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MATHEMATICAL MODEL OF FLEXIBLE LINK DYNAMICS IN MARINE TETHERED SYSTEMS CONSIDERING TORSION AND ITS INFLUENCE ON TENSION FORCE

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ABSTRACT

The rigidity in bending of a flexible link (is an important characteristic that should be considered during regular service conditions. The tension and bending with torsion of wire ropes are also significant factors. This study proposed a method to calculate the vectors of the generalised forces of bending of flexible links. One of the causes of torsional stresses in the power plant of underwater tethered systems is the interaction with ship equipment, such as spiral winding on the winch drum, friction on the flanges of the pulleys or winch drums, and bends on various blocks and rolls that cause torsion. The source of torsional stresses in the FL may also be related to manufacturing, storage, transportation, and its placement on the ship's winch drums. Torsion can lead to a decrease in the tensile strength due to load redistribution between power elements, or even a violation of their structure. In some cases, torsion significantly affects the movement of the underwater tethered system as a whole. The development of a mathematical model to describe the marine tethered systems dynamics, taking into account the effect of torsion, is important and relevant. The mathematical model of the marine tethered systems dynamics was improved and solved by accounting for the generalised forces of the torsion rigidity of the flexible link, using an algorithm and computer program. The influence of the bending and torsional rigidity of the FL on its deflection and tensile strength were considered based on the example of two problems. The developed program's working window image shows the simulated parameters and the initial position of the flexible link. The results show that torsion has almost no effect on the shape of the a flexible link's deflection in the X0Z plane, but leads to a deviation from the X0Z plane when calculating the static deflection of the flexible link. When the carrier vessel is stationary and the submersible vehicle has no restrictions on movement and has positive buoyancy, torsion leads to a three-dimensional change in the shape of the flexible link both in the X0Z plane and in the X0Y plane. The tension force of the flexible link along its length is distributed unevenly, and the torsion of the flexible link can lead to significant changes in its shape, the trajectory of towed objects, and the forces acting on the elements of the marine tethered systems

Keywords: underwater tethered system, flexible links, rigidity, submersible vehicle

INTRODUCTION

When calculating or choosing the design of the flexible links (FL) of marine tethered systems (MTS), account must be taken of the conditions in which they will be operated. The FLs of MTS are used in a wide range of operating modes (different depths, currents, a large number of links in the MTS, their mutual influence, etc.). Complex and difficult (extreme) modes of operation of FLs require special study and determination of the forces acting on them, taking into account the nonlinearity of the governing equations, the possibility of losing the stability of equilibrium and the study of the system behaviour in supercritical states.

During the operation of the FLs of an underwater tethered system (UTS), there is the possibility of damage to the tow-cable (TC) as a result of repeated bends in the rollers, blocks, winch drums and elements of the lowering and lifting devices [1, p. 5]. In the process of long-term operation, for example, trawler winches, which are designed for uniform laying of FL on their drums, cease to fulfill their purpose after a certain period of operation due to wear of their friction pairs.

Mathematical models (MM) of the FL dynamics of MTS have been developed and improved, complemented by taking into account the generalised forces of the torsion rigidity of the FL and with the help of an algorithm and computer program which were solved by carrying out mathematical modelling of the MTS dynamics, including the influence of the torsion of the FL.

The study of the torsional rigidity (TR) of FLs in operating conditions thus becomes important. Although in some industries these issues have been previously addressed in the operation of FLs, for example, in towing and in UTS, these issues remain virtually unexplored due to the high cost of field experiments, lack of specialists, and the control and measurement complexity. In addition, the existing mathematical models (MM) describing the dynamics do not take into account the TR or allow it to be used in engineering calculations [2, p. 58].

To date, these studies have not received the necessary development due to the lack of reliable MM, which could, however, be quite simply and effectively implemented in the form of algorithms and programs for numerical solutions to these problems.

The aim of the study is to improve the previously developed MM of the FL dynamics in a MTS, taking into account the effect of the torsional rigidity and torsional forces of the FL on its deflection and tensile strength.

As concerns the research methods used in this study, analysis, synthesis, generalisation, analogy as an epistemological basis of modelling, modelling (study of the original object) by creating and studying a copy (the model) that has common properties with the original, are all used as general scientific research methods.

The object of study is the FL of a MTS (based on the example of a UTS). The subject of this research adds to and improves the previously developed method for the MM of the dynamics of the FL of a MTS by determining the vectors of the generalised torsional forces of the FL.

ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

The elements of a rope (wire) experience tension, bending and contact loads together with torsion [3]. Torsion causes additional stresses that add to the main stress. These stresses have been studied by Glushko [4], Roslik [5, 6], Chukmasov [7], and Jacobson [8].

Glushko [4], considering the helical winding of the rope on the drum, obtained the torsional stress of the helical winding line. By analogy, Roslik [6] obtained the torsional tension of the helix, based on the assumption that it is equal to the torsion of the rope in the pulley system. In the bending of rope on blocks and drums, supplementary tensions appear, which cause axis and twist deformations. Roslik obtained an expression for the definition of the stress of torsion of the rope.

