Dariusz Jakóbczak
Katedra Podstaw Informatyki i Zarządzania
Wydział Elektroniki i Informatyki
Politechnika Koszalińska

Active Object Modeling and Applications via the Method of Hurwitz-Radon Matrices

Keywords: Hurwitz-Radon Matrices, Active Object Modeling

1. Introduction

Object description by its contour is a critical part in many applications of image processing. Thus computer vision and artificial intelligence have a problem: how to model the active shape [1,2] via discrete set of two-dimensional boundary points? Also subject of shape representation and shape description is still opened [3,4]. The author wants to approach a problem of image structure representation by characteristic contour points and not limited to closed curves, but also working on open curves (for example a signature or handwriting). Proposed method relies on active functional modeling of boundary points situated between the basic set of the nodes. The functions that are used in calculations represent whole family of elementary functions: trigonometric, cyclometric, logarithmic, exponential and power function. Nowadays methods apply mainly polynomial functions, for example Bernstein polynomials in Bezier curves, splines and NURBS [5]. Numerical methods for data interpolation are based on polynomial or trigonometric functions, for example Lagrange, Newton, Aitken and Hermite methods. These methods have some weak sides [6] and are not sufficient for object modeling in the situations when the shape cannot be build by polynomials or trigonometric functions. Also trigonometric basis functions in Fourier Series Shape Models are not appropriate for describing all shapes. Model-based vision such as Active Contour Models (called Snakes) or Active Shape Models use the training sets to fit the data and they are applied only for closed curves. In this paper discussed approach is not limited to closed curves and it does not use a training set of some images, but only a set of two-dimensional nodes of the curve. Proposed Active Object Modeling is the functional modeling via any elementary functions and it helps us to fit the contour and to match the shape in object modeling or image analysis. The author presents novel method of flexible modeling and building the image structure for applications in signature and handwriting modeling, curve fitting, object representation and shape geometry.

This paper takes up new method of two-dimensional Active Object Modeling (AOM) by using a family of Hurwitz-Radon matrices. The method of Hurwitz-Radon Matrices (MHR) requires minimal assumptions about object. The only information about shape or curve is the set of at least five nodes. Proposed method of Hurwitz-Radon Matrices (MHR) is applied in curve modeling via different coefficients: sinusoidal, cosinusoidal, tangent, logarithmic, exponential, arc sin, arc cos, arc tan or power. Function for coefficient calculations is chosen individually at each Active Object Modeling and it depends on initial requirements and shape specifications to fit and to match the object. MHR method uses two-dimensional vectors (x,y) for data analysis and curve modeling. Shape of the object is represented by succeeding boundary points $(x_i,y_i) \in \boldsymbol{R}^2$ as follows in MHR method:

1. At least five nodes (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4) and (x_5,y_5) if MHR method is implemented with matrices of dimension $N = 2$;
2. For better modeling nodes ought to be settled at key points of the curve, for example local minimum or maximum and at least one point between two successive local extrema.

Condition 1 is connected with important features of MHR method: MHR version with matrices of dimension $N = 2$ (MHR-2) needs at least five nodes, MHR version with matrices of dimension $N = 4$ (MHR-4) needs at least nine nodes and MHR version with matrices of dimension $N = 8$ (MHR-8) needs at least 17 nodes. Condition 2 means for example the highest point of the object in a particular orientation, convexity changing or curvature extrema. So this paper wants to answer the question: how to model the active object for discrete set of points?

