

Li CHANGYOU  
Wang WEI  
Zhang YIMIN  
Guo SONG  
Li ZHENYUAN  
Qiao CHANGSHUAI

## INDEXING ACCURACY RELIABILITY SENSITIVITY ANALYSIS OF POWER TOOL TURRET

### ANALIZA WRAŻLIWOŚCI WSKAŹNIKA NIEZAWODNOŚCI W ZAKRESIE DOKŁADNOŚCI INDEKSOWANIA GŁOWICY NARZĘDZIOWEJ Z NAPĘDZANYMI NARZĘDZIAMI

*Power tool turret (PTT) is one of the most key parts of CNC (Computer Numerical Control) turning center and CNC turning and milling machining center. Therefore, it is very important for improving these two types of machine tools' reliability to explore the indexing accuracy reliability of PTT and its sensitivity. To analyze the indexing accuracy reliability sensitivity of PTT, the angular displacement error of the rotating gear disc is discussed based on the measurement uncertainty theory. The tooth thickness wear process of the fixed gear disc, the rotating gear disc and the lock gear disc are modeled using Gamma process of which the parameters are estimated. The indexing error of PTT is formulated by employing the BP neural network and validated by the experiment data. Then, the indexing accuracy reliability equation of PTT is derived and its sensitivity to the mean and the standard deviation of random variables or wear stochastic processes is analyzed. The results show that the presented indexing accuracy reliability and its sensitivity of PTT are effective.*

**Keywords:** power tool turret; indexing accuracy; reliability analysis; wear; finite element.

*Głowica narzędziowa z napędzanymi narzędziami (ang. power tool turret, PTT) stanowi jedną z kluczowych części sterowanych komputerowo tokarskich i tokarsko-frezarskich centrów obróbkowych CNC. Dlatego też dla poprawy niezawodności tych dwóch rodzajów obrabiarek istotne jest badanie niezawodności w zakresie dokładności indeksowania głowicy PTT oraz wrażliwości wskaźnika niezawodności. Dla celów analizy wrażliwości wskaźnika niezawodności w zakresie dokładności indeksowania głowicy PTT, omówiono błąd przesunięcia kąтового obrotowej tarczy narzędziowej w oparciu o teorię niepewności pomiaru. Proces zużycia ściernego zębów tarcz narzędziowych: stałej, obrotowej oraz uchwytowej, zamodelowano przy użyciu procesu Gamma o szacunkowych parametrach. Błąd indeksowania PTT obliczono przy zastosowaniu sieci neuronowej BP i zweryfikowano na podstawie danych doświadczalnych. Następnie wyprowadzono równanie niezawodności w zakresie dokładności indeksowania PTT oraz przeprowadzono analizę wrażliwości wskaźnika niezawodności na średnią i odchylenie standardowe zmiennych losowych lub procesów stochastycznych zużycia. Wyniki pokazały skuteczność omawianej analizy niezawodności w zakresie dokładności indeksowania PTT oraz wrażliwości wskaźnika niezawodności.*

**Słowa kluczowe:** głowica narzędziowa z napędzanymi narzędziami, dokładność indeksowania; analiza niezawodności; zużycie; elementy skończone.

#### 1. Introduction

CNC machine tool is the most key equipment in the manufacturing industry. Its output and technical level imply the advancement and competitiveness of the manufacturing industry of one country. The sale of the machine tools manufactured in China is about 31% of the world total amount in 2010 [7]. However, the most of these machine tools belong to the low-level one which own lower machining accuracy and manufacture the simpler mechanical part. For example, the ratios of the high-level, middle-level and low-level CNC machine tools manufactured in China are 2%, 18% and 80% respectively in 2009 [10]. The high-level CNC machine tools used in China are mostly from the other countries. One of seasons causing the situation is the low reliability of the high-level CNC machine tool manufactured in China. Therefore, it is very important to improve the key function unit reliability of the high-level CNC machine tool.

The CNC machine tool reliability has been investigated by a lot of researchers and many fruits have been given. The fruits might be summarized as the following fields. The first is the CNC machine tool reliability assessment based on the fault time data, such as [3, 11, 21, 22]. The main work is to identify the distribution type of the fault time data and estimate the parameters. It was reported that the time between faults of the CNC machine tool could be described by Weibull distribution or exponential distribution well [5, 19, 21]. The reliability assessment of the CNC machine tool was also investigated when the data of the time between faults was collected by the fixed or random time censoring test [2, 20]. For the small sample of the time between faults, the reliability assessment has also been investigated. For example, the neural network was trained by the existed data to generate the new data [6]. The second is to analyze the effect of the CNC machine tool fault mode or the function part on the whole reliability. There was a direct method used by Jia YZ et al [1, 17] where the fault modes

were ranked according to the occurrence frequency and the key fault modes were analyzed to decrease their occurrence rate. The third is CNC machine tool reliability allocation. The new reliability allocation method should be explored because CNC machine tool could not be supposed to be a series or parallel system generally. The new method should be based on the character of the CNC machine tool. For example, the failure rate allocation method was presented by Wang YQ et al [18] to determine the reliability of fifteen subsystems of CNC lathe. Moreover, the allocation method based on the optimization theory was presented by Wang JL et al [16]. The fourth is the reliability design of the mechanical part. There were some documents where the reliability was formulated according to the fault or failure mechanism and calculated by the reliability analysis method. For example, the fatigue strength reliability analysis method was used to analyze the machine tool part [4]. The kinematic error of the machine tool or part was modeled and its reliability was analyzed when the kinematic error included the stochastic variables, such as [3]. The reliability-based optimization design of the ball screw and nut has also been investigated by Sun KZ et al [12]. Here, the goal function was to minimize the volume and the constraint conditions included the contact fatigue and stiffness reliability. The last is the acceleration reliability test and assembly reliability analysis, such as [23, 24].