As Egorov [9] believed, in choosing the strength of the wire for the rope, one cannot be guided only by the calculated tension of the rope. The ropes used in the cable part of towed systems (TS) operate in conditions of vibration, which appear as a result of the action of the hydrodynamic forces that occur when towing ropes in water. It is noted that one of the causes of tensile stresses in the power plant of the UTS is the interaction with ship equipment, in which the spiral winding on the winch drum, friction on the flanges of the pulleys or winch drums, or bends in the blocks and rolls cause torsion. As the source of torsional stresses in the FL, there may be technological reasons related to both the manufacture and storage, transportation and placement on the ship's winch.

Poddubniy, Shamarin and other authors [10] considered the problem of equilibrium in the flow of a heavy FL, the bearing part of which consists of twisted flexible tensile elements and which resists torsional deformations. As an example, the towing of a deep-water vehicle, which consists of a submersible vehicle (SV) and a TC with two-and-half power armour, is considered. It is noted that the torsion of the TC leads to a redistribution of the total tension between the outer and inner layers of the armour so that the wires of the inner layer are more loaded than the outer layers. Most unfavourable is the case of free rotation of the running end of the FL, resulting in an estimated loss of tensile strength that reaches 15%. Torsion can cause a decrease in the tensile strength of the FL due to redistribution of load between the power elements, or may even lead to a violation of their structure. In some cases of torsion, the FL significantly affects the nature of the UTS movement as a whole.

In article [11], the matrix method of analysis of the system was widened to take into account the dynamics reaction of the towed system, using the method of equivalent linearisation and perturbation disturbance keys, of the angles of the towed body. Two examples were considered: the first uses the fundamental limitations of the passive compensation of the tow-rod and the second touches on the use of floating communications for dynamics relief. The FL modeling uses a differential approximation with a local disturbance in the form of the FL. This dynamics model is measured without taking account of the bending rigidity of the FL, and without interval elements of the FL.

Work [12] explores a method of motion control for a towed submersible vehicle (TSV) with movable wings. The TSV is affected by the non-linearity and uncertainty of the position of the flexible towing cable, hydrodynamic forces, parametric fluctuations, and external disturbances. The cable is approximated by the method of concentrated masses, where the number of cable segments determines the order of the system. Direct consideration of the non-linear dynamics is one of the main features of the cited work. However, the effect of hydrodynamic compression on the cable during its spatial movement is not considered. A way out of this situation could be an approach that implies creating a comprehensive model for the description of the UTS dynamics.

Article [13] considers a model of flexible segments adopted for dynamics calculations. In this model, the cable is divided into a certain number of flexible segments and is described by the non-linear equations at moments of the uniform segment movement. In a given example, the dynamics modelling was also performed in a two-coordinate system, which is currently insufficient to describe the dynamics of the spatial motion of the UTS.

Study [14] considers the application of a method of dynamic optimisation of the trajectory of a free-floating cable in the water depth. The model was considered in a three-dimensional system of coordinates, splitting the FL into the interrelated elements. The cable is modelled as a chain of rods connected to each other by hinges with two degrees of freedom, which could describe the bend of the cable in two planes (three coordinates). The cable is considered to be very flexible, but not able to be lengthened. The proposed model provides an opportunity to obtain the motion trajectory of a vessel and a cable, but does not take into consideration the change in the hydrodynamic characteristics of the cable, meaning that it is insufficient to fully study the dynamics of the FL.

In [15], Drag investigated the method of dynamics optimisation of a drill column which was attached by means of anchoring it at one end on the bottom, by analogy with the method proposed in [16]. This model takes into account the loading from bending and torsion of the drill column. However, the approach in this model does not take account of the cable lines.

Paper [17] considers the application of a dynamic optimisation method for a drill column fixed at one end on the seabed. The model presented takes into consideration the loads from the stretching and rotation of the drill column. However, the specified model is unsuitable for use in cable line calculations.

In [18], a lumped-mass method is used to establish the numerical model for evaluating the performance of a mooring line with embedded chains. To validate the numerical model, comparisons of the numerical results with the analytical formulas and the experimental data are conducted. Good agreement of the profile and the tension response is obtained. Then, the effect of the embedded chains on the static and dynamic response of the mooring line is evaluated, and the dynamic behaviour of the mooring system considering the embedded chains for a net cage system is investigated. The results indicate that the soil resistance on the embedded chains should be included to predict the mooring line development and the load on the embedded anchors in the numerical simulations.

The author tried to find information on the study of the torsion and torsional stresses of FLs and the creation of MMs that describe them, but such literature, as can be seen from the review, is virtually non-existent. It is obvious that the topic of developing a MM for the description of the dynamics of the MTS, taking into account the effect of torsion, is important and relevant.