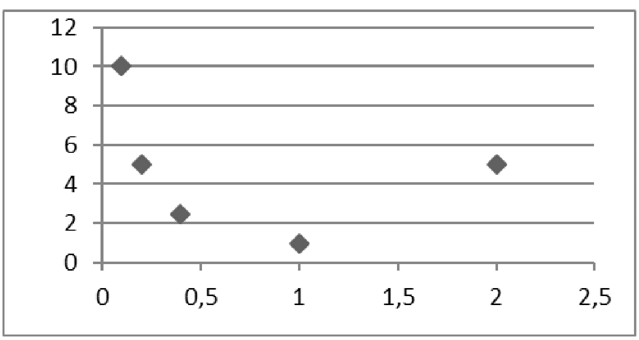

Fig. 1. Nodes of the object before modelin

Coefficients for Active Object Modeling are computed via individual features of the object boundary using power function, sinus, cosine, tangent, logarithm, exponent or arc sin, arc cos, arc tan.

2. Active Object Modeling via MHR

The method of Hurwitz – Radon Matrices (MHR), described in this paper, is computing points between two successive nodes of the curve. Data of Active Object Modeling are interpolated and parameterized for real number $\alpha \in [0;1]$ in the range of two successive nodes. MHR calculations are introduced with square matrices of dimension $N = 2$, 4 or 8. Matrices A_i, $i = 1,2\ldots m$ satisfying

$$A_jA_k+A_kA_j = 0,\ A_j^2 = -I \ \text{ for } j \neq k;\ j, k = 1,2...m$$

are called *a family of Hurwitz - Radon matrices*, discussed by Adolf Hurwitz and Johann Radon separately in 1923. A family of Hurwitz - Radon (HR) matrices [7] are skew-symmetric ($A_i^T = -A_i$), $A_i^{-1} = -A_i$ and only for dimension N=2, 4 or 8 the family of HR matrices consists of N-1 matrices. For $N = 2$:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

For $N = 4$ there are three HR matrices with integer entries:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix},\quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},\quad A_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

For $N = 8$ we have seven HR matrices with elements 0, ±1. So far HR matrices have found applications in Space-Time Block Coding (STBC) [8] and orthogonal design [9], in signal processing [10] and Hamiltonian Neural Nets [11].

How coordinates of curve points are applied in Active Object Modeling? If boundary points have the set of following nodes $\{(x_i,y_i),\ i = 1, 2, \ldots, n\}$ then HR matrices combined with the identity matrix I_N are used to build the orthogonal Hurwitz - Radon Operator (OHR). For points $p_1=(x_1,y_1)$ and $p_2=(x_2,y_2)$ OHR of dimension $N = 2$ is build via matrix M_2:

$$M_2(p_1,p_2) = \frac{1}{x_1^2 + x_2^2}\begin{bmatrix} x_1y_1 + x_2y_2 & x_2y_1 - x_1y_2 \\ x_1y_2 - x_2y_1 & x_1y_1 + x_2y_2 \end{bmatrix} \tag{1}$$

For points $p_1=(x_1,y_1)$, $p_2=(x_2,y_2)$, $p_3=(x_3,y_3)$ and $p_4=(x_4,y_4)$ OHR M_4 of dimension $N = 4$ is introduced:

$$M_4(p_1,p_2,p_3,p_4) = \frac{1}{x_1^2 + x_2^2 + x_3^2 + x_4^2}\begin{bmatrix} u_0 & u_1 & u_2 & u_3 \\ -u_1 & u_0 & -u_3 & u_2 \\ -u_2 & u_3 & u_0 & -u_1 \\ -u_3 & -u_2 & u_1 & u_0 \end{bmatrix} \tag{2}$$

where

$$u_0 = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4, \quad u_1 = -x_1y_2 + x_2y_1 + x_3y_4 - x_4y_3,$$
$$u_2 = -x_1y_3 - x_2y_4 + x_3y_1 + x_4y_2, \quad u_3 = -x_1y_4 + x_2y_3 - x_3y_2 + x_4y_1.$$