Power tool turret (PTT) is one of the most key function parts of CNC turning center and CNC turning and milling machining center. It is mainly from the foreign country in the mainland of China and is being researched by some Chinese companies. But, its performance and reliability are very low. To improve the reliability of PTT manufactured in China, a Key National Science & Technology Special Project has been carried out since 2010. Its objective is to increase the mean time between failures (MTBF) from 1500 to 2250 hours. Moreover, the published document about the PTT reliability has never been found as the best knowledge of the authors. Therefore, the indexing accuracy reliability and its sensitivity will be investigated in this work.

**2. Indexing accuracy reliability and its sensitivity of PTT**

The considered PTT is composed of the power system, the indexing system and the tool disk. It was designed and machined by Shenyang Machine Tool (Group) Co., Ltd of China. Its external view is shown in Fig. 1. The power of the milling is offered by the power system. The indexing system is employed to change the tool according to the machining requirement. Here, the twelve tools can be chosen based on the machined mechanical part and set up to the tool disk before the machining process. The structure of the indexing system is shown in Fig. 2. The indexing system includes gear disc system, hydraulic lock system, transmission and control system. According to the structure characteristic of the considered PTT, the indexing accuracy is determined by the gear discs system and lock force mainly. The gear discs system consists of the rotating gear disc, the fixed gear disc and the lock gear disc. Its three-dimensional drawing is shown in Fig. 3.

**2.1. Distribution and parameter estimation of angular displacement error of rotating gear disc**

The rotating gear disc is fastened to the output shaft of transmission shown in Fig. 3. Therefore, the angular displacement error of rotating gear disc is equal to that of the output shaft of transmission. It is composed of the one-way angular displacement error and backlash angular displacement error of transmission and its detailed effect term is shown in Table 1.

The measurement uncertainty theory is employed to estimate the standard deviation of the angular displacement error of rotating gear disc. The combined measurement uncertainty of the angular displacement error of the rotating gear disc is

$$u(\delta) = \sqrt{u(\delta_\phi)^2 + u(\delta_\varphi)^2} \tag{1}$$

$u(\delta_\varphi)$  and  $u(\delta_\phi)$  are the one-way angular displacement combined measurement uncertainty and the backlash angular displacement combined measurement uncertainty of the transmission system respectively.  $u(\delta_\varphi)$  is calculated by:

$$u(\delta_\varphi) = \sqrt{[u(\delta_{\varphi 42})]^2 + \left[\frac{u(\delta_{\varphi 31})}{i_{34}}\right]^2 + \left[\frac{u(\delta_{\varphi 32})}{i_{34}}\right]^2 + \left[\frac{u(\delta_{\varphi 21})}{i_{34}i_{23}}\right]^2 + \left[\frac{u(\delta_{\varphi 22})}{i_{34}i_{23}}\right]^2 + \left[\frac{u(\delta_{\varphi 11})}{i_{34}i_{23}i_{12}}\right]^2} \tag{2}$$

where  $u(\delta_{\varphi nm})$  is the combined measurement uncertainty of the driving ( $m=1$ ) or driven ( $m=2$ ) gear on the  $n$ th shaft. The first shaft is the output end of the servo motor. The last one ( $n=4$ ) is fasten to the rotating gear disc.  $i_{n'n}$  ( $1 \leq n' \leq 3$ ) is the transmission ratio of the  $n'$ th-stage transmission.  $u(\delta_\phi)$  is

$$u(\delta_\phi) = \sqrt{u^2(\delta_{\varphi 3}) + \frac{u^2(\delta_{\varphi 2})}{i_{34}^2} + \frac{u^2(\delta_{\varphi 1})}{i_{34}^2 i_{23}^2}} \tag{3}$$

$u(\delta_{\varphi i})$  is the backlash combined measurement uncertainty of the  $i$ th-stage transmission. The calculation process of  $u(\delta_{\varphi nm})$  and  $u(\delta_{\varphi i})$  is described in detail in the master thesis wrote by Li ZY (This work is supervised by Zhang YM and Li CY) [9]. Here, the manufacture errors of the gear and the shaft and the shaft radial run-out in running are assumed to follow the normal distribution and are independence mutually. Therefore, the angular displacement error of the rotating gear disc follows the normal distribution  $N(0, u(\delta))$ .