RESULTS OF THE STUDY

The author examined more than 80 articles from research on the existing MMs of MTS. In some articles there was a lack of research on the dynamic interaction between the FL and MTS, as a whole, with obstacles in the water. In many cases, relatively little movement of the FL was modelled with the assistance of partial differential equations. Not one of the examined works made use of generalised coordinates, and the proposed MMs did not model the movement of the FL with great displacements and torsions.

Some articles used the finite element method (FEM) of the FL, which include methods addressing the rigidity of the bars and of the localised mass. As distinct from the method of generalised coordinates, these methods make the major mistake of approximating the form of the FL, which essentially restricts their use in describing the dynamics of a FL with big displacements and especially the fracture of the FL, which inevitably arise in the process of interaction of the FL with obstacles.

In many works, the presence of the FL is not taken into account at all, and some authors assigned the form of the FL a priori. In some works, the authors used simpler and more approximate methods of describing the statics of the FL of the MTS. To simplify the MM, some authors of articles in which they were presented examined the dynamics of the FL without some essential assumptions by neglecting the tension of the FL, its bending rigidity and the rigidity of FL torsion.

In many works, the movement of the FL was examined by the authors in two dimensions. Only in two works did the authors examine major bending of the FL in three coordinates. For improvement of the MM, the Authors of [16] used the method of generalised coordinates, which allows modelling of the motion of the FL with big movements and torsions. As distinct from the examined works, the method of generalised coordinates used in the development of the proposed MM makes such modelling of the motion of the FL with big movements and torsions possible.

Analysis of the existing dynamics models of the FL of a MTS has shown that, in most models, the elements of the FL in the MTS consider the dynamics of the FL at rather small displacements and bends, which testifies to the urgency of developing the proposed mathematical model of the dynamics of the elements of the FL to allow the consideration of big displacements of the FL as a part of the MTS. Previously, the equations of the dynamics of the elements of the FL of the MTS were obtained [16], which makes it possible to describe the significant values of its displacements. Creating the MM of the two related elements of the FL of the MTS allows an algorithm to be developed for calculating the FL dynamics at large displacements. More detailed descriptions of the MM of the FL are listed in [2, 16, 19].

Let us supplement the MM addressing the FL of the MTS through the method of determining the vectors of the generalised torsional forces of the FL. In Fig. 1 bent item FL indicates the angle of rotation of the final cross-section of the FL element

The angle β is determined along the FL centreline ($p \in [0; l]$). Using interpolation,

$$\beta(p) = \beta_0 \cdot (1 - p/l) + \beta_l \cdot p/l, \tag{1}$$

where β_k are the angles of rotation of the cross-sections of the FL at the end points of the element relative to the normal n of the Frenet reference of the centreline [20, 21].

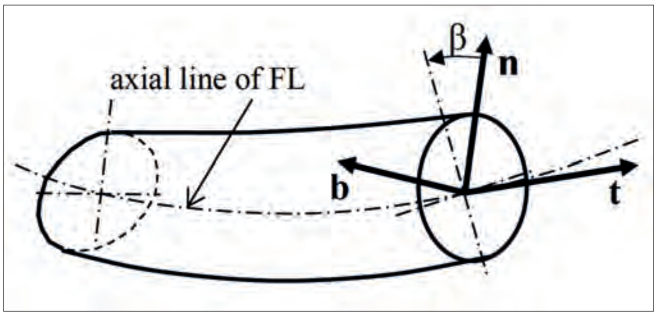

Fig. 1. Rotation of the final cross-section of the FL element relative to the normal axis

Hermitian functions are used as the functions of the FL form:

$$s_1(p) = s_3(l - p) = 1 - 3\xi^2 + 2\xi^3 , \quad (2)$$

$$s_2(p) = s_4(l - p) = l \cdot (\xi - 2\xi^2 + \xi^3) , \quad (3)$$

$$s_3(p) = 3\xi^2 - 2\xi^3 , \quad (4)$$

$$s_4(p) = l \cdot (\xi^3 - \xi^2) , \quad (5)$$

$$\xi = p/l , \quad (6)$$

allowing the shape of the FL element to be approximated by the magnitude and derivative of the radius vector of the FL:

$$x(p) = s_1(p) \cdot x_0 + s_2(p) \cdot x'_0 + s_3(p) \cdot x_l + s_4(p) \cdot x'_l , \quad (7)$$

$$y(p) = s_1(p) \cdot y_0 + s_2(p) \cdot y'_0 + s_3(p) \cdot y_l + s_4(p) \cdot y'_l , \quad (8)$$

$$z(p) = s_1(p) \cdot z_0 + s_2(p) \cdot z'_0 + s_3(p) \cdot z_l + s_4(p) \cdot z'_l , \quad (9)$$

where $x_0, y_0, z_0, x_l, y_l, z_l$ are the coordinates of the final points of the FL element; and $x'_0, y'_0, z'_0, x'_l, y'_l, z'_l$ are the coordinates of the derivative of the final points of the axis lines of the FL element.