For nodes $p_1=(x_1,y_1)$, $p_2=(x_2,y_2)$,… and $p_8=(x_8,y_8)$ OHR M_8 of dimension $N = 8$ is constructed [12] similarly as (1) and (2):

$$M_8(p_1,p_2...p_8) = \frac{1}{\sum_{i=1}^{8} x_i^2}\begin{bmatrix} u_0 & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \\ -u_1 & u_0 & u_3 & -u_2 & u_5 & -u_4 & -u_7 & u_6 \\ -u_2 & -u_3 & u_0 & u_1 & u_6 & u_7 & -u_4 & -u_5 \\ -u_3 & u_2 & -u_1 & u_0 & u_7 & -u_6 & u_5 & -u_4 \\ -u_4 & -u_5 & -u_6 & -u_7 & u_0 & u_1 & u_2 & u_3 \\ -u_5 & u_4 & -u_7 & u_6 & -u_1 & u_0 & -u_3 & u_2 \\ -u_6 & u_7 & u_4 & -u_5 & -u_2 & u_3 & u_0 & -u_1 \\ -u_7 & -u_6 & u_5 & u_4 & -u_3 & -u_2 & u_1 & u_0 \end{bmatrix} \tag{3}$$

where

$$\underline{u} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 \\ -y_2 & y_1 & -y_4 & y_3 & -y_6 & y_5 & y_8 & -y_7 \\ -y_3 & y_4 & y_1 & -y_2 & -y_7 & -y_8 & y_5 & y_6 \\ -y_4 & -y_3 & y_2 & y_1 & -y_8 & y_7 & -y_6 & y_5 \\ -y_5 & y_6 & y_7 & y_8 & y_1 & -y_2 & -y_3 & -y_4 \\ -y_6 & -y_5 & y_8 & -y_7 & y_2 & y_1 & y_4 & -y_3 \\ -y_7 & -y_8 & -y_5 & y_6 & y_3 & -y_4 & y_1 & y_2 \\ -y_8 & y_7 & -y_6 & -y_5 & y_4 & y_3 & -y_2 & y_1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} \tag{4}$$

and $\underline{u} = (u_0, u_1,\dots, u_7)^T$ (4). OHR operators M_N (1)-(3) satisfy the condition of interpolation

$$M_N \cdot \mathbf{x} = \mathbf{y} \tag{5}$$

for $\mathbf{x} = (x_1,x_2\dots,x_N)^T \in \boldsymbol{R}^N$, $\mathbf{x} \neq \mathbf{0}$, $\mathbf{y} = (y_1,y_2\dots,y_N)^T \in \boldsymbol{R}^N$ and N =2, 4 or 8.

2.1 Functional coefficients in MHR Active Object Modeling

Coordinates of points settled between the nodes are computed [13] using described MHR method [14]. Each real number $c \in [a;b]$ is calculated by a convex combination $c = \alpha \cdot a + (1 - \alpha) \cdot b$ for

$$\alpha = \frac{b-c}{b-a} \in [0;1]. \tag{6}$$

The average OHR operator M of dimension $N = 2$, 4 or 8 is build:

$$M = \gamma \cdot A + (1-\gamma) \cdot B \,. \tag{7}$$

The OHR matrix A is constructed (1)-(3) by every second point $p_1=(x_1=a,y_1)$, $p_3=(x_3,y_3),\ldots$ and $p_{2N-1}=(x_{2N-1},y_{2N-1})$:

$$A = M_N(p_1, p_{3,\ldots,} p_{2N-1}).$$

The OHR matrix B is computed (1)-(3) by data $p_2=(x_2=b,y_2)$, $p_4=(x_4,y_4),\ldots$ and $p_{2N}=(x_{2N},y_{2N})$:

$$B = M_N(p_2, p_{4,\ldots,} p_{2N}).$$

Vector of first coordinates C is defined for

$$c_i = \alpha\cdot x_{2i-1} + (1-\alpha)\cdot x_{2i} \quad , \quad i = 1, 2,\ldots, N \tag{8}$$

and $C = [c_1, c_2,\ldots, c_N]^T$. The formula to calculate second coordinates $y(c_i)$ is similar to the interpolation formula (5):

$$Y(C) = M \cdot C \tag{9}$$

where $Y(C) = [y(c_1), y(c_2),\ldots, y(c_N)]^T$. So modeled value of $y(c_i)$ depends on four, eight or sixteen ($2N$) successive nodes, not only two.