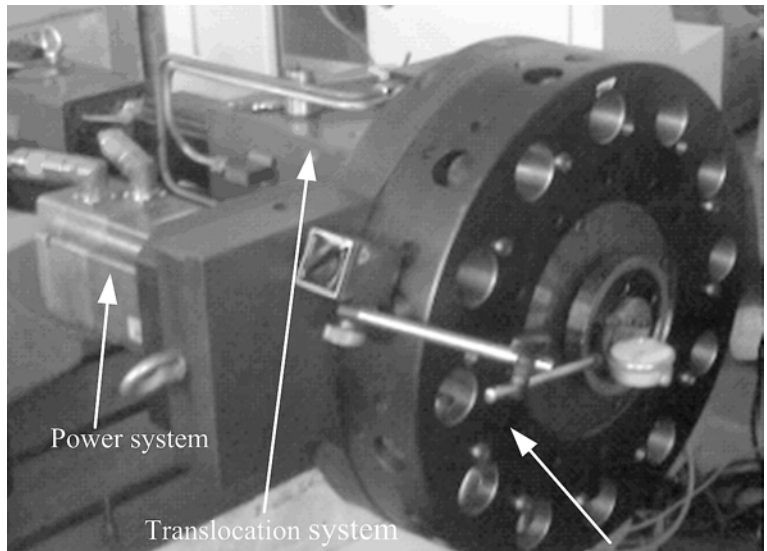


Fig. 1. External view of the considered PTT

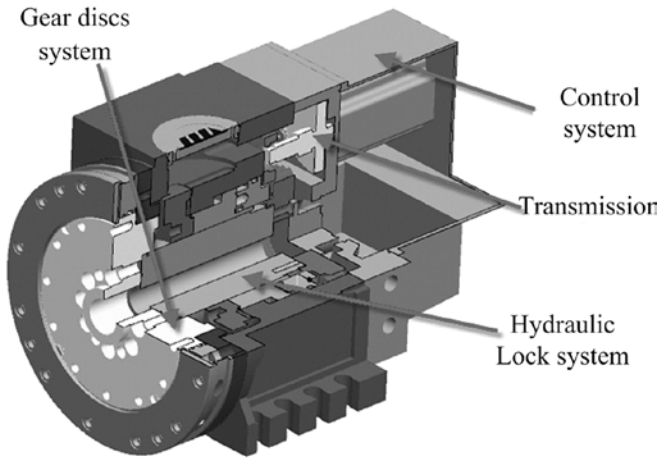


Fig. 2. Structure simplified diagram of indexing system

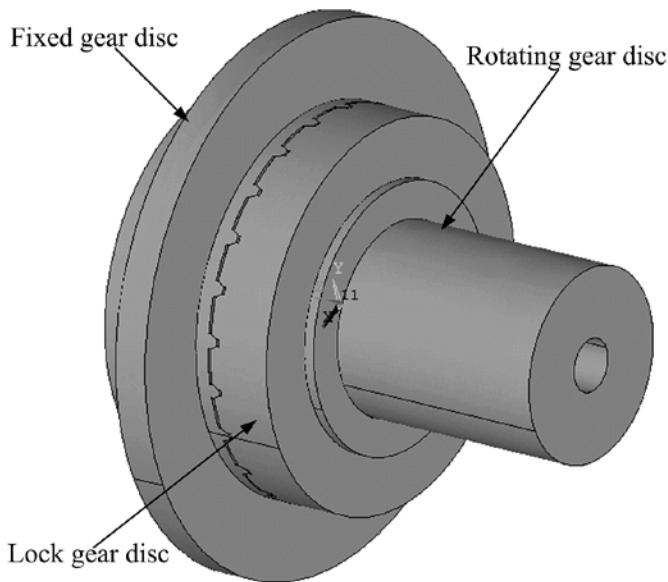


Fig. 3. Three-dimensional drawing of gear discs system

Table 1. Effect term of angular displacement error of rotating gear disc

Angular displacement error of rotating gear disc	One-way rotation error	Gear manufacture error
		Shaft manufacture error
		Gear-Shaft assembly error
	Backlash error	gear pair center distance error
		Gear manufacture error
		Gear-shaft assembly error
		Shaft radial run-out in running

**2.2. Distribution and parameter estimation of other random variables**

The other random variables of the indexing accuracy reliability formulation of the considered PTT are listed in Table2. The radial

direction error is defined as the offset between the axis of the fixed or lock gear disc and the axis of the rotating gear disc. The verticality error describes the offset between the plane of the gear disc and the plane being vertical to the axis of the rotating gear disc. The radial direction error or the verticality error is described by a vector. The vector includes two parameters: mode and direction angle. The error is defined as the different between the designed value and the actual one. In the machining process of the parts of PTT, there are a lot of the stochastic factors, such as the manufacture and the assembly error of the machine tool parts, the environment temperature and vibration, the electric current stochastic fluctuation etc. Therefore, the direction angle of the radial direction error or the verticality error is supposed to follow the uniform distribution  $U(0,2\pi)$ . The others are assumed to follow the normal distribution of which the mean is zero and the standard deviation is estimated by the corresponding B type measurement uncertainty.

**2.3. Random variables wear of PTT**

The indexing accuracy of PTT should deteriorate due to wear of some random variables listed in Table2 among the operating process. The wear process is a continuous-time and continuous-state stochastic process. It is also a monotone increasing stochastic process because it cannot be decreased by itself. For the stochastic deterioration process to be monotonic, we can best consider it as a gamma process [13,14]. Therefore, the Gamma process is employed to formulate the random variable deterioration process in this work.

A Gamma process is a stochastic process with independent, non-negative increment having a gamma distribution with an identical scale parameter. It is a continuous-time and continuous-state stochastic process. Let  $\{X(t), t \geq 0\}$  be a Gamma process. It is with the following properties [8, 15]:

- (1)  $X(0)=0$  with probability one,
- (2)  $X(\tau) - X(t) \sim G(x | v(\tau) - v(t), u), \forall \tau > t \geq 0$ ,
- (3)  $X(t)$  has independent increments.

where  $v(t)$  is the shape function which is a non-decreasing, right-continuous, real-valued function for  $t \geq 0$  with  $v(0)=0, u > 0$  is the scale parameter, and  $G(\cdot)$  is the Gamma distribution.