The vectors of the generalised bending and torsion forces ($\vec{Q}^{\chi}_{i-1}, \vec{Q}^{\chi}_{i_1}, \vec{Q}^{\chi}_{i_2}, \vec{Q}^{\chi}_{i+1}, \vec{Q}^{\tau}_{i-1}, \vec{Q}^{\tau}_{i_1}, \vec{Q}^{\tau}_{i_2}, \vec{Q}^{\tau}_{i+1}$) are included in Eq. (10):

$$\mathbf{M}_1 \cdot \ddot{\vec{e}}_{i-1} + \mathbf{M}_2 \cdot \ddot{\vec{e}}_i + \mathbf{M}_3 \cdot \ddot{\vec{e}}_{i+1} + \mathbf{K}_1 \cdot \vec{e}_{i-1} + \mathbf{K}_2 \cdot \vec{e}_i + \mathbf{K}_3 \cdot \vec{e}_{i+1} + \\ + \vec{Q}^{\chi}_{i-1} + \vec{Q}^{\chi}_{i_1} + \vec{Q}^{\chi}_{i_2} + \vec{Q}^{\chi}_{i+1} + \vec{Q}^{\tau}_{i-1} + \vec{Q}^{\tau}_{i_1} + \vec{Q}^{\tau}_{i_2} + \vec{Q}^{\tau}_{i+1} = \vec{Q}_i , \quad (10)$$

determined by formula (11) for the bending reaction forces of the FL:

$$\vec{Q}^{\chi}_i = EJ \int_0^l \chi \frac{\partial \chi}{\partial \vec{e}_i} dp = \frac{EJ \cdot l}{2} \int_0^1 \frac{\partial \chi^2}{\partial \vec{e}_i} d\xi \quad (11)$$

and by formula (12) for the torsional reaction forces of the FL:

$$\vec{Q}^{\tau}_i = \frac{\partial U^{\tau}}{\partial \vec{e}_i} = G \cdot J_P \int_0^l \tau \frac{\partial \tau}{\partial \vec{e}_i} dp = \frac{G \cdot J_P \cdot l}{2} \int_0^1 \frac{\partial \tau^2}{\partial \vec{e}_i} d\xi . \quad (12)$$

The curvature χ of the centreline with respect to the major axes of inertia of the cross-sectional area with bending stiffness *EJ* is expressed as follows:

$$\chi = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3} . \quad (13)$$

The vectors of the generalised bending and torsion forces ($\vec{Q}^{\chi}_{i-1}, \vec{Q}^{\chi}_{i_1}, \vec{Q}^{\chi}_{i_2}, \vec{Q}^{\chi}_{i+1}, \vec{Q}^{\tau}_{i-1}, \vec{Q}^{\tau}_{i_1}, \vec{Q}^{\tau}_{i_2}, \vec{Q}^{\tau}_{i+1}$) can be calculated by formulas (151) – (158) in reference [10], but the process of determining them is associated with major problems in calculating the derivatives and integrals in analytical form, due to the great complexity of the formulas. This makes this process very time-consuming and does not always enable the integrals to be calculated in analytical form.

To solve this problem, a method for determining the vectors $\vec{Q}^{\chi}_{i-1}, \vec{Q}^{\chi}_{i_1}, \vec{Q}^{\chi}_{i_2}, \vec{Q}^{\chi}_{i+1}, \vec{Q}^{\tau}_{i-1}, \vec{Q}^{\tau}_{i_1}, \vec{Q}^{\tau}_{i_2}, \vec{Q}^{\tau}_{i+1}$ has been developed using numerical methods for calculating the derivatives and integrals included in formulas (11) – (12), by using the vector of generalised coordinates in the nodal points of the FL (14).

Taking account of the introduced matrix equation for the FL element with nodal points i_{-1} and i_1, and also with nodal points i_2 and i_{+1}, the system of equations is rearranged. Taking into account that in the nodal point $i = i_1 \equiv i_2$, the following equality is carried out:

$$\vec{e}_i = \vec{e}_{i_2} = \vec{e}_{i_1} , \quad (14)$$

then the generalised variables at nodal point i are substituted into $\vec{e}_i$ and with the exception of bonds in the nodal point *i* from other equation. The system of equations obtained is defined and correlated with three of the neighbouring nodal points of the FL, but to account for the boundary conditions in this case it is necessary to correct the matrix using Lagrange factors. But defining these complicates the solution of the task and also demands transformation of the matrix of mass near the borders of the FL. In order to secure the impossibility of these problems, the vector of generalised coordinates is written down only to the i-nodal point of the FL:

$$\vec{e}_i = \{\vec{r}_i^{\,0T} \, \vec{r}_i^{\,1T} \, \beta_i\}^T , \quad (15)$$

or, in coordinate form,

$$\vec{e}_i = \{x_i \, y_i \, z_i \, x'_i \, y'_i \, z'_i \, \beta_i\}^T . \quad (16)$$