Key question is dealing with coefficient γ in (7). Coefficient γ is calculated using different functions (power, sinus, cosine, tangent, logarithm, exponent, arc sin, arc cos, arc tan) and choice of function is connected with initial requirements and shape specifications during fitting and matching of the object. Coefficients γ and α (6) are strongly related:

1. $\gamma = 0 \leftrightarrow \alpha = 0$;
2. $\gamma = 1 \leftrightarrow \alpha = 1$;
3. $\gamma \in [0;1]$.

Different values of coefficient γ are connected with implemented functions and positive real number s:

$\gamma=\alpha^s$, $\gamma=sin(\alpha^s\cdot\pi/2)$, $\gamma=sin^s(\alpha\cdot\pi/2)$, $\gamma=1-cos(\alpha^s\cdot\pi/2)$, $\gamma=1-cos^s(\alpha\cdot\pi/2)$, $\gamma=tan(\alpha^s\cdot\pi/4)$, $\gamma=tan^s(\alpha\cdot\pi/4)$, $\gamma=log_2(\alpha^s+1)$, $\gamma=log_2^s(\alpha+1)$, $\gamma=(2^\alpha-1)^s$, $\gamma=2/\pi\cdot arcsin(\alpha^s)$, $\gamma=(2/\pi\cdot arcsin\alpha)^s$, $\gamma=1-2/\pi\cdot arccos(\alpha^s)$, $\gamma=1-(2/\pi\cdot arccos\alpha)^s$, $\gamma=4/\pi\cdot arctan(\alpha^s)$, $\gamma=(4/\pi\cdot arctan\alpha)^s$, $\gamma=ctg(\pi/2-\alpha^s\cdot\pi/4)$, $\gamma=ctg^s(\pi/2-\alpha\cdot\pi/4)$, $\gamma=2-4/\pi\cdot arcctg(\alpha^s)$, $\gamma=(2-4/\pi\cdot arcctg\alpha)^s$.

For example if $s = 1$ then:

basic MHR $\gamma = \alpha$, $\gamma = sin(\alpha \cdot \pi/2)$, $\gamma = 1-cos(\alpha \cdot \pi/2)$, $\gamma = tan(\alpha \cdot \pi/4)$, $\gamma = log_2(\alpha + 1)$, $\gamma = 2^\alpha - 1$, $\gamma = 2/\pi\cdot arcsin(\alpha)$, $\gamma = 1-2/\pi\cdot arccos(\alpha)$, $\gamma = 4/\pi\cdot arctan(\alpha)$.

What is very important, above functions used in γ calculations are strictly monotonic for $\alpha\in [0;1]$, because $\gamma \in [0;1]$ too. Choice of function and parameter s depends on object specifications and individual requirements. Fixing of unknown coordinates for curve points using (6)-(9) is called by author the method of Hurwitz - Radon Matrices (MHR) [15]. Each strictly monotonic function between points (0;0) and (1;1) can be used in Active Object Modeling – not only above functions.

3. Applications of AOM in Handwriting Modeling

Boundary nodes: (0.1;10), (0.2;5), (0.4;2.5), (1;1) and (2;5) from Fig.1 are used in some examples of MHR Active Object Modeling with different γ. These examples are connected with handwriting modeling for letter “*w*”. Points of the object are calculated with matrices of dimension $N = 2$ and $\alpha = 0.1, 0.2,\ldots,0.9$.

<u>Example 1</u>

Sinusoidal modeling with $\gamma = sin(\alpha \cdot \pi/2)$.