Let  $X(t)$  denote the wear of a certain random variable at time  $t, t \geq 0$ . In accordance with the definition of the Gamma process, its probability density function is given by:

$$f_{X(t)}(x) = \frac{u^{v(t)} x^{v(t)-1} \exp(-ux)}{\Gamma(v(t))} I_{[0,\infty)}(x) \tag{4}$$

where  $\Gamma(\cdot)$  is the Gamma function,  $I_A(x) = 1$  for  $x \in A$  and  $I_A(x) = 0$  for  $x \notin A$ . Its expectation and variance are respectively expressed as:

$$E(X(t)) = \frac{v(t)}{u} \tag{5}$$

$$E((X(t) - E(X(t)))^2) = \frac{v(t)}{u^2} \tag{6}$$

Empirical studies show that the expected wear at time  $t$  is often proportional to a power law [15]:

$$E(X(t)) = \frac{ct^b}{u} = at^b \propto t^b \tag{7}$$

where  $a > 0$  (or  $c > 0$ ) and  $b > 0$ . They are estimated by the expectation of Gamma process being equal to the theoretical calculation wear of the corresponding random variable.

Table 2. Random variables and their distribution of PTT

Part	Variable	Unit	Distribution	symbol
lock gear disc	tooth profile-half angle error	°	N(0,0.01112)	x <sub>1</sub>
	tooth direction error	°	N(0,0.01082)	x <sub>2</sub>
	tooth thickness error	m	N(0,0.0001412)	x <sub>3</sub>
	radial direction error	m	N(0,0.00002442)	x <sub>4</sub>
		rad	U(0,2π)	x <sub>5</sub>
	verticality error	m	N(0,0.09322)	x <sub>6</sub>
		rad	U(0,2π)	x <sub>7</sub>
	angular displacement error	°	N(0,0.02842)	x <sub>8</sub>
rotating gear disc	tooth profile half-angle error	°	N(0,0.01112)	x <sub>9</sub>
	tooth direction error	°	N(0,0.01862)	x <sub>10</sub>
	tooth thickness error	m	N(0,0.00012482)	x <sub>11</sub>
	axial error	m	N(0,0.000004562)	x <sub>12</sub>
	verticality error	m	N(0,0.004862)	x <sub>13</sub>
		rad	U(0,2π)	x <sub>14</sub>
	angular displacement error	°	N(0,0.03182)	x <sub>15</sub>
fixed gear disc	tooth profile half-angle error	°	N(0,0.01112)	x <sub>16</sub>
	tooth direction error	°	N(0,0.01932)	x <sub>17</sub>
	tooth thickness error	m	N(0,0.0001412)	x <sub>18</sub>
	radial direction error	m	N(0,0.000007672)	x <sub>19</sub>
		rad	U(0,2π)	x <sub>20</sub>
	axial error	m	N(0,0.000004172)	x <sub>21</sub>
	verticality error	m	N(0,0.00132)	x <sub>22</sub>
		rad	U(0,2π)	x <sub>23</sub>
	angular displacement error	°	N(0,0.02082)	x <sub>24</sub>
lock pressure	MPa	N(6.61899959,0.033095002)	x <sub>25</sub>	

The wear of PTT is due to the relative sliding between two mechanical parts. It is belong to adhesive wear according to the wear theory. Therefore, Archard equation is used to calculate the wear mean of the random variables such as tooth thickness. It is:

$$\Delta V = K \frac{PL}{3H} \tag{8}$$

where  $\Delta V$ ,  $K$ ,  $P$ ,  $L$  and  $H$  are the total wear volume, the wear constant, the force between the friction pair, the relative sliding distance and the yield limit of the material of which the hardness is lower in the friction pair respectively. The total wear volume is proportional to the force between the friction pair from Eq. (8). Therefore, only the tooth thickness wear of the rotating gear disc, the fixed gear disc and the lock gear disc is considered in this work.

$A$  is denoted by the contact surface area between two meshing teeth of the lock gear disc and the rotating gear disc or the fixed gear disc in the locking or loosening process. Then, the wear depth of one tooth is:

$$\Delta \bar{x} = K \frac{PL}{AH} \tag{9}$$

where the unit is meter. In the locking (loosening) process of PTT, the force  $P$  between two contact tooth surfaces changes from zero to the end force value  $P_1$  (from  $P_1$  to zero). The two contact tooth surfaces slide relatively and should deform due to the force  $P$ . The relationship between the stress being vertical to the contact tooth surface and the corresponding strain is:

$$\sigma = E\varepsilon \tag{10}$$

where  $\sigma$ ,  $\varepsilon$  and  $E$  are the stress, the strain and elasticity modulus respectively. The force between two contact tooth surfaces is:

$$P = A\sigma = AE\varepsilon \tag{11}$$

The corresponding tooth thickness deformation of the lock gear disc and the rotating gear disc or the fixed gear disc are  $\Delta s_L = s_L\varepsilon$  and  $\Delta s_{F,R} = s_{F,R}\varepsilon$  respectively.  $s_{F,R} = 6.29\text{mm}$  is the mean tooth thickness of the rotating gear disc or the fixed gear disc.

