27]. Nonlinear behavior of a bubble in water, development of chaos according to the Feigenbaum "scenario", fractal and other characteristics of acoustic turbulence were discussed in [27, 18].

NOISE EMITTED BY ELASTIC STRUCTURES

At least three cases are possible here. First, an elastic structure may oscillate under the effect of external fractal forces. An example of the last may be wall pressure pulsations in TBL. In the second case external forces are regular but the structure (a plate or a shell) has fractal characteristics. Finally, it is possible that an elastic body in a flow of liquid or gas performs nonlinear chaotic oscillations [18].

Sound radiation by a plane layered elastic structure oscillating under the effect of random statistically homogeneous fractal forces was considered in [28]. It was demonstrated that fractal dimension of average intensity of radiation field fluctuations in the far wave field coincides with fractal dimension of external forces. This indicates the opportunity to determine fractal dimension of external forces by the analysis of characteristics of sound radiated by the structure. In this connection one may expect that the acoustic field radiated by a wall oscillating under the effect of wall pressure pulsations in TBL has the singularities spectrum analogous to the singularities spectrum of wall pulsations. The influence of fractal inhomogeneities of a thin elastic plate on the sound field radiated by it was considered in [29].

CONCLUSION

Wave theory is rich with deep results and developed theoretical methods. It is based essentially on models of continuous medium and uses mathematics dealing mostly with "smooth" functions. The concept of physical and statistical fractals allows to characterize quantitatively wave phenomena including those in underwater acoustics in a new way utilizing already developed methods of wave theory. However, application of fractal models may produce essentially new results. This concerns the theory of fractons. This theory solves the problem of wave propagation in inhomogeneous and disordered media which cannot be described by models of continuous media (for example see [4]). Application of this theory to underwater acoustics problems may produce unexpected results for example in the process of development of sound absorbers and sound and vibration insulators. However these problems expect their own researchers.

Here we practically did not consider two important problems, i. e. the theory of fractons and application of R-S analysis, multifractal analysis and wavelets analysis in underwater signal processing.

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