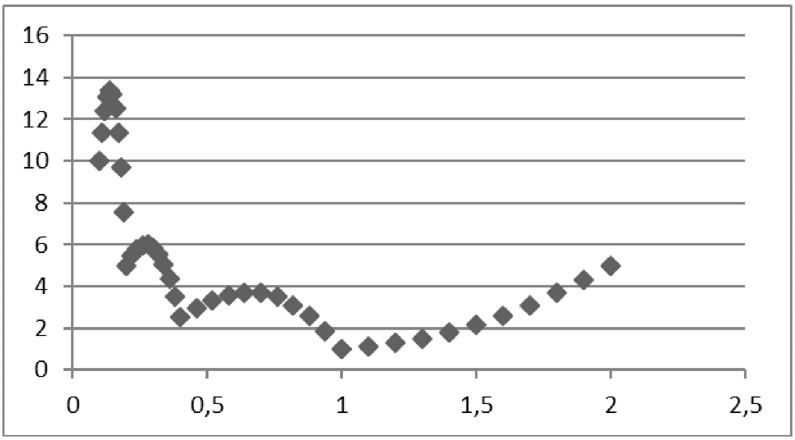

Fig. 2. Sinusoidal modeling with nine reconstructed points between nodes

<u>Example 2</u>

Tangent modeling for $\gamma = tan(\alpha \cdot \pi/4)$.

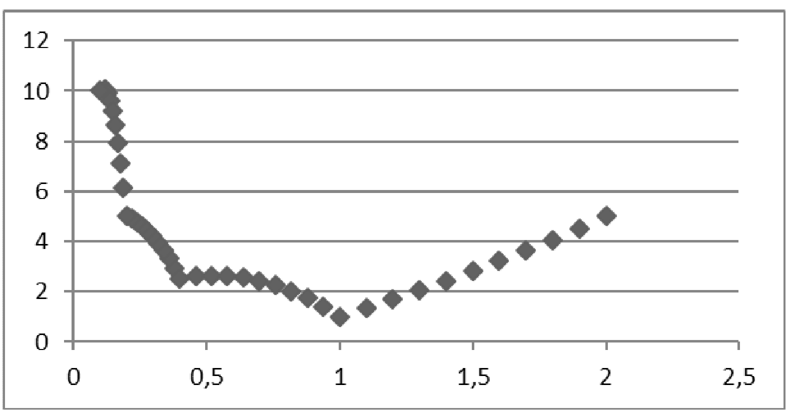

Fig. 3. Tangent modeling with nine interpolated boundary points between nodes

Example 3

Tangent modeling with $\gamma = tan(\alpha^s \cdot \pi/4)$ and $s = 1.5$.

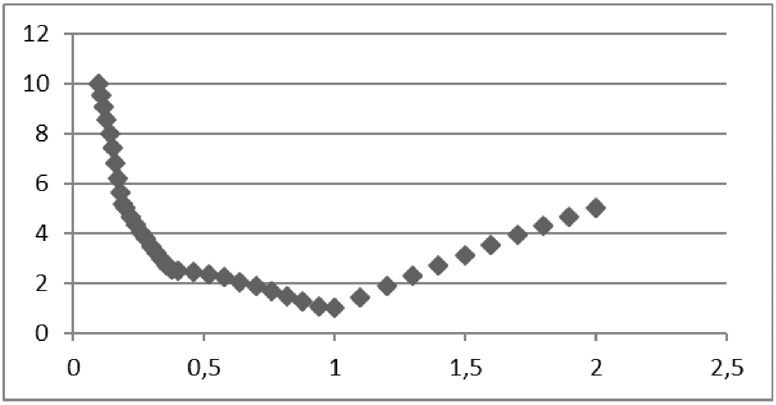

Fig. 4. Tangent modeling with nine recovered curve points between nodes

Example 4

Tangent modeling for $\gamma = tan(\alpha^s \cdot \pi/4)$ and $s = 1.797$.