$$s_L = \frac{\pi(219+160)/2 - 6.29 \cdot 24}{24} = 18.5155\text{mm}$$

is the mean tooth thickness of the lock gear disc. The outside and inner circle diameters of the lock gear disc are 219mm and 160mm respectively. The total tooth number of the gear disc  $z$  is 24. The sliding friction  $P_f = \mu P$  where  $\mu = 0.1$  is the sliding friction coefficient. Then, at the locking (loosening) end (start) time, the force  $P_1$  is:

$$P_1 = \frac{F_y}{z(\mu \cos \alpha + \sin \alpha)} \tag{12}$$

where:

$$F_y = 4 \times 10^6 (\text{pa}) \pi \left( \left( \frac{179.96}{2} \right)^2 - \left( \frac{119.96}{2} \right)^2 \right) \times 10^{-6} (\text{m}^2) = 56533.5881 \text{ N}$$

is the total hydraulic lock force. 179.96 and 119.96 are the diameters of the hydraulic cylinder and the piston rod respectively. Here, the pressure angle  $\alpha$  is 30°. In one locking or loosening process, the total tooth thickness normal deformation of the lock gear disc is:

$$\Delta s_{L1} = \frac{F_y}{z(\mu \cos \alpha + \sin \alpha)} \frac{s_L}{AE} \tag{13}$$

and that of the rotating or fixed gear disc is:

$$\Delta s_{F,R1} = \frac{F_y}{z(\mu \cos \alpha + \sin \alpha)} \frac{s_{F,R}}{AE} \tag{14}$$

Then, in one locking or loosening process, the total relative sliding distance between the lock gear disc and the fixed or rotating gear disc:

$$L_1 = \frac{\Delta s_{F,R1} + \Delta s_{L1}}{\sin(\alpha)} = \frac{F_y (s_{F,R} + s_L)}{AEz \sin(\alpha) (\mu \cos \alpha + \sin \alpha)} \quad (15)$$

$dL$  is denoted by the relative sliding infinitesimal when the force between two contact tooth surfaces is  $P$ . According to Eq. (9), the corresponding tooth thickness wear is:

$$d\Delta \bar{x} = \frac{KP}{AH} dL \quad (16)$$

Eq. (16) is integrated and the total tooth thickness wear of one locking or loosening process of the gear disc is:

$$\Delta \bar{x} = \int_0^{L_1} \frac{KP}{AH} dL \quad (17)$$

The relative sliding distance at any time in the locking or loosening process  $L = \frac{\Delta s_{F,R} + \Delta s_L}{\sin(\alpha)} = 2(\Delta s_{F,R} + \Delta s_L)$ . Then,  $dL$  is:

$$dL = 2d(\Delta s_{F,R} + \Delta s_L) \quad (18)$$

where  $d\Delta s_L = d\varepsilon s_L = s_L d\varepsilon$  and  $d\Delta s_{F,R} = d\varepsilon s_{F,R} = s_{F,R} d\varepsilon$ . Using

$$d\varepsilon = \frac{d\sigma}{E} = \frac{d\left(\frac{P}{A}\right)}{E} = \frac{dP}{EA}, \text{ Eq. (18) is transformed into:}$$

$$dL = 2(s_{F,R} + s_L) \frac{dP}{EA} \quad (19)$$

Then, Eq. (17) is written to:

$$\Delta \bar{x} = \int_0^{P_1} \frac{2KP(s_{F,R} + s_L)}{AH} \frac{dP}{EA} = \frac{KP_1^2 (s_{F,R} + s_L)}{HEA^2} \quad (20)$$

In the considered PTT, the contact surface area between two meshing teeth  $A=136.44\text{mm}^2$ , elasticity modulus  $E=206\text{GPa}$ . The yield limit  $H=685\text{MPa}$  where the material of the gear disc is 20CrMo.  $P_1=4015.6085\text{N}$  by Eq. (12). The wear constant  $K=4.5 \times 10^{-4}$ . Using Eq. (20), the total tooth thickness wear of one locking or loosening process of the gear disc is  $6.8521 \times 10^{-11}$ .

The time spent on one locking or loosening process is 0.3 second from the specified parameter of the considered PTT. The tooth thickness wear is directly proportional to the relative sliding distance. The relative sliding distance is directly proportional to the sliding period of time where the locking or loosening velocity is constant. Therefore, the tooth thickness cumulative wear is directly proportional to the cumulative locking and loosening time  $t$ . The relationship between the tooth thickness cumulative wear and the cumulative locking and loosening time is formulated by:

$$\bar{x}_M = 2.2840 \times 10^{-10} t \quad (21)$$

where the cumulative locking and loosening time  $t$  is different from the servicing time of PTT. The cumulative locking and loosening time  $t$  is the sum of the time spent in locking and loosening the gear discs system. The servicing time is the sum of the indexing time, the ma-

chining time and the downtime. The expectation of Gamma process is assumed to be equal to Eq. (21). That is:

$$\frac{ct^b}{u} = 2.2840 \times 10^{-10} t \quad (22)$$

Then, the parameters of Gamma process  $c=2.2840 \times 10^{-10}$  and  $u=b=1$ .

#### 2.4. Indexing accuracy model of PTT

The indexing error of PTT is estimated by the finite element method. The parametric finite element model of PTT is shown in Fig. 4. It includes twenty five variables listed by Table 2. It could calculate any one angular displacement error of PTT when the sample including twenty five variables is given. For example, when one sample is  $[-3.2099 \times 10^{-2}, 1.3440 \times 10^{-2}, -1.9920 \times 10^{-5}, -2.5084 \times 10^{-5}, 5.2255, -1.4060 \times 10^{-1}, 1.1837, 6.8467 \times 10^{-2}, -3.1164 \times 10^{-2}, 6.5976 \times 10^{-3}, -4.8894 \times 10^{-5}, -5.4007 \times 10^{-6}, -1.0811 \times 10^{-3}, 2.3928 \times 10^{-1}, -2.5873 \times 10^{-2}, -2.6026 \times 10^{-3}, -6.1497 \times 10^{-3}, 6.8805 \times 10^{-5}, 2.3010 \times 10^{-5}, 5.6536, 2.6825 \times 10^{-6}, 2.0399 \times 10^{-3}, 1.9265, -3.4388 \times 10^{-2}, 6.6415 \times 10^{-6}]$ , the absolute indexing error of PTT is equal to  $1.1677 \times 10^{-2}$ .