Function χ_τ included in formula (13) is represented as the ratio of vector products by the formula

$$\chi_\tau = \frac{b^T r'''}{b^T b} = \frac{x'''(y' \cdot z'' - y'' \cdot z') + y'''(z' \cdot x'' - z'' \cdot x') + z'''(x' \cdot y'' - x'' \cdot y')}{(y' \cdot z'' - z' \cdot y'')^2 + (z' \cdot x'' - x' \cdot z'')^2 + (x' \cdot y'' - y' \cdot x'')^2} . \quad (17)$$

Then, in formula (1), the relative angle of torsion of the dihedral section of the FL taking account of the angle of torsion of the dihedral section β is:

$$\tau = \chi_\tau + \partial\beta/\partial p , \quad (18)$$

and may be expressed as

$$\tau = \frac{b^T r'''}{b^T b} + \partial\beta/\partial p . \quad (19)$$

For the derivative of function τ at the generalised coordinates (20), we write the vector of the generalised coordinates only down to the i-nodal point of the FL:

$$\vec{e}_i = \{\vec{r}_i^{\,0T} \, \vec{r}_i^{\,1T} \, \beta_i\}^T , \quad (20)$$

or, in coordinate form,

$$\vec{e}_i = \{x_i\, y_i\, z_i\, x_i'\, y_i'\, z_i'\, \beta_i\}^T\,, \tag{21}$$

and they take the form

$$\frac{\partial \tau}{\partial e_i^T} = -\frac{b^T r'''}{(b^T b)^2}\left\{2\left[\frac{\partial b}{\partial e_i^T}\right]^T b\right\} + + \frac{1}{b^T b}\left\{\left[\frac{\partial b}{\partial e_i^T}\right]^T r''' + \left[\frac{\partial r'''}{\partial e_i^T}\right]^T b\right\}. \tag{22}$$

$$\frac{\partial \tau}{\partial \beta_0} = -\frac{\partial \tau}{\partial \beta_l} = -\frac{1}{l}\,. \tag{23}$$

Taking account of the approximation of the coordinates of the FL and of their derivatives, we define the derivatives and functions within Eq. (22). The integrals (12) were calculated by the Simpson method.

The derivatives and functions that are included in Eq. (11) were calculated similarly. The square of the curvature χ of the axial line with respect to the main axes of inertia of the cross-sectional area with bending stiffness EJ, taking into account the notation of the numerator and denominator of the fraction, has the form

$$\chi^2 = \frac{a}{b} = = \frac{(y'\cdot z'' - z'\cdot y'')^2 + (z'\cdot x'' - x'\cdot z'')^2 + (x'\cdot y'' - y'\cdot x'')^2}{(x'^2 + y'^2 + z'^2)^3} \tag{24}$$

The derivative of the function (24) is determined by the formula:

$$\frac{\partial \chi^2}{\partial \vec{e}_i} = \frac{\frac{\partial a}{\partial \vec{e}_i} b - a \frac{\partial b}{\partial \vec{e}_i}}{b^2}\,. \tag{25}$$

The calculation of the derivatives of formulas (22), (23) and (25) produced a small error in the calculation of the reactions of the FL to bending and torsion, and also the stability of the process of mathematical modeling of the dynamics of the FL and the MTS as a whole.

PRACTICAL VERIFICATION OF SIMULATION RESULTS: THE INFLUENCE OF THE TORSIONAL RIGIDITY OF THE FL ON ITS DEFLECTION AND TENSILE STRENGTH

The influence of the torsional rigidity of the FL on the dynamics of the MTS are considered through the example of two tasks. In both tasks, the initial length of the FL was taken as 100 m. The FL is in the water. The root end of the FL is still. The density of the material of the FL is 7800 kg/m³ and its Young's modulus is $2 \cdot 10^{11}$ Pa. The diameter of the FL is 40 mm and it is divided into 10 elements. The normal coefficient of hydrodynamic resistance of the FL is equal to 1.35, and the tangential is 0.04.

Consider these tasks:

1. calculation of the static deflection of the FL;
2. calculation of the tensile strength: the carrier vessel (CV) is stationary, and the submersible vehicle (SV) has no restrictions on movement and has positive buoyancy.

TASK 1. Calculation of the static deflection of the FL is performed by the method of establishing motion using the developed mathematical model. The root and running ends of the FL at the initial moment of time were motionless, and were on the surface of the sea at a distance of 100 m. The tensile strength of the FL at the initial time is zero. Under the action of gravity, the FL sagged to a steady state. In the absence of torsion of the FL, its deformation occurred only in the X0Z plane.

In the second version of this problem, it was assumed that the root end of the FL was twisted by 10 turns. Torsion of the FL has almost no effect on the shape of the deflection of the FL in the X0Z plane (Fig. 2), but leads to deviation of the FL from the X0Z plane (Fig. 3).