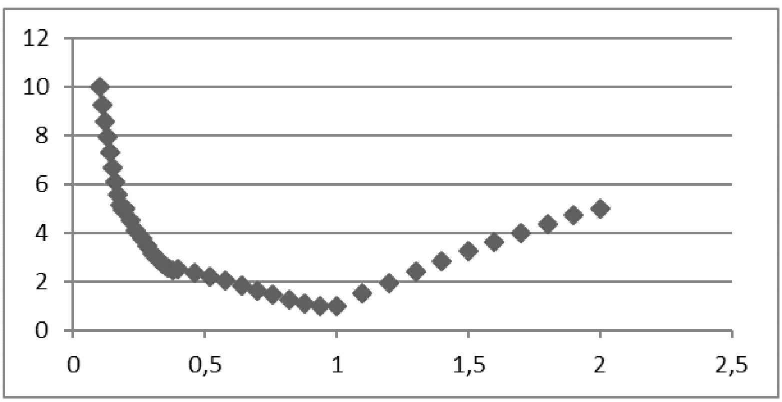

Fig. 5. Tangent modeling with nine reconstructed points between nodes

Example 5

Sinusoidal modeling with $\gamma = sin(\alpha^s \cdot \pi/2)$ and $s = 2.759$.

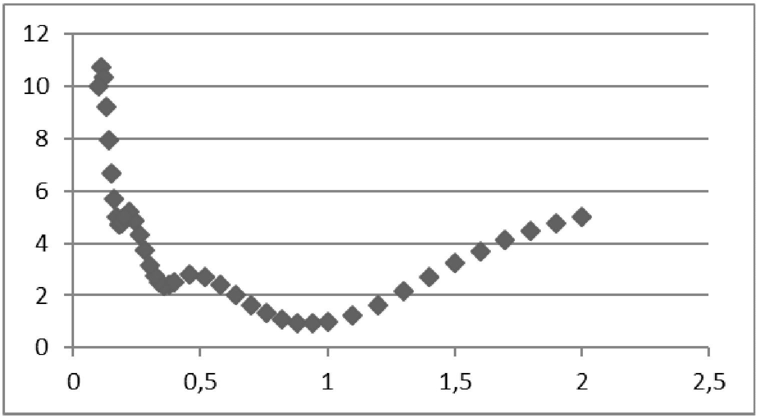

Fig. 6. Sinusoidal modeling with nine interpolated curve points between nodes

Example 6

Power function modeling for $\gamma = \alpha^s$ and $s = 2.1205$.

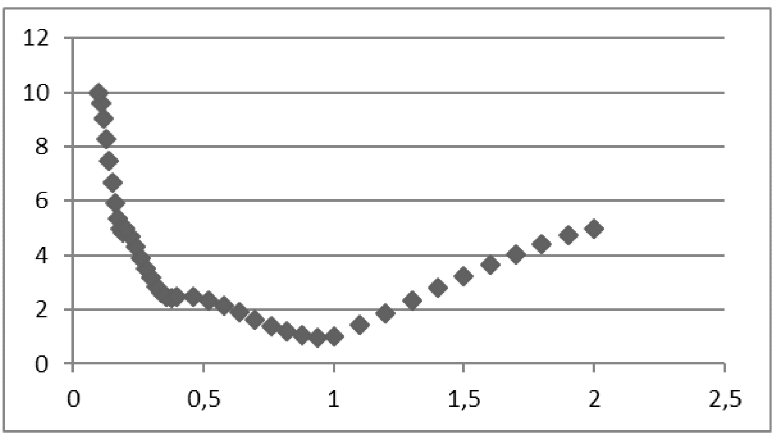

Fig. 7. Power function modeling with nine recovered object points between nodes

Example 7

Logarithmic modeling with $\gamma = log_2(\alpha^s + 1)$ and $s = 2.533$.