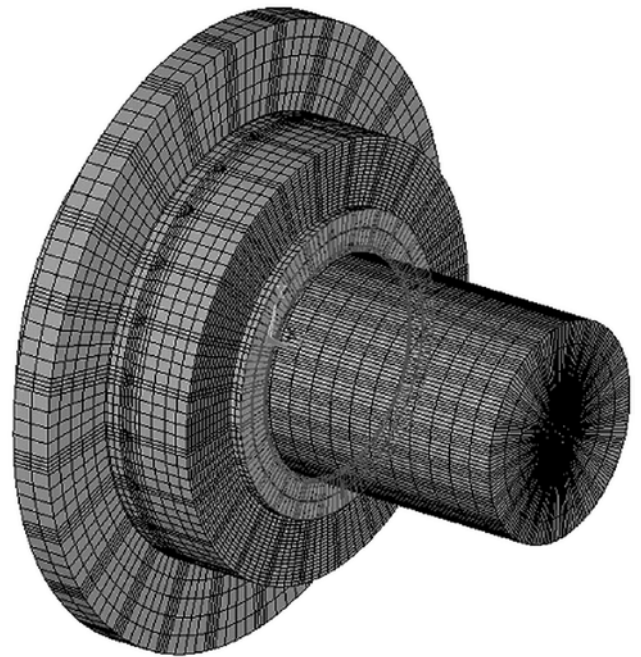


Fig. 4. Parametric finite element model of PTT

The BP neural network is employed to formulate the relationship between the absolute indexing error of PTT and twenty five random variables. The training samples are offered by the parametric finite element model of PTT. The learning result is:

$$y = b_2 + w_2 \frac{1}{1 + \exp[-(w_1 X + b_1)]} = b_2 + \sum_{j=1}^{100} \frac{w_2(j)}{1 + \exp\left[-\sum_{i=1}^{25} w_1(j,i) X(i) - b_1(j)\right]} \quad (23)$$

where  $b_1, w_1, b_2$  and  $w_2$  are the threshold and the weight between the hidden layer and the input layer, the threshold and the weight between the hidden layer and the output layer respectively.  $y$  is the output of

Table 3. Actual measurement values of the absolute indexing error of PTT

station measurement number	1	2	3	4	5	6	7	8	9	10	11	12
clockwise 1	24.6	23.1	25.2	22.6	19.1	19.2	16.9	14.2	12.1	15.8	12.5	19.1
anticlockwise 1	23.5	22.1	19.5	22.2	15.1	18.6	15.2	13.6	11.2	15.1	17.7	11.9
clockwise 2	25.2	23.7	25.8	23.5	20.1	19.8	17	15.9	13.5	17.1	18.7	20.1
anticlockwise 2	22.9	23.4	18.9	22.9	16.3	19.8	14.7	14.8	12.8	16.4	18.4	15.8
clockwise 3	26.5	25.2	27.1	24.6	21	20.7	18.1	16.6	13.8	17.5	19.3	21.5
anticlockwise 3	25.7	24.2	21.2	23.2	16.8	20.6	16.3	15.5	13.5	17.2	19	14.1
clockwise 4	26.9	25.8	28	25.5	22.4	22	19.1	17.5	15	18.3	19.7	22.1
anticlockwise 4	24.9	24.8	20.8	24.3	16.7	20.5	15.3	14.9	13.2	17	17.4	18.9
clockwise 5	28	27	29.3	26.5	23.3	22.6	20.1	18	15.3	19.3	20.6	23.2
anticlockwise 5	24.5	26.2	20.7	23.1	17.3	21.8	17.1	17	14.7	18.3	20.5	13.5

the trained BP neural network and belongs to the set [-1,1]. The input set X is obtained by processing the input training samples from its original range to the range [-1 1]. The actual absolute indexing error (angular displacement error) of PTT is:

$$y' = \frac{y+1}{2}(y_{tmax} - y_{tmin}) + y_{tmin} \quad (24)$$

where  $y_{tmax}$  and  $y_{tmin}$  are the output maximum and the output minimum of the training sample respectively. The twenty five variables are sampled according to their distributions. The samples size is  $25 \times 1000$ . Then, the absolute indexing errors are calculated by Eq. (24). Their mean and standard deviation are 20.9925" and 4.5328 respectively. The 120 actual measurement values of the absolute indexing error of PTT are shown in Table3. Their mean and standard deviation are 19.7142" and 4.2469 respectively. Therefore, the absolute indexing error model of PTT described by Eq. (24) is acceptable.

2.5. Indexing accuracy reliability and its sensitivity of PTT

The indexing accuracy reliability of PTT is estimated by the reliability perturbation method which is proposed by Zhang YM and is described in detail in the document [25].The indexing accuracy reliability could be estimated by:

$$R(\beta(t)) = P[g(\mathbf{X}(t)) > 0] = 1 - F(-\beta(t)) \quad (25)$$

where  $\beta(t)$  is the reliability index,  $g(\mathbf{X}(t))$  is the state function and the function  $F(\bullet)$  is:

$$F(y) = \Phi(y) - \phi(y) \left[ \frac{1}{6} \frac{\theta_g}{\sigma_g^3} H_2(y) + \frac{1}{24} \left( \frac{\eta_g}{\sigma_g^4} - 3 \right) H_3(y) + \frac{1}{72} \left( \frac{\theta_g}{\sigma_g^3} \right)^2 H_5(y) + \dots \right] \quad (26)$$