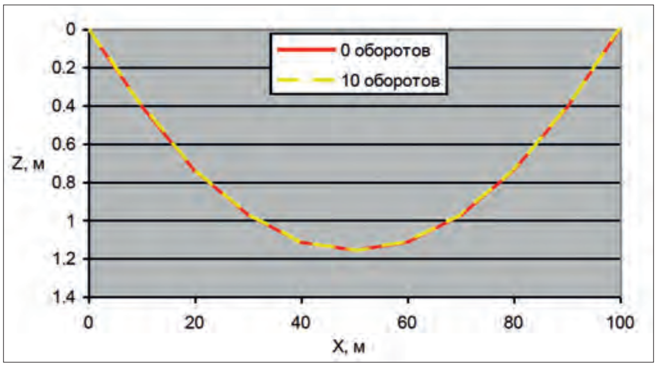

Fig. 2. The final shape of the deflection of FL in the X0Z plane

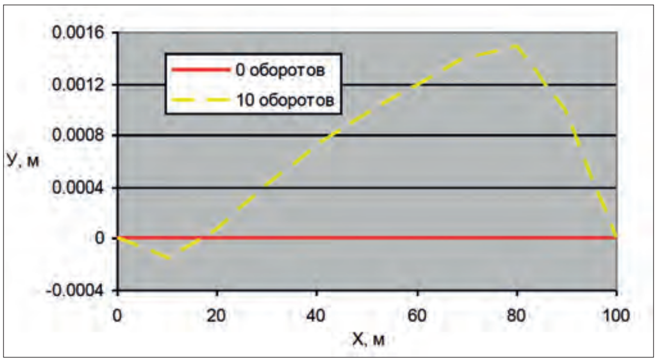

Fig. 3. The final shape of the deflection of FL in the X0Y plane

As a result of the torsion of the FL, even with fixed ends, the FL acquires a three-dimensional shape.

TASK 2. The CV is immobile, and the SV has no restrictions on movement, has positive buoyancy and is connected to the CV at the FL length of 100 m. At the initial moment of time, the FL has no tension and is located along the sea surface. In the course of modelling, the FL sags under the action of gravity and causes the underwater towed vehicle (UTV) to move along the sea surface to the CV. In the absence of FL torsion, the process of FL and SV movement occurs in the X0Z plane (Fig. 4).

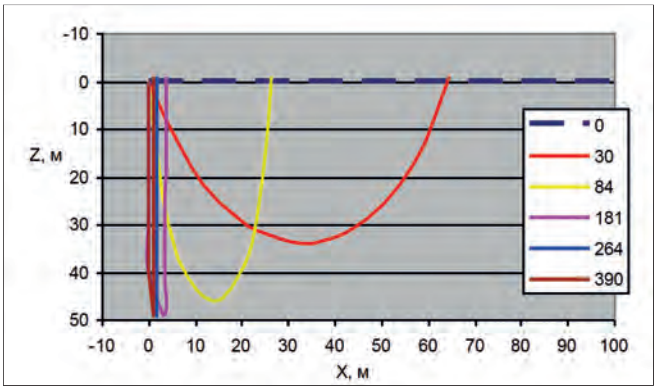

Fig. 4. Form of FL at different points in time (curves are indicated by the corresponding points in time in seconds from the beginning of the process)

The tension force of the FL is distributed unevenly along its length (Fig. 5).

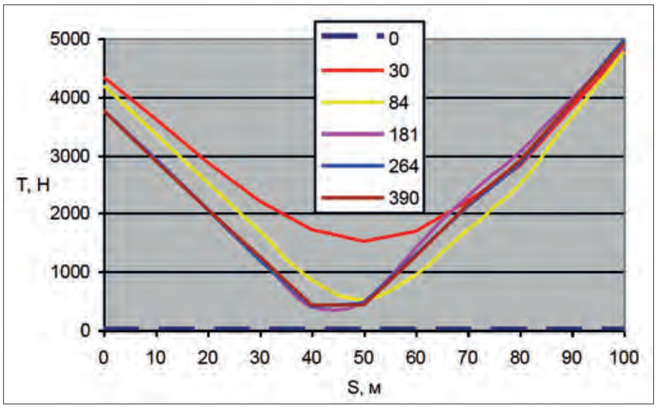

Fig. 5. Tensile strength of FL by its length at different points in time (curves are denoted by the corresponding points in time in seconds from the beginning of the process)

The second version of this task considered the process of release of the FL from the drum of the winch located on the CV, where its root end receives torsion on 10 turns and maintains it in the course of modelling. The angle of rotation of the running end of the FL on the UTV is zero.

Torsion of the FL leads to a three-dimensional change in its shape both in the X0Z plane and in the X0Y plane (Fig. 6 and Fig. 7), respectively.