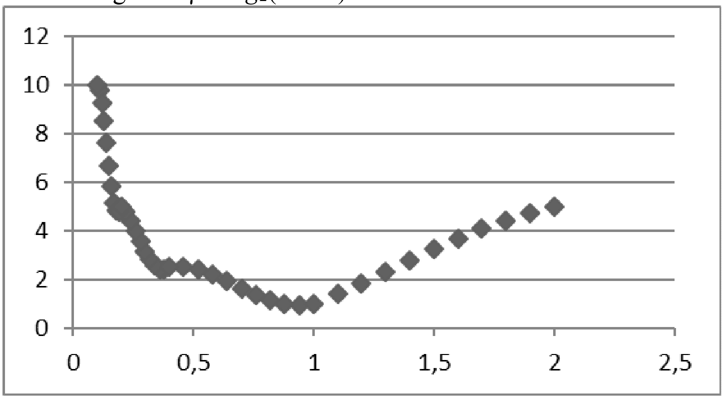

Fig. 8. Logarithmic modeling with nine reconstructed points between nodes

These seven examples demonstrate possibilities of Active Object Modeling for boundary nodes. Reconstructed values and interpolated points, calculated by MHR method, are applied in the process of curve modeling for fitting and matching the object during its analysis. Recovered points can be treated as a part of signature or handwriting and used during different stages of image processing, for example signature modeling, object representation, shape geometry and curve fitting. Every individual signature or handwriting, each letter or number can be modeled by some function for parameter γ. This parameter is treated as characteristic feature of letter or figure.

4. Conclusions

The method of Hurwitz-Radon Matrices (MHR) enables active modeling of two-dimensional shapes using different coefficients γ: sinusoidal, cosinusoidal, tangent, logarithmic, exponential, arc sin, arc cos, arc tan or power function [16]. Function for γ calculations is chosen individually at each Active Object Modeling (AOM) and depends on initial requirements and curve specifications. MHR method leads to shape modeling via discrete set of fixed points. So MHR makes possible the combination of two important problems: interpolation and modeling. Main features of MHR method are:

a) modeling of L points is connected with the computational cost of rank $O(L)$;
b) MHR is well-conditioned method (orthogonal matrices) [17];
c) coefficient γ is crucial in the process of AOM and it is computed individually for each object (contour, letter or figure).

Future works are going to: features of coefficient γ, implementation of MHR and AOM in object recognition [18], shape representation, curve fitting, contour modeling and parameterization [19].

References

1. Mahoor, M.H., Abdel-Mottaleb, M., Nasser Ansari, A.: Improved Active Shape Model for Facial Feature Extraction in Color Images. Journal of Multimedia 1(4), 21-28 (2006)
2. Cootes, T.F., Taylor, C.J., Cooper, D.H., Graham, J.: Active Shape Models-Their Training and Application. Computer Vision and Image Understanding 1(61), 38-59 (1995)
3. Amanatiadis, A., Kaburlasos, V.G., Gasteratos, A., Papadakis, S.E.: Evaluation of Shape Descriptors for Shape-Based Image Retrieval. IET Image Processing 5(5), 493-499 (2011)
4. Zhang, D., Lu, G.: Review of Shape Representation and Description Techniques. Pattern Recognition 1(37), 1-19 (2004)
5. Schumaker, L.L.: Spline Functions: Basic Theory. Cambridge Mathematical Library (2007)
6. Dahlquist, G., Bjoerck, A.: Numerical Methods. Prentice Hall, New York (1974)
7. Eckmann, B.: Topology, Algebra, Analysis- Relations and Missing Links. Notices of the American Mathematical Society 5(46), 520-527 (1999)
8. Citko, W., Jakóbczak, D., Sieńko, W.: On Hurwitz - Radon Matrices Based Signal Processing. Workshop Signal Processing at Poznan University of Technology (2005)
9. Tarokh, V., Jafarkhani, H., Calderbank, R.: Space-Time Block Codes from Orthogonal Designs. IEEE Transactions on Information Theory 5(45), 1456-1467 (1999)
10. Sieńko, W., Citko, W., Wilamowski, B.: Hamiltonian Neural Nets as a Universal Signal Processor. 28th Annual Conference of the IEEE Industrial Electronics Society IECON (2002)
11. Sieńko, W., Citko, W.: Hamiltonian Neural Net Based Signal Processing. The International Conference on Signal and Electronic System ICSES (2002)
12. Jakóbczak, D.: 2D and 3D Image Modeling Using Hurwitz-Radon Matrices. Polish Journal of Environmental Studies 4A(16), 104-107 (2007)
13. Jakóbczak, D.: Shape Representation and Shape Coefficients via Method of Hurwitz-Radon Matrices. Lecture Notes in Computer Science 6374 (Computer Vision and Graphics: Proc. ICCVG 2010, Part I), Springer-Verlag Berlin Heidelberg, 411-419 (2010)
14. Jakóbczak, D.: Curve Interpolation Using Hurwitz-Radon Matrices. Polish Journal of Environmental Studies 3B(18), 126-130 (2009)
15. Jakóbczak, D.: Application of Hurwitz-Radon Matrices in Shape Representation. In: Banaszak, Z., Świć, A. (eds.) Applied Computer Science: Modelling of