$\Phi(\bullet)$  and  $\phi(\bullet)$  are the cumulative probability function and the probability density function of the standard normal distribution respectively.  $\sigma_g$ ,  $\theta_g$  and  $\eta_g$  the standard deviation, the third and the fourth order center moment of the state function.  $H_j(\bullet)$  is the  $j$ th Hermite polynomial. If  $R(\beta(t))$  is more than 1 using Eq. (25), it is estimated by:

$$R^*(\beta(t)) = R(\beta(t)) - \frac{R(\beta(t)) - \Phi(\beta(t))}{\{1 + [R(\beta(t)) - \Phi(\beta(t))] \beta(t)\}^{\beta(t)}} \quad (27)$$

The indexing accuracy reliability curve of the considered PTT is shown in Fig. 5. The star is the results of Monte Carlo simulation. The solid curve is the calculated values of the reliability perturbation method. It could be seen that the different between Monte Carlo simulation and the reliability perturbation method is very small. Moreover, the reliability calculated by the reliability perturbation method is slightly less than zero among the period of time where the reliability decreases from positive number to zero in Fig. 5. It might be a numerical computation error.

The indexing accuracy reliability sensitivity of PTT to the mean and standard deviation of the random variables listed in Table2 and the wear processes could be calculated by Eq. (28) and (29) respectively:

$$\frac{\partial R(t)}{\partial \bar{\mathbf{X}}^T(t)} = \frac{\partial R(\beta(t))}{\partial \beta(t)} \frac{\partial \beta(t)}{\partial \mu_g(t)} \frac{\partial \mu_g(t)}{\partial \bar{\mathbf{X}}^T(t)} \quad (28)$$

$$\frac{\partial R}{\partial \text{Var}(\mathbf{X}(t))} = \left[ \frac{\partial R(\beta(t))}{\partial \beta(t)} \frac{\partial \beta(t)}{\partial \sigma_g(t)} + \frac{\partial R(\beta(t))}{\partial \sigma_g(t)} \right] \frac{\partial \sigma_g(t)}{\partial \text{Var}(\mathbf{X}(t))} \quad (29)$$

The detail description about Eq. (28) and (29) is introduced in [25]. The indexing accuracy reliability sensitivity of the considered PTT to the random variable's and the wear process's means is shown in Fig. 6. The sensitivity to the standard deviations is described in Fig. 7. Here, the subfigures (1), (2), ..., (25) are the sensitivity curves to the means or the standard deviations of  $x_1, x_2, \dots, x_{25}$  respectively. The sensitivity to the means or the standard deviations of the tooth thickness wear processes of the lock gear disc, the rotating gear disc and the fixed gear disc are shown by the subfigures (26), (27) and (28) in Fig. 6 or Fig. 7 respectively.

From Fig. 6, it could be seen that (1) the sensitivity to the mean of some random variables is less than or equal to zero. These random variables include the direction angle of the radial direction error and the angular displacement error of the lock gear disc, the verticality error's mode and the angular displacement error of the rotating gear disc, the mode and the direction angle of the verticality error of the fixed gear disc and the tooth thickness wear of any one gear disc. The sensitivity of other random variables is more than or equal to zero. (2) With the increase of the cumulative locking and loosening time, the variation trend of the absolute sensitivity to the mean of all random variables increases gradually and then decreases to zero. (3) The fluctuation range of the sensitivity to the mean of the rotating gear disc of the fixed gear disc is the largest.