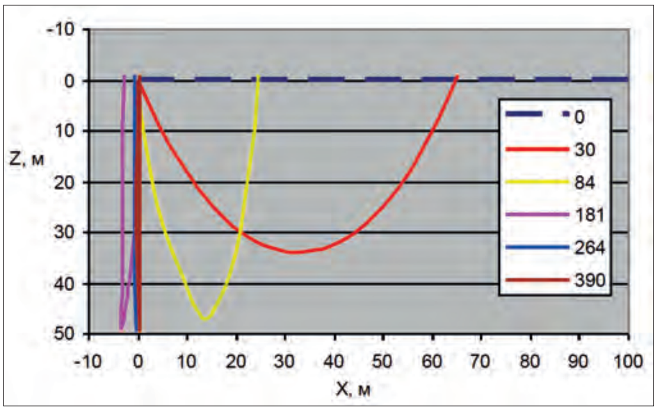

Fig. 6. Form of FL at different times

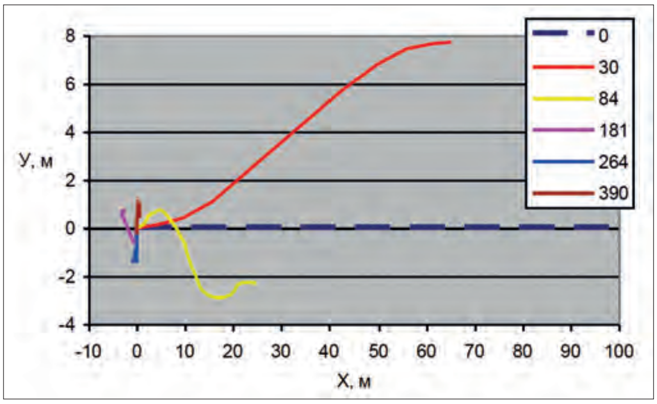

Fig. 7. Form of FL at different times

The tension force of the FL is distributed unevenly along its length (Fig. 8).

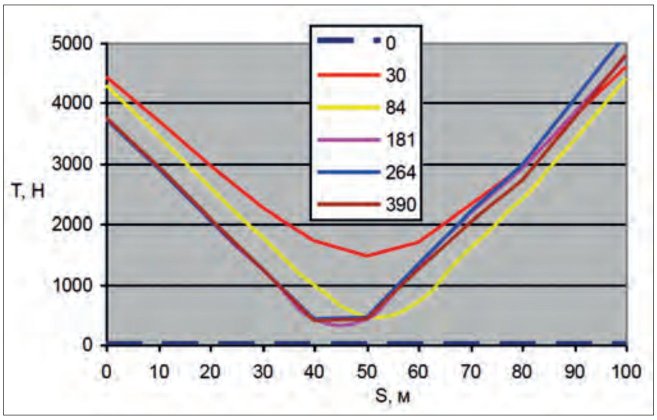

Fig. 8. Tensile strength of FL along its length at different points in time (curves are indicated by the corresponding points in time in seconds from the beginning of the process)

The maximum deviation of the FL and the SV from the X0Z plane is 7.7 m 30 seconds after the start of simulation. In the following moments of time, projection of the FL on the X0Y plane significantly changes its shape in the process of approaching the UTV to the CV, crossing the X0Z plane many times, as well as circulating around the CV (Z axis).

In the Z0X plane, the form of the FL after its rotation is also markedly different from its shape in the absence of torsion (Fig. 9).

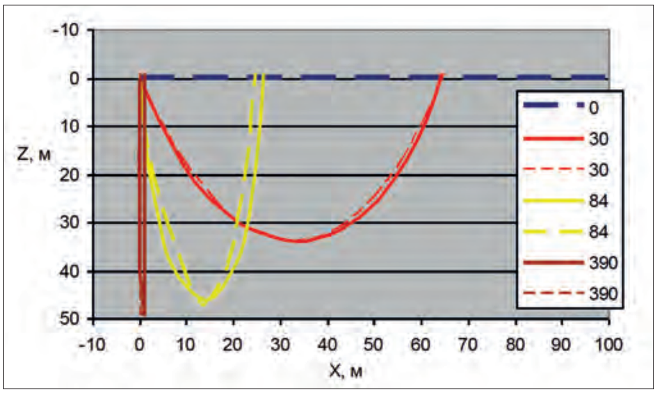

Fig. 9. Form of FL at different points in time (curves are denoted by the corresponding points in time in seconds from the beginning of the process, the dashed lines correspond to the torsion FL)

To a lesser extent, torsion of the FL affects the distribution of tensile force along its length (Fig. 10).

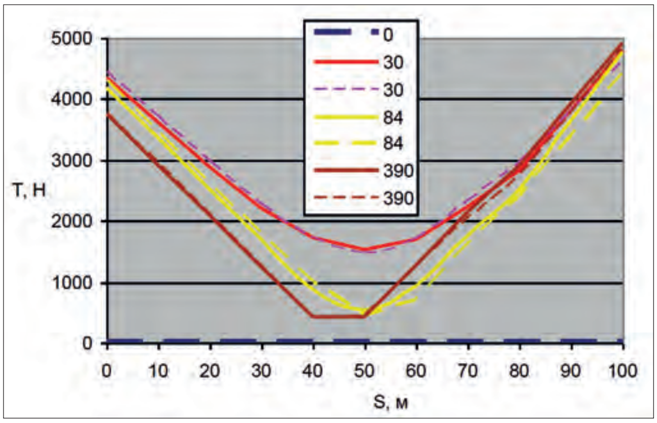

Fig. 10. Tensile strength of FL by its length at different points in time (curves are denoted by the corresponding moments of time in seconds from the beginning of the process, dashed lines correspond to torsion of FL)

Based on these data, we can conclude that, for correct modelling of the dynamics of the FL, taking into account its torsion, it is necessary to carry out a three-dimensional statement of the problem. In this case, assumptions that simplify the idea of the movement of the FL only in the X0Z plane can lead to qualitatively and quantitatively incorrect conclusions.