Production Processes 1(6), pp. 63-74. Lublin University of Technology Press, Lublin (2010)

16. Jakóbczak, D.: Object Modeling Using Method of Hurwitz-Radon Matrices of Rank k. In: Wolski, W., Borawski, M. (eds.) Computer Graphics: Selected Issues, pp. 79-90. University of Szczecin Press, Szczecin (2010)
17. Jakóbczak, D.: Implementation of Hurwitz-Radon Matrices in Shape Representation. In: Choraś, R.S. (ed.) Advances in Intelligent and Soft Computing 84, Image Processing and Communications: Challenges 2, pp. 39-50. Springer-Verlag, Berlin Heidelberg (2010)
18. Jakóbczak, D.: Object Recognition via Contour Points Reconstruction Using Hurwitz-Radon Matrices. In: Józefczyk, J., Orski, D. (eds.) Knowledge-Based Intelligent System Advancements: Systemic and Cybernetic Approaches, pp. 87-107. IGI Global, Hershey PA, USA (2011)
19. Jakóbczak, D.: Curve Parameterization and Curvature via Method of Hurwitz-Radon Matrices. Image Processing & Communications- An International Journal 1-2(16), 49-56 (2011)

Abstract

Artificial intelligence and computer vision need methods for object modeling having discrete set of boundary points. A novel method of Hurwitz-Radon Matrices (MHR) is used in shape modeling. Proposed method is based on the family of Hurwitz-Radon (HR) matrices which possess columns composed of orthogonal vectors. Two-dimensional active curve is modeling via different functions: sinus, cosine, tangent, logarithm, exponent, arc sin, arc cos, arc tan and power function. It is shown how to build the orthogonal matrix OHR operator and how to use it in a process of object modeling.

Streszczenie

Matematyka i jej zastosowania wymagają odpowiednich metod modelowania oraz interpolacji danych. Autorska metoda Macierzy Hurwitza-Radona (MHR) jest sposobem modelowania krzywej 2D. Oparta jest ona na rodzinie macierzy Hurwitza-Radona, których kluczową cechą jest ortogonalność kolumn. Dwuwymiarowe dane są interpolowane z wykorzystaniem różnych funkcji rozkładu prawdopodobieństwa: potęgowych, wielomianowych, wykładniczych, logarytmicznych, trygonometrycznych, cyklometrycznych. W pracy pokazano budowę ortogonalnego operatora macierzowego i jego wykorzystanie w rekonstrukcji i modelowaniu danych.

Słowa kluczowe: macierze Hurwitza-Radona, aktywne modelowanie obiektów