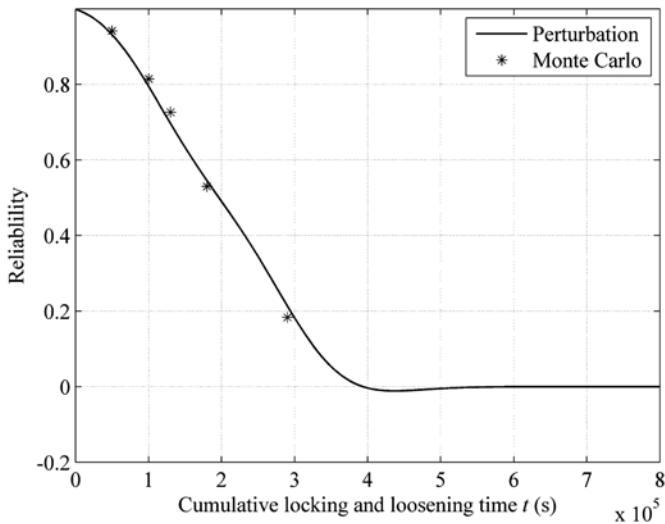


Fig. 5. Indexing accuracy reliability curve of the considered PTT

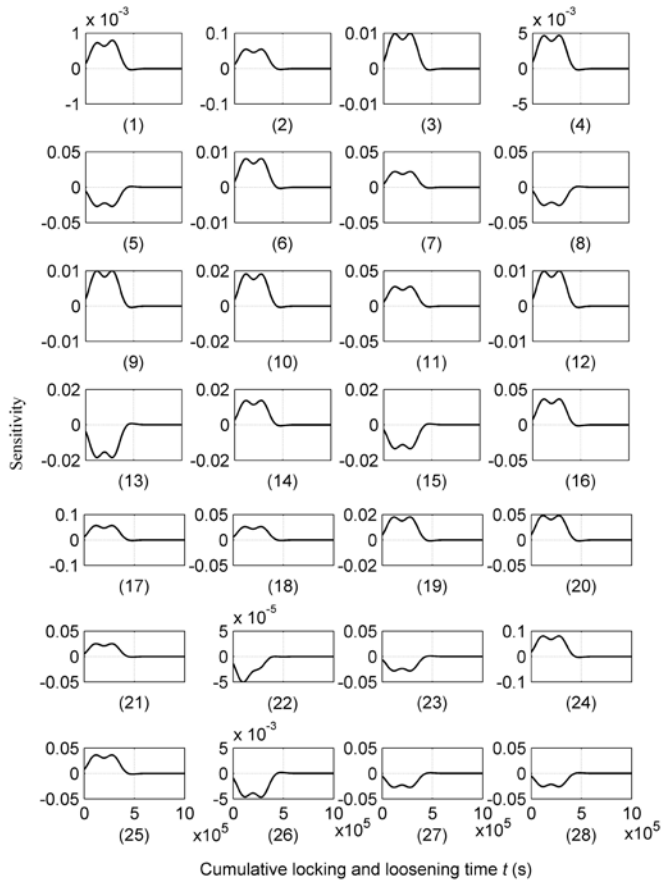


Fig. 6. Indexing accuracy reliability sensitivity of the considered PTT to the mean of the random variables and their deterioration processes

From Fig. 7, it could be seen that (1) With the increase of the cumulative locking and loosening time, the variation trend of the sensitivity to the standard deviation of all random variables decreases to the negative maximum firstly, gradually increases to the positive maximum subsequently and then decreases to zero finally. (2) The fluctuation range of the sensitivity to the standard deviation of the rotating gear disc of the fixed gear disc is the largest too.

According to the above calculated results, the indexing accuracy reliability of the considered PTT could be improved by increasing (decreasing) the mean of one of the random variables and the wear

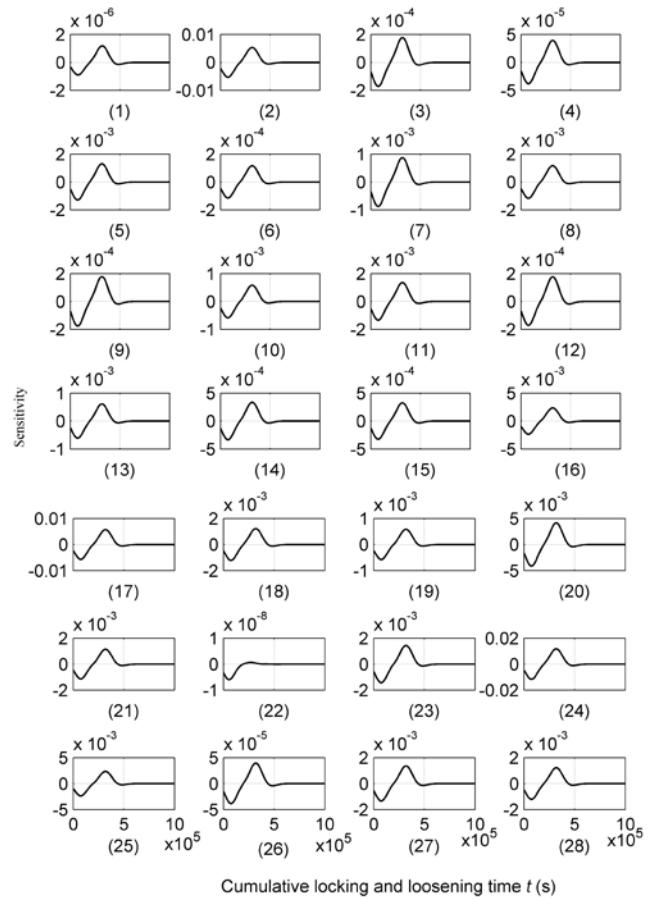


Fig. 7. Indexing accuracy reliability sensitivity of the considered PTT to the standard deviation of the random variables and their deterioration processes

stochastic processes to which the sensitivity is less (more) than or equal to zero when all parameters of other random variables are invariant. It implies that the corresponding design values, machining accuracy or wear should be decreased (increased) to improve the indexing accuracy reliability of the considered PTT. For example, the verticality of the fixed gear disc, which includes mode and direction angle, should be decreased and the tooth thickness of the gear disc should be increased.

If increase (decrease) the standard deviation of any one random variable or wear stochastic process when all parameters of other random variables are invariant, the indexing accuracy reliability of the considered PTT should be improved among the service early stage and should be reduced among the service middle stage. It means that the machining accuracy or wear should be improved (reduced) to increase (decrease) the standard deviation.

### 3. Conclusions

The indexing accuracy reliability and its sensitivity of PTT were presented in this work. The deterministic indexing error formulation including twenty five variables was formulated by the finite element method and BP neural network. The indexing accuracy reliability estimated by the reliability perturbation method was compared with Monte Carlo simulation and is acceptable. According to the indexing accuracy reliability sensitivity, the scheme was proposed to improve the indexing accuracy reliability of the considered PTT. A section of the scheme has been adopted and the reliability of the considered PTT has been improved observably.

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**LI CHANGYOU**

**Wang WEI**

**Zhang YIMIN**

**Li ZHENYUAN**

**Qiao CHANGSHUAI**

Box NO.319, NO. 3-11, Wenhua Road, Heping District  
Shenyang, P. R. China. Postal Code: 110004

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**Guo SONG**

No.9, Hunnan East Road, Hunnan New District

Shenyang City, Liaoning, P.R.China. Postal Code: 110168

E-mails: 243465096@qq.com, ymzhang@mail.neu.edu.cn,  
guosong@sjzu.edu.cn, 772664104@qq.com,  
727873785@qq.com

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