DISCUSSION OF RESULTS

The created MM of the element dynamics of the FL makes it possible to take into account large movements of the FL as part of the MTS and take into account also the following:

1. The movement of the CV, which is determined by the following factors: sea surface turbulence; kinematic characteristics of CV motion, and sea current velocity [16];
2. Design features of the FL affecting the functional characteristics of the MTS and which are determined by the following factors: the length and change in the length of the FL in the process of moving the CV; the elasticity and strength of the FL; positive or negative buoyancy of the FL, as well as cargo, floats and buoys associated with it; the hydrodynamic resistance forces of the FL in the process of its movement in water; and forces acting on the root and running ends of the FL [16];
3. The movement of the UTV, which is determined by the following factors: the mass and buoyancy of the UTV; the relative location of the UTV in relation to the VC and the kinematic characteristics of its motion; the forces of hydrodynamic resistance of the SV in the course of its movement in water;
4. The impact of obstacles on the movement of the UTV and FL, which is determined by the following factors: the location of obstacles in the water; the size of the obstacles; kinematic characteristics of the movement of obstacles [22–24].

The mathematical modelling of two related elements of the FL of the MTS made it possible to develop an algorithm for calculating the dynamics of the FL during large movements and to solve some problems which were not considered in the existing MM [25]:

1. To determine the change of form of the FL and forces of its tension in the process of manoeuvring of the CV and UTV, taking into account sea waves, underwater currents, wind loads on the CV, the sea depth and its changes in a given water area, and the mass and elastic properties of the FL;
2. To determine the relative position of the CV and SV in the process of their manoeuvring;
3. To determine the modes of manoeuvring of the MTS, which leads to the formation of loops ("pegs") on the FL (usually formed on the stretched FL when there is a "slack" and the presence of torque, which depends on the tension of the FL);
4. To determine the regimes of manoeuvring of that MTS that reduce the vibration of bad flow of FL at the inflow;
5. To determine the tensile, bending [26] and torsional forces in the FL;
6. To determine the system of equations describing the dynamics of the element under load and rotation;
7. To develop an algorithm for modelling the dynamics of the FL, which makes it possible to perform calculations of the dynamics of the FL of the MTS.

By means of practical tests of the modelling results on the influence of the torsion rigidity of the FL on its flexure and tension force, the conclusion may be reached that, for correct modelling of the dynamics of the FL accounting for its torsion, a complete three-dimensional formulation of the problem is necessary. In this case, assumptions taken in earlier work that simplify the movement of the FL as occurring only in the X0Z plane mean that qualitatively and quantitatively accurate conclusions cannot be obtained.

With the developed MM of the dynamics of the FL and the algorithm established, the computer program description of the dynamics of the FL of the MTS will allow the designer of the MTS, which includes the FL, to more efficiently and quickly design almost all classes of MTS [27]. The originality of the MM of the FL dynamics is that the modelled dynamics of the MTS with the FL includes not only equations for the FL, but also the equation of the dynamics of the CV and towed SV, the motion of which determines the boundary conditions in the nodes of the FL, with numbers i = 0 and i = N.

CONCLUSIONS

Based on the study, the following main conclusions can be drawn:

1. Torsion of the FL can lead to a significant change in the shape of the FL and the trajectory of towed objects, as well as changes in the forces acting on the elements of the MTS.
2. The advanced MM, as well as the algorithm and computer program make it possible to perform mathematical modelling of the dynamics of the MTS, taking into account the torsion of the FL.

3. For correct modelling of the FL dynamics, taking into account its rotation, it is necessary to establish a three-dimensional statement of the problem. In this case, simplifying the assumptions about the FL moving only in the X0Z plane can lead to qualitatively and quantitatively incorrect conclusions.

Thus the generalised forces of torsion of the FL in Eq. (10) are used, which, together with Eq. (12), allow the possibility of taking into account the influence of the torsional rigidity of the FL in its dynamics.

Examples of modelling of the FL dynamics which were examined show the possibility of registering the rigidity of torsion of the FL in an improved MM, and also its essential influence on the functional characteristics of the MTS.

The scientific novelty of this MM of the dynamics of the FL of the MTS is that it is complex, allowing us to study the FL taking into account its stretching, bending and torsion and operating conditions as a part of practically all classes of MTS.

Thus the aim of research is achieved: a method that allows us to complete the previously developed MM of the dynamics of the FL of the MTS and account for the rigidity of the FL is proposed.

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